UPSEE 2019 Paper-1 Set-AA
Physics

1. A wire has a mass $(0.1 \pm 0.001)g$, radius $(0.5 \pm 0.005)mm$ and length $(10 \pm 0.1)cm$. The maximum percentage error in the measurement of its density is

(A) 1%

(B) 2%

(C) 3%

(D) 4%

2. A body slides down a frictionless inclined plane starting from rest. If S_n and S_{n+1} be the distance travelled by the body during n^{th} and $(n+1)^{th}$ seconds, then the ratio $\frac{S_{n+1}}{S_n}$ is

(A) $\frac{2n-1}{2n+1}$

(B) $\frac{2n}{2n+1}$

(C) $\frac{2n+1}{2n-1}$

(D) $\frac{2n}{2n-1}$

3. The trajectory of a projectile in a vertical plane is $y=\alpha x-\beta x^2$, where α and β are constants and x and y are respectively the horizontal and vertical distances of the projectile from the point of projection. The maximum height attended by projectile is

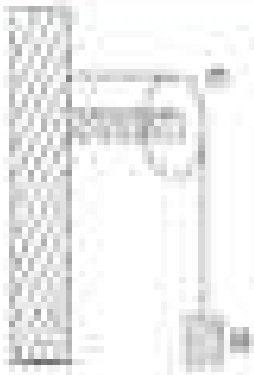
(A) $\frac{\alpha}{\beta}$

(B) $\frac{\alpha}{2\beta}$

(C) $\frac{\beta}{\alpha}$

(D) $\frac{\beta}{2\alpha}$

4. A string of negligible mass passing over a clamped pulley of mass m supports a body of mass M as shown in figure. The force exerted by the clamp on the pulley is



(A) $\sqrt{(M+m)+MQg}$

(B) $\sqrt{(M+m2+m2g}$

(C) $\sqrt{2Mg}$)

(D) $\sqrt{2mg}$)

5. The linear momentum of a particle moving in x–y plane under the influence of a force is given $\vec{p}(t)=A(\hat{i}\cos bt-\hat{j}\sin bt)$ where A and b are

angle between the force and momentum is

(A) 0°

(B) 45°

(C) 60°

(D) 90°

6. According to Kepler's second law, the line joining the planet to sun sweeps out equal area in equal intervals of time. It is a consequence of law of conservation of (A) Linear momentum (B) Energy (C) Angular momentum (D) All of the above

7. A particle moves in a straight line with its retardation proportional to its displacement. The loss of its kinetic energy for any displacement x is proportional to

(A) $\frac{1}{x}$

(B) x

(C) x^2

(D) e^x

8. A disc of mass M and radius R is rolling with angular speed ω on a horizontal plane as shown in figure. The magnitude of the angular momentum of the disc about the origin O is

(A) $\frac{1}{2}MR^2\omega$

(B) $\frac{1}{3}MR^2\omega$

(C) $MR^2\omega$

(D) $2MR^2\omega$

9. The time period of a simple pendulum is T. If its point of suspension is moved upward according to relation $y=Kt^2$ where K is a constant then its new time period T1

(A) is equal to T

(B) is greater than T

(C) is less than T

(D) is infinity

10. Surface tension of a liquid, with increase in its temperature,

(A) increases

(B) decreases

(C) remains the same

(D) first increases and then decreases

11. The dimensional formula of Reynold's number is same as

(A) coefficient of viscosity

(B) coefficient of friction

(C) universal gravitational constant

(D) velocity of light

12. A ball falling in a lake of depth 200m shows 0.1% decrease in its volume at the bottom. The bulk modulus of the material of the ball is

(A) $1.96\times10^{9}N/m^2$

(B) $1.96\times10^{13}N/m^2$

(C) $1.96\times10^{-9}N/m^2$

(D) $1.96\times10^{-7}N/m^2$

13. One mole of a monoatomic gas $\left(\gamma = \frac{5}{3}\right)$ is mixed with one mole of a diatomic gas $\left(\gamma = \frac{7}{5}\right)$. The value of γ for the mixture is

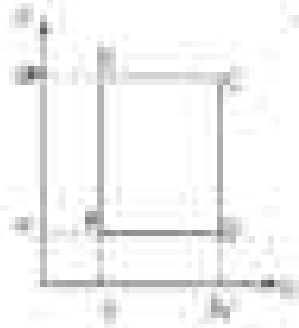
(A) 1.40

(B) 1.50

(C) 1.53

(D) 3.0

14. An ideal monoatomic gas is taken round the cycle ABCDA as shown in P-V diagram. The work done during the cycle is:



(A) PV

(B) 2PV

(C) $\frac{PV}{2}$

(D) 0

15. Two stars emit maximum radiation at wavelength 4000Å and 5000Å respectively. The ratio of their temperatures is

(A) 1:2

(B) 2:1

(C) 2:3

(D) 3:2

16. A carnot engine takes 800 calories of heat at 500K and rejects 150 calories of heat to the sink. The temperature of the sink is

(A) 1000K

(B) 750K

(C) 600K

(D) 250K

17. The rate of transfer of heat is maximum in

(A) Conduction

(B) Convection

(C) Radiation

(D) None of above

18. The latent heat of ice is 80 Cal/gm. The change in entropy when 10 gram of ice at 0°C is converted into water of same temperature is

(A) 0.293Cal/K

(B) 2.93Cal/K

(C) 80Cal/K

(D) 8Cal/K

19. A police car with a siren of frequency 8KHz is moving with uniform velocity of 20m/sec towards a tall building which reflects the sound waves. If speed of sound in air be 320m/sec, the frequency of siren heard by car driver is.

(A) 7.1KHz

(B) 8.8KHz

(C) 9.2KHz

(D) 10.1KHz

20. The phase difference between two waves a [illegible] and a2=Asin(wt is

(A) $\frac{\pi}{2}$

(B) $\frac{\pi}{4}$

(C) $\frac{\pi}{6}$

(D) π

21. The velocity of sound waves in a medium does not depend on

(A) temperature

(B) pressure

(C) humidity

(D) direction of air

22. The number of beats heard per second, by three sound sources of equal intensities and frequencies of 300, 301 and 302Hz, is

(A) 4

(B) 3

(C) 2

(D) 1

23. A steady current flows through a metallic conductor of non-uniform area of cross section. Along the length of conductor:

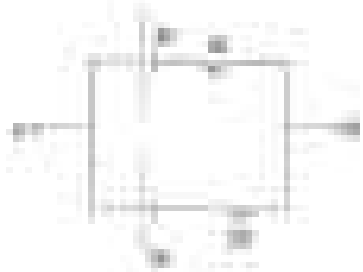
(A) only current is constant

(B) only drift speed is constant

(C) both current and drift speed are constant

(D) neither current nor drift speed is constant.

24. Two batteries of EMF 6V and 3V with internal resistances 1Ω and 2Ω respectively are connected as shown in figure. The potential difference across A and B is:



(A) 9 V

(B) 5

(C) 4

(D) 3V

V

25. A parallel plate capacitor is charged and the charging battery is then disconnected. A dielectric slab is now introduced between the plates of the capacitor. Which of the following is correct?

(A) potential difference across capacitor remains constant

(B) capacitance of the capacitor remains constant

(C) energy associated with capacitor increases.

(D) energy associated with capacitor decreases.

26. An electric bulb is rated as 200V–200W. The power consumed by the bulb when operated at 100V is

(A) 25W

(B) 50W

(C) 75W

(D) 100W

27. Displacement current is caused due to

(A) a time varying electric field

(B) a constant electric field

(C) free electrons flow

(D) all of the above

28. The dipole moment of a dipole formed by a proton and electron at a distance of 1mm is

(A) 1.6×10−19c-m (B)

1.6×10−29c-m (C)

1.6×10−39c-m (D)

1.6×10−19c-m

29. The resistance of a platinum wire is 5Ω at 0°C. If its temperature coefficient of resistance is 0.004/°C then its resistance at 60°C temperature will be

(A) 12Ω

(B) 7Ω

(C) 370Ω

(D) 2800Ω

30. The temperature, above which a ferromagnetic material becomes paramagnetic, is called.

(A) Critical temperature

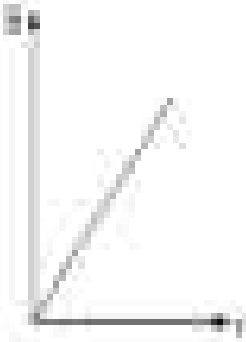
(B) Neutral temperature

(C) Temperature of inversion

(D) Curie temperature

31. Which of the following graph shows the variation of magnetic induction B with distance r from a current carrying long wire?

(A)



(B)

(C)

(D)

32. An electron having charge 'e' is moving with a constant speed v along a circle of radius r. Its magnetic moment will be

(A) evr

(B) $\frac{evr}{2}$

(C) 2πevr

(D) Zero

33. You are given an ammeter, a galvanometer and a voltmeter. From these, the device having maximum resistance is:

(A) ammeter

(B) galvanometer

(C) voltmeter

(D) all will have the same resistance

34. The unit of self inductance is

(A) Joule/Ampere

(B) Volt/Ampere

(C) Volt-Ampere/Second

(D) Volt-Second/Ampere

35. Eddy currents are produced when

(A) a metal is kept in varying magnetic field

(B) a metal is kept in steady magnetic field

(C) a circular coil is placed in a magnetic field

(D) a current is passed through a circular coil.

36. Lenz's law is consequence of the law of conservation of

(A) Charge

(B) Momentum

(C) Mass

(D) Energy

37. Two coherent monochromatic light beams of intensities ratio 1:4 are superposed. The ratio of maximum and minimum intensities in the resulting beam will be:

(A) 9:1

(B) 6:3

(C) 25:9

(D) 9:25

38. A convex lens of focal length 10cm and refractive index 1.5 is dipped in a liquid of refractive index 1.7. It will behave as

(A) a convex lens of focal length 10cm

(B) a convex lens of focal length 35cm

(C) a concave lens of focal length 10cm

(D) a concave lens of focal length 35cm

39. Two beams of red and violet colour are made to pass separately through a prism with angle of prism 60°. In the position of minimum deviation, the angle of refraction will be:

(A) 60° for both colours

(B) 30° for both colours

(C) greater for violet colour

(D) greater for red colour

40. The resolving power of telescope can be increased by

(A) increasing the diameter of object

(B) increasing the wavelength of light used

(C) decreasing the diameter of objective

(D) decreasing the frequency of light used

41. 'Lumen' is the unit of

(A) Luminous flux

(B) Luminous intensity

(C) Illuminance

(D) Light frequency

42. When an unpolarised beam of light of intensity I_0 is incident on a polaroid, the intensity of transmitted light is

(A) I_0

(B) $\frac{I_0}{2}$

(C) $\frac{I_0}{4}$

(D) $\frac{I_0}{8}$

43. An achromatic combination of lens is formed by joining

(A) two convex lenses

(B) two concave lens

(C) one convex lens and one concave lens

(D) one convex lens and one plain mirror

44. A metal surface of work function 3eV is illuminated by photons of energy 2eV. The kinetic energy of emitted photo-electrons will be

(A) 1eV

(B) 2eV

(C) 3eV

(D) 5

45. The potential difference applied to an X-ray tube is 5kV and the current through it is 3.2mA. Then the number of electrons striking the target per second is

(A) 5×10^{16}

(B) 2×10^{16}

(C) 1×10^{17}

(D) 4×10^{18}

46. The mass density of a nucleus varies with mass number A as

(A) 42

(B) 43

(C) 83

(D) 4 - 1

47. The half life time of a radioactive sample is 5minutes. The amount of substance decayed in 20minutes will be

(A) 93.75%

(B) 75%

(C) 25%

(D) 6.25%

48. The depletion layer of an unbiased P-N junction consists of

(A) only electron

(B) only holes

(C) both electrons and holes

(D) neither electrons nor holes.

49. Which of the following is not the property of laser beams?

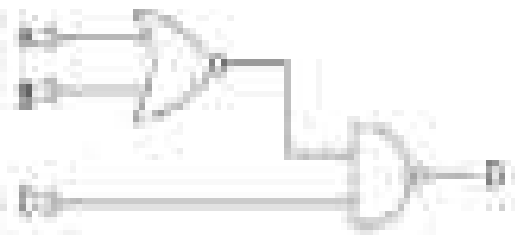
(A) Highly intense

(B) Monochromatic

(C) Directional

(D) Incoherent

50. For the given combination of gates in figure, the logic states of inputs are A=B=1 and C=0, then the logic state of output D is:



(A) 0

(B) 1

(C) 2

(D) 3

UPSEE 2019 Paper-1 Set-AA
Chemistry

61. The reaction

$P_4 + 3NaOH + 3H_2O \rightarrow 3NaH_2PO_2 + PH_3$ is an example of

(A) disproportionation reaction

(B) neutralization reaction

(C) double decomposition reaction

(D) pyrolytic reaction

62. For the following three reactions (I), (II) and (III), equilibrium constants are given

(I) $CO(g) + H_2O(g) \rightleftharpoons CO_2(g) + H_2(g)$; K1

(II) $CH_4(g) + 2H_2O(g) \rightleftharpoons CO_2(g) + 4H_2(g)$; K2

(III) $CH_4($

Which of the following relation is correct?

(A) K3 = K2/K1

(B) K1 = K3

(C) K2K3 = K1

(D) K3 = K1/K2

63: Three electrolytic cells A, B, C containing solutions of $ZnSO_4$, $AgNO_3$ and $CuSO_4$ respectively are connected in series. A steady current of 1.5 amperes was passed through them until 1.45 g of silver deposited at the cathode of cell B. What mass of Cu and Zn were deposited.

(A) Zn=0.44 gm , Cu=63.5 gm

(B) Zn=65.4 gm ; Cu=63.5 gm

(C) Zn=0.44 gm ; Cu=0.427 gm

(D) Zn=1.45 gm ; Cu=1.45 gm

64: Which of the following does not have linear shape?

