BITSAT - Paper 2023 Shift 1

Solved Paper

Question 1

An object moves with speed v_1, v_2 and v_3 along a line segment AB, BC and CD respectively as shown in figure. Where AB = BC and AD = 3 AB, then average speed of the object will be:

Options:

A. $\frac{v_1v_2v_3}{3(v_1v_2 + v_2v_3 + v_3v_1)}$

B. $\frac{3v_1v_2v_3}{(v_1v_2 + v_2v_3 + v_3v_1)}$

C. $\frac{(r^2 + r_2 + r_3)}{3}$

D. $\frac{(v_1 + v_2 + v_3)}{3v_1v_2v_3}$

Answer: B

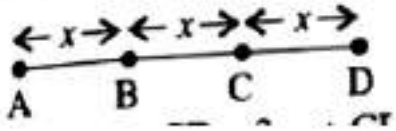
Solution:

Solution:

Consider,

AB = x

BC = x



$2x + CD = 3x \Rightarrow CD = 3x - 2x = x$

Average speed of the object $\langle v \rangle$

$= \frac{\text{Total distance}}{\text{Total time}}$

$\langle v \rangle = \frac{3x}{\frac{x}{v_1} + \frac{x}{v_2} + \frac{x}{v_3}} = \frac{3v_1v_2v_3}{v_2v_3 + v_1v_3 + v_1v_2}$

Question 2

The effect of increase in temperature on the number of electrons in

conduction band (n_e) and resistance of a semiconductor will be as:

Options:

A. Both n_e and resistance decrease

B. Both n_e and resistance increase

C. n_e increases, resistance decreases

D. n_e decreases, resistance increases

Answer: C

Solution:

Solution:

When temperature increases, more electrons excite to conduction band and hence conductivity increases, therefore resistance decreases.

Question 3

A radio-active material is reduced to 1 / 8 of its original amount in 3 days. If 8×10^{-3} kg of the material is left after 5 days. The initial amount of the material is

Options:

A. 700 gm

B. 900 gm

C. 475 gm

D. 256 gm

Answer: D

Solution:

Solution:

$N = N_0\left(\frac{1}{2}\right)^n \quad N = \frac{N_0}{8}$

$\frac{N_0}{8} = N_0\left(\frac{1}{2}\right)^n \Rightarrow \left(\frac{1}{2}\right)^3 = \left(\frac{1}{2}\right)^n$

$n = 3$

3 half lives = 3 days

1 half life = 1 day

5 days = 5 half life

$N = N_0\left(\frac{1}{2}\right)^n \Rightarrow 8 \times 10^{-3} = N_0\left(\frac{1}{2}\right)^5$

$\Rightarrow N_0 = 2^5 \times 8 \times 10^{-3} = 256\,\text{gm}$

Question 4

A 12.5 eV electron beam is used to bombard gaseous hydrogen at room temperature. The number of spectral lines emitted will be:

Options:

A. 2

B. 1

C. 3

D. 4

Answer: C

Solution:

Solution:

If we assume electron in hydrogen atom takes energy 12.09 eV from the incoming radiation, the maximum excited state of electron will be $n = 3$. So, number of spectral lines is $\frac{3(3-1)}{2} = 3$.

Here we assume some part of energy $12.5\,\text{eV} - 12.09\,\text{eV} = 0.41\,\text{eV}$ get lost due to collision.

Question 5

If 1000 droplets of water of surface tension 0.07 N / m. having same radius 1 mm each, combine to from a single drop. In the process the released surface energy is-

$\left(\text{Take } \pi = \frac{22}{7}\right)$

Options:

A. $7.92 \times 10^{-6}\text{J}$

B. $7.92 \times 10^{-4}\text{J}$

C. 9.68×10^{-4}J

D. 8.8×10^{-5}J

Answer: B

Solution:

Solution:

We have

$V_f = V_i$

$\Rightarrow \frac{4}{3}\pi r_f^3 = 1000 \times \frac{4}{3}\pi r_i^3 \Rightarrow r_f^3 = 1000 r_i^3$

$\Rightarrow r_f = 10 r_i$

So, released energy

= Initial surface energy − final surface energy

$= 1000 \times T \times 4\pi r_i^2 - T \times 4\pi r_f^2$

$= 4\pi T(1000 r_i^2 - r_f^2)$

$= 4\pi \times 0.07(1000 r_i^2 - 100 r_i^2)$

$= 4\pi \times 0.07 \times 900 r_i^2$

$= 4\pi \times 63 \times 10^{-6} = 7.92 \times 10^{-4}$J

Question 6

The force between two small charged spheres having charges of 1×10^{-7}C and 2×10^{-7}C placed 20 cm apart in air is

Options:

A. 4.5×10^{-2}N

B. 4.5×10^{-3}N

C. 5.4×10^{-2}N

D. 5.4×10^{-3}N

Answer: B

Solution:

Solution:

Here, $q_1 = 1 \times 10^{-7}$C, q_2 and 2×10^{-7}C,

$r = 20\,\text{cm} = 20 \times 10^{-2}\text{m}$

$$F = \frac{q_1 q_2}{4\pi\varepsilon_0 r^2} = \frac{9 \times 10^9 \times 1 \times 10^{-7} \times 2 \times 10^{-7}}{(20 \times 10^{-2})^2}$$

$= 4.5 \times 10^{-3}$N

Question 7

The work done in placing a charge of 8×10^{-18} coulomb on a condenser of capacity 100 microfarad is

Options:

A. 3.1×10^{-26} joule

B. 4×10^{-10} joule

C. 32×10^{-32} joule

D. 16×10^{-32} joule

Answer: C

Solution:

Solution:

$$\text{Work done} = \frac{1}{2}\frac{q^2}{C} = \frac{(8 \times 10^{-18})^2}{2 \times 100 \times 10^{-6}} = 32 \times 10^{-32}\text{J}$$

Question 8

The resistance of a wire is 5Ω. It's new resistance in ohm if stretched to 5 times of its original length will be :

Options:

A. 625

B. 5

C. 125

D. 25

Answer: C

Solution:

Solution:

Let resistance of a wire R and length l.

$$R = \frac{\rho \ell}{A} = 5\Omega$$

∴ Volume of wire is constant in stretching

$$V_i = V_f \Rightarrow A_i \ell_i = A_f \ell_f$$

$$A\ell = A'(5\ell) \Rightarrow A' = \frac{A}{5}$$

$$R_f = \frac{\rho \ell_f}{A_f} = \frac{\rho(5\ell)}{\left(\frac{A}{5}\right)} = 25\left(\frac{\rho \ell}{A}\right) = 25 \times 5 = 125\Omega$$

Question 9

A charge particle is moving in a uniform magnetic field $\left(2\hat{i} + 3\hat{j}\right)$T. If it has an acceleration of $\left(\alpha\hat{i} - 4\hat{j}\right)$m / s^2, then the value of α will be :

Options:

A. 3

B. 6

C. 12

D. 2

Answer: B

Solution:

Solution:

(b) Given that uniform magnetic field, $\vec{B} = (2\hat{i} + 3\hat{j})T$

Acceleration $\vec{a} = (\alpha\hat{i} - 4\hat{j})\text{m}/s^2$

We know that

$$F = q(\vec{v} \times \vec{B}) \Rightarrow ma = q(\vec{v} \times \vec{B})$$

Here, $\vec{a} \perp \vec{B}$, so, $\vec{a} \cdot \vec{B} = 0$

$$(\alpha\hat{i} - 4\hat{j})(2\hat{i} + 3\hat{j}) = 0 \Rightarrow 2\alpha - 12 = 0 \Rightarrow \alpha = 6$$

Question 10

Proton (p) and electron (e) will have same de-Broglie wavelength when

the ratio of their momentum is (assume, $m_n = 1849m_e$)

Options:

A. 1:43

B. 43 : 1

C. 1 : 1849

D. 1 : 1

Answer: D

Solution:

Solution:

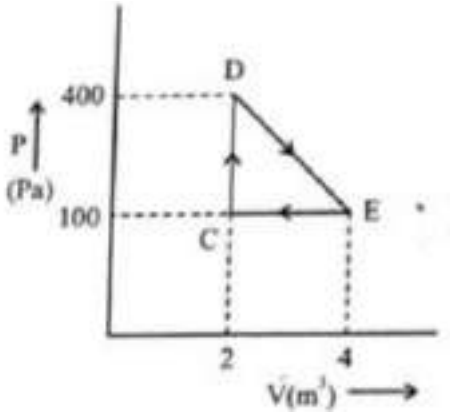
De Broglie wavelength is $\lambda = \frac{h}{mv}$

$$\lambda_p = \lambda_e \Rightarrow \frac{h}{m_p v_p} = \frac{h}{m_e v_e}$$

$$m_e v_e = m_p v_p \Rightarrow p_e = p_p \quad \therefore \frac{p_p}{p_e} = \frac{1}{1}$$

Question 11

A thermodynamic system is taken through cyclic process. The total work done in the process is :



Options:

A. 100J

B. 300J

C. Zero

D. 200J

Answer: B

Solution:

Solution:

Work done = Area under the curve

$$\Rightarrow W = \frac{1}{2} \times (4-2) \times (400-100) = \frac{1}{2}(2) \times 300$$

$$W = 300J$$

Question 12

In a reflecting telescope, a secondary mirror is used to:

Options:

A. reduce the problem of mechanical support

B. remove spherical aberration

C. make chromatic aberration zero

D. move the eyepiece outside the telescopic tube

Answer: D

Solution:

Solution:

To redirect the light that enters the telescope to the eyepiece or camera. The primary mirror of a reflecting telescopes gathers the light and reflects towards the secondary mirror which then reflect the light towards the eyepiece allowing the observer to see image.

It has advantage of a large focal length in a short telescope.

Question 13

The magnetic moment of an electron (e) revolving in an orbit around nucleus with an orbital angular momentum is given by:

Options:

A. $\vec{\mu}_L = \frac{e\vec{L}}{2m}$

B. $\vec{\mu}_L = -\frac{e\vec{L}}{2m}$

C. $\vec{\mu}_l = -\frac{e\vec{L}}{m}$

D. $\vec{\mu}_l = \frac{2e\vec{L}}{m}$

Answer: A

Solution:

Solution:

As $\vec{M} = \vec{IA}$

$\Rightarrow \left|\vec{M}\right| = \frac{e}{\frac{2\pi R}{v}}\pi R^2 \quad \left[\because I = \frac{Q}{T} = \frac{e}{\frac{2\pi R}{v}}\right]$

$\Rightarrow \left|\vec{M}\right| = \frac{1}{2}evR \Rightarrow \left|\vec{M}\right| = \frac{mvR}{1}\cdot\frac{e}{2m}$

$\Rightarrow \left|\vec{M}\right| = \frac{eL}{2m} \Rightarrow \left|\vec{M}\right| = -\frac{e\vec{L}}{2m}$

[$\because$ Here $\vec{M}$ and $\vec{L}$ will always be opposite]

Question 14

The ratio of intensities at two points P and Q on the screen in a Young's double slit experiment where phase difference between two wave of same amplitude are $\frac{\pi}{3}$ and $\frac{\pi}{2}$, respectively are

Options:

A. 1 : 3

B. 3 : 1

C. 3 : 2

D. 2 : 3

Answer: C

Solution:

Solution:

Intensity at a point in Young's double slit experiment is given by

$I = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\varphi$

Here $I_1 = I_2 = I_0$ (say)

At P

$\therefore I_p = I_0 + I_0 + 2I_0\cos\frac{\pi}{3} = 2I_0 + 2I_0 \times \frac{1}{2} = 3I_0$

At Q

$I_Q = I_0 + I_0 + 2I_0\cos 90^\circ = 2I_0$

$\frac{I_P}{I_Q} = \frac{3}{2}$

Question 15

A bicycle tyre is filled with air having pressure of 270 kPa at 27˚C. The approximate pressure of the air in the tyre when the temperature increases to 36˚C is

Options:

A. 270 kPa

B. 262 KPa

C. 278 kPa

D. 360 kPa

Answer: C

Solution:

Solution:

From the ideal gas equation $PV = nRT$

Here, volume is constant $\therefore \frac{P_1}{T_1} = \frac{P_2}{T_2}$

Here, $T_1 = 27 + 273 = 300K$

$P_1 = 270\,kPa$

$T_2 = 36 + 273 = 309K$

$\Rightarrow P_2 = \frac{P_1}{T_1} \times T_2 = \frac{270 \times (309)}{300} = 278\,kPa.$

Question 16

A particle executes SHM of amplitude A. The distance from the mean position when it's kinetic energy becomes equal to its potential energy is

Options:

A. $\sqrt{2A}$

B. 2A

C. $\frac{1}{\sqrt{2}}A$

D. $\frac{1}{2}A$

Answer: C

Solution:

Solution:

Let the distance from the mean position is X.

