AMU

Engineering Entrance Exam

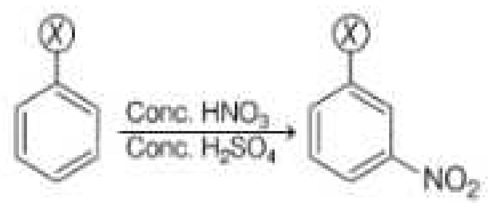
Solved Paper 2021

Chemistry

1. Which is not a purification technique for organic compounds?

(a) Spectroscopy (b) Chromatography
(c) Crystallisation (d) Distillation

2.



What is 'X' in the above reaction?

(a) —Cl (b) —OH
(c) $—NO_2$ (d) —NHR

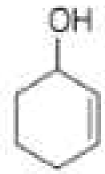
3. Which of the following undergoes Hell-Volhard-Zelinsky reaction?

(a) CH_3CH_2COOH
(b) CCl_3COOH
(c) C_6H_5COOH
(d) $(CH_3)_3CCOOH$

4. Which one of the following conformations of cyclohexane is the least stable?

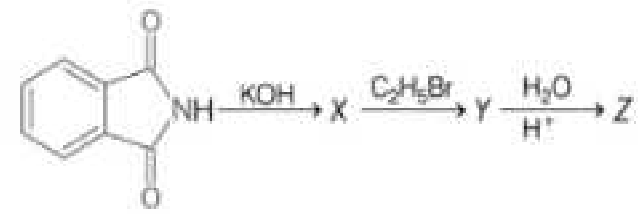
(a) Half-chair (b) Boat
(c) Twisted-boat (d) Chair

5. IUPAC name of the following compound is

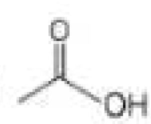
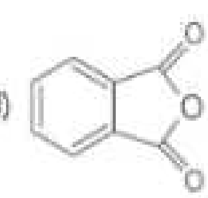


(a) 3-hydroxy cyclohexene
(b) cyclohex-2-en-1-ol
(c) cyclohex-5-en-1-ol
(d) cyclohex-1-en-3-ol

6. The product 'Z' in the following reaction is



(a)  (b) 

(c)  (d) 

7. In a constant volume calorimeter (bomb calorimeter), 10.5 g of C_2H_4 gas was burnt in excess of oxygen at 300 K. The temperature of the calorimeter was found to increase from 300 K to 300.50 K due to combustion process (given that heat capacity of the calorimeter is 10.5 kcal/K). What is the heat of combustion of the gas?
(Consider $R = 2$ cal/mol K)

(a) − 14 kcal/mol (b) − 14.6 kcal/mol
(c) − 15.2 kcal/mol (d) − 12.8 kcal/mol

8. 5.6 dm^3 of an unknown gas at STP requires 50.0 J of heat to raise its temperature by 10°C at constant volume. The atomicity of gas is
($R = 8.314\ J\ K^{-1}\ mol^{-1}$)

(a) 1.71 (b) 1.83
(c) 1.415 (d) 1.09

9. When 1 g liquid naphthalene $[C_{10}H_8]$ solidifies, 100 J heat is evolved. Calculate the enthalpy of solidification of naphthalene.

(a) – 128 kJ/mol (b) – 128 kJ/mol
(c) + 12.8 kJ/mol (d) – 12.8 kJ/mol

10. Ammonium hydrogen sulphide $[NH_4HS(s)]$ dissociates to $NH_3(g)$ and $H_2S(g)$. If the observed pressure at equilibrium is 3.0 atm at 380 K, the equilibrium constant, K_p of the reaction is

(a) 3.0 (b) 1.5
(c) 2.25 (d) 4.5

11. A mixture of SO_3, SO_2 and O_2 gases are present in a 10 L flask at a temperature at which $K_C = 100$ for the reaction,

$$SO_2(g) + O_2(g) \rightleftharpoons SO_3(g)$$

If the number of moles of SO_3 in the flask is twice the number of moles of SO_2. The number of moles of oxygen present are

(a) 0.04 (b) 0.40
(c) 0.025 (d) 0.60

12. A compound is formed by two elements *M* and *N*. The element *M* form ccp and atoms of *N* occupy 2/3 of the tetrahedral voids. What is the formula of the compound?