(A) I_3^-

(B) I^+_3

(C) XeF_2

(D) ClO^-

66: The correct decreasing order of the boiling points of compounds H_2O, HF and NH_3 is

(A) HF>H_2O>NH_3

(B) H_2O>HF>NH_3

(C) $NH_3 + HF \rightarrow H_2O$

(D) $NH_3 + H_2O \rightarrow HF$

86. What is the maximum volume of water required to dissolve 2g of calcium sulphate at 298K? K_{sp} of $CaSO_4$ is 9.0×10^{-6}

(A) 2.45 L

(B) 4.08 L

(C) 4.90 L

(D) 3.00 L

87. What is the correct volume of equilibrium constant for the following reaction at 600 K if the values of ΔH° is 77.5 kJ mol^{-1} and ΔS° = 135 JK^{-1}mol^{-1}

$2NOCl_{(g)} \rightleftharpoons 2NO_{(g)} + Cl_{2(g)}$

(A) 8.545×10^{-4}

(B) 8.545×10^{-2}

(C) 8.314

(D) 136

68. Acetic acid dissociates 1.3%. What will be the pH of M/10 solution of the acid.

(A) 3.886

(B) 2.056

(C) 3.000

(D) 2.056

69. In which compound does H show O. N of –1?

(A) SiH4

(B) AlH3

(C) NH3

(D) CaH2

60. Which of the following oxides would be reduced by C?

Al2O3, MgO, ZnO, CaO, FeO, PbO

(A) Al2O3, PbO and CaO

(B) ZnO, Fe2O3 and PbO

(C) Fe2O3, MgO and Al2O3

(D) MgO, CaO and Al2O3

61. Titration curve if a strong base is titrated with strong acid is

(A) 

(B) 

(C) 

(D) 

62. Nernst equation is related to

(A) Vapour pressure of droplets of liquids

(B) Temperature of a liquid

(C) Adsorption of liquid on solids

(D) None of the above

63. Nylon 66 is formed by

(A) Free radical addition polymer

(B) Ionic addition polymerization

(C) Condensation polymerization

(D) All of the above

64. Which of the following is the correct IUPAC name of the following structure

(A) 3-chloro-2 methyl cyclohexanol

(B) 2-methyl-6 chloro cyclohexanol

(C) 1-chloro-4 methyl cyclohexanol

(D) 5-chloro-2 methyl cyclohexanol

65. Average atomic weight of an element M is 51.7. If two isotopes of M, M50 and M52 are present, then percentage of occurrence of M50 in nature will be.

(A) 85%

(B) 15%

(C) 60%

(D) 100%

66. The following graph shows how

changes with the initial reactant concentration(). The order of the reaction will be

(A) 0

(B) 1

(C) 2

(D) 3

67. Which of the following is not permissible arrangement of electrons in an atom?

(A) n=3,l=2,m=-2,s=-1/2

(B) n=4,l=0,m=0,s=-1/2

(C) n=5,l=3,m=0,s=-1/2

(D) n=3,l=2,m=-3,s=-1/2

68. The molecules of which of the following gases has the highest speed?

[A] O_2 at 0°C

[B] N_2 at 1000°C

[C] CH_4 at 298K

[D] H_2 at −60°C

69. The vapour density of a mixture containing NO_2 and N_2O_4 is 38.3 at 27°C. What will be the mole of NO_2 in 100 mole mixture?

[A] 70.6 mole

[B] 33.48 mole

[C] 60 mole

[D] 46 mole

70. The enthalpy change of a reaction does not depend on

(A) State of reactants and products

(B) Nature of reactants and products

(C) Different intermediate reactions

(D) Initial and final enthalpy of a reaction.

71. In the following sequence of reactions, the product Y will be

(A) 

(B) 

(C) 

(D) 

72. The reaction of P4 with X leads selectively to P4O6. The X is

(A) dry O2

(B) a mixture of O2 and N2

(C) moist O2

(D) O2 in the presence of aqueous NaOH

73. Bleaching powder contains a salt of an oxoacid as one of its components. The anhydride of that oxoacid is

(A) Cl2O

(B) C_3H_7O

(C) C_3H_8

(D) C_3H_6

74. Which is the following is correct statement:

(A) Starch is polymer of β-glucose

(B) Amylose is a component of cellulose

(C) Proteins are composed of only one type of amino acid.

(D) In cyclic structure of fructose, there are four carbons and one oxygen atom.

75. The correct composition of malachite is

(A) $CuFeS_2$

(B) $CuCO_3$

(C) $CuCO_3 \cdot Cu(OH)_2$

(D) $Cu(OH)_2$

76. IUPAC name of the compound

(A) ethoxy-methanone

(B) ethyl 3-methyl propanoate

(C) ethoxy-propanone

(D) 2- methyl-ethoxy propanone

77. Which shows highest magnetic moment?

(A) Cr^{2+}

(B) Mn^{2+}

(C) Cu^{2+}

(D) CO^{2+}

78. In the Sandmeyer's reaction – N ≡ N – X group of diazonium salt is replaced by

(A) Halide group

(B) Nitro group

(C) $-SH^{+}$ group

(D) $-NH^{+}NO_2$ group

79. Which of the following carbohydrates are branched polymer of glucose

(A) Amylose

(B) Amylopectin

(C) Cellulose

(D) Glycogen

80. Which of the following acid has the lowest PKa value?

(A) 

(B) Cl–CH2–CH2–COOH

(C) CCl3COOH

(D) CHCl2COOH

81. Which of the following alcohol is dehydrated most easily by concentrated H2SO4

(A) p-O2NC6H4CH(OH)CH3

(B) p-ClC6H4CH(OH)CH3

(C) p-CH3OC6H4CH(OH)CH3

(D) C6H4CH(OH)CH3

82. Which of the carbonyl compound will be most polar

(A) 

(B) 

(C) 

(D) 

83. The product (C) obtained from sequence of reaction will be

(A) 

(B) 

(C) 

(D) 

84. Which one of the following is most stable:

(A) 

(B) 

(C) 

(D)

85. Liquid hydrocarbons can be converted to a mixture of gaseous hydrocarbons by :-

(A) Oxidation

(B) Distillation under reduced pressure

(C) Cracking

(D) Vaporization

86. Tautomerism is exhibited by-

(A) 

(B) 

(C) 

(D) 

87. Most hazardous metal pollutants of automobile exhaust is

(A) Hg

(B) Cd

(C) Pb

(D) Cu

88. The wavelength associated with a golf ball weighing 200g and moving with a speed of 5m/h is of the order

(A) 10^{-10} m

(B) 10–30m

(C) 10–30 m

(D) 1045m

89. Silicones are :

(A) Organometallic compound

(B) Compounds obtained from silica

(C) Compounds obtained by hydrolysis of organo chloro silane

(D) Macromolecules prepared from silicates

90. 20% solution of urea is isotonic with 6% solution of non-ionic solute 'x'. What will be the atomic mass of solute 'x'.

[A] 4gmol-1

[B] 60gmol-1

[C] 36gmol-1

[D] 32gmol-1

91. Which is the correct order of size of O, O^{2-}, F^- and F

(A) O^{2-}>O>F->F

(B) O>O^{2-}>F>F^-

(C) Cl > F > F > Cl

(D) Cl > F > Cl > F

92. Covalent molecules are usually held in a crystal structure by

(A) Dipole-dipole attraction

(B) Electrostatic attraction

(C) Van der Waal's attraction

(D) Hydrogen bond

93. The normal dehydrating agent, used in a laboratory is

(A) $MgCO_3$

(B) CaF_2

(C) NaCl

(D) $CaCl_2$

94. B_2H_6 reacts with $(CH_3)_3N$ to produce

(A) $BH_3 \leftarrow (CH_3)_3$

(B) $B_2H_5{}^+ N(CH_3)_3CH_3 \; BH_4{}^-$

(C) $(CH_3)_3N^+ \; BH_3$

(D) $B_2H_5{}^+ (CH_3)_3CH_3 BH_3$

95. What is 'X' in the following reaction

(A) 

(B) 

(C) 

(D) 

96. The number of structural isomers for C_6H_{14}

(A) 3

(B) 4

(C) 5

(D) 6

97. Find the major product (Considering E as the electrophile) when the following substrate is subjected to electrophilic aromatic substitution

(A)

(B)

(C)

(D)

98. The second order Bragg diffraction of X-rays with wavelength of 2.00Å from a set of parallel planes in a crystal occurs at 60°. The distance between the scattering planes in the crystal is

(A) 5.75Å

(B) 2.00Å

(C) 4.00Å

(D) 2.30Å

99. Gold sol is not

(A) a macromolecular colloid

(B) a lyophobic colloid

(C) a multimolecular colloid

(D) negatively charged colloid

100. Carbon monoxide forms volatile compound with

(A) Ni

(B) Cu

(C) Al

(D) S

UPSEE 2019 Paper-1 Set-AA
Mathematics

101. If the parabola $y = -x^2 - 2x + k$ touches the parabola $y = -\frac{1}{2}x^2 - 4x + 3$ then the value of k is

(A) 1

(B) 2

(C) 3

(D) 4

102. If $f(x) = \frac{1}{x-1}$ & $g(x) = \frac{x-2}{x+2}$, then the domain of $(fog)(x)$ is

(A) $\{x : x \neq -2\}$

(B) $\{x : x \neq 1\}$

(C) $\{x : x \neq 1, -2\}$

(D) $\{x : x \neq 0, -1, 1\}$

103. If $x = \frac{\pi}{3} - 2$, then the value of $x - \frac{1}{x}$ is

$\frac{\pi}{3}$

[A] 3

[B] −2

[C] 1

[D] −3

104. Let $z=x+iy$ be a complex number satisfying the following equation $z-(2+i)=\overline{z}-4$. Which of the following options describes the above equation?

[A] $y=\frac{1}{2}x-\frac{3}{2}i$

[B] $y=2x-3i$

[C] $y=\frac{1}{2}x+\frac{3}{2}i$

[D] $y=2x-3i$

106. Let $z=x+iy$ be a complex number. The equation $\begin{vmatrix} z+1 & i \\ \bar{z} & 1 \end{vmatrix}=0$ is

[A] $x^2+y^2+y=0$

[B] $x^2-x+y^2=0$

[C] $x^2+x-y^2=0$

[D] $x^2+x+y^2=0$

106. The imaginary part of $\frac{(1+i)^n}{(1-i)^{n-2}} + \frac{(1-i)^n}{(1+i)^{n-2}}$ is

(A) 0

(B) $\frac{1}{32}$

(C) $\frac{1}{32}$

(D) $\frac{1}{32}$

107. Let x be an $n \times 1$ matrix. Let 0 and I be the zero, and identity matrices of order n respectively. Define $x^{T} = \frac{x^{T}}{x^{T}x}$ is the transpose of x.

(A) $PZ = P = Z$

(B) $PZ = P = I$

(C) $PZ = P = 0$

(D) $PZ = P = Z$

108. Let A be a 2×2 matrix such that $A\begin{bmatrix}1\\1\end{bmatrix}=-\frac{1}{2}\begin{bmatrix}1\\1\end{bmatrix}$ and $\begin{bmatrix}0&1\end{bmatrix}A=\frac{1}{2}\begin{bmatrix}-1&1\end{bmatrix}$. If 0 and I denote the zero and identity matrices of order 2, respectively, then which of the following options is CORRECT?

[A] $P_0-P_6+P_4+P_2=0$

[B] $P_0+P_6-P_4+P_2=I$

[C] $P_0+P_6+P_4-P_2=2I$

[D] $P_0-P_6-P_4-P_2=0$

109. If $\det\begin{bmatrix}1&1&2\\ t&12&4\\ t^2&12&1+12\end{bmatrix}=0$, then the values of t are

[A] $1,2,\frac{1}{3}$

[B] $-1,2,\frac{1}{2}$

[C] $1,-2,\frac{1}{2}$

[D] $1,2,-\frac{1}{3}$

110. Let $x^2+\alpha x+\beta=0$ be the equation whose roots are the negatives of the roots of $x^2+7x-2=0$ then the value of $\alpha+\beta$ is

(A) 5

(B) −5

(C) 9

(D) −9

111. If the roots of $4k^2-(5k+1)x+5k=0$ differ by unity then the sum of all possible values of k is

(A) $\frac{11}{5}$

(B) $\frac{12}{5}$

(C) $\frac{13}{5}$

(D) $\frac{14}{5}$

112. Let $a_1, a_2, a_3, \ldots$ be an arithmetic progression with non-zero common difference. It is given that $\sum_{i=1}^{11} a_i = 63$ and $a_k = a_1 + a_2$ for some k. The value of k is

[A] 6

[B] 7

[C] 8

[D] 9

113. Let S be the set of all right angled triangles with integer sides forming consecutive terms of an arithmetic progression. The number of triangles in S with perimeter less than 30 is

[A] 0

[B] 1

[C] 2

[D] 3

114. It is given that $\lim_{x \to 0} \frac{e^{(a+b)x} - bx - 1}{x^2} = 2$. Then the value of $a+b$ is

[A] 1

[B] 2

[C] 3

[D] 4

115. The sum of the intercepts on the axes of the tangent to the curve $x^{\frac{1}{3}} + y^{\frac{1}{3}} = 3a$ at $(4,1)$ is

[A] 3

[B] 5

[C] 7

[D] 9

116. If $f(x)=(x-1)^2(x+1)^3$ then the function f

(A) a local maximum at $x=$

(B) a local minimum at $x=$

(C) a local minimum at $x=-1$

(D) a local maximum at $x=-2$

117. If the function $f(x)=\begin{cases} 2x+2 & \text{if } x<2 \\ ax^2+bx+1 & \text{if } 2\le x\le 3 \\ 2x+a+b & \text{if } x>3 \end{cases}$ is continuous, then the values of (a,b) is

(A) $\left(\frac{1}{2},\frac{1}{2}\right)$

(B) $\left(\frac{1}{2},\frac{1}{3}\right)$

(C) $\left(\frac{1}{3},\frac{1}{3}\right)$

(D) $\left(\frac{1}{3},\frac{1}{2}\right)$

118. Suppose f is differentiable function such that $f(g(x))=2$ and $f'(x)=1+(f(x))^2$. Then the value of $g'(2)$ is

(A) $\frac{1}{17}$

(B) $\frac{2}{17}$

(C) $\frac{3}{17}$

(D) $\frac{4}{17}$

119. The area bounded by the graphs of $f(x) = x^2 - 2x^2$ and $g(x) = 2x2$ is

(A) $\frac{121}{17}$

(B) $\frac{124}{16}$

(C) $\frac{120}{16}$

(D) $\frac{121}{16}$

120. If the line $y =$ divides the region bounded by the curves $y=x2$ and y^{β} into regions of equal area, then the value of λ is $=$ 9

(A) $\frac{3}{\cdot 4}$

(B) $\frac{6}{\cdot 4}$

(C) $\frac{7}{4}$

(D) $\frac{9}{4}$

121. If $[\cdot]$ denotes the greatest integer less than or equal to y [illegible], then the value of the integral $\int$ [illegible]