Given KE = PE

So, $\frac{1}{2}M\omega^2(A^2-x^2)=\frac{1}{2}M\omega^2x^2\ A^2-x^2=x^2\Rightarrow A^2=2\times2$

$\therefore x=\pm\frac{A}{\sqrt{2}}$

Question 17

Electric field in a certain region is given by $\vec{E}=\left(\frac{A}{x^2}\hat{i}+\frac{B}{y^3}\hat{j}\right)$. The SI unit of A and B are:

Options:

A. Nm^3C^{-1}; Nm^2C^{-1}

B. Nm^2C^{-1}; Nm^3C^{-1}

C. Nm^3C; Nm^2C

D. Nm^2C; Nm^3C

Answer: B

Solution:

Solution:

Electric field in a certain region is given by,

$\vec{E}=\frac{A}{x^2}\hat{i}+\frac{B}{y^3}\hat{j}$

$\left[\frac{A}{x^2}\right]=NC^{-1}\Rightarrow[A]=Nm^2C^{-1}$

$\left[\frac{B}{y^3}\right]=NC^{-1}\Rightarrow[B]=Nm^3C^{-1}$

Question 18

At any instant the velocity of a particle of mass 500g is $\left(2t\hat{i}+3t^2\hat{j}\right)ms^{-1}$. If the force acting on the particle at t = 1s is $\left(\hat{i}+x\hat{j}\right)N$. Then the value of x will be:

Options:

A. 3

B. 4

C. 6

D. 2

Answer: A

Solution:

Solution:

Mass of particle,

$m = 500g = 0.5kg$

velocity of a particle,

$\vec{v} = 2t\hat{i} + 3t^2\hat{j}$

$\vec{a} = \frac{d\vec{v}}{dt} = 2\hat{i} + 6t\hat{j}$

at $t = 1$, $\vec{a} = 2\hat{i} + 6\hat{j}$

Force acting on the particle,

$\vec{F} = m\vec{a} = 0.5(2\hat{i} + 6\hat{j}) = \hat{i} + 3\hat{j}$

$\vec{F} = \hat{i} + x\hat{j}$

Hence $x = 3$

Question 19

A particle of mass m moving with velocity v collides with a stationary particle of mass 2m. After collision, they stick together and continue to move together with velocity

Options:

A. v

B. $\frac{v}{2}$

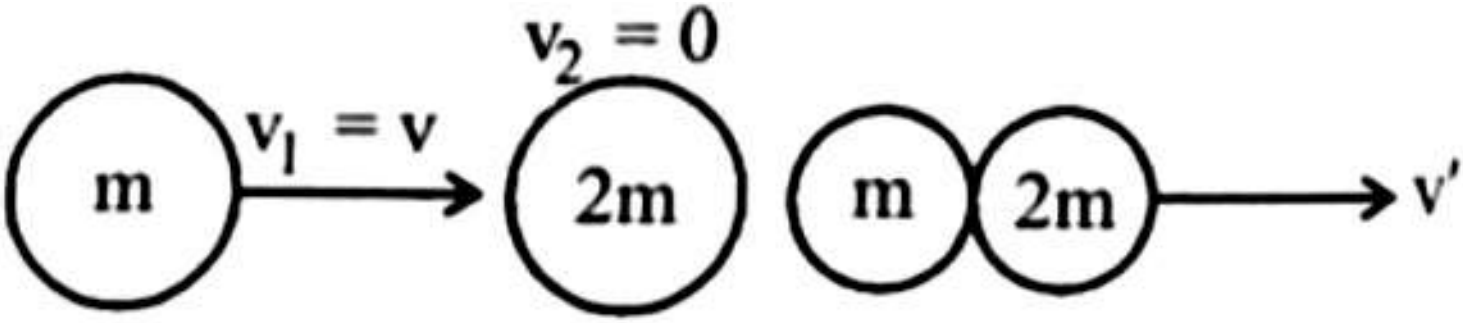
C. $\frac{v}{3}$

D. $\frac{V}{4}$

Answer: C

Solution:

Solution:



Applying conservation of linear momentum

$$\Rightarrow \vec{P}_i = \vec{P}_f (\because P = mv)$$

$$mv_1 + 2mv_2 = (m + 2m)v'$$

$$mv + 2m \times 0 = (3m)v'$$

$$\Rightarrow mv = 3mv' \Rightarrow v' = \frac{v}{3}$$

Question 20

Which of the following Maxwell's equations is valid for time varying conditions but not valid for static conditions :

Options:

A. $\oint \vec{B} \cdot \vec{dl} = \mu_0 I$

B. $\oint \vec{E} \cdot \vec{dl} = 0$

C. $\oint \vec{E} \cdot \vec{dl} = -\frac{\partial \varphi_B}{\partial t}$

D. $\oint \vec{D} \cdot \vec{dA} = Q$

Answer: C

Solution:

Solution:

For time varying condition Maxwell's equation, $\oint \vec{E} \cdot \vec{dl} = -\frac{d\varphi_B}{dt}$

Question 21

In an LC oscillator, if values of inductance and capacitance become twice and eight times, respectively, then the resonant frequency of

oscillator becomes x times its initial resonant frequency ω_0. The value of x is:

Options:

A. 1 / 4

B. 16

C. 1 / 16

D. 4

Answer: A

Solution:

Solution:

The resonance frequency of LC oscillations circuit is

$$\omega = \frac{1}{\sqrt{LC}} \Rightarrow L' \rightarrow 2L$$

$$C' \rightarrow 8C$$

$$\omega = \frac{1}{\sqrt{2L \times 8C}} = \frac{1}{4\sqrt{LC}} 9\omega_0 = \frac{1}{\sqrt{LC}}$$

$$\omega = \frac{\omega_0}{4} \text{ So, } x = \frac{1}{4}$$

Question 22

A conducting loop of radius $\frac{10}{\sqrt{\pi}}$ cm is placed perpendicular to a uniform magnetic field of 0.5T. The magnetic field is decreased to zero in 0.5 s at a steady rate. The induced emf in the circular loop at 0.25s is:

Options:

A. emf = 1 mV

B. emf = 10 mV

C. emf = 100 mV

D. emf = 5 mV

Answer: B

Solution:

Solution:

As $\varepsilon_{|t=0.5\,sec} = -\dfrac{d\varphi}{dt}$

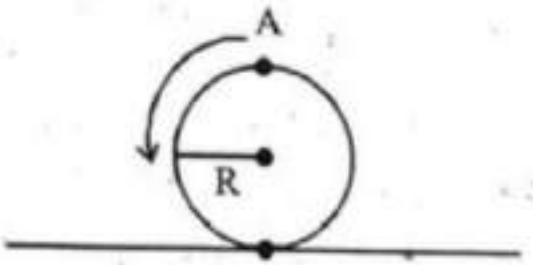
$= -A\dfrac{dB}{dt}$ $[\because \theta = 0^\circ \Rightarrow \cos\theta = 1]$

$= -\pi \times \left(\dfrac{10}{\sqrt{\pi}}\right)^2 \times 10^{-4} \times \dfrac{0-0.5}{0.5} = 10^{-2}\text{V} = 10\,\text{mV}$

As $\dfrac{dB}{dt}$ = constant ⇒ Induced emf will not change with time. So, $e|_{0.5\,sec} = e|_{0.25\,sec} = 10\,\text{mV}$

Question 23

A disc is rolling without slipping on a surface. The radius of the disc is R. At t = 0, the top most point on the disc is A as shown in figure. When the disc completes half of its rotation, the displacement of point A from its initial position is



Options:

A. $R\sqrt{(\pi^2 + 4)}$

B. $R\sqrt{(\pi^2 + 1)}$

C. 2R

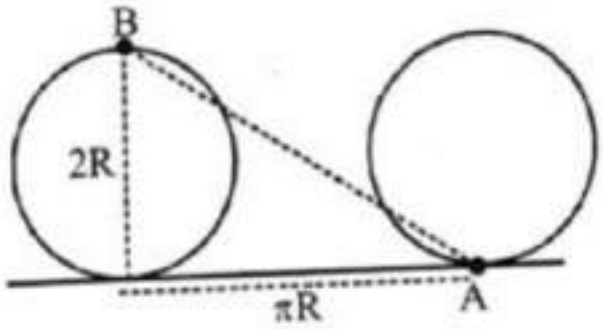
D. $2R\sqrt{(1 + 4\pi^2)}$

Answer: A

Solution:

Solution:

From figure,



Displacement,

$BA = \sqrt{(2R)^2 + (\pi R)^2} = R\sqrt{4 + \pi^2}$

Question 24

Two planets A and B of radii R and 1.5R have densities ρ and ρ / 2 respectively. The ratio of acceleration due to gravity at the surface of B to A is :

Options:

A. 2 : 3

B. 2 : 1

C. 3 : 4

D. 4 : 3

Answer: C

Solution:

Solution:

Acceleration due to gravity,

$$g = \frac{GM}{R^2} = \frac{4}{3}\pi G\rho R$$

$$\therefore \frac{g_2}{g_1} = \frac{\rho_2}{\rho_1} \times \frac{R_2}{R_1} = \frac{1}{2} \times 1.5 = \frac{3}{4}$$

Question 25

A 100m long wire having cross-sectional area $6.25 \times 10^{-4} m^2$ and Young's modulus is 10^{10} Nm^{-2} is subjected to a load of 250N, then the elongation in the wire will be :

Options:

A. 6.25×10^{-3}m

B. 4×10^{-4}m

C. 6.25×10^{-6}m

D. 4×10^{-3}m

Answer: D

Solution:

Solution:

$$\Delta \ell = \frac{F\ell}{YA} = \frac{250 \times 100}{10^{10} \times 6.25 \times 10^{-4}} = 40 \times 10^{-4}\text{m}$$

$$= 4 \times 10^{-3}\text{m}$$

Question 26

The ratio of speed of sound in hydrogen gas to the speed of sound in oxygen gas at the same temperature is:

Options:

A. 4 : 1

B. 1 : 2

C. 1 : 4

D. 1 : 1

Answer: A

Solution:

Solution:

Given $M_{H_2} = 2; M_{O_2} = 32$

Speed of sound, $v = \sqrt{\frac{\gamma RT}{M}}$

$$\Rightarrow v \propto \frac{1}{\sqrt{M}}$$

$$\therefore \frac{v_{H_2}}{v_{O_2}} = \sqrt{\frac{M_{O_2}}{M_{H_2}}} = \sqrt{\frac{32}{2}} = 4 : 1$$

Question 27

The free space inside a current carrying toroid is filled with a material of susceptibility 2×10^{-2}. The percentage increase in the value of magnetic field inside the toroid will be

Options:

A. 2%

B. 0.2%

C. 0.1%

D. 1%

Answer: A

Solution:

Solution:

Given,

Susceptibility of material, $\chi_m = 2 \times 10^{-2}$

Using $\mu_r = 1 + \chi_m = 1 + 0.02 = 1.02$

$B_{final} = \mu_r B_0$ (here, B_0 = initial magnetic field)

% increase in magnetic field

$$= \frac{B_{final} - B_0}{B_0} \times 100 = \frac{\mu_r B_0 - B_0 \times 100}{B_0}$$

$$= \frac{(\chi + 1) - 1 \times 100}{1} = 0.02 \times 100 = 2\%$$

Question 28

The ratio of average electric energy density and total average energy density of electromagnetic wave is :

Options:

A. 2

B. $\frac{1}{2}$

C. 1

D. 3

Answer: B

Solution:

Solution:

We have $\frac{U_E}{U_T} = \frac{U_E}{U_E + U_B} = \frac{U_E}{2U_E} = \frac{1}{2}$

$$\left[\because U_E = U_B = \frac{1}{2} E_0 E_0^2 = \frac{B_0^2}{2\mu_0} \right]$$

Question 29

In a Young's double slit experiment, the intensities at two points, for the path difference $\frac{\lambda}{4}$ and $\frac{\lambda}{3}$ (λ being the wavelength of light used) are I_1 and I_2 respectively. If I_0 denotes the intensity produced by each one of the individual slits, then $\frac{I_1 + I_2}{I_0}$ = ...

Options:

A. 3

B. 5

C. 7

D. 10

Answer: A

Solution:

Solution:

Resultant intensity in Young's double slit experiment

$$I = 4I_0\cos^2\left(\frac{\Delta\varphi}{2}\right)$$

For path difference $\frac{\lambda}{4}$ phase difference,

$$\Delta\varphi = \frac{2\pi}{\lambda}\times\frac{\lambda}{4} = \frac{\pi}{4}$$

$$\therefore I_1 = 4I_0\cos^2\left(\frac{\pi}{4}\right) = 2I_0$$

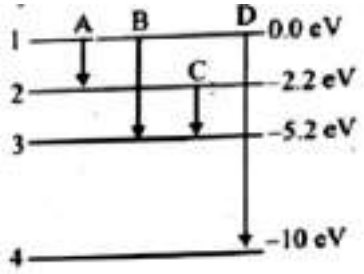
For path difference $\frac{\lambda}{3}$

$$I_2 = 4I_0\cos^2\left(\frac{2\pi}{\lambda}\times\frac{\lambda}{3}\right) = I_0$$

$$\therefore \frac{I_1 + I_2}{I_0} = 3$$

Question 30

The energy levels of an atom is shown is figure. Which one of these transitions will result in the emission of a photon of wavelength 124.1 nm ? Given ($h = 6.62 \times 10^{-34}$ Js)



Options:

A. B

B. A

C. C

D. D

Answer: D

Solution:

Solution:

As $E(eV) = \frac{1240}{\lambda(nm)} = \frac{1240}{124.1} \simeq 10\,eV$ Only is transition (D), the energy gap is $10\,eV$ So, option (d) is correct

$$K_a = 0.001\left(\frac{\alpha^2}{1-\alpha}\right) = \frac{0.001 \times \left(\frac{2}{19}\right)^2}{1-\left(\frac{2}{19}\right)}$$

Question 31

Frenkel and Schottky defects are :

Options:

A. nucleus defects

B. non-crystal defects

C. crystal defects

D. nuclear defects

Answer: C

Solution:

Solution:

Frenkel and Schottky defects are crystal defects. It arises due to dislodgement of cation or anion from their places in the crystal lattice.

Question 32

The Bohr orbit radius for the hydrogen atom (n = 1) is approximately 0.530Å. The radius for the first excited state (n = 2) orbit is (in Å)

Options:

A. 0.13

B. 1.06

C. 4.77

D. 2.12

Answer: D

Solution:

Solution:

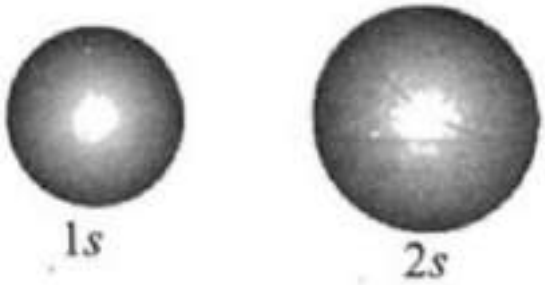
Given : Radius of hydrogen atom $= 0.530\ \mathring{A}$, Number of excited state (n) = 2 and atomic number of hydrogen atom (Z) = 1. We know that the Bohr radius

$$(r) = \frac{n^2}{Z} \times \text{radius of atom} = \frac{(2)^2}{1} \times 0.530$$

$$= 4 \times 0.530 = 2.12\mathring{A}$$

Question 33

The probability density plots of 1s and 2s orbitals are given in figure.'