(a) M_2N_3 (b) M_3N_4
(c) M_4N_3 (d) M_3N_2

13. Time taken for a zero order reaction to complete 3/4th of the reaction is

(a) $t_{3/4} = 1.5\, t_{1/2}$
(b) $t_{3/4} = \frac{2}{3} t_{1/2}$
(c) $t_{3/4} = \frac{3}{4} t_{1/2}$
(d) $t_{3/4} = 0.5\, t_{1/2}$

14. If $E^\circ_{Fe^{2+}/Fe}$ and $E^\circ_{Fe^{3+}/Fe^{2+}}$ are – 0.44 V and 0.77 V, respectively. The $E^\circ_{Fe^{3+}/Fe}$ will be

(a) 0.40 V (b) 0.11 V
(c) 0.33 V (d) 0.04 V

15. A steam engine operates between 500 K and 300 K under high pressure. What is the minimum amount of heat that must be withdrawn from heat reservoir to obtain 500 J of work?

(a) 200 J (b) 1250 J
(c) 2000 J (d) 1000 J

16. A non-ideal solution of components *A* and *B* show positive deviation from Raoult's law. For this mixture

(a) $\Delta H_{mix} > 0$ and $\Delta V_{mix} > 0$
(b) $\Delta H_{mix} < 0$ and $\Delta V_{mix} < 0$
(c) $\Delta H_{mix} = 0$ and $\Delta V_{mix} = 0$
(d) $\Delta H_{mix} = 0$ and $\Delta V_{mix} > 0$

17. CO_2 upon heating with carbon at high temperature is reduced to carbon monoxide

$$CO_2(g) + C_2(s) \rightleftharpoons 2CO(g)$$

K_p for the reaction is 1.90 atm. The total pressure at equilibrium was found to be 2.0 atm. What are the partial pressure of CO_2 and CO?

(a) 0.78 atm, 1.22 atm
(b) 1.22 atm, 0.78 atm
(c) 0.88 atm, 1.12 atm
(d) 0.92 atm, 1.36 atm

18. Which of the following is correct statement?

(a) $BeSO_4$ is insoluble but BeO is soluble in water.
(b) KI is more soluble than LiI in ethanol.
(c) KO_2 is diamagnetic.
(d) BaO is soluble but $BaSO_4$ is insoluble in water.

19. When 30.0 g of a non-volatile solute having the general molecular formula $C_nH_{2n}O_n$ is dissolved in 1.0 kg of water, the solution freezes at $-0.93°C$.
What will be the value of *n* in the formula?
(Given that, for water $K_f = 1.86$ K kg mol^{-1})

(a) 1 (b) 2
(c) 3 (d) 1.5

20. The rearrangement of methyl isonitrile (CH_3NC) to acetonitrile (CH_3CN) is a first-order reaction and has a rate constant of $5.11 \times 10^{-5} s^{-1}$ at 472 K.

$$CH_3—N \equiv C \longrightarrow CH_3—C \equiv N$$

If the initial concentration of CH_3NC is 0.0340 M, what is the molarity of CH_3NC after 2.00 h?

(a) 0.003749 M
(b) 0.0435 M
(c) $e^{-3.749}$ M
(d) $e^{-0.0435}$ M

21. The molal freezing point depression constant for benzene (C_6H_6) is 4.90 K kg mol^{-1}. Selenium exists as a polymer of the type Se_x. When 3.26 g of selenium is dissolved in 2.26 g of benzene, the observed freezing point is 0.112°C lower than that of pure benzene. The molecular formula of selenium is
(Atomic mass of Se = 78.8 g mol^{-1}) :

(a) Se_4 (b) Se_2
(c) Se_6 (d) Se_8

22. H_2S is passed into 1 dm^3 of a solution containing 0.1 mole of Zn^{2+} and 0.01 mole of Cu^{2+} till the sulphide ion concentration reaches 8.1×10^{-19} moles. Which one of the following statements is true?

[K_{sp} of ZnS and CuS are 3×10^{-22} and 8×10^{-36}, respectively]

(a) Only CuS precipitates
(b) Only ZnS precipitates
(c) Both CuS and ZnS precipitates
(d) No precipitation occurs

23. The IUPAC name of the coordination complex $[PdI_2(NOO)_2(H_2O)_2]$ is

(a) dihydroxodiaquaiodonitrito-N-palladium (IV)
(b) dihydroxodiiodonitrito-N-palladium (IV)
(c) diaquadiiodonitrito-O-palladium (IV)
(d) diaquadiiodonitrito-N-palladium (IV)

24. Identify X and Y in the following reaction.

$$Cu^{+} \begin{cases} \xrightarrow[\text{Dilute}]{HNO_3} Cu(NO_3)_2 + H_2O + X \\ \xrightarrow[\text{(Conc.)}]{HNO_3} Cu(NO_3)_2 + H_2O + Y \end{cases}$$

(a) X is NO but Y is NO_2
(b) X is NO_2 but Y is NO
(c) X is N_2O but Y is N_2O_4
(d) X is NO_2 but Y is N_2O