(A) 8

(B) 9

(C) 10

(D) 11

122. If $I = \int_0^{\pi} \cos x + \sin x \, dx$ then $\int \ln(\cos^{I} x + \sin x)$ equals

(A) $\frac{\pi I}{4}$

(B) $\frac{3I}{\pi}$

(C) $\frac{4I}{\pi}$

(D)

123. The differential equations representing a family of circles having centre on the y - axis and radius 4 is

(A) $\left(\frac{dy}{dx}\right)^2 + \frac{x^2}{(x^2-2)} = 0$

(B) $\left(\frac{dy}{dx}\right)^2 + \frac{x^2}{(x^2-4)} = 0$

(C) $x^2\left(\frac{dy}{dx}\right)^2 + (y-4)^2 = 0$

(D) $\left(\frac{dy}{dx}\right)^2 + \frac{x^2}{(y-4)^2} = 0$

124. If $y(x)$ satisfies equation $\frac{d^2y}{dx^2} - 4x^2 = 0$ and $y(0)=0$, then $y(1)$ is

(A) $\frac{1}{2}$

(B) 1

(C) $\frac{4}{3}$

(D) $\frac{5}{3}$

125. If $y(x)$ satisfies equation $\frac{dy}{dx}+y=x$ and $y(0)=1$, then $y(x)$ $\frac{3}{2+e^{x}}$ then the maximum value of $y(x)$ is

(A) 1

(B) 2

(C) 3

(D) 4

126. The equation $x^2+xy-12y^2-13x+6y+5=0$ represents

(A) a pair of straight lines through the origin

(B) a pair of perpendicular straight lines

(C) a pair of parallel straight lines

(D) a pair of straight lines not passing through the origin, neither parallel nor perpendicular

127. The normal at the point (2,3) to the circle

$x^2+y^2-2x-4y+3=0$ intersects the circle $x^2+y^2=1$ at points P and Q. The area of the circle with PQ as diameter is

(A) $\frac{\pi}{2}$

(B) π

(C) 2π

(D) $\frac{3\pi}{2}$

128. The circle $x^2+y^2+2x+2by+1=0$ and $x^2+y^2+2ax-8y+12=0$ intersect orthogonally. Also circles $x^2+y^2+p(x-y+1)=0$ and $p(x^2+y^2)+x-y=1$ inter

orthogonally. Then k^2 equals

(A) $\frac{1}{4}$

(B) $\frac{1}{2}$

(C) 2

(D) 4

120. The common tangent to the parabolas $y^2=32x$ and $x^2=128y$ intersects the coordinate axes at points P and Q respectively. Then length of PQ is

[A] 213

[B] 313

[C] 513

[D] 613

130. Tangent to the ellipse $\frac{x^2}{a}+y^2=$ 1 at the point P$\left(\frac{1}{\sqrt{}}, 2, \frac{1}{2\sqrt{}}\right)$ touches a circle $x^2+y^2=r^2$ at the point Q. Then the length of PQ is

[A] $\frac{1}{10}$

[B] $\frac{3}{10}$

[C] $\frac{7}{10}$

[D] $\frac{11}{10}$

131. If the line $3x + 4y = 7$ is a normal at a point $P=(x1,y1)$ of the hyperbola $3x^2 - 4y^2 = 1$ then the distance of P from the origin is

[A] $\frac{\sqrt{89}}{12}$

[B] $\frac{\sqrt{337}}{12}$

[C] $\frac{\sqrt{433}}{12}$

[D] $\frac{\sqrt{527}}{12}$

132. If p, q, r are all distinct real numbers and the vectors $a\hat{i} + a\hat{j} + (1+a)\hat{k}$, $q\hat{i} + q2\hat{j} + (1+q3)\hat{k}$ and $r\hat{i} + r2\hat{j} + (1+r3)\hat{k}$ are coplanar, Then pqr equals

[A] 2

[B] 1

[C] -1

[D] $\frac{1}{2}$

133. Let $\vec{a},\vec{b},\vec{c}$ be three unit vectors such that $\vec{a}+\vec{b}+\vec{c}$ and $\vec{c}$ are non parallel, then the angles which $\vec{b}$ makes with $\vec{a}$ and $\vec{c}$ respectively

[A] 30°&60°

[B] 60°&90°

[C] 30°&90°

[D] 90°&60°

134. Let $\vec{a}=\hat{i}+\hat{j}+\hat{k}$ and $\vec{b}=\hat{i}-\hat{j}$. Then the angle between the vectors $\vec{a}+\vec{b}$ and $\vec{b}$ and $\vec{a}$ and $(\vec{a}-\vec{b})$ is

[A] 30°

[B] 45°

[C] 60°

[D] 90°

135. Let $\vec{a}$ and $\vec{b}$ be two vectors with $|\vec{a}|=1,|\vec{b}|=\sqrt{2}$ and $\vec{a}\cdot\vec{b}=2$. The value of $|\vec{a}\times\vec{b}|$ is

[A] 16

[B] 18

[C] 30

[D] 24

136. The value of $20C_0 + \frac{20}{21}C_1 + \frac{20}{31}C_2 + \cdots + \frac{20}{41}C_{20}$ is

[A] $\frac{2041}{20}C_{20}$

[B] $\frac{2141}{20}C_{20}$

[C] $\frac{2041}{21}C_{21}$

[D] $\frac{2141}{21}C_{19}$

137. The ratio of coefficients of 9th and 7th terms in the expansion of $(1+x)^n$ is 9:7. Then the coefficient of 4th term is

[A] 396

[B] 466

[C] 630

[D] 645

138. Six balls are placed randomly into six cells. Then the probability that exactly one cell remains empty is

(A) $\frac{29}{216}$

(B) $\frac{39}{216}$

(C) $\frac{15}{108}$

(D) $\frac{31}{108}$

129. A pair of fair dice is tossed repeatedly until a sum of four or an odd sum appears. Then the probability that a sum of four appears first is

(A) $\frac{1}{3}$

(B) $\frac{2}{5}$

(C) $\frac{3}{8}$

(D) $\frac{1}{7}$

140. Let A and B be two events with $P(A) = 0.36$, $P(B) = 0.36$, $P(A \cap B) = 0.60$. Then $P(A|B^c)$

(A) $\frac{1}{4}$

(B) $\frac{3}{8}$

(C) $\frac{1}{2}$

(D) $\frac{5}{8}$

141. Rural and urban students are equally likely to get admission in a college. If 100 students get admission, then the probability that more rural students get admission than urban students is

(A) $\sum_{k=51}^{100} {}^{100}C_k \left(\frac{1}{2}\right)^{100}$

(B) $\sum_{k=51}^{100} {}^{100}C_k \left(\frac{1}{2}\right)^{99}$

(C) $1 - \sum_{k=51}^{100} {}^{100}C_k \left(\frac{1}{2}\right)^{100}$

(C) 

142. The value of $\sin 20^\circ \sin 40^\circ \sin 80^\circ$ equals

(A) $\frac{3}{2}$

(B) $\frac{\sqrt{3}}{4}$

(C) $\frac{3}{8}$

(D) $\frac{\sqrt{3}}{8}$

141. The sum of all the solutions of the equation $\cos 3\theta = \sin 2\theta$ in

the interval $\left[\frac{\pi}{2}, \pi\right]$ is

(A) $\frac{11\pi}{5}$

(B) $\frac{17\pi}{3}$

(C) $\frac{11\pi}{4}$

(D) $\frac{5\pi}{12}$

144. In ΔABC, $\angle B = 90^\circ$ and perpendicular from B on AC intersects it at D. If $AC = 4BD$, then the smallest angle of ΔABC is

(A) $\frac{\pi}{12}$

(B) $\frac{\pi}{8}$

(C) $\frac{\pi}{6}$

(D) $\frac{\pi}{5}$

145. A solution of the equation $\tan^{-1} 2x + \tan^{-1} 3x = \frac{\pi}{4}$ is

(A) 1

(B) $\frac{1}{3}$

(C) $\frac{1}{2}$

[D] $\frac{1}{4}$

146. The angle of elevation from a window to the top of a flag is 60° and the angle of depression to the base of the flag is 30°. The horizontal distance of the window from the flag is 6 meters. Then the height of the flag is

[A] 35meters

[B] 43meters

[C] 83meters

[D] 163meters

147. A car travels 200 km in 2 hours and travels 240 km in next 3 hours. If the acceleration is constant then the distance it will travel in the next one hour is

[A] 48km

[B] 64km

[C] 72km

[D] 84km

148. A ball is thrown vertically upwards. It is at the same height h after 10 seconds and 20 seconds. Then the value of h is (assume $g=9.8 m/s^2$)

[A] 270 meters

[B] 360 meters

[C] 490 meters

[D] 980 meters

149. Two forces P and $2P$ are inclined at 120° with each other. If their resultant makes an angle α with their bisector then the value of α (in degree) is

[A] 15°

[B] 30°

[C] 45°

[D] 60°

150. Two forces, of equal magnitude F, are inclined at angle α with each other. If the resultant is also F, then the angle α (in degree) is

[A] 30°

[B] 45°

[illegible] 60°

[illegible] 120°

UPSEE 2019 Paper-1 Set-AA
Answer key

1	D	31	D	61	C	91	D	121	A
2	C	32	B	62	A	92	C	122	C
3	B	33	C	63	C	93	D	123	B
4	A	34	D	64	B	94	C	124	A
5	D	35	A	65	B	95	C	125	A
6	C	36	D	66	C	96	C	126	D
7	C	37	A	67	D	97	C	127	A
8	D	38	D	68	D	98	D	128	D
9	C	39	B	69	B	99	A	129	D
10	B	40	A	70	C	100	A	130	B
11	B	41	A	71	C	101	A	131	B
12	A	42	C	72	B	102	A	132	C
13	B	43	C	73	A	103	D	133	D
14	A	44	D	74	A	104	A	134	B
15	D	45	B	75	C	105	A	135	D
16	D	46	C	76	B	106	D	136	C
17	C	47	A	77	B	107	C	137	B
18	D	48	D	78	A	108	C	138	C
19	C	49	D	79	B	109	D	139	D
20	B	50	B	80	C	110	D	140	B
21	B	51	A	81	C	111	D	141	D
22	D	52	D	82	D	112	C	142	C
23	A	53	C	83	C	113	C	143	A
24	B	54	B	84	D	114	D	144	A
25	D	55	D	85	C	115	D	145	D
26	A	56	C	86	B	116	A	146	C

27	A	57	A	87	C	117	A	147	B
28	C	58	A	88	C	118	D	148	D
29	A	59	D	89	C	119	A	149	B
30	D	60	B	90	C	120	D	150	D

UPSEE 2019 Paper-1 Set-AA
Solutions – Physics

1. Sol:

Mass of the wire $m=(0.1\pm0.001)g$, radius $r=(0.5\pm0.005)mm$ and length $l=(10\pm0.1)cm$

The density of wire is given by the relation,

$$\rho=\frac{m}{v}$$

$$=\frac{m}{\pi r^2 l} \qquad (v=\pi r^2 l)$$

Therefore, the percentage error in the density is,

$$\frac{\Delta\rho}{\rho}\times100\% = \frac{\Delta m}{m}+2\frac{\Delta r}{r}+\frac{\Delta l}{l}\times100\%$$

$$=\left(\frac{0.001}{0.1}+2\times\frac{0.005}{0.5}+\frac{0.1}{10}\right)\times100\%$$

$$=(0.01+0.02+0.01)\times100\%$$

$$=4\%$$

Hence, the maximum percentage error in the measurement of density is 4%.

2. Sol:

The distance travelled by an object during the nth second is given by the relation,

$$s_n = u + \frac{a}{2}(2n-1) \qquad (1)$$

Where, u is the initial velocity and a is the acceleration of the object.

Since the body starts from rest therefore, u=0

So, the equation (1) becomes,

$$s_n = \frac{a}{2}(2n-1) \qquad (2)$$

Now, the distance travelled during (n+3)th second will be

$$s_{n+3} = \frac{a}{2}(2(n+3)-1)$$

$$s_{n+3} = \frac{a}{2}(2n+1) \qquad (3)$$

Dividing equation (3) by (2), we get

$$\frac{s_{n+3}}{s_n} = \frac{\frac{a}{2}(2n+1)}{\frac{a}{2}(2n-1)}$$

$$= \frac{(2n+1)}{(2n-1)}$$

3.3c)

The trajectory of the projectile is given as,

$y = \alpha x - \beta x^2$

At the maximum height, the trajectory will be parallel to the horizontal axis. It means the slope of the line will be zero.

$$\frac{dy}{dx} = 0$$

$$\frac{d}{dx}(\alpha x - \beta x^2)$$

$$\alpha - 2\beta x = 0$$

$$x = \frac{\alpha}{2\beta}$$

Now, the maximum height attained by the projectile is,

$$y_{max} = \frac{\alpha}{2\beta}\left(\frac{\alpha}{2\beta}\right) - \beta\left(\frac{\alpha}{2\beta}\right)^2$$

$$= \frac{\alpha^2}{2\beta} - \frac{\alpha^2}{4\beta}$$

$$= \frac{\alpha^2}{4\beta}$$

Hence, the maximum height attained by the projectile is $\frac{\alpha^2}{4\beta}$

Remark: In the question maximum height attained by the projectile is asked but in the options, the horizontal distance at maximum height is given.

4. Sol:

The force exerted by the clamp on the pulley is the resultant of the tension force in the string and the weight of the pulley and the body.

The tension in the string is,

$T = Mg$

The weight of the pulley and the body is,

$W = (M + m)g$

The vector diagram for the two forces is shown below.

Therefore, the resultant force exerted by the clamp on the pulley is,

F = W2 + T2

= ((M+m)g+(Mg)2

= (M + m)2 + M2g

5. Sol:

The linear momentum of the particle is given as,

$$\vec{p}(t)=A(\hat{i}\cos bt-\hat{j}\sin bt)$$

The force exerted on the particle is,

$$\vec{F}(t)=\frac{d\vec{p}(t)}{dt}$$

$$=\frac{d}{dt}\{A(\hat{i}\cos bt)\}$$

$$=Ab(-\hat{i}\sin bt)$$

Taking the dot product of $\vec{F}$ and $\vec{p}$, we get:

$$\vec{F}\cdot\vec{p}=(Ab(-\hat{i}\sin bt))(\hat{j}\cos bt+A(\hat{i}\cos bt$$

Fp cosθ = −A2b sin btcosbt + 0 + 0 + A2b sin btcosbt

Fp cosθ = 0

cosθ = 0

θ = 90°

Hence, the angle between the force and the momentum is 90°.