The density of dots in a region represents the probability density of finding electrons in the region.
On the basis of above diagram which of the following statements is incorrect?

Options:

A. 1s and 2s orbitals are spherical in shape.

B. The probability of finding the electron is maximum near the nucleus.

C. The probability of finding the electron at a given distance is equal in all directions.

D. The probability density of electrons for 2s orbital decreases uniformly as distance from the nucleus increases.

Answer: D

Solution:

Solution:

The probability density of electrons in $2s$ orbital first increases then decreases and after that it increases again as distance increases from nucleus.

Question 34

Element with electronic configuration $1s^2 2s^2 2p^6 3s^2 3p^6 3d^{10} 4s^2 4p^6 4d^{10} 5s^2 5p^3$ belongs to the following group of the periodic table

Options:

A. 5 th

B. 15 th

C. 3rd

D. 17 th

Answer: B

Solution:

Solution:

Its valence shell has 5 electrons (ns^2, np^3). It belongs to 15 th group of the periodic table.

Question 35

Which of the following pairs will form the most stable ionic bond?

Options:

A. Na and Cl

B. Mg and F

C. Li and F

D. Na and F

Answer: B

Solution:

Solution:

The stability of the ionic bond depends upon the lattice energy which is expected to be more between Mg and F due to +2 charge on Mg atom.

Question 36

How much ethyl alcohol must be added to 1 litre of water so that the solution will freeze at 14˚C ?
(K_f. for water = 1.86˚C / mol)

Options:

A. 7.5 mol

B. 8.5 mol

C. 9.5 mol

D. 10.5 mol

Answer: A

Solution:

Solution:

7.5 mol

$$\Delta T_f = K_f m$$

$$\Delta T_f = K_f \frac{n_2 \times 1000}{w_1}$$

$$\Rightarrow 14 = 1.86 \times \frac{n_2 \times 1000}{1000}$$

$$n_2 = 7.5 \text{ mol}$$

Question 37

The conductivity of a weak acid HA of concentration 0.001 mol L^{-1} is $2.0 \times 10^{-5} Scm^{-1}$.
If Λ_m°(HA) = $190 Scm^2 mol^{-1}$, the ionization constant (K_a) of HA is equal to _____ 10^{-6}. ×

Options:

A. 24

B. 48

C. 12

D. 45

Answer: C

Solution:

Solution:

$$\Lambda_m = 1000 \times \frac{\kappa}{M}$$

$$= 1000 \times \frac{2 \times 10^{-5}}{0.001} = 20 Scm^2 mol^{-1}$$

$$\Rightarrow \alpha = \frac{\Lambda_m}{\Lambda_m^\circ} = \frac{20}{190} = \left(\frac{2}{19}\right)$$

$$\underset{0.001(1-\alpha)}{HA} \rightleftharpoons \underset{0.001\alpha}{H^+} + \underset{0.001\alpha}{A^-}$$

$$K_a = 0.001\left(\frac{\alpha^2}{1-\alpha}\right) = \frac{0.001 \times \left(\frac{2}{19}\right)^2}{1-\left(\frac{2}{19}\right)}$$

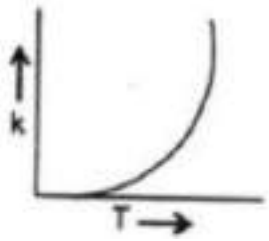
$$= 12.3 \times 10^{-6}$$

Question 38

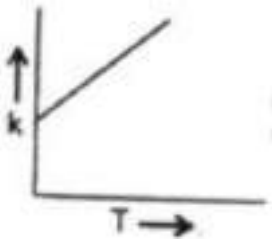
Plots showing the variation of the rate constant (k) with temperature (T) are given below. The plot that follows Arrhenius equation is

Options:

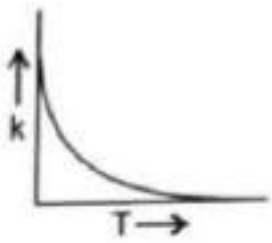
A.



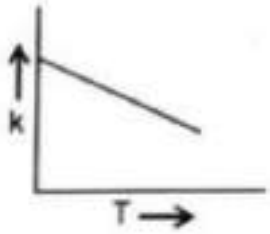
B.



C.



D.



Answer: A

Solution:

Solution:

As per Arrhenius equation $\left(k = Ae^{-E_a/RT}\right)$, the rate constant increases exponentially with temperature.

Question 39

Which of the following method is used for coagulation of the sol?

Options:

A. By mixing two oppositely charged sols.

B. By electrophoresis.

C. By addition of electrolytes.

D. All of the above.

Answer: D

Question 40

The reaction that does NOT take place in a blast furnace between 900K to 1500K temperature range during extraction of iron is :

Options:

A. $Fe_2O_3 + CO \rightarrow 2\,FeO + CO_2$

B. $FeO + CO \rightarrow Fe + CO_2$

C. $C + CO_2 \rightarrow 2\,CO$

D. $CaO + SiO_2 \rightarrow CaSiO_3$

Answer: A

Solution:

Solution:

At 900 – 1500K (higher temperature range in the blast furnace)

Reaction which take place are:

$C + CO_2 \rightarrow 2CO$

$FeO + CO \rightarrow Fe + CO_2$

$CaO + SiO_2 \rightarrow CaSiO_3$ (Slag formation)

$Fe_2O_3 + CO \rightarrow 2FeO + CO_2$ at 500 – 800K.

Question 41

Kinetic theory of gases proves

Options:

A. only Boyle's law

B. only Charles' law

C. only Avogadro's law

D. all of these

Answer: D

Solution:

Solution:

Kinetic theory of gases proves all the given gas laws.

Question 42

If enthalpies of formation of $C_2H_4(g)$, $CO_2(g)$ and $H_2O(l)$ at 25˚C and 1 atm pressure are 52 , 394 and −286 kJ / mol respectively, the change in enthalpy for combustion of C_2H_4 is equal to

Options:

A. $-141.2\,kJ / mol$

B. $-1412\,kJ / mol$

C. $+14.2\,kJ / mol$

D. $+1412\,kJ / mol$

Answer: B

Solution:

Solution:

Enthalpy of formation of C_2H_4, CO_2 and H_2O are 52, −394 and −286 kJ/mol respectively. (Given)

The reaction is

$$C_2H_4 + 3O_2 \rightarrow 2CO_2 + 2H_2O$$

change in enthalpy,

$$(\Delta H) = \Delta H_{\text{products}} - \Delta H_{\text{reactants}}$$

$$= 2 \times (-394) + 2 \times (-286) - (52 + 0)$$

$$= -1412\ \text{kJ/mol}$$

Question 43

The photochemical smog does not generally contain:

Options:

A. NO

B. SO_2

C. NO_2

D. HCHO

Answer: C

Solution:

Solution:

Photochemical smog contains nitrogen dioxide (NO2), Ozone (O3), PAN (peroxyacetylnitrate), and compounds containing −CHO group.

Question 44

Geometrical isomerism is not shown by

Options:

A. $CH_3CH_2\overset{CH_3}{\overset{|}{C}} = \underset{CH_3}{\underset{|}{C}}CH_2CH_3$

B. $C_2H_5 - \underset{H}{\underset{|}{C}} = \underset{H}{\underset{|}{C}} - CH_2I$

C. $CH_2 = C(Cl)CH_3$

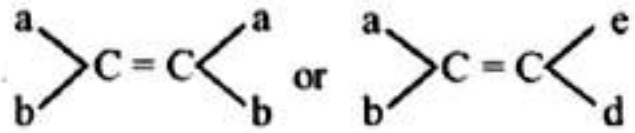
D. $CH_3 - CH = CH - CH = CH_2$

Answer: C

Solution:

Solution:

The condition for geometrical isomerism is



$CH_2 - C(Cl)CH_3$ does not follow above mention condition.

Question 45

For the separation of two immiscible liquids which method is used?

Options:

A. Chromatography

B. Fractionating column

C. Fractional distillation

D. Separating funnel

Answer: D

Solution:

Solution:

Separating funnel is used when the two liquids are immiscible.

Question 46

What is x in the following reaction?
$Al(s) + NaOH(aq) + H_2O(l) \rightarrow x + H_2(g)$

Options:

A. $Na_2[Al(OH)_4]^-$

B. $Na^+[Al(OH)_4]^-$

C. $Na_2[Al(OH)_6]^-$

D. $Na^+[Al(OH)_6]^-$

Answer: B

Solution:

Solution:

$$2Al(s) + 2NaOH(aq) + 6H_2O(l) \rightarrow$$

$$2Na^+[Al(OH)_4]^-(aq) + 3H_2(g)$$

Question 47

Which of the following will precipitate first when aqueous solution containing sulphate ions are added?

Options:

A. Mg^{2+}

B. Ca^{2+}

C. Sr^{2+}

D. Ba^{2+}

Answer: D

Solution:

Solution:

Down the group solubility of sulphate decreases. Thus, Ba^{2+} ions will precipitate out most easily.

Question 48

Ionic hydrides reacts with water to give

Options:

A. acidic solutions

B. hydride ions

C. basic solutions

D. electrons

Answer: C

Solution:

Solution:

Ionic hydrides give the basic solution when it reacts with water, e.g.,

$LiH + H_2O \rightarrow LiOH + H_2$

Question 49

The drug used as an antidepressant is

Options:

A. Luminol

B. Tofranil

C. Mescaline

D. Sulphadiazine

Answer: B

Solution:

Solution:

Tofranil is used for the treatment of antidepressant.

Question 50

Melamine plastic crockery is a copolymer of:

Options:

A. HCHO and melamine

B. HCHO and ethylene

C. melamine and ethylene

D. None of these

Answer: A

Solution:

Solution:

Melamine plastic crockery is a copolymer of HCHO and Melamine.

Question 51

The helical structure of protein is stabilized by

Options:

A. dipeptide bonds

B. hydrogen bonds

C. ether bonds

D. peptide bonds

Answer: B

Solution:

Solution:

The α-helix structure is formed when the chain of α-amino acids coils as a right handed screw (called α-helix) because of the formation of hydrogen bonds between amide groups of the same peptide chain, i.e., NH group in one unit is linked to carbonyl oxygen of the third unit by hydrogen bonding. This hydrogen bonding between different units is responsible for holding helix in a position.

Question 52

Which of the following factors affect the basic strength of amine?
(i) Inductive effect
(ii) Steric hinderance
(iii) Solvation effect
(iv) Solubility in organic solvents.

Options:

A. (i) and (iv)

B. (i), (ii) and (iii)

C. (ii) and (iii)

D. (ii) and (iv)

Answer: B

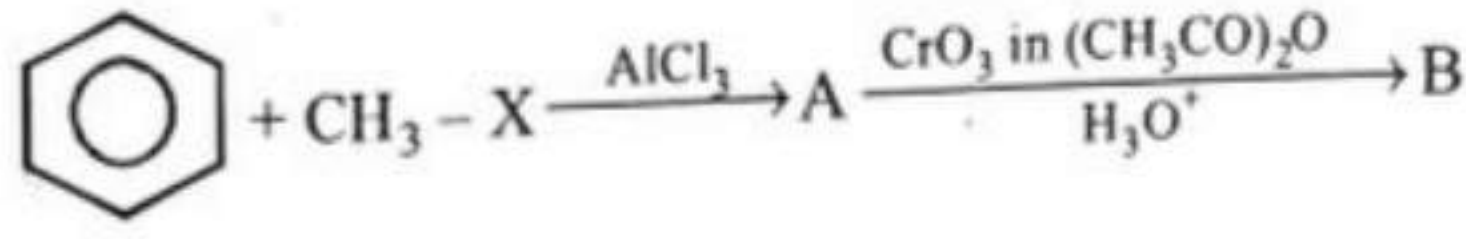
Solution:

Solution:

Inductive effect, steric hinderance and solvation effect the basic strength of amines.