25. The complex similar to glucose solution towards electrical conductance is

(a) $[Cr(H_2O)_6]Cl_3$ (b) $[Co(NH_3)_4Cl_2]Cl$
(c) $[Co(NH_3)_3Cl_3]$ (d) $[Cr(H_2O)_5Cl]Cl_2$

26. How many phosphorous oxygen bonds are present in cyclotrimetaphosphoric acid?

(a) 12 (b) 9
(c) 6 (d) 4

27. The vapour phase refining of nickel is done by

(a) van Arkel method
(b) Mond process
(c) Hall-Heroult process
(d) None of the above

28. Which of the following elements of group 15 has the least tendency to exhibit – 3 oxidation state?

(a) P (b) Bi
(c) Sb (d) As

29. Which of the following statements is incorrect for nitrogen?

(a) Exhibits oxidation state from + 1 to + 4
(b) Exhibits unique ability to form $p\pi - p\pi$ multiple bonds.
(c) Exhibits inert-pair effect.
(d) Exhibits maximum tendency for – 3 oxidation state.

30. Which of the following oxides of group 15 show purely acidic character?

(a) N_2O_3 (b) As_2O_3
(c) Sb_2O_3 (d) Bi_2O_3

31. The oxidation state of antimony in zinc antimonide is

(a) + 4 (b) − 3
(c) − 1 (d) + 3

32. A decreases in the electronegativity in group 16 down the group implies

(a) a decrease in atomic size/radii.
(b) an increase in ionisation enthalpy.
(c) an increase in metallic character.
(d) None of the above

33. When PCl_5 is changed to PCl_6^- ion, the *d*-orbital involved in sp^3d^2 hybridisation of P is

(a) $d_{x^2-y^2}$ (b) d_{xy}
(c) d_{z^2} (d) d_{xz} and d_{yz}

34. Which of the following species is not paramagnetic?

(a) O_2 (b) S_2
(c) S_2^+ (d) O_2^{2+}

35. Which of the following conditions does not apply to large scale manufacture of ammonia by Haber's process?

(a) High pressure
(b) Use of iron oxide as catalyst
(c) Use of small amount of K_2O and Al_2O_3
(d) Low temperature

36. Which form of sulphur dioxide decolourises acidified $KMnO_4$?

(a) Gaseous SO_2 (b) Liquid SO_2
(c) Aqueous SO_2 (d) None of these

37. The correct order of boiling points of hydrides of group 15 elements is

(a) $BiH_3 > SbH_3 > NH_3 > AsH_3 > PH_3$
(b) $NH_3 > PH_3 > AsH_3 > SbH_3 > BiH_3$
(c) $PH_3 > AsH_3 > BiH_3 > SbH_3 > NH_3$
(d) $SbH_3 > NH_3 > BiH_3 > PH_3 > AsH_3$

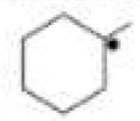
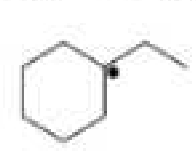
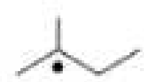
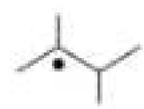
38. The zinc ore sphalerite has the composition

(a) ZnO (b) $ZnCO_3$
(c) ZnS (d) $Zn(OH)_2$

39. Zone refining process can not be applied for the metal :

(a) germanium (b) gallium
(c) indium (d) aluminium

40. The maximum number of hyperconjugating structures are shown by

(a) 
(b) 
(c) 
(d) 

41. Which of the following has the highest nucleophilicity?

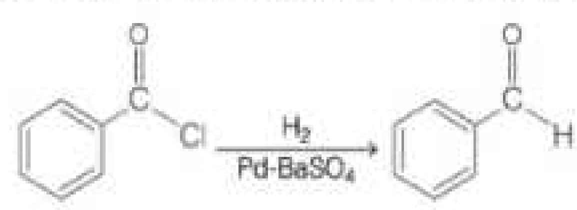
(a) F^- (b) H_3C^-
(c) HO^- (d) H_2N^-

42. The final product in the following sequence of reactions is

$$H-C\equiv C-H \xrightarrow{NaNH_2} A \xrightarrow{CH_3I} B \xrightarrow[H_3O^+]{Hg^{2+}} C$$

(a) CH_3CH_2CHO (b) CH_3CHO
(c) CH_3CH_2COOH (d) CH_3COCH_3

43. The name of the following reaction is

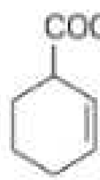


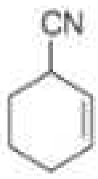
(a) Etard reaction
(b) Stephen reaction
(c) Rosenmund reaction
(d) Gattermann-Koch reaction

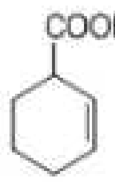
44. 