6. Sol:

The angular momentum is given as,

$L = mvr$

Where, m is the mass of the planet, v is the orbital speed of the planet and r is the radius of the orbit.

Since, mass of the planet is constant and angular momentum is conserved, when the planet is farther from the sun (radius is larger), the orbital speed must be smaller and when the planet is closer to the sun (radius is smaller), the orbital speed must be greater.

Therefore, the planet sweeps out equal area in equal intervals of time.

Hence, Kepler's second law is a consequence of conservation of angular momentum.

7. Sol:

Let a be the retardation of the particle. It is given that the retardation is proportional to the displacement x.

a x

$$\frac{dv}{dt} = -kx \qquad \text{(k is proportionality constant)}$$

$$\frac{dx}{dt} = -kx$$

$$vdv = -kxdx$$

Let for a displacement from 0 to x the velocity changes from v2 to v2.

Integrating the above equation, we get

$$\int_{v_1}^{v_2} vdv = -k\int_0^x xdx$$

$$\frac{v_2^2}{2} - \frac{v_1^2}{2} = -\frac{1}{2}kx2$$

$$\frac{1}{2}m(v2^2) = \frac{1}{2}mkx2$$

$$\Delta K = -\frac{1}{2}mkx2$$

$$\Delta K \quad x2$$

Hence, the change in kinetic energy is proportional to x2

8. Soln

The angular momentum of the disc about the origin is the sum of the angular momentum of the centre of mass and the angular momentum of the body about centre of mass.

$L = I_{COM} + L_{cm}$

$= MR^2 + \omega$ (1)

Where, v is the linear speed of the disc and I is the moment of inertia of the disc.

The relation between the angular velocity and the linear velocity is given as,

$v = R\omega$

The moment of inertia of the disc is given as,

$I = \frac{1}{2} MR^2$

Substituting the values in equation (1), we get

$L = MR(R\omega) + \frac{1}{2} MR^2 \omega$

$= \frac{3}{2} MR^2 \omega$

9. Sol:

The time period of a simple pendulum is given by the relation,

$T = 2\pi \sqrt{\frac{l}{g}}$

Where, l is the length of the pendulum and g is the acceleration due to gravity.

The upward displacement is given as,

$y = \lambda t^2$

Differentiate the above equation twice with respect to time

$$\frac{dy}{dt} = 2\lambda t$$

$$\frac{d^2y}{dt^2} = 2\lambda$$

$$a_y = 2\lambda$$

The new time period of the pendulum is,

$$T_1 = 2\pi \sqrt{\frac{l}{g + a_y}}$$

$$= 2\pi \sqrt{\frac{l}{g + 2\lambda}}$$

Hence, it is clear from the above equation that new time period of pendulum is less than T.

10. So:

When the temperature of the liquid increases the kinetic energy of the molecules of the liquid also increases.

The surface tension of the liquid is the measure of intermolecular attraction. The intermolecular attraction

decreases with the increase in temperature due to increased kinetic energy of molecules.

Hence, the surface tension decreases with the increase in temperature of liquid.

11. Sol:

The Reynold's number is given by the formula,

$$Re = \frac{vL}{\nu}$$

Where, v is the velocity of the fluid, L is the characteristic linear dimension and ν is the kinematic viscosity.

Dimensional formula of Reynold's number is,

$$Re = \frac{(m)(M)(m)}{(m)(s)}$$

[illegible]

Thus, Reynold's number is a dimensionless quantity.

From the given options, only coefficient of friction is dimensionless quantity.

Hence, the dimensional formula of Reynold's number is same as coefficient of friction.

12. Sol:

Change in pressure is given as,

$\Delta P = \rho g h$

$= (1000\ kg/m^3)(9.8\ m/s^2$

$= 1.96 \times 10^6\ N/m^2$

The bulk modulus of the material of the ball is,

$B = \frac{\Delta p}{\Delta v / v}$

$= \frac{1.96 \times 10^6\ N/m^2}{\frac{0.32}{100}}$

$= 1.96 \times 10^8\ N/m^2$

Hence, the bulk modulus of the material of the ball is $1.96 \times 10^8 N/m^2$.

13. Sol:

The value of γ for the mixture of gases is given by the relation,

$\frac{n_1 + n_2}{\gamma_{mix} - 1} = \frac{n_1}{\gamma_1 - 1} + \frac{n_2}{\gamma_2}$

Where, n1 and n2 are the number of moles of the two gases and

γ_{mix} is the specific heat ratio for mixture. Therefore,

$$\frac{1}{\gamma_{mix}-1}=\frac{1}{\frac{5}{2}-1}+\frac{1}{\frac{7}{5}-1}$$

$$\frac{2}{\gamma_{mix}-1}=\frac{3}{2}+\frac{5}{2}$$

$$\frac{2}{\gamma_{mix}-1}=4$$

$$\gamma_{mix}-1=\frac{2}{4}$$

$$\gamma_{mix}-1=0.50$$

$$\gamma_{mix}=1.50$$

Hence, the value of γ for the mixture is 1.50.

14. Sol:

The work done during the cycle is,

Work done = Area enclosed

$$=(2P-P)(2V-V)$$

$$=PV$$

15. Sol:

Given: Wavelength, $\lambda_{max1}=4000\text{Å}$ and $\lambda_{max2}=6000\text{Å}$

According to Wien's displacement law,

$\lambda_{max} \propto \frac{1}{T}$

Therefore,

$\frac{\lambda_{max}}{\lambda_{max}} = \frac{T}{T}$

$\frac{4000\,\text{Å}}{6000\,\text{Å}} = \frac{T}{T}$

$\frac{4000\,\text{Å}}{6000\,\text{Å}} = \frac{T}{T}$

Hence, the ratio of their temperatures is 3:2.

16. Set:

Given: Temperature, T1=300K, Energy Q1=500cal and Q2=150cal

The efficiency of the carnot engine is given by the relation,

$\eta = 1 - \frac{Q_2}{Q_1} = 1 - \frac{T_2}{T_1}$

Therefore,

$$\frac{Q_2}{Q_1} = \frac{T_2}{T_1}$$

$$Q_2 = T_2\left(\frac{Q_1}{T_1}\right)$$

$$= (500K)\left(\frac{150\ cal}{300\ cal}\right)$$

$$= 250K$$

Hence, the temperature of the cold is 200K.

17. Sol:

Since the radiation moves with the speed of light therefore, it is the fastest mode of heat transfer.

18. Sol:

Given: Latent heat of ice, L_{ice}=80Cal/gm, mass of ice, m=1.0gm and temperature T=273K

The change in entropy is given by the relation,

$$\Delta S = \frac{Q}{T}$$

$$= \frac{(10gm)(80Cal/gm)}{273K}$$

$$= 2.93\ Cal/K$$

Hence, the change in entropy of ice is 2.93Cal/K.

15; Sol:

Given, velocity of car v_s=30m/sec, frequency, f = 8kHz and the velocity of sound v_s=320m/sec.

The apparent frequency is given by the relation,

$$f' = f\left(\frac{v - v_o}{v - v_s}\right)$$

$$= (8kHz)\left(\frac{320m/sec - 20\ m/sec}{320\ m/sec - 30\ m/sec}\right)$$

$$= (8kHz)\left(\frac{340}{290}\right)$$

$$= 9.1\ kHz$$

Hence, the frequency of siren heard by car driver is 9.1 kHz.

20. Sol:

The two waves are,

$x_1 = A\sin\omega t$ and $x_2 = A\sin\left(\omega t + \frac{\pi}{3}\right)$

Therefore, the phase difference is,

$$\Delta\varphi = \frac{\pi}{3} - 0$$

$$= \frac{\pi}{3}$$

Hence, the phase difference between the two waves is $\frac{\pi}{3}$.

21. Sol:

The velocity of sound wave is given by the relation,

$$v = \sqrt{\frac{B}{\rho}}$$

When the pressure P of the gas (medium) changes, the density ρ of the gas will also be change by the same factor to compensate the change.

Hence, the velocity of sound is independent on the pressure.

22. Sol:

The displacements of three waves are,

$y_1=a\sin 2\pi(300t)$

$y_2=a\sin 2\pi(301t)$

$y_3=a\sin 2\pi(302t)$

The resultant displacement due to all three waves is,

$$y=a\sin 2\pi(300t)+a\sin 2\pi(301t)+a\sin 2\pi(302t)$$
$$=2a\sin 2\pi(301t)\cos 2\pi t+a\sin 2\pi(301t)$$
$$=a(2\cos 2\pi t+1)\sin 2\pi(301t)$$

The resultant amplitude is,

$A=a(2\cos 2\pi t+1)$

For maximum amplitude A_m

$2\pi ft = -1$

$2\pi ft = 2n\pi + \frac{2\pi}{3}$

$t = n + \frac{1}{3}$

$t = \frac{1}{3}, \frac{4}{3}, \frac{7}{3}, \frac{10}{3}, \ldots$

This shows that maxima occur every one second.

For minimum amplitude A,

$\cos 2\pi ft = 1$

$2\pi ft = 2n\pi$

$t = 0, 1, 2, 3, \ldots$

This shows that minima also occur every one second.

Hence, the number of beat heard per second is one.

(ii) Set:

The current in a conductor does not depend on the area of cross section. Therefore, it remains constant.

The current in a conductor is given by the relation,

$I = neAv$

Therefore, the drift velocity is,

$$v_d = \frac{I}{neA}$$

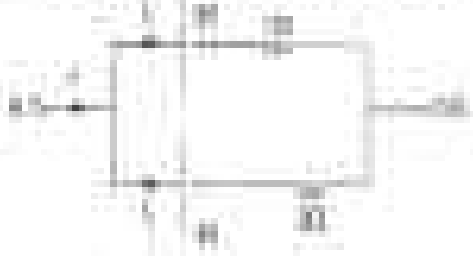
$$v_d \propto \frac{I}{A}$$

Thus, the drift velocity is inversely proportional to area of cross section.

Hence, only current is constant in a conductor of non uniform cross section area.

24. Sol:

The current in the circuit is shown below.



The current I_1 is,

$$I_1 = \frac{4\,V}{(1+1)\Omega}$$

$$= 2\,A$$

The current I_2 is,

$$R = (2+2)\,\Omega$$

$$= \quad \Omega$$

Total current in the circuit is,

$$I = I_1 + I_2$$

$$= 3\text{A} + \frac{3}{4}\text{A}$$

$$= \frac{15}{4}\text{A}$$

Total resistance in the circuit is,

$$\frac{1}{R} = \frac{1}{2} + \frac{1}{4}$$

$$= \frac{2+1}{4}$$

$$= \frac{4}{3}\,\Omega$$

$$R = \quad \Omega$$

The potential difference across A and B is,

$$V = IR$$

$$= \left(\frac{15}{4}\right)\text{A} \times \left(\frac{4}{3}\right)\Omega$$

$$= 5\text{V}$$

Hence, the potential difference across A and B is 5 V.

Ans:

Let the charge stored on the capacitor be Q.

The initial capacitance of the capacitor is given by the relation,

$$C = \frac{\varepsilon_0 A}{d}$$

As the capacitor is disconnected from the battery, the charge on the capacitor remains same.

The potential difference across the capacitor is,

$$V = \frac{Q}{C}$$

The energy associated with capacitor is given by the relation,

$$E = \frac{1}{2} CV^2$$

The capacitance of the capacitor when dielectric slab is introduced between the plates,

$$C' = \frac{k\varepsilon_0 A}{d}$$

$$= kC$$

Where, k is the dielectric constant of the slab.

Since $k > 1$, therefore, $C' > C$

Now, the new potential difference across is,

$$V' = \frac{Q}{C'} = \frac{Q}{kC} = \frac{V}{k}$$

Since k>1, therefore, V'<V

The energy associated with capacitor after the insertion of slab is,

$$E' = \frac{1}{2}(kC)V^2 = \frac{1}{2}kCV^2 = kE$$

Since k>1, therefore, E'>E

Hence, energy accociated with capacitor decreases when a slab is inserted inside the capacitor.

2k. Sol:

Given: Voltage V1=200V, Power P1=100W and voltage V2=100V

The power consumed by the bulb is given by the relation,

$P = \frac{V^2}{R}$

Since, resistance of the bulb remains constant therefore,

$P \propto V^2$

$\frac{P_2}{P_1} = \left(\frac{V_2}{V_1}\right)^2$

$P_2 = \left(\frac{100V}{200V}\right)^2 \times 100W$

$= \frac{1}{4} \times 100W$

$= 25W$

Hence, the power consumed by the bulb is 25W.

27. Ans:

The displacement current is given by the relation,

$I_{disp} = \frac{d\phi_E}{dt}$

Above equation shows that displacement current is associated with the change in electric flux and hence, with the change in electric field.

Hence, displacement current is caused by a time varying electric field.

28. Ans:

The dipole moment of a dipole is given by the relation,

$p = qd$

Where, q is the magnitude of charge and d is the separation between the charges.

Substituting the values, we get

$p = (1.6 \times 10^{-19}\ C)(1\ m)$

$= 1.6 \times 10^{-19}\ C\text{-}m$

Hence, the dipole moment of the dipole is 1.6×10^{-28} C-m.

29. Sol:

Given: Resistance $R0 = 100\Omega$, temperature coefficient of resistance $\alpha = 0.0045/°C$ and change in temperature $\Delta T = 60°C$

The resistance of a conductor depends on temperature by the relation,

$R = R0\ (1 + \alpha\Delta T)$

$= (100\Omega)(1 + (0.0045/°C)(60°C))$

$= (100\Omega)(1 + 0.27)$

$= 127\Omega$

Hence, the resistance of the wire at 60°C is 127Ω.

30. Sol:

A ferromagnetic material becomes a paramagnetic when it is heated above the Curie temperature.

31. Sol:

The magnetic field due to a long current carrying wire is given by the relation,

$$B = \frac{\mu_0 I}{2\pi r}$$

The above equation shows that magnetic field is inversely proportional to the distance r.

$$B \propto \frac{1}{r}$$

Hence, the graph for the magnetic induction due to long current carrying wire is,

32. Sol:

The current due to orbital motion of electron is,

$$i = \frac{e}{T} = \frac{e}{2\pi r/v} = \frac{ev}{2\pi r}$$

The magnetic moment associated with the electron is,

$$\mu = iA = \left(\frac{ev}{2\pi r}\right)(\pi r^2) = \frac{evr}{2}$$

33. Sol:

Voltmeter is connected in parallel across the resistor whose voltage is to be measured. To measure the voltage across the resistor accurately, all the current should pass through it and no current should pass through the voltmeter. Thus, the resistance of the voltmeter should be very high. Hence, voltmeter has the maximum resistance.