Question 53

Find out B in the given reactions



Options:

A. acetophenone

B. benzaldehyde

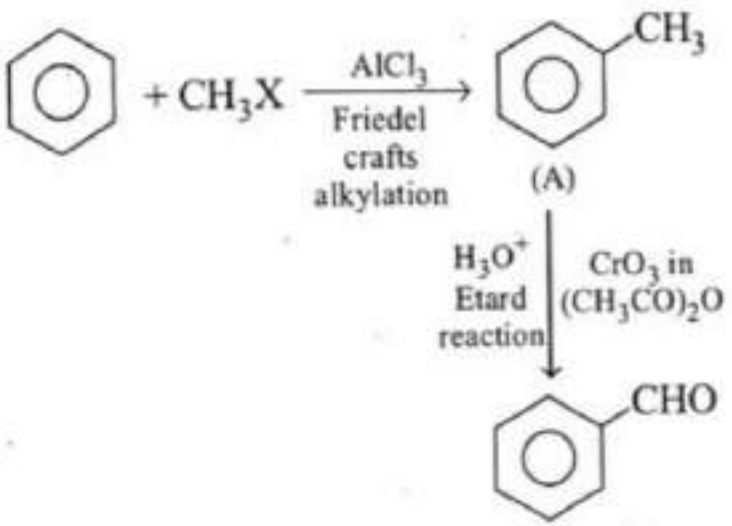
C. cyclohexyl carbaldehyde

D. benzoic acid

Answer: B

Solution:

Solution:

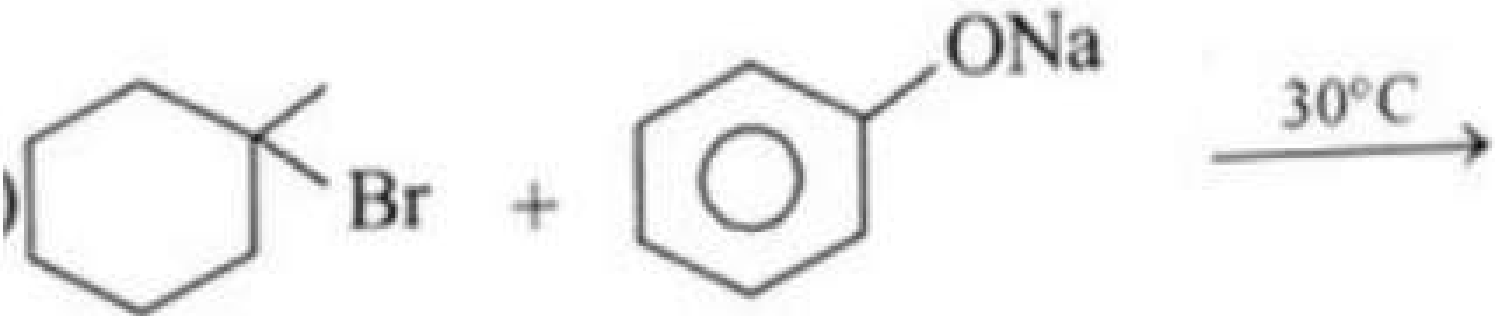


Question 54

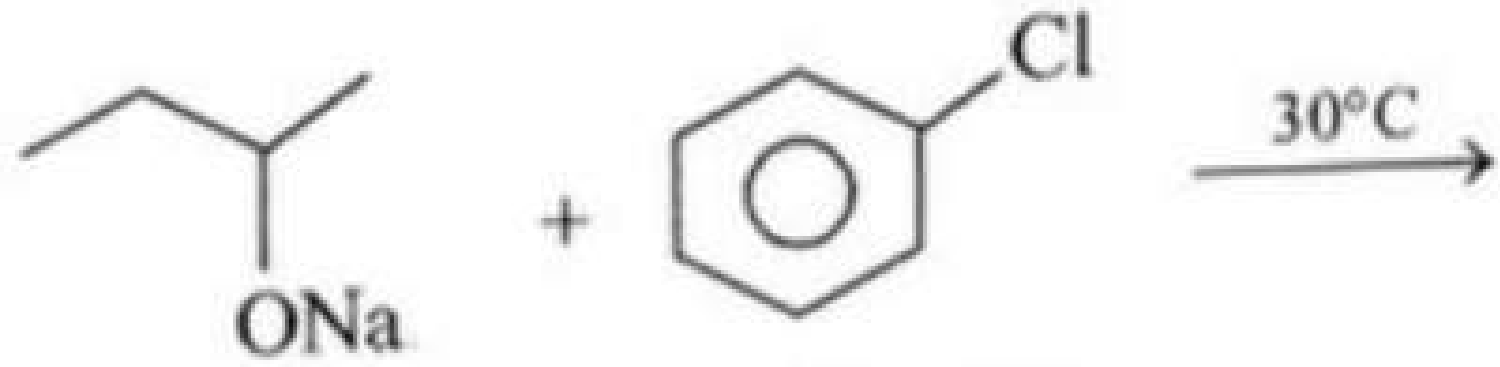
Which method is useful for the synthesis of ether?

Options:

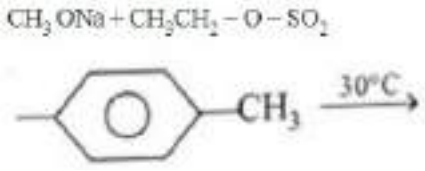
A.



B.



C.



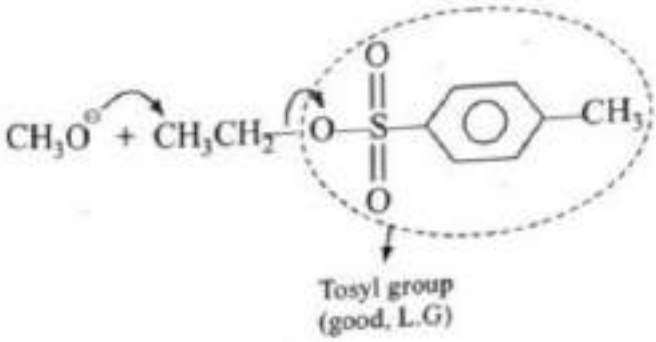
D. $CH_3CH - OH \xrightarrow[[443K]]{H_2SO_4}$

Answer: C

Solution:

Solution:

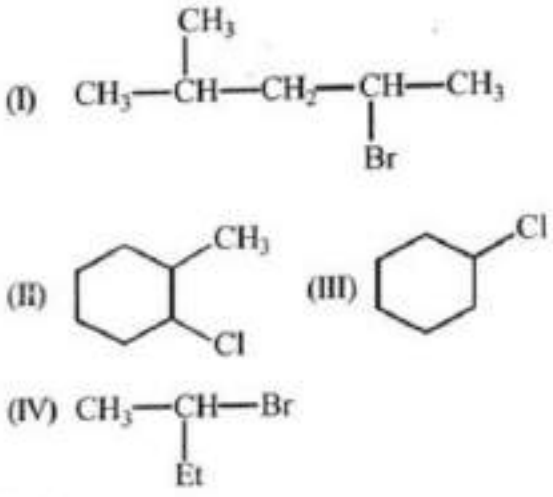
At 443K compound in (d) will produce propene. In (a) alkene will be produced as tertiary halide and strong base favours elimination. In (b) reaction is not possible at room temperature as due to resonance C − Cl bond has partial double bond character which is very difficult to break.



$\rightarrow OTs^{\ominus} + CH_3OCH_2CH_3$

Question 55

Among the given halides, which one will give same product in both S_N1 and S_N2 reactions.



Options:

A. (III) only

B. (I) and (II)

C. (III) and (IV)

D. (I), (III) and (IV)

Answer: C

Solution:

Solution:

S_N2 and S_N1 same, if $C^{\oplus}$ not rearrange.

Question 56

Among the ligands NH_3, en, $CN^-$and CO the correct order of their increasing field strength, is :

Options:

A. NH_3< en <CN^- < CO

B. CN^- < NH_3 < NH_3< en

C. en <CN^- < NH_3 < CO

D. CO < NH_3< en <CN^-

Answer: A

Solution:

Solution:

Ligands can be arranged in a series in the orders of increasing field strength as given below:

Weak field ligands :

$I^- < Br^- < S^{2-} < SCN^- < Cl^- < N_3^-, F^-$

$< Urea, OH^- <$ oxalate

Strong field ligands

$O^{--} < H_2O < NCS^- < EDTA < Py, NH_3 <$

$en = SO_3^- <$ bipy, Phen $< NO_2^- < CH_3^-$

$< C_6H_5^- < CN^- < CO$

Such a series is termed as spectrochemical series. It is an experimentally determined series based on the absorption of light by complexes with different ligands.

Question 57

S − S bond is not present in

Options:

A. $S_2O_4^{2-}$

B. $S_2O_5^{2-}$

C. $S_2O_3^{2-}$

D. $S_2O_7^{2-}$

Answer: D

Solution:

Solution:

Due to some backbonding by sidewise overlapping between d-orbitals of metal and p orbital of carbon, the $Fe-C$ bond in $[Fe(CO)_5]$ hasboth σ and π character.

Question 58

In the laboratory, manganese (II) salt is oxidised to permanganate ion in aqueous solution by

Options:

A. hydrogen peroxide

B. conc. nitric acid

C. peroxy disulphate

D. dichromate

Answer: C

Solution:

Solution:

In laboratory, manganese (II) ion salt is oxidised to permanganate ion in aqueous solution by peroxodisulphate.

$$2Mn^{2+} + \underset{\text{peroxodisulphate ion}}{5S_2O_8^{2-}} + 8H_2O \rightarrow$$

$$2MnO_4^- + 10SO_4^{2-} + 16H^+$$

Question 59

Which one of the following molecular hydrides acts as a Lewis acid?

Options:

A. NH_3

B. H_2O

C. B_2H_6

D. CH_4

Answer: C

Solution:

Solution:

Boron in B_2H_6 is electron deficient

Question 60

Electrode potential data are given below:

$Fe^{+3}(aq) + e^- \rightarrow Fe^{+2}(aq)$; $E^\circ = +0.77V$

$Al^{3+}(aq) + 3e^- \rightarrow Al_{(s)}$; $E^\circ = -1.66V$

$Br_2(aq) + 2e^- \rightarrow 2Br^-(aq)$; $E^\circ = +1.08V$

Based on the data, the reducing power of Fe^{2+}, Al and Br^-will increase in the order

Options:

A. $Br^- < Fe^{2+} < Al$

B. $Fe^{2+} < Al < Br^-$

C. $Al < Br^- < Fe^{2+}$

D. $Al < Fe^{2+} < Br$

Answer: A

Solution:

Solution:

	Fe	Al	Br
E°_{Red}	0.77	−1.66	1.08
E°_{Oxi}	−0.77	1.66	−1.08

Hence, reducing power Al > Fe^{2+} > Br^-

Question 61

fricassee

Options:

A. grill

B. decorate

C. stew

D. to baste

Answer: C

Solution:

Solution:

Question 62

RETRIBUTION

Options:

A. compensation

B. forgiveness

C. contempt

D. grudge

Answer: B

Solution:

Solution:
Retribution is punishment, contempt is feeling of disgust and grudge is an illfeeling.

Question 63

SUMPTUOUS

Options:

A. irritable

B. meagre

C. fancy

D. sad

Answer: B

Solution:

Solution:
Sumptuous means sufficient or more in quantity, meagre means very little.

Question 64

Rajeev failed in the examination because his answers were not _____ to the questions asked

Options:

A. allusive

B. pertinent

C. revealing

D. referential

Answer: B

Solution:

Solution:
Pertinent means relevant.

Question 65

Choose the correct words to complete the sentence:
It was _____ cold _____ we couldn't go out.

Options:

A. so, that

B. too, to

C. neither, nor

D. either, or

Answer: A

Solution:

Solution:
Cold and couldn't go out are suggestive that it was very cold because of which we couldn't go out. So the word substitute for very is 'so' and because is 'that'.

Question 66

Faced with the
P : traditional culture in the pre-independence India
Q : challenge of the intrusion of colonial culture and ideology
R : developed during the nineteenth century
S : at attempt to reinvigorate traditional institutions and realize the potential of
Which one of the following is the correct sequence?

Options:

A. P − R − Q − S

B. Q-S - P - R

C. P − S − Q − R

D. Q − R − P − S

Answer: B

Solution:

Solution:
Faced with the challenge of the intrusion of colonial culture and ideology at attempt to reinvigorate traditional institutions and realize the potential of traditional culture in the pre-independence India developed during the nineteenth century.

Question 67

A diversified use
P : as a heating or power generation fuel by converting gas into
Q : adding a new dimension to the traditional use of gas
R : of natural gas is emerging
S : amongst other products, high quality diesel transportation fuel virtually free of sulphur
Which one of the following is the correct sequence?

Options:

A. R − P − Q − S

B. S − Q − P − R

C. R − Q − P − S

D. S − P − Q − R

Answer: C

Solution:

Solution:
A diversified use of natural gas is emerging adding a new dimension to the traditional use of gas as a heating or power generation fuel by converting gas into amongst other products, high quality diesel transportation fuel virtually free of sulphur.

Question 68

Music is often linked to _____.

Options:

A. anger

B. mood

C. anxiety

D. happiness

Answer: B

Solution:

Solution:
Music is often linked to mood. A certain song can make us feel happy, sad, energetic, or relaxed.

Question 69

How is music an important part of life?

Options:

A. It makes us feel different emotions

B. It makes us sad

C. It helps in our daily activities

D. It helps us in remembering things

Answer: A

Solution:

Solution:
Music helps one feel different emotions. Based on the mood, a certain song can make us feel happy, sad, energetic, or relaxed.

Question 70

Which of the statements is true?

Options:

A. All forms of music may heal wounds

B. All forms of music may have good effect

C. All forms of music may be sooting

D. All forms of music may have therapeutic effects

Answer: D

Solution:

Solution:
All forms of music may have therapeutic effects, although music from one's own culture may be most effective.

Question 71

On the following questions, select the related word/letters/from the attractive
MASTER: OCUVGT ::LABOUR:?