Product 'R' is

(a) 

(b) 

(c) 

(d) 

45. Which enzyme converts glucose and fructose into ethanol?

(a) Diastase (b) Invertase
(c) Zymase (d) Maltase

46. The most reactive compound towards S_N2 displacement reaction is

(a) 1-bromobutane
(b) 1-bromo-2, 2-dimethylpropane
(c) 1-bromo-2-methybutane
(d) 1-bromo-3-methylbutane

47. When an acid 'X' is treated with aqueous $NaHCO_3$, CO_2 gas is not liberated. What is 'X'?

(a) Benzonic acid
(b) Picric acid
(c) Lactic acid
(d) Oxalic acid

48. Identify 'S' in the following

$$C_6H_5NH_2 \xrightarrow[HCl]{NaNO_2} P \xrightarrow{CuCN} Q \xrightarrow[Ni]{H_2} R \xrightarrow{HNO_2} S$$

(a) $C_6H_5CH_2NH_2$ (b) $C_6H_5NHCH_2CH_3$
(c) $C_6H_5CH_2OH$ (d) C_6H_5NHOH

49. Heroin is a derivative of

(a) cocaine (b) morphine
(c) caffeine (d) nicotine

50. The base present in DNA, but not in RNA is

(a) adenine (b) guanine
(c) thymine (d) uracil

Physics

51. A carrier wave of peak voltage 15 V is used to transmit a message signal. Find the peak voltage of the modulating signal in order to have a modulation index of 65%.

(a) 5.7 V (b) 6.3 V
(c) 9.7 V (d) 11.2 V

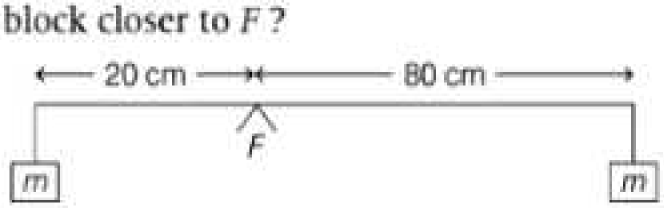
52. A beam of light consisting of two wavelengths, 650 nm and 520 nm, is used to obtain interference fringes in a Young's double slit experiment. What is the least distance from the central maximum where the bright fringes due to both the wavelengths coincide?

(a) 0.83 mm (b) 1.23 mm
(c) 1.56 mm (d) 1.71 mm

53. The magnitude of an unbalanced force on a 10 kg object increases at a constant rate from zero to 50 N in 4.0 s, causing the initially stationary object to move. What is the object's speed at the end of 4.0 s?

(a) 10 m/s (b) 15 m/s
(c) 20 m/s (d) 25 m/s

54. Two blocks, each of mass m, are suspended from the ends of a rigid massless rod (as shown). The rod is held horizontally on the fulcrum F and then released. What is the magnitude of the initial acceleration of the block closer to F?



(a) 0.8 m/s^2 (b) 1.7 m/s^2
(c) 2.5 m/s^2 (d) 3.1 m/s^2

55. A hollow sphere of inner radius 8 cm and outer radius 9 cm floats half-submerged in a liquid of density 800 kg/m^3. Find the density of the material of the sphere.

(a) 800 kg/m^3 (b) 1000 kg/m^3
(c) 1300 kg/m^3 (d) 1400 kg/m^3

56. In an accelerator, protons move along a circular path of diameter 23 m in an evacuated chamber, whose residual gas is at 295 K and 1.0×10^{-6} torr pressure. Find the mean free path of the gas molecules if the molecular diameter is 2.0×10^{-8} cm.

(a) 0.8 m (b) 10 m
(c) 86 m (d) 172 m

57. An infinite non-conducting sheet has a surface density 0.10 $\mu C/m^2$ on one side. How far apart are equipotential surfaces whose potentials differ by 50 V?

(a) 8.8 mm (b) 6.5 mm
(c) 4.6 mm (d) 3.1 mm

58. A circular loop of wire having a radius of 8 cm carries a current of 0.20 A. A unit vector parallel to the dipole moment μ of the loop is given by $(0.6\hat{\mathbf{i}} - 0.8\hat{\mathbf{j}})$. If the loop is located in a uniform magnetic field $\mathbf{B} = (0.25\text{ T})\,\hat{\mathbf{i}} + (0.30\text{ T})\,\hat{\mathbf{k}}$, find the magnetic potential energy of the loop.

(a) -6.0×10^{-4} J
(b) 6.0×10^{-4} J
(c) -8.0×10^{-5} J
(d) 8.0×10^{-5} J

59. Two electrons in lithium $(Z = 3)$ have the quantum numbers $n = 1, l = 0, m_l = 0, m_s = \pm\frac{1}{2}$. What quantum numbers can the third electron have if the atom is to be in its ground state?