34. Sol:

The induced emf in a coil is proportional to the rate of change of current in the coil.

$$V \propto \frac{dI}{dt}$$

$$V = -L\frac{dI}{dt}$$

Where, L is the self inductance.

The unit of self inductance is,

$$\text{Unit of } L = \frac{\text{Volt-sec}}{\text{Amp}}$$

35. Sol:

Eddy currents are produced in conductor due to change in magnetic flux. The magnetic flux can be changed when a conductor is moving through a magnetic field, or when the magnetic field surrounding a stationary conductor is varying. Hence, eddy currents are produced when a metal is kept in varying magnetic field.

36. Sol:

The Lenz's law state that the polarity of the induced emf is such that it opposes the change in magnetic flux that causes it. The Lenz's law is a consequence of the conservation of energy. The mechanical work done to move the magnet is converted into electrical energy.

37. Sol:

Let the intensity of two waves be I and 4I.

The resultant intensity for two coherent sources is given by the relation,

$I=I_1+I_2+2\sqrt{I_1 I_2}\cos\phi$

Where, ϕ is the phase difference between the two waves.

The resultant intensity will be maximum, when $\cos\phi=1$

$I_{max}=I+4I+2\sqrt{I\cdot 4I}$

$=I+4I+4I$

$=9I$

The resultant intensity will be minimum, when $\cos\phi=-1$

$I_{min}=I+4I-2\sqrt{I\cdot 4I}$

$=I+4I-4I$

$=I$

The ratio of maximum and minimum intensities is,

$$L_{air} = 9l$$
$$L_{air} = 1$$
$$= 9$$
$$1$$

Hence, the ratio of wavelength and resistance elements is 9:1.

38. Sol.

Given: focal length fa=10cm, refractive index ng=2.5 and

refractive index nl=1.75

The focal length in liquid is given as,

$$f_l = \frac{n_g - 1}{n_l - 1} f_a$$

$$f_l = \frac{n_g - 1}{n_l - 1} \times f_a$$

Substituting the values, we get

$$f_l = \frac{2.5 - 1}{1.75 - 1} \times (10cm)$$
$$= \frac{1.5}{0.75} \times (10cm)$$
$$= 2 \times (10cm)$$
$$= 20cm$$

The negative sign indicates that the lens will behave like a concave lens.

Hence, the lens will behave like as a concave lens of focal length 36cm.

39. Sol:

For minimum deviation, the incident angle and the refracted angle both are same and equal to half the prism angle for all the colours.

Therefore, for minimum deviation,

$i1 = i2 = \frac{A}{2}$

$= \frac{60^\circ}{2}$

$= 30^\circ$

Hence, the angle of refraction will be 30° for both the colours.

40. Sol:

The resolving power of a telescope is given by the relation,

$\text{Resolving power} = \frac{d}{1.22\lambda}$

This shows that resolving power is directly proportional to the diameter of the objective.

Hence, resolving power of a telescope can be increased by increasing the diameter of the objective.

41. Sol:

'Lumen' is the international unit of luminous flux.

42. Sol:

When a beam of unpolarised light is passes through a polaroid, its intensity becomes half.

Hence, the intensity of transmitted light is $\frac{I_0}{2}$.

43. Sol:

In an achromatic combination of lenses, two lenses are combined to form images of different colours at the same point. This can be done by combining a pair convex lens and one concave lens.

44. Sol:

Given: work function, $\phi = 3eV$ and energy of photon, $E_{ph} = 1eV$

The kinetic energy of emitted photo-electrons is given by the relation,

$K = E_{ph} - \phi$

$= 2eV - 3eV$

$= -1eV$

The negative sign indicates that no photo-electrons ejected from the metal surface and hence they have no kinetic energy.

45. (a)

The current through X-ray tube is given by the relation,

$I = \frac{q}{t} = \frac{ne}{t}$

$n = \frac{It}{e}$

$= \frac{(3.2 \times 10^{-3}\ A)(1s)}{1.6 \times 10^{-19}\ C}$

$= 2 \times 10^{16}$

Hence, the number of electrons striking the target per second is 2×10^{16}.

46. (a)

The nuclear radius is given by the relation,

$$r = r_0 A^{\frac{1}{3}}$$

Where, r_0 is the Bohr radius and A is the mass number.

Let m be the mass of a nucleon.

The mass density of the nucleus is,

$$\rho = \frac{M}{V}$$

$$= \frac{mA}{\frac{4}{3}\pi r^3}$$

$$= \frac{3mA}{4\pi r_0^3 A}$$

$$= \frac{3m}{4\pi r_0^3}$$

Hence, the mass density of nucleus is independent on mass number.

47. Sol:

Given: Half life ($t_{1/2}$) = 6min and total time t = 30min

Total number of half lives are,

$$n = \frac{20\ \text{min}}{5\ \text{min}}$$

$$= 4$$

The fraction of the substance remains undecayed is

$$\frac{N}{N0} = \left(\frac{1}{2}\right)^{n}$$

$$= \left(\frac{1}{2}\right)^{4}$$

$$= \frac{1}{16}$$

The fraction of the substance decayed will be;

$$\text{Fraction decayed} = 1 - \frac{N}{N_0}$$

$$= 1 - \frac{1}{16}$$

$$= \frac{15}{16}$$

$$= 93.75\%$$

Hence, the amount of substance decayed in 20 min will be 93.75%

48. Sol:

Depletion layer is formed when a P-type material is joined with an N-type material. The free electrons in the N-type material near the junction deplayed the holes in the P-type material. So, a charge free layer is created near the junction that consists of negative and positive immobile ions. Hence, the depletion layer of an unbiased P-N junction is free from electrons and holes.

49. Sol:

A laser beam has the following properties:

(1). It is highly intense in nature

(2). It is highly directional

(3). It is monochromatic

(4). It is coherent

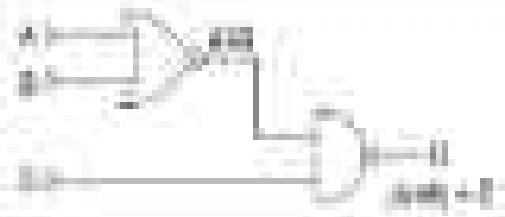
50. Sol:

The output O for the given combination of gates is,

$O = A + B \cdot C$

$= A + B + C$

$= (A + B) + C$



A	B	A+B	C	D
1	1	0	0	1

Hence, the logic state of output D is 1.

UPSEE 2019 Paper-1 Set-AA
Solutions – Chemistry

61. A redox reaction in which a reactant gets converted into two or more than two different products is known as disproportionation reaction. In the given reaction, $P_4 + 3NaOH + 3H_2O \rightarrow PH_3 + 3NaH_2PO_2$, phosphorus is making two different compounds. Therefore, it is a disproportionation reaction.

62. The given three reactions (i), (ii), and (iii) possessing the equilibrium constants are shown below.

(i) $CO(g) + H_2O(g) \rightleftharpoons CO_2(g) + H_2(g)$ ...K1

(ii) $CH_4(g) + 2H_2O(g) \rightleftharpoons CO_2(g) + 4H_2(g)$ ...K2

(iii) $CH_4($

For reaction (i), the equilibrium constant is expressed as follows.

$$K_1 = \frac{[CO_2][H_2]}{[CO][H_2O]} \quad \text{...(i)}$$

For reaction (ii), the equilibrium constant is expressed as follows.

$$K3 = \frac{[Cu^{2+}][H_2]^2}{[Cu(NH_3)_4^{2+}][NH_3]^4} \quad \ldots (3)$$

For reaction (3), the equilibrium constant is expressed as follows:

$$K3 = \frac{[Cu^{2+}][H_2]^2}{[Cu^{2+}][NH_3]^4} \quad \ldots (3)$$

According to equations, (1), (2) and (3), the resultant equilibrium constant expression is K3 = K1 × K2.

(c). The ionization equation for silver is given below:

$Ag^+ + e^- \rightarrow Ag(108g)$

That means 108g of Ag gets deposited through charge of 96487 C. So, 1.45 g of Ag gets deposited through charge of

$$\frac{96487 \, C \times 1.45}{108} = 1295.43C$$

The given current is 5.5 A, so the time required is calculated as,

$t = \frac{E}{I}$

$= \frac{1296.48}{1.5}$

$= 864\ s$ or $864\ min$

$= 14.4\ min$

The reaction of copper is shown below.

$Cu2+ + 2e^- \rightarrow Cu(63.5g)$

That means the total charge deposited by 63.5g copper is

2×96487 and 2×96487 deposits the amount of copper as

$\frac{63.5 \times 1296.48}{2 \times 96487} = 0.427g$

The reaction of zinc is shown below.

$Zn2+ + 2e^- \rightarrow Zn(65.4g)$

That means the total charge deposited by 65.4g zinc is

2×96487 and 2×96487 deposits the amount of zinc as

$\frac{65.4 \times 1296.48}{2 \times 96487} = 0.44g$

64. The ion, CH_3^- does not have a linear shape as it contains three atoms attached to central carbon atom. It has trigonal planar shape as shown below.

66. The correct decreasing order of the boiling points of compounds H_2O, HF and NH_3 is $H_2O > HF > NH_3$ because the hydrogen bonding in H_2O is highly stronger than the hydrogen bonding in HF and NH_3. The boiling point of HF is higher than that of NH_3 because HF contains three lone pairs of electrons on fluorine, so it has more strong hydrogen bond whereas, NH_3 contains only one lone pair on nitrogen atom.

56. The amount of calcium sulphate is 2g at 298K and Ksp for $CaSO_4$ is 9.0×10^{-6}. The molar mass of $CaSO_4$ is 136g/mol. The ionic equation of $CaSO_4$ is shown below.

$CaSO_{4(s)} \rightleftharpoons Ca^{2+}_{(aq)} + SO_4^{2-}{}_{(aq)}$

The expression for K_{sp} is,

K_{sp} [illegible]

The value of solubility, s is calculated as,

$K_{sp} = s^2$

$9.0 \times 10^{-6} = s2$

$s2 = 3.00 \times 10^{-3}$ mol/L

The total solubility in grams/Liter is calculated as,

3.00×10^{-3} mol/L $\times$ 136 g/mol = 0.41 g/L

Thus, 0.41g of CaSO4 is dissolved in 1 L of water, so 2g of CaSO4 is dissolved in 2g/0.41g/L = 4.9L of water.

67. The given reaction is as follows.

2NOCl(g) $\rightleftharpoons$ 2NO(g) + Cl2(g)

The reaction occurs at 400 K at the values of ΔH° equals to 77.5 kJ mol^{-1} or 77500 J mol^{-1} and ΔS° = 136.36 $JK^{-1}mol^{-1}$.

The value of ΔG° is calculated below.

$\Delta G^\circ = \Delta H^\circ - T\Delta S^\circ$

= 77500 J mol^{-1} − 400 K × 136.36 J K^{-1} mol^{-1}

= [illegible] 77500 mol^{-1} − 54500 mol^{-1}

= [illegible] J mol^{-1}

The value equilibrium constant is calculated as,

$\Delta G^{\circ} = -2.303\ RT \log K$

$23500 = -2.303 \times 8.314 \times 400 \log K$

$\log K = \frac{-23500}{2.303 \times 8.314 \times 400}$

$\log K = -3.06834$

The value of K is calculated as,

$\log K = -3.06834$

$K = 10^{-3.06834}$

$= 8.545 \times 10^{-4}$

58. The dissociation of Acetic acid is 1.3%. The acetic acid solution is N/10 solution of the acid that is 0.1 N of acetic acid solution. So, the hydrogen concentration in 0.1 N of 1.3 % solution is 0.0013N.

So, the pH of the solution is calculated as,

$pH = \log[H^+]$

$pH = \log[0.0013N]$

$= 2.886$

59. In compound, CaH_2, calcium has +2 oxidation number. The oxidation number of hydrogen is supposed to be x and it is calculated below:

$1 + 2x = 0$

$2x = -2$

$x = -1$

So, in CaH_2, the oxidation number of hydrogen is -1.

60. The compounds, ZnO, Fe_2O_3 and PbO would be reduced by carbon, C as shown below:

$ZnO + C \rightarrow Zn + CO$

$PbO + C \rightarrow Pb + CO$

$Fe_2O_3 + 3C \rightarrow 2Fe + 3CO$

61. The titration curve if a strong base is titrated with strong acid is shown below:

In the titration curve of strong acid and base, the equivalence point is obtained by the above shown curve.

62. The Kelvin equation is related to vapor pressure of droplets of liquids which occurs because of the liquid vapor interface. The Kelvin equation is given below.

$$\ln\frac{p}{p_0^{sat}} = \frac{2\gamma V_m}{rRT}$$

63. Nylon 66 is formed by the condensation polymerization between hexamethylene diamine and adipic acid.

64. The given structure is,

The IUPAC name of the given structure is 5-chloro-2-methyl cyclohexanol.

65. If the relative abundance of M^{81}Br is supposed to be x and the relative abundance of M^{79}Br is supposed to be (1-x) then the percentage of occurrence of M^{81}Br in nature is calculated below

$$\text{Weight} = \frac{W1x + W2(1-x)}{x + (1-x)}$$

$$61. \quad = \frac{80 \times x + 62 \times (1-x)}{1}$$

$$? \quad = 80x + 62 - 62x$$

$$61. \quad x = 0.38$$

?

66. For the second order reaction, the half life equation is given as follows.

$$t(1/2) = \frac{1}{k[A]_0}$$

So, the graph given by second order half life is shown below.

67. The arrangement of electrons in an atom which is not permissible is n=3,l=2,m=-3,s=-1/2, because if l=2 then m=-3. So, this set of setting is not permissible arrangement of electrons in an atom.

68. The molecules of H2 gas at −0°C gas has the highest speed because root mean velocity is inversely proportional to the molecular mass and among all the molecules, H2 gas lowest molecular mass. Therefore, the velocity of H2 gas will be higher.

69. The vapour density of a mixture containing NO2 and N2O4 is 38.3 at 27°C. The moles of NO2 is supposed to be x and the number of moles of N2O4 is 100-x. The expression for vapor density is given below.

$$\text{Vapor density} = \frac{\text{Molar mass of NO}_2/\text{N}_2\text{O}_4 \text{ mixture}}{2}$$

Molar mass of NO2/N2O4 mixture = vapor density × 2

Molar mass of NO2/N2O4 mixture = 38.3 × 2

Molar mass of NO2/N2O4 mixture 76.6 g mol−1

The mole of NO2 in 100 mole mixture is calculated below.

$$x(\text{molar mass of NO2 gas}) + (1-x)(\text{molar mass of N2O4 gas}) = 76.6$$

$$x(46) + (1-x)(92) = 76.6$$

x = 33.43

70. The enthalpy change of a reaction does not depend on the different intermediate reactions because according to the Hess's law, the complete amount of enthalpy in a reaction does not depend upon the various steps involved in that reaction.