Options:

A. NCDQWT

B. HDERWT

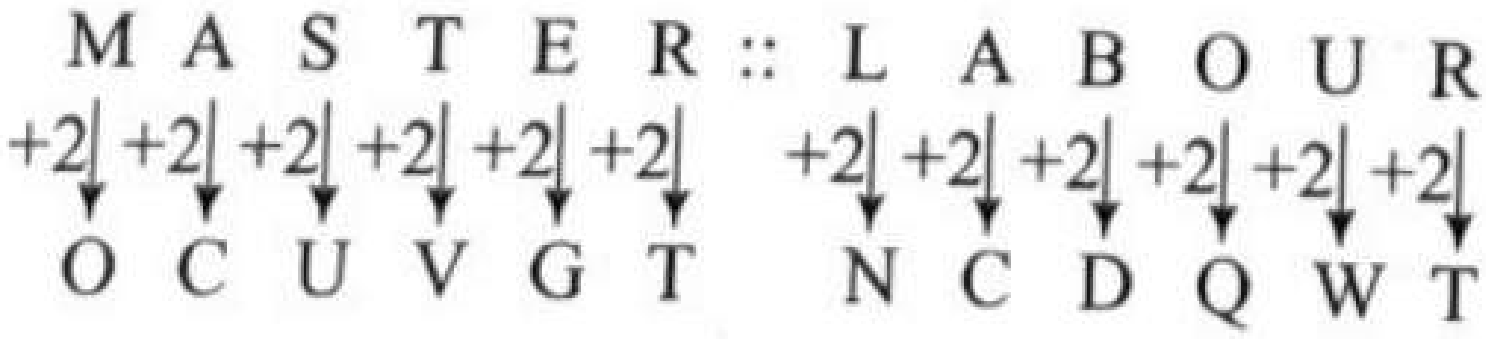
C. NBECRWT

D. NEDRWT

Answer: A

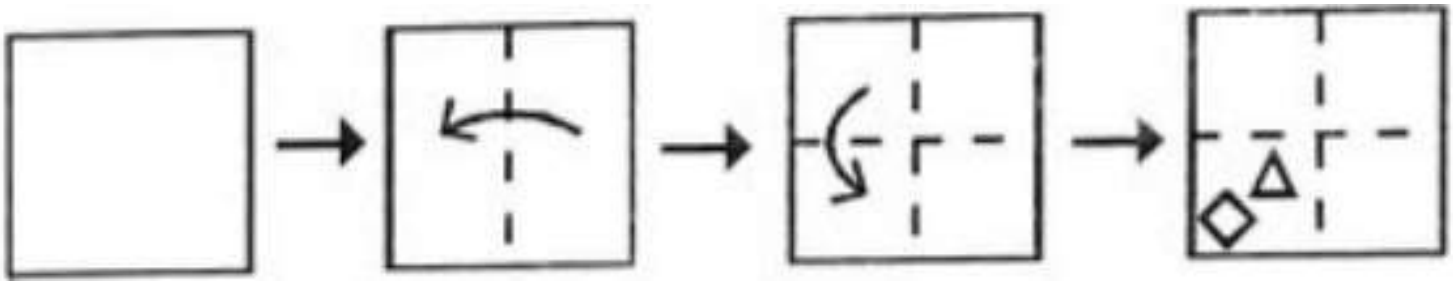
Solution:

Solution:



Question 72

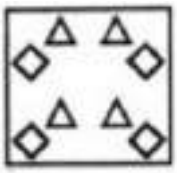
The sequence of folding a paper and the manner in which the folded paper has been cut is shown in the following figures. How would this paper look when unfolded?



Options:

A.

B.



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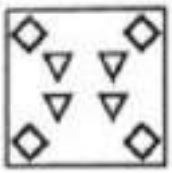
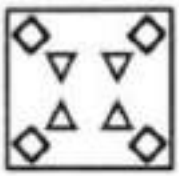


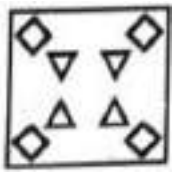
D.



Answer: D

Solution:

Solution:



Question 73

In a given code SISTER is coded as 535301 . UNCLE as 84670 and BOY as 129 . How is RUSTIC written in that code?

Options:

A. 633185

B. 185336

C. 363815

D. 581363

Answer: B

Solution:

Solution:
In this code the alphabets are coded as follows
SISTER UNCLE BOY
535301 84670 129
If we apply this method, the code comes out to be 185336

Question 74

Daya has a brother, Anil. Daya is the son of Chandra. Bimal is Chandra's father. In terms of relationship, what is Anil of Bimal?

Options:

A. Son

B. Grandson

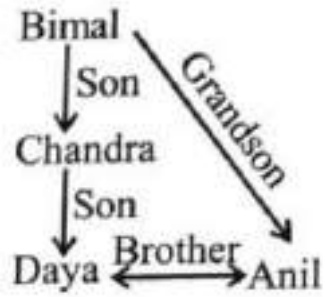
C. Brother

D. Grandfather

Answer: B

Solution:

Solution:



Question 75

Find the odd word pair among the given four word pairs.

Options:

A. Error : Accurate

B. Careless: Casual

C. Strength : Lethargy

D. Gloomy: Cheerful

Answer: B

Solution:

Solution:
Except option (b) all given pairs are synonyms to each other.

Question 76

Which letter will come at the place of question mark (?)

Options:

A. U

B. V

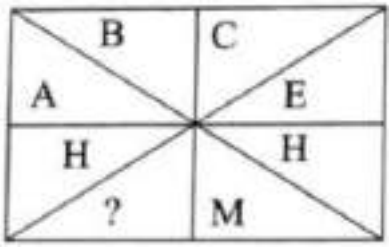
C. W

D. X

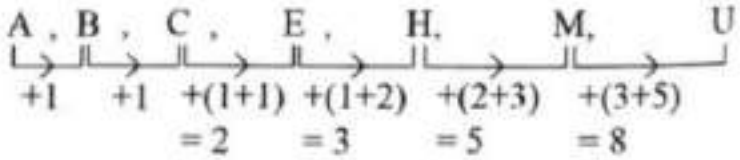
Answer: A

Solution:

Solution:



Pattern is, each next term is at the gap of sum of gaps of first two continuous terms.
Like:



Question 77

Arrange the following words as per order in the dictionary.
1. Flunching

2. Fluntlock
3. Flunpites
4. Fluntlocks
5. Flunchers

Options:

A. 1, 5, 2, 4, 3

B. 5, 1, 2, 4, 3

C. 5, 1, 3, 2, 4

D. 5, 1, 3, 4, 2

Answer: C

Solution:

Solution:
Flunchers '! Flunching '! Flunpites '! Fluntlock '! Fluntlocks

---

Question 78

Two statements are given followed by three conclusions numbered I, II and III. Assuming the statements to be true, even if they seem to be at variance with commonly known facts, decide which of the conclusions logically follow(s) from the statements.
Statements:
All utensils are spoons.
All bowls are spoons.
Conclusions:
I. No utensil is a bowls.
II. Some utensils are bowls
III. No spoon is a utensil.

Options:

A. Only conclusions I follows

B. Conclusions I and III follow

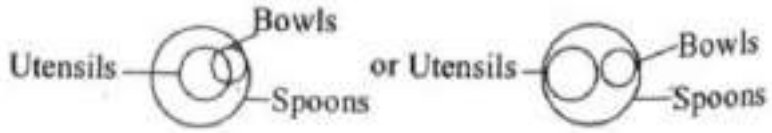
C. Either conclusion I or II follows

D. Only conclusion III follows

Answer: C

Solution:

Solution:



Hence, either conclusion I or II follows.

Question 79

In this question, a word has been given following by four other words, one of which cannot be formed by using the letters of the given word. Find this word.
'CHEMOTHERAPY'

Options:

A. HECTARE

B. MOTHER

C. THEATER

D. FATHER

Answer: D

Solution:

Solution:
FATHER

Question 80

Which one set of letters when sequentially placed at the gaps in the given letter series would complete it?
fgg _____ gff _____ f _____ gfg _____ fgfo

Options:

A. fggf

B. ccfc

C. fgfg

D. ffgg

Answer: A

Solution:

Solution:
The sequence is:
f ggxf / gf f / f gf / gxf f g / f
So, option (a) is correct.

Question 81

Select the option in which the numbers are related in the same way as are the numbers in the given set.
(9, 217, 8)

Options:

A. (4, 37, 3)

B. (2, 76, 5)

C. (5, 625, 6)

D. (3, 49, 2)

Answer: A

Solution:

Solution:
As, (9, 217, 8)
$9 \times 8 \Rightarrow 72 \times 3 + 1 = 217$
Similarly, (4, 37, 3)
$4 \times 3 = 12 \times 3 + 1 = 37$

Question 82

Find the next term in the following series: X24C, V22E, T20G, _____

Options:

A. RI 19

B. R19I

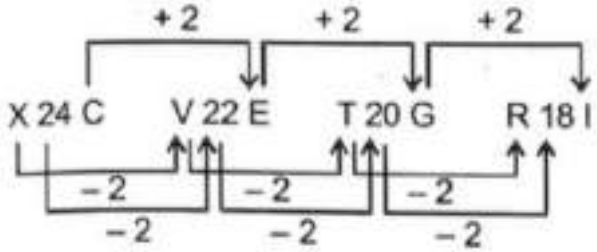
C. R18I

D. RI 18

Answer: C

Solution:

Solution:



Question 83

In the following question, Select the related number that will correct the place of question mark.
108 : 11664: : 112 : ?

Options:

A. 12504

B. 12544

C. 13644

D. 17644

Answer: B

Solution:

Solution:
$108^2 = 11664$
Similarly, $112^2 = 12544$

Question 84

Which number pair is odd among the given four number Pair

Options:

A. $123 - 321$

B. $456 - 654$

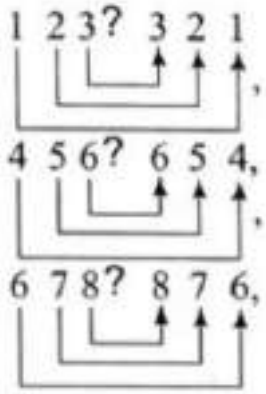
C. $789 - 978$

D. $678 - 876$

Answer: C

Solution:

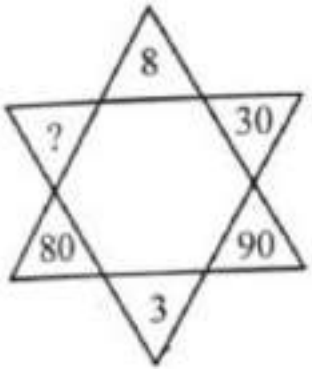
Solution:



but 789 − 978 does not follow this rule.

Question 85

In the questions, select the missing number from the given responses.



Options:

A. 20

B. 15

C. 40

D. 10

Answer: C

Solution:

Solution:

Question 86

Find the Missing Number: 2, 12, 36, 80, 150, ?

Options:

A. 195

B. 210

C. 252

D. 258

Answer: C

Solution:

Solution:
Given series is: 2, 12, 36, 80, 150
$2 \rightarrow (1) + (1)^3$
$12 \rightarrow (2)^2 + (2)^2$
$36 \rightarrow (3)^2 + (3)^3$
$80 \rightarrow (4)^2 + (4)^3$
$150 \rightarrow (5)^2 + (5)^3$
$? \rightarrow (6)^2 + (6)^3 = 36 + 216$
$= 252$

Question 87

If 'when' means ' x ', 'you' means ' ÷ ' 'come' means '-' and 'will' means '+', then what will be the value of " 8 when 12 will 16 you 2 come 10" = ?

Options:

A. 45

B. 94

C. 96

D. 112

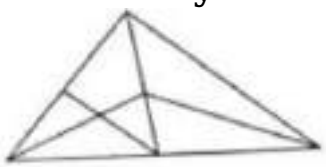
Answer: B

Solution:

Solution:
When →×
You →÷
Come →−
Will →+
8 When 12 Will 16 You 2 Come 10 = ?
Using the correct symbols, we have
$= 8 \times 12 + 16 \div 2 - 10$
$= 96 + 8 - 10$
$= 104 - 10 = 94$

Question 88

How many triangles are there in the following figure?



Options:

A. 11

B. 13

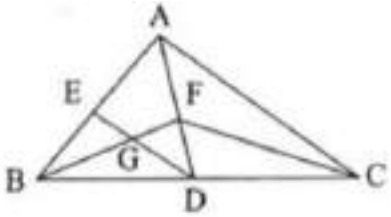
C. 9

D. 15

Answer: B

Solution:

Solution:



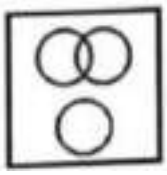
The triangles are:
△ABC; △ABD; △ADC; △AFC
ΔFDC; ΔAFB; ΔFDB; ΔFBC
△GBD; ΔADE; ΔGBE; ΔFDG
△DBE

Question 89

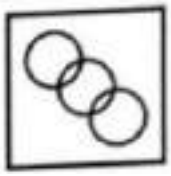
Identify the Venn diagram that best represents the relationship among classes given below. Profit, dividend and Bonus

Options:

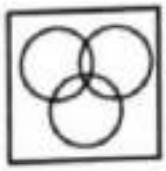
A.



B.



C.



D.

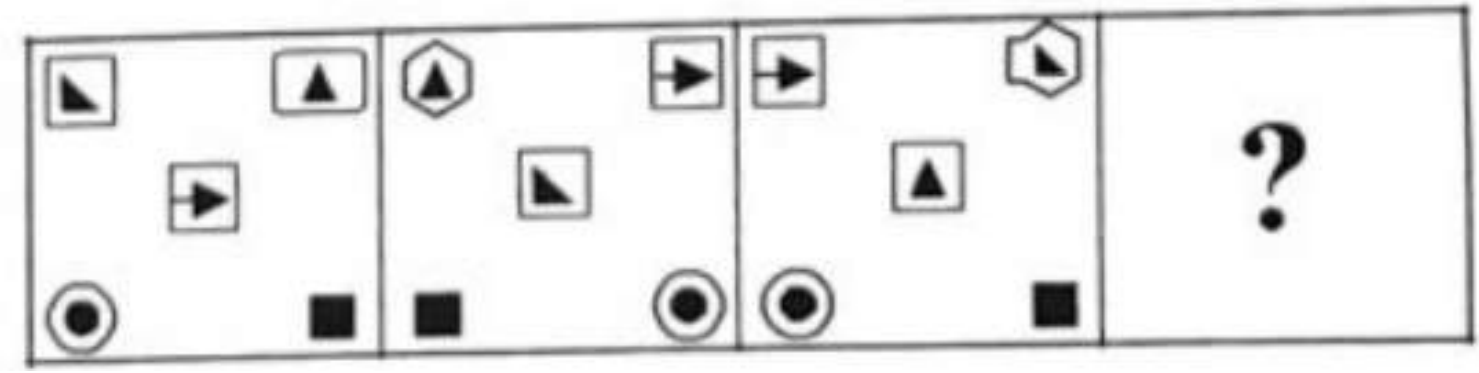
Answer: D

Solution:

Solution:
Bonus and dividend are different from each other but both these are part of profit.