(a) $\left(1, 0, 0, \pm\frac{1}{2}\right)$ (b) $\left(2, 1, 0, \pm\frac{1}{2}\right)$
(c) $\left(2, 0, 0, \pm\frac{1}{2}\right)$ (d) $\left(2, 1, 1, \pm\frac{1}{2}\right)$

60. Which of the following fusion reactions will not result in the net release of energy?

(a) $^{6}Li + {^{6}Li}$ (b) $^{12}C + {^{12}C}$
(c) $^{20}Ne + {^{20}Ne}$ (d) $^{35}Cl + {^{35}Cl}$

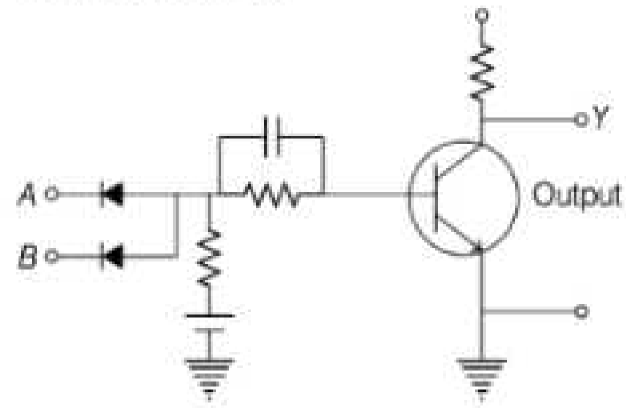
61. At what rate is hydrogen being consumed in the core of the sun by the $p - p$ cycle? (Given, 26.2 MeV of thermal energy is produced when 4 protons are consumed; power of the sun $= 3.9 \times 10^{26}$ W)

(a) 6.2×10^{11} kg/s (b) 3.9×10^{10} kg/s
(c) 1.6×10^{9} kg/s (d) 0.6×10^{8} kg/s

62. What is the probability that a quantum state whose energy is 0.10 eV above the Fermi energy will be occupied? Take a sample temperature of 800 K.

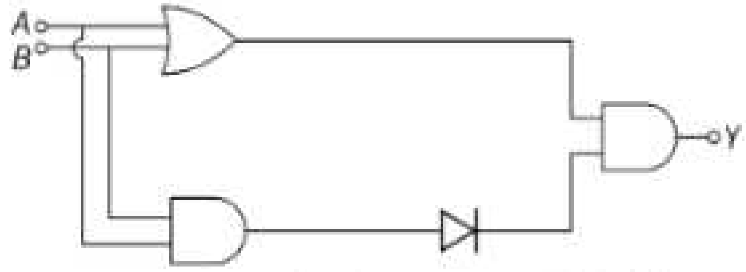
(a) ~ 5% (b) ~ 10%
(c) ~20% (d) ~ 30%

63. Which logic gate is achieved in the following circuit?



(a) OR (b) NOT
(c) AND (d) NAND

64. The Boolean expression for the following logic gate is



(a) $Y = (A + B)(\overline{A \cdot B})$ (b) $Y = A\overline{B} + B\overline{A}$
(c) $Y = (\overline{A + B})(AB)$ (d) $Y = \overline{AB}$

65. Which of the following has the dimensional formula $[ML^{-1}T^{-1}]$?

(a) Surface tension
(b) Thermal conductivity
(c) Universal constant of gravitation
(d) Coefficient of viscosity

66. A particle starts from rest and is moving with a constant acceleration. After 10 s, the speed is found to be 100 m/s and one second later the speed becomes 150 m/s. Find the distance travelled during 11^{th} second.

(a) 96 m (b) 125 m
(c) 145 m (d) 162 m

67. The range of a projectile projected at an angle of 15° is 50 m. If it is projected with the same speed at an angle of 45°, its range will be

(a) 30 m (b) 50 m
(c) 70 m (d) 100 m

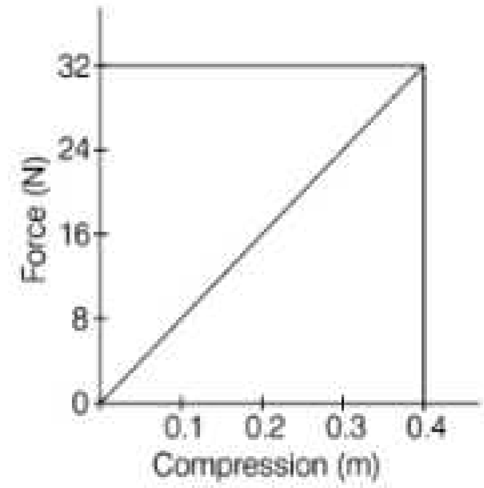
68. A particle of mass 10 g is moving under the influence of a force $(5\hat{\mathbf{i}} + 2.5\,\hat{\mathbf{j}})$ N. If the particle starts from rest, find its position at time $t = 5$ s.