71. The complete reaction is shown below

72. The reaction of P4 with X leads selectively to P4O6. The X is a mixture of O2 and N2 because this mixture is used to control the oxidation rate of P4.

73. The formula for bleaching powder is Ca(OCl)Cl, so it poss the acid of HOCl. The anhydride of that oxoacid, HOCl which is present in bleaching powder, is Cl2O as shown below.

$Cl_2O + H_2O \rightarrow 2HOCl$

74. Starch is a polymer made up of amylose and amylopectin in which the 2-glucose. So, this statement is wrong.

75. The composition of malachite is $CuCO_3 \cdot Cu(OH)_2$. It is the f ore utilized to form copper.

76. The given structure is,

The IUPAC name of this structure is ethyl-2-methyl propanoate because its parent chain contains three carbon and a ketone group, C=O.

77. The bivalent metal ion which has maximum number of unpaired electrons is Mn^{2+} among the given first transition series elements that corresponds to the maximum highest magnetic moment. Mn^{2+} she 5 unpaired electrons.

78. In the Sandmeyer's reaction $-N{\equiv}N-X$ group of diazonium salt is replaced by halide group as shown below.

79. The carbohydrate, amylopectin is a branched polymer of glucose. Amylopectin consists of α-1,6-glycosidic linkage at the branch side.

80. The lowest pK_a value is possessed by the compound, CCl_3COOH that is 0.66. This compound has three chlorine atom attached to it due to which this compound has very high value of K_a. Greater the value of K_a, lesser will be the pK_a value of a compound. Therefore, CCl_3COOH has the lowest pK_a value.

81. The alcohol, *p*-$CH_3OC_6H_4CH(OH)CH_3$ is dehydrated most easily by concentrated H_2SO_4 because it contains methoxy group which is less electronwithdrawing as compared to NO_2 group.

82. The compound which contains electronwithdrawing group or electron donating groups forms a stable carbocation and can be dehydrated easily.

82. The compound which is most polar is shown below.

The above shown compound is most polar because in this compound carbon is attached to hydrogen atoms and an oxygen atom. The electronegativity difference between carbon-hydrogen and carbon-oxygen is very high due to which this compound is highly polar.

83. The complete reaction is shown below.

Therefore, option (2) is correct.

84. The most stable carbanion is $CH_3-\overset{\oplus}{O}$ because it is highly stabilized by resonance.

85. The higher hydrocarbons are present as liquid and lower grade hydrocarbons are present as gases. After the cracking the liquid hydrocarbons, a mixture of gaseous hydrocarbons are obtained.

86. The tautomerism is exhibited by the compound shown below.

This compound shows tautomerism because in this compound, electronegative nitrogen atom is directly attached to carbon atoms and carbon atom also attached with hydrogen atoms.

87. The most hazardous metal pollutants of automobile exhaust is lead, Pb. It adds upto 98% of pollution in the environment.

88. The wavelength associated with a golf ball weighing 200g and moving with a speed of 5 m/h is of the order a calculated below.

Wavelength $= \frac{h}{mv}$

$= \frac{6.626\times10^{-34}}{200\times5}$ m

$= 6.626\times10^{-36}$ m

Therefore, the wavelength is 10^{-35}m.

89. Silicones are the compounds obtained by hydrolysis of organo chloro silane.

90. As 10% solution of urea is isotonic with 6% solution of non-volatile solute 'x', so, the atomic mass of solute 'x' is calculated below:

$$\frac{W_{solute}}{m_{solute}} = \frac{W}{m}$$

$$\frac{10}{60\ g/mol \times \frac{100}{100}} = \frac{6}{m \times \frac{100}{100}}$$

$$m = \frac{60\times6}{10}\ g/mol$$

$$= 36\ g/mol$$

91. The correct order of size of O, O2- , F- and F is

O2- > F- > O > F

The reason for this order is that the atomic radius decreases while moving across the period. So, the radius of oxygen is higher than fluorine. The species, O2-, F- are isoelectronic. The ionic radii is inversely proportional to effective nuclear charge which is higher in F- as compared to O2-.

92. Covalent molecules are usually held in a crystal structure by Van-der Waal's forces of attraction while the crystal structure.

93. The normal dehydrating agent, used in a laboratory is calcium chloride, CaCl2. It can easily absorb the water.

94. B2H6 reacts with (CH3)3N to produce [illegible]BH

below:

B2H6 + 2N(CH3)3 → 2[illegible]

96. The compound A is shown below:

96. The number of structural isomers for C_6H_{14} is five which are n-hexane, 2-methylpentane, 3-methylpentane, 2,3-dimethylbutane and 2,2-dimethylbutane.

97. The given substrate is shown below.

The major product if the substrate is subjected to electrophilic aromatic substitution is shown below.

98. The wavelength of second order Bragg diffraction of X-rays is 2.004 from a set of parallel planes in a crystal takes place at 60°. The distance between the scattering planes in the crystal is calculated below.

$$n\lambda = 2d\sin\theta$$

$$2^{nd} \text{ order: } 1.00\text{ Å} = 2 \times d \times \sin 60^{\circ}$$

$$1.00\text{ Å} = 2 \times d \times \frac{\sqrt{3}}{2}$$

$$d = 1.15\text{ Å}$$

99. Solid ice is not a macromolecular solid. They are the ionic particles.

100. Carbon monoxide forms volatile compound with nickel. Nickel forms a volatile compound, on heating under a stream of CO as shown below.

$$Ni + 4CO \xrightarrow{330-350^{\circ}C} [Ni(CO)_4]$$

UPSEE 2019 Paper-1 Set-AA
Solutions – Mathematics

101. Consider the parabola,

$y = -x^2 - 2x + k$ ... (I)

And,

$y = -\frac{x^2}{2} - 4x - 3$ ... (II)

Equate equations (I) and (II)

$-x^2 - 2x + k = -\frac{1}{2}x^2 - 4x - 3$

$-\frac{x^2}{2} + 2x + k + 3 = 0$

$x^2 - 4x + 2(3 + k) =$

Since the parabolas intersect each other, so the roots will be coincident.

Equate discriminant to zero

$D = b^2 - 4ac$

$0 = (-4)^2 - 8(3 + k)$

$8(3 + k) = 8$

$k = 1$

102. Consider the function,

$f(x) = x^{\frac{1}{2}}$

And,

$g(x) = \frac{1}{x+1}$

So,

$(f \circ g)(x) = f(g(x))$

$= f\left(\frac{1}{x+1}\right)$

$= \frac{1}{\left(\frac{1}{x+1}\right)^{\frac{1}{2}}}$

$= \frac{1}{\frac{1}{(x+1)^{\frac{1}{2}}}}$

$= (x+1)^{\frac{1}{2}}$

The domain of $(f \circ g)(x)$ is determined as,

$x + 1 \geq 0$

$x \geq -1$

The domain is,

$x \in [-1, \infty)$

10. Consider the function,

$y = \frac{1}{x}$ ——— (i)

Now,

$$x = \frac{1}{2}(z + \bar{z})$$

$$= \frac{1}{2}(x + iy + x - iy)$$

$$= \frac{1}{2}(2x)$$

$$= x$$

Substitute x for x in the expression (i):

$$\left(\frac{z-1}{z+1}\right)^2 = \frac{x^2 - 2x + 1}{x^2 + 2x + 1}$$

$$= \frac{x^2 + 1 - 2x}{x^2 + 1 + 2x}$$

$$= \frac{(x-1)^2}{(x+1)^2}$$

$$= \left(\frac{x-1}{x+1}\right)^2$$

$$= \left(\frac{z-1}{z+1}\right)^2$$

104. Consider the equation,

$$z - (2+i) = \mathrm{Re}(z) - 4$$

The above equation can be solved as,

$$(x-2)^2+(y-1)^2=(6-4)^2$$

$$(y-1)^2+(6-4)^2=(x-2)^2$$

$$y^2-2y+1+36-8y+y^2=x^2-4x+4$$

$$y=1\pm\frac{2}{3}x$$

106. Consider the equation:

$$\sin\left(\frac{\pi}{2}x+1\right)=\pi$$

The above equation is solved as,

$$\sin\left(\frac{\pi}{2}x+1\right)=\sin\pi$$

$$\frac{\pi}{2}x+1=\pi$$

$$\frac{\pi}{2}x=\pi-1$$

$$x=\frac{2}{\pi}(\pi-1)$$

$$\sin\left(\frac{\pi}{2}x+1\right)=\pi$$

$$\sin\left(\frac{\pi}{2}x+1\right)=\sin\pi$$

$$\frac{\pi}{2}x+1=\pi$$

$$\frac{\pi}{2}x=\pi-1$$

$$x=\frac{2}{\pi}(\pi-1)$$

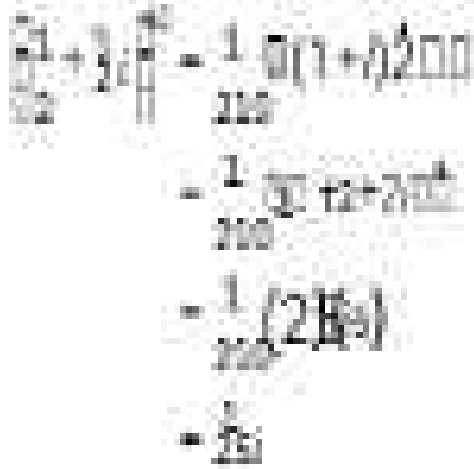
Solve further,

$$\sin\left(\frac{\pi}{2}x+1\right)=\sin\pi$$

$$x^2+y^2+x+y=0$$

106. Consider the expression:

Solve the above expression as,



Therefore, the imaginary part is,

107. Since, the division of matrix by a matrix is not defined. Hence, there correct solution cannot be determined.

108. Consider a 2×2 matrix,

$P =$ 

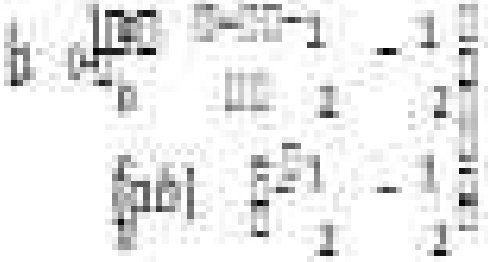
Also,

$$\begin{bmatrix} 1 & 0 \end{bmatrix} P = -\frac{1}{2}\begin{bmatrix} 1 & 2 \end{bmatrix} \qquad \text{— (I)}$$

And,

$$\begin{bmatrix} 0 & 1 \end{bmatrix} P = -\frac{1}{2}\begin{bmatrix} -1 & 2 \end{bmatrix} \qquad \text{— (II)}$$

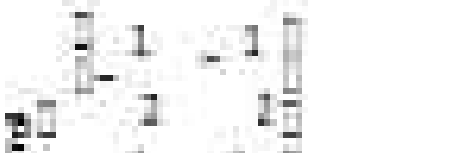
Substitute  for P in equation (I).



So,

Substitute  for P in equation (II).

Thus, the matrix is,



And,

Therefore,

10) Consider the determinant

Apply operation R2→R2−4R1

$$\begin{vmatrix} 1 & 1 & 2 \\ 0 & 2 & 5 \\ 4 & 12 & 1+12 \end{vmatrix} = 0$$

Solve the determinant as follows,

$$1(2x^2 - 5 + 26) - 4) = 0$$

$$2x^2 - 8x^2 + 7 + 1 = 0$$

$$(x-1)(2x+1)(x-3) = 0$$

Therefore,

$$x = \frac{-1}{2}, \quad 1$$

110. Consider the equation

$x^2+\alpha x+\beta=0$ ——(I)

And,

$x^2+3x-2=0$ ——(II)

Substitute $-x$ for x in equation (II),

$$(-x)^2 + 7(-x) - 2 = 0$$

$$x^2 - 7x - 2 = 0$$ ——(III)

Equation (I) and (III) are now identical.

Therefore,

$\alpha = -7$

$\beta = -2$

So,

$\alpha + \beta = -7 - 2$

$= -9$

111. Consider the equation:

$4k2-(5k+1)x+5k=0$ ——(i)

Let α and β be the roots of the equation (i).

So,

$\alpha+\beta = \frac{5k+1}{4}$

And,

$\alpha\beta = \frac{5k}{4}$

Now the roots differ by 1.

$\alpha-\beta=1$

So,

$(\alpha+\beta)2 - 4\alpha\beta$

$\left(\frac{5k+1}{4}\right)^2 - 4\left(\frac{5k}{4}\right) = 1^2$

$5k2 - 14k - 30 = 0$

Solve the quadratic equation for k.

$k = 8$;

Sum of the possible values of k,

$$k_1 + k_2 = 8 + \frac{16}{3} = \frac{40}{3}$$

112. The sum of the AP is

$a_4 + a_5 + a_{12} = 63$

The common difference of the AP is not equal to zero.

The AP can be rewritten as,

$$(a + 3d) + (a + 4d) + (a + 11d) = 63$$

$$3a + 18d = 63$$

$$a + 6d = 21$$

$$a_7 = 21$$

So,

$k = 8$

113. Let the sides of the triangle be $a - d$, a, $a + d$.

So,

$$(a-b)^2+c^2=(a+c)^2$$

$$a^2+b^2-2ab+c^2=a^2+c^2+2ac$$

$$a(a-4d)=0$$

Thus,

$a=4d$

Now,

$$3d^2+4d^2+5^2=0$$

$$5^2=5^2$$

$$d=1$$

Therefore, the sides of the triangles are 3, 4, 5 and 4, 4, 10 Such two triangles are not possible.