Question 90

Select the figure from among the given options that can replace the question mark (?) in the following series.



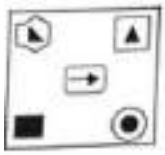
Options:

A.

B.

C.

D.



Answer: A

Solution:

Solution:
Figure given in option (a) can replace the question mark.

Question 91

If $\sec^2\theta = \frac{4}{3}$, then the general value of θ is

Options:

A. $2n\pi \pm \frac{\pi}{6}$

B. $n\pi \pm \frac{\pi}{6}$

C. $2n\pi \pm \frac{\pi}{3}$

D. $n\pi \pm \frac{\pi}{3}$

Answer: B

Solution:

Solution:

We have $\sec^2\theta = \frac{4}{3} \Rightarrow \cos^2\theta = \frac{3}{4}$

$\Rightarrow \cos^2\theta = \cos^2\left(\frac{\pi}{6}\right) \Rightarrow$

$\theta = n\pi \pm \frac{\pi}{6}$.....

Question 92

Number of words from the letters of the words BHARAT in which B and H will never come together is

Options:

A. 210

B. 240

C. 422

D. 400

Answer: B

Solution:

Solution:

There are 6 letters in the word BHARAT, 2 of them are identical. Hence total number of words = 6! / 2! = 360

Number of words in which B and H come together

$= \frac{5!2!}{2!} = 120$

Question 93

The ratio in which the YZ-plane divide the line segment formed by joining the points (−2, 4, 7) and (3, −5, 8) is 2 : m. The value of m is

Options:

A. 2

B. 3

C. 4

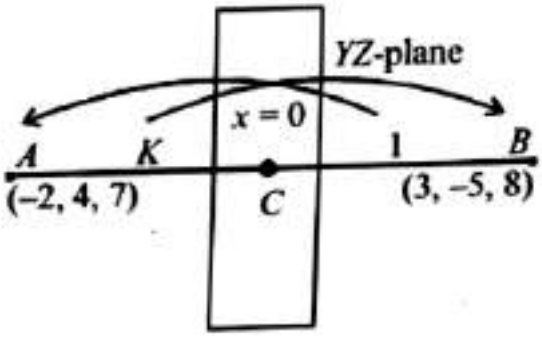
D. 1

Answer: B

Solution:

Solution:

Let the points be $A(-2, 4, 7)$ and $B(3, -5, 8)$ on YZ-plane, x-coordinate $= 0$.



Let the ratio be $K : 1$.

The coordinates of C

are $\left(\frac{3K-2}{K+1}, \frac{-5K+4}{K+1}, \frac{8K+7}{K+1}\right)$

Clearly $\frac{3K-2}{K+1} = 0 \Rightarrow 3K = 2 \Rightarrow K = \frac{2}{3}$

Hence required ratio is 2 : 3.

Question 94

A set A has 3 elements and another set B has 6 elements. Then

Options:

A. $3 \leq n(A \cup B) \leq 6$

B. $3 \leq n(A \cup B) \leq 9$

C. $6 \leq n(A \cup B) \leq 9$

D. $0 \leq n(A \cup B) \leq 9$

Answer: C

Solution:

Solution:

We have

$\min n(A \cup B) = \max\{n(A), n(B)\} = \max\{3, 6\} = 6$

$\text{Max } n(A \cup B) = n(A) + n(B) = 9;$

$\therefore 6 \leq n(A \cup B) \leq 9$

Question 95

For all $n \in N$, the sum of $\frac{n^5}{5} + \frac{n^3}{3} + \frac{7n}{15}$ is

Options:

A. a negative integer

B. a whole number

C. a real number

D. a natural number

Answer: D

Solution:

Solution:

Let the statement P(n) be defined as

$P(n): \frac{n^5}{5}+\frac{n^3}{3}+\frac{7n}{15}$ is a natural number for all $n \in N$.

Step I : For $n = 1$, $P(1): \frac{1}{5}+\frac{1}{3}+\frac{7}{15}=1 \in N$

Hence, it is true for $n = 1$.

Step II : Let it is true for $n = k$,

i.e. $\frac{k^5}{5}+\frac{k^3}{3}+\frac{7k}{15}=\lambda \in N \ldots$ (i)

Step III : For $n = k + 1$

$$\frac{(k+1)^5}{5}+\frac{(k+1)^3}{3}+\frac{7(k+1)}{15}$$

$$=\frac{1}{5}(k^5+5k^4+10k^3+10k^2+5k+1)$$

$$+\frac{1}{3}(k^3+3k^2+3k+1)+\frac{7}{15}k+\frac{7}{15}$$

$$=\left(\frac{k^5}{5}+\frac{k^3}{3}+\frac{7}{15}k\right)+(k^4+2k^3+3k^2+2k)$$

$$+\frac{1}{5}+\frac{1}{3}+\frac{7}{15}$$

$$=\lambda+k^4+2k^3+3k^2+2k+1$$

[using equation (i)]

which is a natural number, since $\lambda k \in N$. Therefore, $P(k + 1)$ is true, when $P(k)$ is true, Hence, from the principle of mathematical induction, the statement is true for all natural numbers n.

Question 96

The roots of the given equation $(p - q)x^2 + (q - r)x + (r - p) = 0$ are :

Options:

A. $\frac{p-q}{r-p}$, 1

B. $\frac{q-r}{p-q}$, 1

C. $\frac{r-p}{p-q}$, 1

D. None of these

Answer: C

Solution:

Solution:

Given equation is

$(p-q)x^2+(q-r)x+(r-p)=0$

By using formula for finding the roots viz. $\frac{-b\pm\sqrt{b^2-4ac}}{2a}$, we get

$x=\frac{(r-q)\pm\sqrt{(q-r)^2-4(r-p)(p-q)}}{2(p-q)}$

$\Rightarrow x=\frac{(r-q)\pm(q+r-2p)}{2(p-q)}=\frac{r-p}{p-q},1$

Question 97

What is the angle between the two straight lines $y = (2 - \sqrt{3})x + 5$ and $y = (2 + \sqrt{3})x - 7$?

Options:

A. 60˚

B. 45˚

C. 30˚

D. 15˚

Answer: A

Solution:

Solution:

The given lines are:

$y=(2-\sqrt{3})x+5$ and $y=(2+\sqrt{3})x-7$

Therefore, slope of first line $=m_1=2-\sqrt{3}$ and slope of second line $=m_2=2+\sqrt{3}$

$\therefore \tan\theta=\left|\frac{m_2-m_1}{1+m_1m_2}\right|=\left|\frac{2+\sqrt{3}-2+\sqrt{3}}{1+(4-3)}\right|$

$=\left|\frac{2\sqrt{3}}{2}\right|=\sqrt{3}=\tan\frac{\pi}{3}\Rightarrow\theta=\frac{\pi}{3}=60^\circ$

Question 98

The range of the function f (x) = $\sqrt{3x^2 - 4x + 5}$ is

Options:

A. $\left(-\infty, \sqrt{\frac{11}{3}}\right]$

B. $\left(-\infty, \sqrt{\frac{11}{5}}\right)$

C. $\left[\sqrt{\frac{11}{3}}, \infty\right)$

D. $\left(\sqrt{\frac{11}{5}}, \infty\right)$

Answer: C

Solution:

Solution:

$f(x)$ is defined if $3x^2-4x+5 \geq 0$

$\Rightarrow 3\left[x^2-\frac{4}{3}x+\frac{5}{3}\right] \geq 0 \Rightarrow 3\left[\left(x-\frac{2}{3}\right)^2+\frac{11}{9}\right] \geq 0$

Which is true for all real $x \therefore$ Domain of $f(x) = (-\infty, \infty)$

Let $y = \sqrt{3x^2-4x+5}$

$\Rightarrow y^2 = 3x^2-4x+5$ i.e. $3x^2-4x+(5-y^2)=0$

For x to be real, $16-12(5-y^2) \geq 0 \Rightarrow y \geq \sqrt{\frac{11}{3}}$

$\therefore$ Range of $y = \left[\sqrt{\frac{11}{3}}, \infty\right)$

Question 99

If f (x) = $\frac{x}{\sqrt{1+x^2}}$, then (fof of) (x) is

Options:

A. $\frac{3x}{\sqrt{1+x^2}}$

B. $\frac{x}{\sqrt{1+3x^2}}$

C. $\frac{3x}{\sqrt{1-x^2}}$

D. None of these

Answer: B

Solution:

Solution:

$$f(x) = \frac{x}{\sqrt{1+x^2}};$$

$$\text{fof} = \frac{\frac{x}{\sqrt{1+x^2}}}{\sqrt{1+\frac{x^2}{1+x^2}}} = \frac{x}{\sqrt{2x^2+1}}$$

$$\text{fofof} = \frac{\frac{x}{\sqrt{2x^2+1}}}{\sqrt{1+\frac{x^2}{2x^2+1}}} = \frac{x}{\sqrt{1+3x^2}}$$

Question 100

The derivative of e^{x^3} with respect to log x is

Options:

A. e^{x^3}

B. $3x^2 2e^{x^3}$

C. $3x^3 e^{x^3}$

D. $3x^3 e^{x^3} + 3x^2$

Answer: C

Solution:

Solution:

Let $y = e^{x^3}, z = \log x$

On differentiating w.r.t.x, we get

$$\frac{dy}{dx} = e^{x^3}(3x^2) = 3x^2 e^{x^3} \text{ and } \frac{dz}{dx} = \frac{1}{x}$$

$$\therefore \frac{dy}{dz} = \frac{\frac{dy}{dx}}{\frac{dz}{dx}} = \frac{3x^2 e^{x^3}}{\left(\frac{1}{x}\right)} = 3x^3 e^{x^3}$$

Question 101

If the coordinates of the points A and B be (3, 3) and (7, 6), then the length of the portion of the line AB intercepted between the axes is

Options:

A. $\frac{5}{4}$

B. $\frac{\sqrt{10}}{4}$

C. $\frac{\sqrt{13}}{3}$

D. None of these

Answer: A

Solution:

Solution:

Equation of line AB is $y-3=\frac{6-3}{7-3}(x-3)$

$\Rightarrow 3x-4y+3=0 \Rightarrow \frac{x}{-1}+\frac{y}{3/4}=1$

Hence required length is $\sqrt{(-1)^2+\left(\frac{3}{4}\right)^2}=\frac{5}{4}$.

Question 102

Solution of $2^x + 2^{|x|} \geq 2\sqrt{2}$ is

Options:

A. $(-\infty, \log_2(\sqrt{2}+1))$

B. $(0, \infty)$

C. $\left(\frac{1}{2}, \log_2(\sqrt{2}-1)\right)$

D. $(-\infty, \log_2(\sqrt{2}-1)] \cup \left[\frac{1}{2}, \infty\right)$

Answer: D

Solution:

Solution:

$2^x + 2^{|x|} \geq 2\sqrt{2}$ . . . (i)

Case I: $x \geq 0$, then Eq. (i) becomes

$2^x + 2^x \geq 2\sqrt{2}$

$\Rightarrow 2^x \geq \sqrt{2} \Rightarrow x \geq \frac{1}{2}$

Case II: $x < 0$, then Eq. (i) becomes

$2^x + 2^{-x} \geq 2\sqrt{2}$

$\Rightarrow t + \frac{1}{t} \geq 2\sqrt{2}$, where $2^x = t$

$\rightarrow t^2 - 2\sqrt{2}t + 1 \geq 0$

$\rightarrow x \leq \log_2(\sqrt{2} - 1)$

Also, $0 < \sqrt{2} - 1 < 1$, $\log_2(\sqrt{2} - 1) < 0$.

∴ The solution is

$(-\infty, \log_2(\sqrt{2} - 1))] \cup \left[\frac{1}{2}, \infty\right)$.

Question 103

If y = $\sqrt{\left(\frac{1+\cos 2\theta}{1-\cos 2\theta}\right)}$, then $\frac{dy}{d\theta}$ at θ = $\frac{3\pi}{4}$ is :

Options:

A. -2

B. 2

C. \pm 2

D. None of these

Answer: A

Solution:

Solution:

$$y = \sqrt{\frac{1+\cos 2\theta}{1-\cos 2\theta}}$$

$$\Rightarrow y = \sqrt{\frac{2\cos^2\theta}{2\sin^2\theta}} = \sqrt{\cot^2\theta}$$

$$\Rightarrow y = \cot\theta$$

Differentiate w.r.t. " θ ", we get: $\frac{dy}{d\theta} = -\text{cosec}^2\theta$

Now, $\left(\frac{dy}{d\theta}\right)_{\theta=\frac{3\pi}{4}} = -\text{cosec}^2\left(\frac{3\pi}{4}\right)$

$$= -\text{cosec}^2\left(\pi - \frac{\pi}{4}\right) = -\text{cosec}^2\frac{\pi}{4} = -2$$

Question 104

The number of solutions of $\frac{dy}{dx} = \frac{y+1}{x-1}$, when y(1) = 2 is

Options:

A. none

B. one

C. two

D. infinite

Answer: B

Solution:

Solution:

Since, $\frac{dy}{dx} = \frac{y+1}{x-1}$

$$\Rightarrow -\frac{dy}{y+1} = \frac{dx}{x-1}$$

After integrating on both sides, we have

$$\log(y+1) = \log(x-1) - \log C$$

$$C(y+1) = (x-1)$$

$$C = \frac{x-1}{y+1}$$

If $x = 1$, then $y = 2$, so $C = 0$.

Therefore, $x - 1 = 0$

Hence, there is only one solution.