(a) $(6250\hat{\mathbf{i}} + 3125\hat{\mathbf{j}})$ m
(b) $(3125\hat{\mathbf{i}} + 6250\hat{\mathbf{j}})$ m
(c) $(12500\hat{\mathbf{i}} + 6250\hat{\mathbf{j}})$ m
(d) $(6250\hat{\mathbf{i}} + 12500\hat{\mathbf{j}})$ m

69. The average mass of a rain drop is 3×10^{-5} kg and its terminal velocity is 9 m/s. Find the energy transferred by rain to each square metre of the surface at a place which receives 100 cm of rain in a year.

(a) 1.09×10^{2} J
(b) 2.3×10^{3} J
(c) 4.1×10^{4} J
(d) 5.5×10^{5} J

70. Figure shows a force-compression curve of a spring. A body of mass 5.0 kg moving with a velocity of 8 m/s hits the spring. Find the compression produced in the spring when the body hits it.

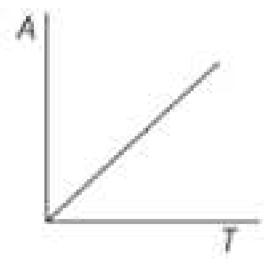


(a) 2 m (b) 1.8 m
(c) 1.4 m (d) 0.8 m

71. A body of mass 0.5 kg moves along X-axis with velocity $v = ax^{3/2}$, where $\alpha = 5\ m^{-1/2}s^{-1}$. Find the work done by the net force during its displacement from $x = 0$ to $x = 2$m.

(a) 20 J (b) 30 J
(c) 40 J (d) 50 J

72. Which of the following represents the areal velocity (A) versus time (T) graph for a planet?

(a)


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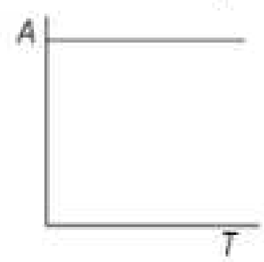

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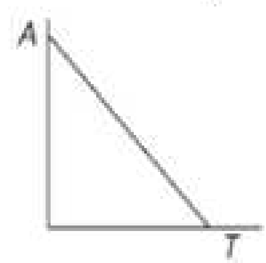
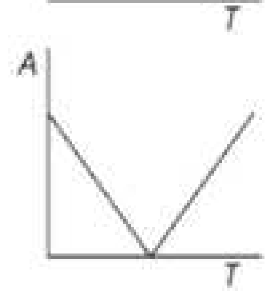


(d)


73. The density of a non-uniform rod of length 1.0 m is given by $\rho(x) = a(1 + bx^2)$, where a and b are constants and $0 \le x \le 1$. The centre of mass (CM) of the rod will be at

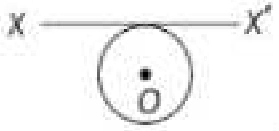
(a) $\frac{3(2+b)}{4(3+b)}$ (b) $\frac{4(2+b)}{3(3+b)}$

(c) $\frac{3(3+b)}{4(2+b)}$ (d) $\frac{4(3+b)}{3(2+b)}$

74. Which of the following statements is not correct?

(a) During rolling, the force of friction acts in the same direction as the direction of motion of the centre of mass of the body.
(b) The instantaneous speed of the point of contact during rolling is zero.
(c) The instantaneous acceleration of the point of contact during rolling is zero.
(d) For perfect rolling motion, work done against friction is zero.

75. A thin wire of length L and uniform linear mass density ρ is bent into a circular loop (as shown). The moment of inertia of the loop about the axis XX' is



(a) $\frac{\rho L^3}{8\pi^2}$ (b) $\frac{\rho L^3}{16\pi^2}$

(c) $\frac{5\rho L^3}{16\pi^2}$ (d) $\frac{3\rho L^3}{8\pi^2}$

76. A body of mass m is projected at an angle θ with X-axis, with an initial speed v_0 in xy-plane. At the time $t = \frac{v_0 \sin\theta}{g}$, the angular momentum of the body is

(a) $\frac{1}{2} mgv_0 t^2 \cos\theta\,\hat{i}$ (b) $-mgv_0 t^2 \cos\theta\,\hat{j}$

(c) $mg\,v_0 t \cos\theta\,\hat{k}$ (d) $-\frac{1}{2} mgv_0 t^2 \cos\theta\,\hat{k}$

77. Which of the following is not correct for artificial satellites?

(a) They are used to study different regions of atmosphere.
(b) They are used for information about weather forecast.
(c) They are not used to study the shape of the planets.
(d) They are used to study the radiations coming from sun and outer space.