114. Consider the limit

$$\lim_{x\to 0}\frac{e^{\sin x}-1-x}{x^2}$$

The above limit is solved as,

$$\lim_{x\to 0}\frac{1+(\sin x)+\frac{(\sin x)^2}{2!}+\cdots-\sin x-1}{x^2}=\frac{1}{2}$$

The above condition is possible only when,

$x = 0$

And,

$\frac{x}{2} = 2$

Therefore,

$x = 2$

$y = 2$

So,

$x + y = 2 + 2$

$= 4$

116. Consider the curve,

$x^2 \quad y = 3$

Differentiate the above equation.

$$\frac{1}{2}\times\frac{1}{x}\frac{dy}{dx}=0$$

$$\frac{dy}{dx}=-\frac{y}{x}$$

$$\left[\frac{dy}{dx}\right]_{(4,2)}=-\frac{1}{2}$$

$$m=-\frac{1}{2}$$

Therefore, the equation of tangent is,

$$y-1=-\frac{1}{2}(x-4)$$

$$x+2y=6$$

$$\frac{x}{6}+\frac{y}{3}=1$$

The x intercept is 6 and y intercept is 3.

Sum of intercept,

Sum=6+3

$=9$

116. Consider the function,

$$f(x)=(x-1)2(x+1)3$$

Differentiate with respect to x.

$$f'(x) = 2(x+1)(x-1) + (x+1)^2 = 3(x-1)^2$$

$$= (x-1)(x+1)[2(x+1) + 3(x-1)]$$

$$= (x-1)(x+1)(5x-1)$$

Therefore, the function has a local maxima at $x = \frac{1}{5}$.

117. Consider the piecewise function.

$$f(x) = \begin{cases} x+2 & x \le 1 \\ ax^2 + bx + 3 & 1 < x < 3 \\ 2x + a + b & x \ge 3 \end{cases}$$

The function is continuous then,

$$\lim_{x \to 1^-} f(x) = \lim_{x \to 1^+} f(x)$$

$$\lim_{x \to 1^-} x + 2 = \lim_{x \to 1^+} ax^2 + bx + 3$$

$$3 = a + b + 3$$

$$a + b = 1 \qquad \text{---(1)}$$

Also,

$$\lim_{x \to 3^-} f(x) = \lim_{x \to 3^+} f(x)$$

$$\lim_{x \to 3^-} ax^2 + bx + 3 = \lim_{x \to 3^+} 2x + a + b$$

$$9a + 3b + 3 = 6 + a + b$$

$$8a + 2b = 3 \qquad \text{---(2)}$$

Solve equation (i) and (ii) for the value of a and b.

$$a = \frac{1}{2}$$

$$b = \frac{1}{2}$$

Therefore,

$$a^2 + b^2 = \left(\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^2$$

$$= \frac{1}{4} + \frac{1}{4}$$

$$= \frac{2}{4}$$

$$= \frac{1}{2}$$

11i. Consider the function

$$f(g(x)) = x^2 \qquad \text{---(i)}$$

Differentiate equation (i) with respect to x.

$f'(g(x))g'(x) = 2x$

$$g'(x) = \frac{2x}{f'(g(x))}$$

$$= \frac{2x}{1 + f(x)2}$$

$$= \frac{2x}{1 + x^4}$$

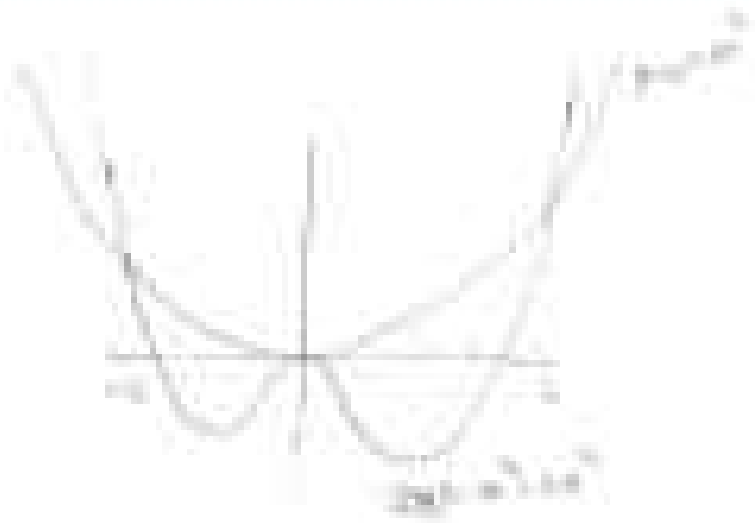
Substitute 2 for x in the above equation.

$$g'(2) = \frac{2(2)}{1 + 24}$$

$$= \frac{4}{1 + 16}$$

$$= \frac{4}{17}$$

119. Consider the figure shown depicting the required area.



The area of the shaded region is calculated as,

$$A = \int_{a}^{b} g(x) - f(x)\,dx$$

$$= \int_{a}^{b} x^2 - x\,dx$$

$$= 2\int_{0}^{1} x^2 - x\,dx$$

$$= 2\left[\frac{x^3}{3} - \frac{x^2}{2}\right]_{0}^{1}$$

Solve further,

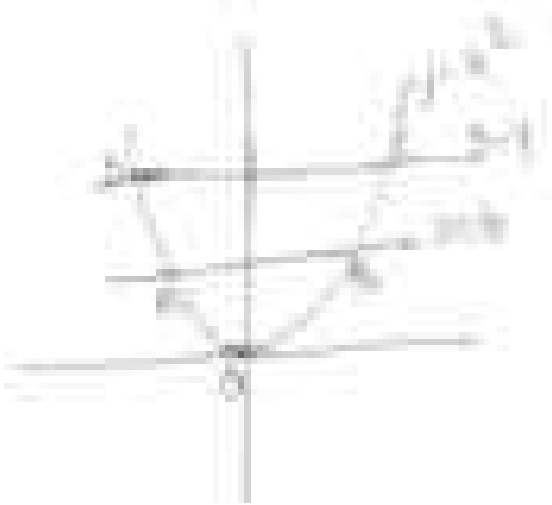
$$A = 2\left(\frac{1}{3} - \frac{1}{2}\right)$$

$$= 2\left(\frac{2-3}{6}\right)$$

$$= 2\left(-\frac{1}{6}\right)$$

$$= -\frac{1}{3}$$

120. Consider the figure shown below.



Now the area of region OAB is equal to the area of region ABCD.

Therefore,

$$\int_0^{b^2} \sqrt{y}\,dy = \int_b^1 \sqrt{y}\,dy$$

$$\frac{2}{3}\left[y^{3/2}\right]_0^{b^2} = \frac{2}{3}\left[y^{3/2}\right]_b^1$$

$$b^3 = 1 - b^3$$

$$2b^3 = 1$$

$$b = \left(\frac{1}{2}\right)^{1/3}$$

Simplify further,

$$I = \frac{\pi}{4}$$

121. Consider the integral,

$$I = \int_{-1}^{1} \frac{1}{1+x^2}\,dx$$

The above integral is solved as,

$$I = \int_{-1}^{0} \frac{1}{1+x^2}\,dx + \int_{0}^{1} \frac{1}{1+x^2}\,dx$$

$$= 0 + \int_{0}^{1} \frac{1}{1+x^2}\,dx + \int_{0}^{1} \frac{1}{1+x^2}\,dx$$

$$= 0 + [\tan^{-1}x]_0^1 + [\tan^{-1}x]_0^1$$

$$= \left[\frac{\pi}{4} - 0\right] + \left[\frac{\pi}{4} - 0\right]$$

$$= \frac{\pi}{2} + \frac{\pi}{2}$$

$$= \pi$$

122. Consider the integral,

$$I = \int_{\pi/4}^{\pi/2} \frac{\cos x}{\sin x}\,dx$$

Simplify the above integral as,

$$I = \int_{\pi/2}^{\pi} \frac{1}{\sin x + \cos x}\,dx$$

$$I = \int_{\pi/2}^{\pi} \frac{1}{\sin x + \cos x}\,dx$$

$$I = \frac{\pi}{2}$$

123. Let the center of the circle be $(0,a)$.

So, the equation of circle is,

$$(x-0)^2+(y-a)^2=4^2 \quad \text{---(i)}$$

Differentiate on both sides of the equation.

$$2x + 2(y-a)\frac{dy}{dx} = 0$$

$$(y-a) = -\frac{x}{dy/dx}$$

Substitute $-\frac{x}{dy/dx}$ for $(y-a)$ in equation (i).

124. Consider the differential equation,

$$(1-x^2)\frac{dy}{dx}-2xy-4x^2=0$$

Simplify the above equation.

$$\frac{dy}{dx}+\frac{2x}{1-x^2}y=\frac{4x^2}{1-x^2}$$

Calculate the integrating factor.

$$IF=e^{\int P\,dx}$$

$$=e^{\int \frac{2x}{1-x^2}dx}$$

$$=1+x^2$$

The solution of the equation is,

$$y(1+x) = \int_{1+x}^{x+2} 2\,f(1,z)\,dz$$

$$y(1+z) = \int 4\,dz$$

$$y(1+x) = \square + C$$

$$y = \square$$

At $y(0)$, the value of C is $C=0$.

Substitute 1 for x in the above expression.

$$y(1) = \frac{4(1)}{1+12}$$

$$= \frac{4}{3(2)}$$

$$= \frac{2}{3}$$

120. Consider the differential equation.

$$(1+x)\frac{dy}{dx} + yx = 1$$

Rewrite the above expression as,

$$\frac{dy}{dx} + \frac{ex}{1+ex}y = \frac{1}{1+e^x}$$

Calculate the integrating factor:

$$IF = e^{\int \frac{ex}{1+ex} dx}$$

$$= e^{\ln(1+ex)}$$

$$= 1+ex$$

The solution of the equation is calculated as,

$$y(1+ex) = \int \frac{1}{1+ex}(1+ex)dx$$

$$y(1+ex) = x + C \qquad \text{---(i)}$$

Now,

$$y(1) = \frac{2}{1+e}$$

So,

$$\frac{2}{1+e}(1+e) = 1 + C$$

$$C = 1$$

$$C = e$$

Substitute 1 for C in equation (i).

$$y(1+ex) = x + 2$$

$$y = \frac{x+2}{1+ex}$$

Now, $\frac{dy}{dx} = 0$ and $x = 0$.

$y_{max} = 1^{\frac{2}{1}} \cdot 1$

$= 2$

$= 2$

$= 1$

126. Consider the equation:

$6x^2 + xy - 12y^2 - 13x + 6y + 6 = 0$

Here,

$a=6, b=-12, c=6, h = \frac{1}{2}, g = \frac{-13}{2}$ and $f = 3$.

For the given equation,

$$\begin{vmatrix} a & h & g \\ h & b & f \\ g & f & c \end{vmatrix} = 0$$

$$\begin{vmatrix} 1 & -13 & \\ -6 & 2 & \\ 2 & -11 & 2 \\ 2 & & 3 \\ -13 & & \\ 2 & 3 & 6 \end{vmatrix} = 0$$

$$6(12) - (9)(-3) + 5(-39 + 19.5) - 6.5(\quad) = 0$$

$$0 = 0$$

Thus, the given equation represents a pair of lines

Now,

$c \neq 0$

$f \neq 0$

So no line intersects through origin.

Now,

$$a + b \neq 0$$

$$6 + (-12) \neq$$

$$-6 \neq 0$$

So, the lines are not perpendicular.

Now,

$k2 = ab$

$\frac{1}{4} = (k)(-12)$

$k = -\frac{1}{48}$

So, the lines are not parallel.

127. The normal passes through the centre. Hence the equation of normal at (2,3) is,

$$y - 3 = \frac{3}{2}(x - 2)$$

$$x - y + 1 = 0$$

The normal clearly intersect the $x2 - y^2 = 1$ at $P(0,-1)$ and $Q(-1,0)$.

Hence, the area of the circle with PQ as diameter is,

$$A = \pi\left(\frac{1}{2}\right)^2$$

$$= \frac{\pi}{2}$$

128. Apply the condition for orthogonal intersection in first pair of circle.

Therefore,

$k = 2$

$p = \frac{1}{2}$

$= 4$

129. Consider the parabola

$y^2 = 12x$

The tangent for the above parabola can be taken as,

$$y=mx+\frac{\bar{a}}{m} \quad \text{---(i)}$$

Equation (i) also represent the tangent for $x2=-10y$.

So,

$$x^2=-10\left(y+\frac{a}{m}\right)$$

$$x^2+10mx+\frac{8(10a)}{m}=0$$

Calculate the discriminant.

$$(10m)^2-\frac{4(8)(10a)}{m}=0$$

$$m=\frac{-2}{\bar{a}}$$

So;

$$y=-x\frac{2}{a}+\bar{a} \quad \text{---(ii)}$$

Thus, the coordinates of P is $(-10,0)$ and the coordinates is $(0,-12)$.

Therefore, the length of PQ is,

$PQ=\sqrt{(12)}$

130. Consider the ellipse

$$\frac{x^2}{4}+y^2=1$$

The tangent to the ellipse at point P $\left(2\cos\frac{\pi}{4}, \sin\frac{\pi}{4}\right)$

$$\frac{x}{4}\cdot 2+y\cdot\frac{1}{2}=1$$

$$2x+2y-4=0 \qquad \ldots(i)$$

The line is also tangent to the circle,

$x^2+y^2=r^2$

So,

$$\frac{|0+0-4|}{\sqrt{2^2+2^2}}=r$$

$$r=\frac{4}{2\sqrt{2}}$$

So the equation of circle is,

$$x^2+y^2=\left(\frac{4}{2\sqrt{2}}\right)^2$$

$$x^2+y^2=\frac{16}{8}$$

$$x^2+y^2=\frac{8}{4}$$

The length of PQ (length of tangent to circle from P) is,

$$PQ = \sqrt{(2)^2 + \frac{1}{2}^2 - \frac{8}{10}}$$

120. Consider the curve,

$3x^2 - 4y^2 = 1$.

Differentiate the above equation with respect to x.

$$6x - \frac{8y\,dy}{dx} = 0$$

$$\frac{dy}{dx} = \frac{3x}{4y}$$

Slope of normal at point on the curve is,

$$m_{normal} = \frac{-1}{\left(\frac{dy}{dx}\right)_{(x_1, y_1)}}$$

$$= -\frac{4y_1}{3x_1}$$

So,

$$4y_1 - 3x_1 = 0 \qquad \text{---(i)}$$

Also, the points (x1,y1) lies on the line 3x+4.

So,

3x1+4y1=7 ..... (II)

Solve equation (I) and (II).