Question 105

The probability of getting sum more than 7 when a pair of dice are thrown is:

Options:

A. $\frac{7}{36}$

B. $\frac{5}{12}$

C. $\frac{7}{12}$

D. None of these

Answer: B

Solution:

Solution:

Here $n(S) = 6^2 = 36$

Let E be the event "getting sum more than 7" i.e. sum of pair of dice = 8, 9, 10, 11, 12

i.e. $E = \{ (2,6)\ (3,5)\ (4,4)\ (5,3)\ (6,2)\ (3,6)\ (4,5)\ (5,4)\ (6,3)\ (4,6)\ (5,5)\ (6,4)\ (5,6)\ (6,5)\ (6,6) \}$

$\therefore n(E) = 15. \therefore$ Req. prob $= \frac{n(E)}{n(S)} = \frac{15}{36} = \frac{5}{12}$

Question 106

The probability that a card drawn from a pack of 52 cards will be a diamond or king is:

Options:

A. $\frac{1}{52}$

B. $\frac{2}{13}$

C. $\frac{4}{13}$

D. $\frac{1}{13}$

Answer: C

Solution:

Solution:

Total no. of cards = 52

13 cards are diamonds and 4 cards are king. There is only one card which is a king of diamond.

$\therefore P$ (card is diamond) $= \frac{13}{52}$, P (card is king)

$= \frac{4}{52}$

P (card is king of diamond) $= \frac{1}{52}$

$\therefore$ P (card is diamond or king)

$= \frac{13}{52} + \frac{4}{52} - \frac{1}{52} = \frac{16}{52} = \frac{4}{13}$

Question 107

If A = $\begin{bmatrix} 0 & 2 \\ 3 & -4 \end{bmatrix}$ and kA = $\begin{bmatrix} 0 & 3a \\ 2b & 24 \end{bmatrix}$, then the values of k, a and b respectively are:

Options:

A. $-6, -12, -18$

B. $-6, -4, -9$

C. $-6, 4, 9$

D. $-6, 12, 18$

Answer: B

Solution:

Solution:

$$kA = \begin{bmatrix} 0 & 2k \\ 3k & -4k \end{bmatrix} = \begin{bmatrix} 0 & 3a \\ 2b & 24 \end{bmatrix}$$

$\Rightarrow k = -6, a = -4$ and $b = -9$

Question 108

If the eccentricity and length of latus rectum of a hyperbola are $\frac{\sqrt{13}}{3}$ and $\frac{10}{3}$ units respectively, then what is the length of the transverse axis?

Options:

A. $\frac{7}{2}$ unit

B. 12 unit

C. $\frac{15}{2}$ unit

D. $\frac{15}{4}$ unit

Answer: C

Solution:

Solution:

Length of latus rectum of a hyperbola is $2b^2/a$ where a is the half of the distance between two vertex of the hyperbola.

Latus rectum $= \frac{2b^2}{a} = \frac{10}{3}$ or, $b^2 = \frac{5a}{3}$ . . . (i)

In case of hyperbola, $b^2 = a^2(e^2 - 1)$ . . . (ii)

Putting value of b^2 from equation (i) and $e = \frac{\sqrt{13}}{3}$ in equation (ii). $\frac{5a}{3} = a^2\left(\frac{13}{9} - 1\right)$ or,

$\frac{5a}{3} = \frac{4a^2}{9}$

$\Rightarrow 4a^2 - 15a = 0$ or, $a(4 - 15a) = 0$

$a \neq 0$, hence, $a = \frac{15}{4}$

Question 109

If the sum of an infinite GP a, ar, ar^2, ar^3, ... is 15 and the sum of the squares of its each term is 150 , then the sum of ar^2, a^4, ar^6, ... is :

Options:

A. $\frac{5}{2}$

B. $\frac{1}{2}$

C. $\frac{25}{2}$

D. $\frac{9}{2}$

Answer: B

Solution:

Solution:

Sum of infinite terms of series $a + ar + ar^2 +$

$.... = 15 \ldots$ (i)

$\therefore \frac{a}{1-r} = 15$

Sequence formed by square of terms:

$\text{Sum} = \frac{a^2}{1-r^2} = 150$

$a^2 + a^2r^2 + \ldots - = 150$

$\Rightarrow \frac{a}{1-r} \cdot \frac{a}{1+r} = 15 \Rightarrow 15 \cdot \frac{a}{1+r} = 150$

$\Rightarrow \frac{a}{1+r} = 10 \ldots$ (ii)

On dividing equation (i) by (ii)

$\frac{1+r}{1-r} = \frac{15}{10}$

or $r = \frac{1}{5} \rightarrow a = 12$

Now series : $ar^2 + ar^4 + ar^6 + \ldots$

$\text{Sum} = \frac{ar^2}{1-r^2} = \frac{12 \cdot \left(\frac{1}{25}\right)}{1 - \frac{1}{25}} = \frac{1}{2}$

Question 110

The interval in which the function f (x) = $\frac{4x^2 + 1}{x}$ is decreasing is :

Options:

A. $\left(-\frac{1}{2}, \frac{1}{2}\right)$

B. $\left[-\frac{1}{2}, \frac{1}{2}\right]$

C. $(-1, 1)$

D. $[-1, 1]$

Answer: A

Solution:

Solution:

Given $f(x) = \frac{4x^2+1}{x}$ Thus $f'(x) = 4 - \frac{1}{x^2} f(x)$ will be decreasing if $f'(x) < 0$

Thus $4 - \frac{1}{x^2} < 0 \Rightarrow \frac{1}{x^2} > 4 \Rightarrow \frac{-1}{2} < x < \frac{1}{2}$ Thus interval in which $f(x)$ is decreasing, is $\left(-\frac{1}{2}, \frac{1}{2}\right)$

Question 111

If $\int \frac{e^x(1+\sin x)\,dx}{1+\cos x} = e^x f(x) + C$, then f (x) is equal to

Options:

A. $\sin \frac{x}{2}$

B. $\cos \frac{x}{2}$

C. $\tan \frac{x}{2}$

D. $\log \frac{x}{2}$

Answer: C

Solution:

Solution:

$$\int e^x \frac{(1+\sin x)}{(1+\cos x)} dx$$

$$= \int e^x \left[\frac{1}{2} \sec^2 \frac{x}{2} + \tan \frac{x}{2} \right] dx$$

$$= \frac{1}{2} \int e^x \sec^2 \frac{x}{2} dx + \int e^x \tan \frac{x}{2} dx$$

$$= e^x \tan \frac{x}{2} + C$$

But $I = e^x f(x) + C$ (given)

$$\therefore f(x) = \tan \frac{x}{2}$$

Question 112

The curve given by x + y = e^{xy} has a tangent parallel to the Y-axis at the point

Options:

A. (0, 1)

B. (1, 0)

C. (1, 1)

D. None of these

Answer: B

Solution:

Solution:

$\because x+y=e^{xy}$

Differentiating w.r.t. x, we get

$1+\frac{dy}{dx}=e^{xy}\left[y+x\frac{dy}{dx}\right]$

$\Rightarrow \frac{dy}{dx}(1-xe^{xy})=ye^{xy}-1$

$\Rightarrow \frac{dy}{dx}=\frac{ye^{xy}-1}{1-xe^{xy}} \because \frac{dy}{dx}=\infty,$

as tangent is parallel to Y-axis

$\Rightarrow 1-xe^{xy}=0$

$\therefore xe^{xy}=1$

This holds, when $x = 1$ and $y = 0$

Question 113

The area enclosed between the curve $y = \log_e(x + e)$ and the coordinate axes is

Options:

A. 1

B. 2

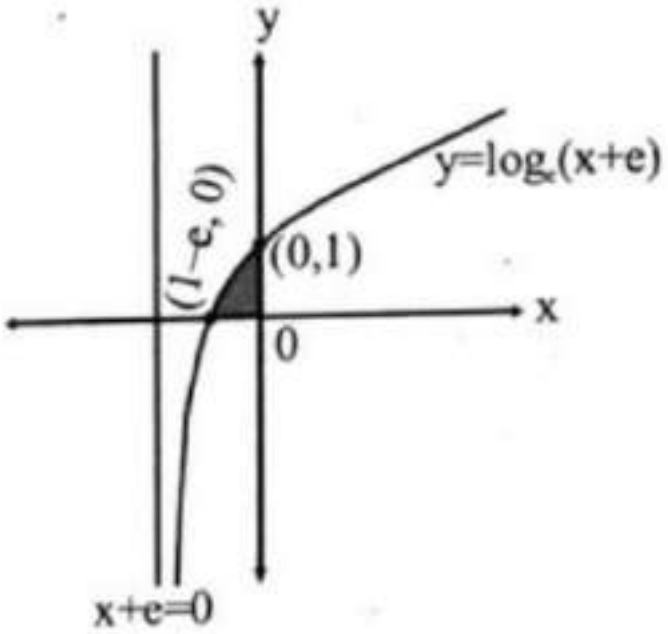
C. 3

D. 4

Answer: A

Solution:

Solution:



Required area $A = \int_{1-e}^{0} y\,dx = \int_{1-e}^{0} \log_e(x+e)\,dx$ put $x+e=t \Rightarrow dx = dt$ also when $x = 1-e, t = 1$ and when $x = 0, t = e$

$$\therefore A = \int_{1}^{e} \log_e t\,dt = [t\log_e t - t]_1^e$$

$$e - e - 0 + 1 = 1$$

Question 114

If $\vec{a} = \hat{i} + \hat{j} + \hat{k}$, $\vec{a} \cdot \vec{b} = 1$ and $\vec{a} \times \vec{b} = \vec{j} - \vec{k}$, then $\vec{b}$ is

Options:

A. $\hat{i} - \hat{j} + \hat{k}$

B. $2\hat{j} - \hat{k}$

C. $2\hat{i}$

D. $\hat{i}$

Answer: D

Solution:

Solution:

Given $\vec{a} = \hat{i} + \hat{j} + \hat{k}$, $\vec{a} \cdot \vec{b} = 1$

Question 115

$\lim_{x \to 0} \frac{|\sin x|}{x}$ is equal to

Options:

A. 1

B. -1

C. Does not exist

D. None of these

Answer: C

Solution:

Solution:

Let $I = \lim_{x \to 0} \frac{[\sin x]}{x}$; RHL $= \lim_{x \to 0^+} \frac{\sin x}{x} = 1$

LHL $= \lim_{x \to 0^-} \left(\frac{-\sin x}{x} \right) = -\left(\lim_{x \to 0^-} \frac{\sin x}{x} \right) = -1$

As, LHL $\neq$ RHL Hence, limit I does not exist.

Question 116

The lines $\frac{x-2}{1} = \frac{y-3}{1} = \frac{z-4}{-k}$ and $\frac{x-1}{k} = \frac{y-4}{2} = \frac{z-5}{1}$ are coplanar if

Options:

A. k = 3 or -2

B. k = 0 or -1

C. k = 1 or -1

D. k = 0 or -3

Answer: D

Solution:

Solution:

Two planes are coplanar if

$$\begin{vmatrix} x_2-x_1 & y_2-y_1 & z_2-z_1 \\ l_1 & m_1 & n_1 \\ l_2 & m_2 & n_2 \end{vmatrix} = 0$$

$$\begin{vmatrix} 1 & -1 & -1 \\ 1 & 1 & -k \\ k & 2 & 1 \end{vmatrix} = 0$$

Applying $C_2 \to C_2 + C_1, C_3 \to C_3 + C_1$

$$\begin{vmatrix} 1 & 0 & 0 \\ 1 & 2 & 1-k \\ k & k+2 & 1+k \end{vmatrix} = 0$$

$$\Rightarrow 1[2+2k-(k+2)(1-k)] = 0$$

$$\Rightarrow 2+2k-(-k^2-k+2) = 0$$

$$k^2+3k = 0 \Rightarrow k(k+3) = 0$$

Question 117

Negation of the Boolean expression p ⇔ (q ⇒ p) is

Options:

A. (~p) ∧ q

B. (p) ∧ (~q)

C. (~p) ∨ (~q)

D. (~p) ∧ (~q)

Answer: D

Solution:

Solution:

Given expression is $p \Leftrightarrow (q \Rightarrow p)$

$\sim (p \leftrightarrow (q \rightarrow p))$

$\sim (p \leftrightarrow q) = (p \wedge \sim q) \vee (q \wedge \sim p)$

$\sim (p \leftrightarrow (q \rightarrow p))$

$= (p \wedge \sim (q \rightarrow p)) \vee ((q \rightarrow p) \wedge \sim p)$

$(p \wedge \sim (q \rightarrow p)) = p \wedge (q \wedge \sim p)$

$= (p \wedge \sim p) \wedge q = c$

$(q \rightarrow p) \wedge \sim p = (\sim q \vee p) \wedge \sim p$

$= \sim p \wedge (\sim q \vee p)$

$= (\sim p \wedge \sim q) \vee (\sim p \wedge p) = \sim p \wedge \sim q$

$\sim (p \leftrightarrow (q \rightarrow p)) = c \vee (\sim p \wedge \sim q) = \sim p \wedge \sim q$

Question 118

The maximum value of z = 5x + 2y subject to constraints x + y ≤ 7, x + 2y ≤ 10, x, y ≥ 0

Options:

A. 10

B. 26

C. 35

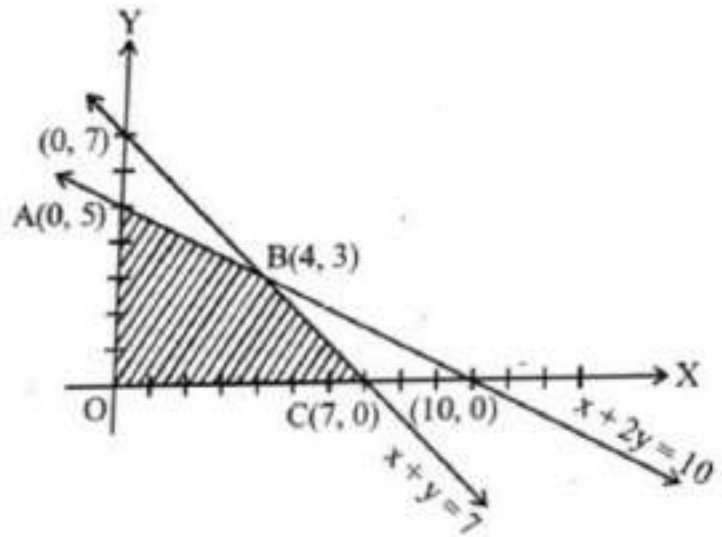
D. 70

Answer: C

Solution:

Solution:

Change the inequalities into equations and draw the graph of lines, thus we get the required feasible region as shown below.