78. On taking a solid ball from the surface to the bottom of a lake of 200 m depth, the volume of the ball is reduced by 0.1%. Find the bulk modulus of the material of the ball.

(a) $2 \times 10^7\ N/m^2$
(b) $1.6 \times 10^8\ N/m^2$
(c) $2.0 \times 10^9\ N/m^2$
(d) $0.6 \times 10^{10}\ N/m^2$

79. Two plates, each of cross-sectional area A, have thicknesses L and $2L$ and thermal conductivities K and $2K$, respectively, are joined to form a single plate of thickness $3L$. If the temperatures of the free surfaces are T_1 and T_2, what is their equivalent thermal conductivity?

(a) $3K$ (b) K
(c) $K/2$ (d) $3K/2$

80. The work of 146 kJ is performed in order to compress 1.0 kilomole of a gas adiabatically and in this process the temperature of the gas increases by 7°C. The gas is

(a) diatomic
(b) triatomic
(c) a mixture of monoatomic and diatomic
(d) monoatomic

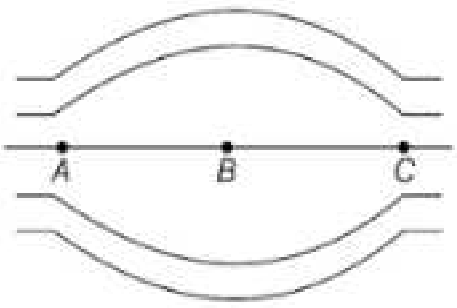
81. A particle free to move along the X-axis has potential energy given by $U(x) = K[1 - e^{-x^2}]$; $-\infty < x < \infty$. What is the nature of oscillation for small displacements near $x = 0$?

(a) SHM (b) Non-periodic
(c) Periodic but not SHM (d) None of these

82. The fundamental frequency of a 1.5 m long stretched steel wire is 175 Hz. Find the percentage change in the frequency of wire if tension in the wire is increased by 3%. (Density of steel $= 7.8 \times 10^3\ kg/m^3$)

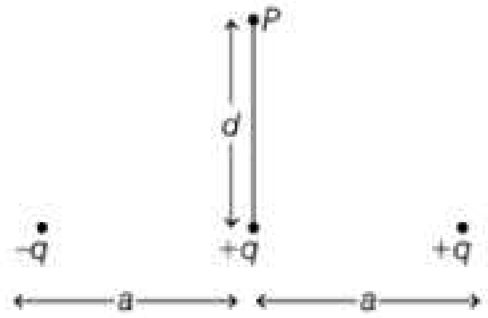
(a) 0.8% (b) 1.5%
(c) 2.1% (d) 3.0%

83. Figure shows some electric field lines corresponding to an electric field. If E_A, E_B and E_C are the electric field intensities at points A, B and C respectively, then



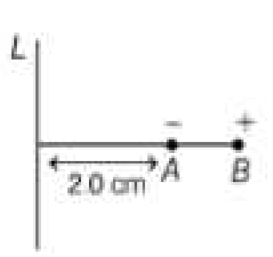
(a) $E_A > E_B > E_C$ (b) $E_A = E_B = E_C$
(c) $E_A = E_C > E_B$ (d) $E_A = E_C < E_B$

84. The magnitude of the electric field at the point P in the configuration (as shown) for $d >> a$ is



(a) $\frac{q}{4\pi \varepsilon_0 d^3}$ (b) $\frac{2qa}{4\pi \varepsilon_0 d^3}$
(c) $\frac{q}{4\pi \varepsilon_0 d^3}\sqrt{d^2 + 4a^2}$ (d) $\frac{q}{4\pi \varepsilon_0 d^3}\sqrt{4d^2 + a^2}$

85. As shown, L is a long line charge of linear charge density 4.0×10^{-4} C/m and AB is an electric dipole consists of charges $\pm 2.0 \times 10^{-8}$ C separated by a distance of 2.0×10^{-3} m. Find the force acting on the dipole.



(a) 0.6 N towards L
(b) 0.6 N away from L
(c) 0.8 N towards L
(d) 0.8 N away from L

86. Suppose the four coloured-bands on the resistor are brown , yellow, green and gold as read from left to right. The value of the resistance is

(a) (1.4 ± 0.07) MΩ (b) (1.2 ± 0.05) MΩ
(c) (1.1 ± 0.03) MΩ (d) (1.0 ± 0.06) MΩ

87. In an electric circuit, a resistor develops 200 J of thermal energy in 5.0 s when a current of 1.0 A is passed through it. If the current is increased to 3.0 A, what will be the energy developed in 5.0 s?