$x1 = \frac{4}{5}$

$y1 = -\frac{3}{5}$

Therefore, the distance from the origin is calculated as,

$OA = \sqrt{\left(\frac{4}{5}\right)^2 + \left(-\frac{3}{5}\right)^2}$

$= \frac{\sqrt{25}}{12}$

132. The given vectors are coplanar.

$$\begin{vmatrix} p & pq & 1+pqr \\ q & q^2 & 1+pqr \\ r & rq & 1+pqr \end{vmatrix} = 0$$

$$\begin{vmatrix} pq & 1 & p \\ q^2 & q & q \\ rq & 1 & r \end{vmatrix} = 0$$

$$(1+pqr)\begin{vmatrix} 1 & p & p \\ 1 & q & q \\ 1 & r & r \end{vmatrix} = 0$$

Since p, q, r are distinct, hence,

$$1+pqr=0$$

$$pqr=-1$$

133. Consider the vector,

$$\vec{b}\times(\vec{c}\times\vec{a})=\vec{c}$$

$$(\vec{b}\cdot\vec{a})\vec{c}-(\vec{b}\cdot\vec{c})\vec{a}=\vec{c}$$

Now, $\vec{a}$ and $\vec{c}$ are not parallel.

Therefore,

$$\vec{a}\cdot\vec{b} = |\vec{a}||\vec{b}|\cos 60^\circ$$

And,

$$\vec{a}\cdot\vec{c} = |\vec{a}||\vec{c}|\cos 60^\circ$$

The angle between $\vec{a}$ and $\vec{b}$ is 60° and between $\vec{a}$ and $\vec{c}$ is 60°.

13. Let $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$

Substitute $-\hat{i}+\hat{j}$ for $\vec{a}$ and $3\hat{i}+\hat{k}$ for $\vec{b}$ in the above equation.

$$\vec{c} = 4(-\hat{i}+\hat{j}) + (3\hat{i}+\hat{k})$$

$$= -3\hat{i}+4\hat{j}+\hat{k}$$

Let $\vec{a} = \frac{1}{2}(\vec{b}+\vec{c})$

Substitute $-\hat{i}+4\hat{j}+\hat{k}$ for $\vec{b}$ and $2\hat{i}+3\hat{j}$ for $\vec{c}$ in the above equation.

$$\vec{a} = \frac{1}{2}((2\hat{i}+3\hat{j}) + (-\hat{i}+4\hat{j}+\hat{k}))$$

$$= \frac{1}{2}(\hat{i}+7\hat{j}+\hat{k})$$

$$= \frac{1}{2}\hat{i}+\frac{7}{2}\hat{j}+\frac{1}{2}\hat{k}$$

The angle between and is calculated as,

$$\cos\theta = \frac{\vec{a}\cdot\vec{b}}{|\vec{a}||\vec{b}|}$$

$$= \frac{(-3\hat{i}+\hat{j})\cdot(-6\hat{i}-6\hat{j})}{\sqrt{(-3)^2+1^2}\sqrt{(-6)^2+(-6)^2}}$$

$$= \frac{18-6}{\sqrt{10}\sqrt{72}}$$

$$= \frac{12}{\sqrt{720}}$$

$$= \frac{1}{\sqrt{5}}$$

Solve further,

$$\theta = \cos^{-1}\frac{1}{\sqrt{5}} = \frac{\pi}{2}$$

$$= 63.4^\circ$$

136. Write the expression for

$$\vec{a}+\vec{b} = 2\left(\vec{a}+\frac{\vec{b}}{2}\right)-\vec{a}-\frac{\vec{b}}{2} \quad \text{---(1)}$$

Substitute 13 for , 13 for and 132 for in equation

(1).

$$\frac{\square}{\square}+\frac{\square}{\square}=2(132+192)-222$$

$$=2(530)-484$$

$$=1060-484$$

$$=576$$

Solve further,

$$\frac{\square}{\square}+\frac{\square}{\square}=576$$

$$=24$$

136. Consider the expression

$$\frac{\square}{\square}+\frac{\square}{\square}+\frac{\square}{\square}$$

The above expression is solved as,

$$\frac{\square}{\square}+\frac{\square}{\square}+\frac{\square}{\square}=\frac{\square}{\square}+\frac{\square}{\square}+\frac{\square}{\square}+\frac{\square}{\square}$$

$$=\square\left(\frac{\square}{\square}+\frac{\square}{\square}+\frac{\square}{\square}+\frac{\square}{\square}\right)$$

$$=\square\left(\frac{\square}{\square}+\frac{\square}{\square}+\frac{\square}{\square}+\frac{\square}{\square}\right)$$

$$=\square\left(\frac{\square}{\square}+\frac{\square}{\square}\right)$$

Solve further,

$$210 = \frac{21!}{11!} + \frac{22!}{2!} + 11! + \frac{60!}{40!} = 210\left(\frac{21!}{11!} + \frac{22!}{2!} + 11! + \frac{60!}{40!}\right)$$

$$= 210\left(\frac{21!}{11!}\right)$$

$$= 210\left(\frac{\quad}{\quad}\right)$$

137. Calculate the ratio of the 6th and the 7th term.

Coeff of $T_6 = {}^{8}C_5$

Coeff of T_7

$$\frac{a}{7} = \frac{8!}{5!(8-5)!}$$

$$\frac{7}{7} = \frac{7!}{(7-5)!}$$

$$\frac{a}{7} = \frac{6!}{5!(6-5)!}$$

$$\frac{7}{7} = \frac{6!}{(6-5)!}$$

$$\frac{a}{7} = \frac{(n-6)!(n-7)!}{(n-5)!(n-7)!}$$

Solve further,

$$(n-6)(n-7) = 7$$

$$n^2 - 7n - 6n + 42 - 42 = 2$$

$$n(n-13) - 2(n-13) = 0$$

$$(n-13)(n-2) = 0$$

$$n = 13, 2$$

So,

$n=15$

The coefficient of the fourth term is,

$\text{Coeff of } T_4 = {}^{15}C_3$

$= 455$

158. Number of ways in which six balls can be put in six cells is 6^6 ways.

The favorable cases are $6C_5 \times 2C_1 \times 5!$

The probability is calculated as,

$p = \frac{{}^6C_5 \times {}^2C_1 \times 5!}{6^6}$

$= \frac{720}{46656}$

159. The probability of obtaining sum as four is $\frac{3}{36}$

The probability of obtaining sum as odd is $\frac{1}{2}$

The probability of sum neither odd nor four is $1 - \frac{3}{36} - \frac{1}{2}$

$= \frac{18}{36} - \frac{1}{2} - \frac{1}{2}$

Therefore, the required probability is,

$$p = \frac{1}{12} + \frac{5}{12} - \frac{1}{12} + \frac{5}{12} - \frac{5}{12} + \frac{1}{12}$$

$$= \frac{11}{12}$$

$$= 1 - \frac{1}{12}$$

$$= \frac{11}{12}$$

14) Calculate the value of $P(A)$

$$P(A) = 1 - 0.55$$

$$= 0.45$$

Calculate the value of $P(A \cap B)$.

$$P(A \cap B) = P(A) + P(B) - P(A \cup B)$$

$$= 0.4 + 0.35 - 0.6$$

$$= 0.15$$

Calculate the value of $P(A \mid BC)$

$$P(A \mid BC) = \frac{P(A \cap BC)}{P(BC)}$$

$$= \frac{P(A) - P(A \cap B)}{1 - P(B)}$$

$$= \frac{0.45 - 0.15}{1 - 0.35}$$

$$= \frac{3}{13}$$

141. Let the number of rural students be x.

The probability of success is equal to the probability of failure.

So,

$$p = \frac{1}{2}$$

The probability that more rural students get admission than urban students is calculated as,

$$P(X > 50) = {}^{100}C_{51}\left(\frac{1}{2}\right)^{51}\left(\frac{1}{2}\right)^{49} + {}^{100}C_{52}\left(\frac{1}{2}\right)^{52}\left(\frac{1}{2}\right)^{48} + \ldots + {}^{100}C_{100}\left(\frac{1}{2}\right)^{100}\left(\frac{1}{2}\right)^{0}$$

$$= \frac{1}{2^{100}}\left({}^{100}C_{51} + {}^{100}C_{52} + \ldots + {}^{100}C_{100}\right)$$

$$= \frac{1}{2^{100}}\left({}^{100}C_{0} + {}^{100}C_{1} + {}^{100}C_{2} + \ldots + {}^{100}C_{50}\right)$$

Solve further,

$$P(X > 50) = \frac{1}{2^{100}}\left(2^{99} - {}^{100}C_{50}\right)$$

$$= \frac{1}{2}\left(\frac{2^{100}}{2^{100}} - \frac{{}^{100}C_{50}}{2^{100}}\right)$$

142. Calculate the value of $\sin 20^\circ \sin 40^\circ \sin 60^\circ$

$$\sin 20^\circ \sin 40^\circ \sin 60^\circ = \sin 20^\circ \sin(60^\circ - 20^\circ)\sin(60^\circ + 20^\circ)$$

$$= \sin 20^\circ \left(\sin 60^\circ \cos 20^\circ - \cos 60^\circ \sin 20^\circ\right)\left(\sin 60^\circ \cos 20^\circ + \cos 60^\circ \sin 20^\circ\right)$$

$$= \sin 20^\circ \left(\frac{3}{4}\cos^2 20^\circ - \frac{1}{4}\sin^2 20^\circ\right)$$

$$= \sin 20^\circ \left(\frac{3\cos^2 20^\circ - \sin^2 20^\circ}{4}\right)$$

Solve further,

$$\sin 20^\circ \sin 40^\circ \sin 60^\circ = \frac{1}{4}\sin 20^\circ \left(3\cos^2 20^\circ - \sin^2 20^\circ\right)$$

$$= \frac{1}{4}\sin 20^\circ \left[3(1 - \sin^2 20^\circ) - \sin^2 20^\circ\right]$$

$$= \frac{1}{4}\sin 20^\circ \left(3 - 3\sin^2 20^\circ - \sin^2 20^\circ\right)$$

$$= \frac{\left(3\sin 20^\circ - 4\sin^3 20^\circ\right)}{4}$$

Solve further,

$$\sin 20^\circ \sin 40^\circ \sin 60^\circ = \frac{[\sin 60]^\circ}{4}$$

$$= \frac{\frac{\sqrt{3}}{2}}{4}$$

$$= \frac{\sqrt{3}}{8}$$

140. Consider the given equation:

cos3θ=sin2θ

Rewrite the above equation as;

$$\cos 3\theta = \cos\left(\frac{\pi}{2} - 2\theta\right)$$

$$3\theta = 2n\pi \pm \left(\frac{\pi}{2} - 2\theta\right)$$

So,

$$3\theta = 2n\pi + \left(\frac{\pi}{2} - 2\theta\right)$$

$$\theta = \frac{2n\pi}{5} + \frac{\pi}{10}$$

And,

$$3\theta = 2n\pi - \left(\frac{\pi}{2} - 2\theta\right)$$

$$\theta = \frac{2n\pi}{1} - \frac{\pi}{2}$$

So the solution in the interval $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ are,

$$-\frac{\pi}{2}, \frac{\pi}{10}, \frac{\pi}{2}, \frac{3\pi}{10}$$

The sum of the solutions set is,

$$\text{Sum} = -\frac{\pi}{2} + \frac{\pi}{10} + \frac{\pi}{2} - \frac{3\pi}{10}$$

$$= -\pi$$

b

144. Consider the figure shown below.

From the figure

$$\sin\theta = \frac{r}{a}$$

And

$$\cos\theta = \frac{b}{4r}$$

Thus,

$$\sin^2\theta\cos^2\theta = \left(\frac{1}{2}\right)^2$$

$$\sin^2\theta\cos^2\theta = \frac{1}{4}$$

$$2\sin^2\theta\cos^2\theta = \frac{1}{2}$$

$$\sin 2\theta = \frac{1}{2}$$

Solve further,

$$2\theta = \sin^{-1}\left(\frac{1}{2}\right)$$

$$2\theta = \frac{\pi}{6}$$

$$\theta = \frac{\pi}{12}$$

145. Consider the equation.

$$\tan^{-1}3x + \tan^{-1}3x = \frac{\pi}{4}$$

The above equation is solved as follows.

$$\tan^{-1}\left(\frac{1}{1+2x(3x)}\right)+\tan^{-1}\left(\frac{3x}{1-6x^2}\right)=\frac{\pi}{4}$$

$$\tan^{-1}\left(\frac{1}{1+6x^2}\right)+\tan^{-1}\left(\frac{3x}{1-6x^2}\right)=\frac{\pi}{4}$$

$$\tan^{-1}\left(\frac{\frac{1}{1+6x^2}+\frac{3x}{1-6x^2}}{1-\frac{1}{1+6x^2}\cdot\frac{3x}{1-6x^2}}\right)=\frac{\pi}{4}$$

$$\frac{1-6x^2+3x+18x^3}{1-6x^2-3x+18x^3}=1$$

$$1-6x^2+3x+18x^3=1-6x^2-3x+18x^3$$

$$6x+18x^3=0$$

$$18x^3+6x=0$$

$$(x+1)(6x-1)=0$$

Since the value of x cannot be negative.

Therefore,

$$x=\frac{1}{6}$$

146. Consider the figure shown below.

From the above figure,

$$\tan 60^\circ=\frac{h}{6}$$

$$h=6\tan 60^\circ$$

$$=6\sqrt{3}$$

And,

$s + 200 + 36 = (100 \times 6) + \frac{1}{2}(6)(6)^2$

$s + 0 = 648 - 144$

$s = 504 - 440$

$= 64km$

0

160. Let the speed at which the ball is thrown up be u.

$h = ut + \frac{1}{2}at^2$

$h = 10u + \frac{1}{2}(-9.8)(10^2$ ..... (I)

And,

$h = ut + \frac{1}{2}at^2$

$h = 20u + \frac{1}{2}(-9.8)(20^2)$ ..... (II)

Solve equation (I) and (II) for the value of h.

$h = 900m$

140. Let the angle made by the resultant with vector whose magnitude is F be β.

$$\tan\beta = \frac{2P\sin 120^\circ}{P+2P\cos 120^\circ}$$

$$\beta = \tan^{-1}\left(\frac{\sqrt{3}}{1-1}\right)$$

$$= 90^\circ$$

Therefore,

$$\alpha = 90^\circ - 60^\circ$$

$$= 30^\circ$$

100. Let the two forces be F_1 and F_2.

So, the resultant force is $F_1 + F_2$.

Also,

$$F_1 - F_2 = \frac{F}{2}$$

$$= F/2$$

$$= F$$

Now,

$$F_1^2 + F_2^2 = F_1^2 + F_2^2 + 2F_1F_2\cos\theta$$

$$F^2 = 2F^2 + 2F^2\cos\theta$$

$$\cos\theta = \frac{1}{2}$$

$$= -$$

Solve further,

$$\alpha = \cos^{-1}\left(\frac{1}{2}\right)$$

$$= 120^{\circ}$$