The region bounded by the vertices A(0, 5), B(4, 3), C(7, 0).

The objective function is maximum at $C(7, 0)$ and $\text{Max}\, z = 5 \times 7 + 2 \times 0 = 35$

Question 119

Find the mean deviation about the mean for the data 4, 7, 8, 9, 10, 12, 13, 17

Options:

A. 3

B. 24

C. 10

D. 8

Answer: A

Solution:

Solution:

Arithmetic mean $\bar{x}$ of 4, 7, 8, 9, 10, 12, 13, 17 is

$$\bar{x} = \frac{4+7+8+9+10+12+13+17}{8} = \frac{80}{8} = 10$$

$$\Sigma|x_i - \bar{x}| = 6+3+2+1+0+2+3+7 = 24$$

$\therefore$ Mean deviation about mean

$$= \text{M.D.}(\bar{x}) = \frac{\Sigma|x_i - \bar{x}|}{n} = \frac{24}{8} = 3$$

Question 120

Bag P contains 6 red and 4 blue balls and bag Q contains 5 red and 6 blue balls. A ball is transferred from bag P to bag Q and then a ball is drawn from bag Q. What is the probability that the ball drawn is blue?

Options:

A. $\frac{7}{15}$

B. $\frac{8}{15}$

C. $\frac{4}{19}$

D. $\frac{8}{19}$

Answer: B

Solution:

Solution:

Let E_1, E_2 and A be the events defined as follows:
E_1 = red ball is transferred from bag P to bag Q
E_2 = blue ball is transferred from bag P to bag Q
A = the ball drawn from bag Q is blue
As the bag P contains 6 red and 4 blue balls,

$$P(E_1) = \frac{6}{10} = \frac{3}{5} \text{ and } P(E_2) = \frac{4}{10} = \frac{2}{5}$$

Note that E_1 and E_2 are mutually exclusive and exhaustive events.
When E_1 has occurred i.e., a red ball has already been transferred from bag P to Q, then bag Q will contain 6 red and 6 blue balls, So, $P(A \mid E_1) = 6/12 = 1/2$

When E_2 has occurred i.e., a blue ball has already been transferred from bag P to Q, then bag Q will contain 5 red and 7 blue balls, So, $P(A \mid E_2) = 7/12$

By using law of total probability, we get

$$P(A) = P(E_1)P(A \mid E_1) + P(E_2)P(A \mid E_2)$$

$$= \frac{3}{5} \times \frac{1}{2} + \frac{2}{5} \times \frac{7}{12} = \frac{8}{15}$$

Question 121

$\tan^{-1}\left(\frac{1}{4}\right) + \tan^{-1}\left(\frac{2}{9}\right)$ is equal to

Options:

A. $\frac{1}{2}\cos^{-1}\left(\frac{3}{5}\right)$

B. $\frac{1}{2}\sin^{-1}\left(\frac{3}{5}\right)$

C. $\frac{1}{2}\tan^{-1}\left(\frac{3}{5}\right)$

D. $\tan^{-1}\left(\frac{1}{2}\right)$

Answer: D

Solution:

Solution:

$$\tan^{-1}\left(\frac{1}{4}\right) + \tan^{-1}\left(\frac{2}{9}\right) = \tan^{-1}\left\{\frac{\frac{1}{4}+\frac{2}{9}}{1-\frac{1}{4}\times\frac{2}{9}}\right\}$$

$$= \tan^{-1}\left\{\frac{9+8}{36-2}\right\} = \tan^{-1}\left(\frac{1}{2}\right)$$

Question 122

The middle term in the expansion of $\left(\frac{10}{x} + \frac{x}{10}\right)^{10}$ is

Options:

A. $^{10}C_5$

B. $^{10}C_6$

C. $^{10}C_5 \frac{1}{x^{10}}$

D. $^{10}C_5 x^{10}$

Answer: A

Solution:

Solution:

General term $= T_{r+1}$

$= {}^{10}C_r \left(\frac{10}{x}\right)^{10-r} \cdot \left(\frac{x}{10}\right)^r$

Here $n = 10$, which is an even number.

Now, $\left[\frac{10}{2} + 1\right]^{th}$ term i.e. 6^{th} term is the middle term.

Hence, middle term $= T_6$

$T_{5+1} = {}^{10}C_5 \left(\frac{10}{x}\right)^{10-5} \left(\frac{x}{10}\right)^5$

$= {}^{10}C_5 \left(\frac{10}{x}\right)^5 \left(\frac{x}{10}\right)^5 = {}^{10}C_5.$

Question 123

The equation of a common tangent to the parabolas $y = x^2$ and $y = -(x-2)^2$ is

Options:

A. $y = 4(x - 2)$

B. $y = 4(x - 1)$

C. $y = 4(x + 1)$

D. $y = 4(x + 2)$

Answer: B

Solution:

Solution:

Equation of tangent of parabola $y = x^2$ be

$tx = y + at^2 \ldots$ (i)

$y = tx - \frac{t^2}{4}$

Solve with $y = -(x-2)^2$

$tx - \frac{t^2}{4} = -(x-2)^2$

$x^2 + x(t-4) - \frac{t^2}{4} + 4 = 0$

Here, Discriminant $= 0$.

$(t-4)^2 - 4 \cdot \left(4 - \frac{t^2}{4}\right) = 0 \Rightarrow t^2 - 4t = 0 \Rightarrow t = 0$

or $t = 4$

Put value of t in eq. (i), then $y = 4(x-1)$.

Question 124

A circle touches both the y-axis and the line x+ y = 0. Then the locus of its center is

Options:

A. $y = \sqrt{2}x$

B. $x = \sqrt{2}y$

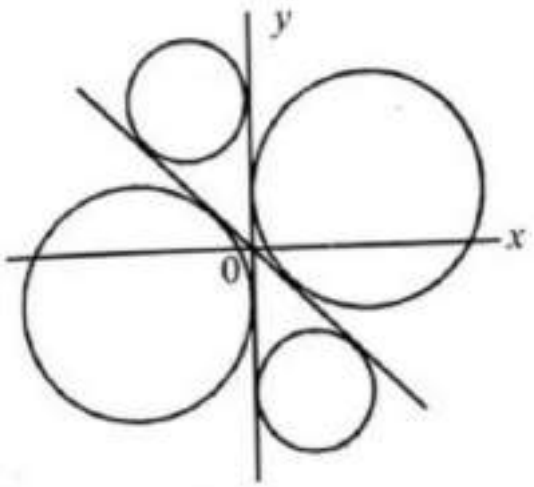
C. $y^2 - x^2 = 2xy$

D. $x^2 - y^2 = 2xy$

Answer: D

Solution:

Solution:



Let (h, k) is centre of circle

$$\left|\frac{h-k}{\sqrt{2}}\right| = |h| \Rightarrow h^2 - k^2 + 2hk = 0$$

Question 125

The function $f(x) = \tan^{-1}(\sin x + \cos x)$ is an increasing function in

Options:

A. $\left(\frac{\pi}{4}, \frac{\pi}{2}\right)$

B. $\left(-\frac{\pi}{2}, \frac{\pi}{4}\right)$

C. $\left(0, \frac{\pi}{2}\right)$

D. $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

Answer: B

Solution:

Solution:

Since, $f(x) = \tan^{-1}(\sin x + \cos x)$

$$\therefore f'(x) = \frac{1}{1+(\sin x + \cos x)^2}(\cos x - \sin x)$$

$$= \frac{\sqrt{2}\cos\left(x + \frac{\pi}{4}\right)}{1+(\sin x + \cos x)^2}$$

$f(x)$ is increasing if $f'(x) > 0 \Rightarrow \cos\left(x + \frac{\pi}{4}\right) > 0$

$$\Rightarrow -\frac{\pi}{2} < x + \frac{\pi}{4} < \frac{\pi}{2} \Rightarrow -\frac{3\pi}{2} < x + \frac{\pi}{4}$$

Question 126

$i^{57} + \frac{1}{i^{25}}$, when simplified has the value

Options:

A. 0

B. 2i

C. $-2i$

D. 2

Answer: A

Solution:

Solution:

$$i^{57}+\frac{1}{i^{25}}=(i^4)^{14}\cdot i+\frac{1}{(i^4)^6\cdot i}=i+\frac{1}{i}=i-i=0$$

Question 127

If one root of the equation $x^2 + px + 12 = 0$ is 4 , while the equation $x^2 + px + q = 0$ has equal roots, then the value of ' q ' is

Options:

A. 4

B. 12

C. 3

D. $\frac{49}{4}$

Answer: D

Solution:

Solution:

4 is a root of $x^2+px+12=0$

$\Rightarrow 16+4p+12=0\Rightarrow p=-7$

Now, the equation $x^2+px+q=0$ has equal roots.

$\therefore p^2-4q=0\Rightarrow q=\frac{p^2}{4}=\frac{49}{4}$

So $I=\int_0^1 e^t\,dt=[e^t]_0^1=e-1$

Question 128

$\int \frac{x+3}{(x+4)^2} e^x dx$ is equal to

Options:

A. $e^x\left(\frac{1}{x+4}\right)+C$

B. $e^{-x}\left(\frac{1}{x+4}\right)+C$

C. $e^{-x}\left(\frac{1}{x-4}\right)+C$

D. $e^{2x}\left(\frac{1}{x-4}\right)+C$

Answer: A

Solution:

Solution:

Suppose,

$$I=\int \frac{x+3}{(x+4)^2}e^x dx=\int \frac{(x+4)1}{(x+4)^2}e^x dx$$

$$=\int \frac{e^x}{(x+4)}-\int \frac{e^x}{(x+4)^2}dx$$

$$=\int e^x\left(\frac{1}{(x+4)}-\frac{1}{(x+4)^2}\right)dx$$

$$=e^x\frac{1}{(x+4)}+C$$

$$[\int e^x\{f(x)+f'(x)\}dx=e^x f(x)+C]$$

Question 129

The shortest distance between the lines $\frac{x-3}{2}=\frac{y-2}{3}=\frac{z-1}{-1}$ and $\frac{x+3}{2}=\frac{y-6}{1}=\frac{z-5}{3}$ is :

Options:

A. $\frac{18}{\sqrt{5}}$

B. $\frac{22}{3\sqrt{5}}$

C. $\frac{46}{3\sqrt{5}}$

D. $6\sqrt{3}$

Answer: A

Solution:

Solution:

$\frac{x+3}{2}=\frac{y-6}{1}=\frac{z-5}{3}$

Lines passes through the points

$\vec{a}_1=(3,2,1)$ and $\vec{a}_2=(-3,6,5)$.

$\vec{b}_1=2\hat{i}+3\hat{j}-\hat{k}$

$\vec{b}_1=2\hat{i}+\hat{j}-3k,\ \vec{a}_2-\vec{a}_1=6\hat{i}-4j-4\hat{k}$

Shortest distance $=\frac{\left|(\vec{a}_2-\vec{a}_1)\,(\vec{b}_1\times\vec{b}_2)\right|}{\left|(\vec{b}_1\times\vec{b}_2)\right|}$

$\vec{b}_1\times\vec{b}_2=\begin{vmatrix}\hat{i}&\hat{j}&\hat{k}\\2&3&-1\\2&1&3\end{vmatrix}=10\hat{i}-8\hat{j}-4\hat{k}$

$(\vec{a}_2-\vec{a}_1)\cdot(\vec{b}_1\times\vec{b}_2)=60-32+16=108$

$\left|\vec{b}_1\times\vec{b}_2\right|=\sqrt{100+64+16}=\sqrt{180}$

$S.D=\frac{108}{\sqrt{180}}=\frac{108}{6\sqrt{5}}=\frac{18}{\sqrt{5}}$

Question 130

If P(B) = $\frac{3}{5}$, P(A | B) = $\frac{1}{2}$ and P(A ∪ B) = $\frac{4}{5}$, then P(A ∪ B)′ + P(A′ ∪ B) =

Options:

A. $\frac{1}{5}$

B. $\frac{4}{5}$

C. $\frac{1}{2}$

D. 1

Answer: D

Solution:

Solution:

$P(B) = \frac{3}{5}, P(A \mid B) = \frac{1}{2}$ and

$P(A \cup B) = \frac{4}{5}$

$P(A \cap B) = P(A \mid B)P(B) = \frac{1}{2} \cdot \frac{3}{5} = \frac{3}{10}$

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$P(A) = \frac{4}{5} - \frac{3}{10} = \frac{1}{2}$ $P(A') = 1 - P(A) = \frac{1}{2}$

We know, $P(A \cap B) + P(A' \cap B) = P(B)$

[as $A \cap B$ and $A' \cap B$ are mutually exclusive events]

$\Rightarrow \frac{3}{10} + P(A' \cap B) = \frac{3}{5}$

$\Rightarrow P(A' \cap B) = \frac{3}{5} - \frac{3}{10} = \frac{3}{10}$

Now, $P(A' \cup B) = P(A') + P(B) - P(A' \cap B)$

$= \frac{1}{2} + \frac{3}{5} - \frac{3}{10} = \frac{5+6-3}{10} = \frac{4}{5}$

$P((A \cup B)') = 1 - P(A \cup B) = 1 - \frac{4}{5} = \frac{1}{5}$

$\therefore P((A \cup B)') + P(A' \cup B) = \frac{1}{5} + \frac{4}{5} = 1$