(a) 800 J (b) 1200 J
(c) 1600 J (d) 1800 J

88. A 10.0 μF capacitor having a charge of 40 μC is discharged through a wire of resistance 2.5 Ω. The heat dissipated in the wire between 25 to 50 μs is (Given, $e^{-1} = 0.37$)

(a) 4.7 μJ (b) 9.4 μJ
(c) 12.6 μJ (d) 14.2 μJ

89. Which of the following properties corresponds to diamagnetic substances?

(a) Magnetic susceptibility is a small but positive quantity.
(b) The relative permeability is slightly more than 1.
(c) Magnetic susceptibility is a small but negative quantity.
(d) The relative permeability is of the order of thousands.

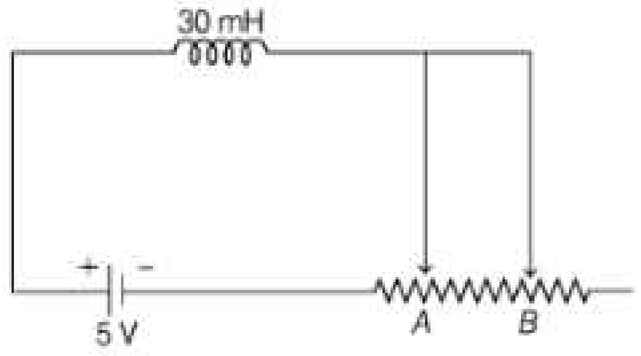
90. Radiation corresponding to the transition $n = 4$ to $n = 2$ in H-atom falls on a metallic surface whose work-function is 1.9 eV. What is the maximum kinetic energy of the photoelectrons?

(a) 0.35 eV (b) 0.43 eV
(c) 0.51 eV (d) 0.65 eV

91. Which of the following relations is correct between the frequencies ν_{K_β}, ν_{K_α} and ν_{L_α} of K_β, K_α and L_α X-rays, respectively of the same material?

(a) $\nu_{K_\beta} = \nu_{K_\alpha} + \nu_{L_\alpha}$
(b) $\nu_{L_\alpha} = \nu_{K_\beta} + \nu_{K_\alpha}$
(c) $\nu_{K_\beta} = \nu_{K_\alpha} - \nu_{L_\alpha}$
(d) $\nu_{L_\alpha} = (\nu_{K_\alpha} + \nu_{K_\beta})/2$

92. In the following circuit, the sliding contact *A* may be pulled towards right. If the resistance in the circuit is 10.0 Ω when the sliding contact was at *A*, then the current (i) for the sliding contact at *B* will be



(a) $i = 0.5$ A (b) $i < 0.5$ A
(c) $i > 0.5$ A (d) $i = 0$

93. An inductance coil stores 20 J of magnetic field energy and dissipates energy as heat at the rate of 200 W when a current of 2 A is passed through it. What is the time constant of the circuit when this inductance coil is joined across an ideal battery ?

(a) 0.5 s (b) 0.3 s
(c) 0.2 s (d) 0.10 s

94. The energy stored in 90 cm length of a laser beam operating at 3 mW is

(a) 3×10^{-10} J (b) 6×10^{-11} J
(c) 9×10^{-12} J (d) 1.2×10^{-14} J

95. After absorbing a photon of wavelength 242 nm, a moving hydrogen atom stops. Find the speed of hydrogen atom.

(a) 1.6 m/s (b) 2.4 m/s
(c) 3.4 m/s (d) 4.9 m/s

96. Find the energy needed to remove the electron from a hydrogen-like atom, if its excitation energy in the first excited state is 41 eV.

(a) 48 eV (b) 54 eV
(c) 61 eV (d) 70 eV

97. In which of the following mass number regions, the nuclei are most stable?

(a) $2 < A < 12$ (b) $16 < A < 40$
(c) $50 < A < 80$ (d) $100 < A < 240$

98. Which of the following properties cannot be related to α-rays?

(a) Its penetrating power is low
(b) α-rays coming from radioactive materials travels at 10^8 m/s
(c) α-ray cannot produce scintillation when it strikes fluorescent material
(d) It causes ionisation in gases

99. For a light of wavelength 6000 Å, coming from a distant star, what is the limit of resolution of a telescope whose objective has a diameter of 200 inch?

(a) 1.4×10^{-7} rad
(b) 2.9×10^{-7} rad
(c) 3.6×10^{-8} rad
(d) 4.5×10^{-8} rad

100. The Zener diode can be used as

(a) a half-wave rectifier
(b) a full-wave rectifier
(c) a voltage regulator
(d) an amplifier

Mathematics

101. Area enclosed by $|y| = |\sin x|$ between $x = \frac{\pi}{2}$ to $x = \frac{3\pi}{2}$ is

(a) 0 (b) 2
(c) 4 (d) 2π

102. Let $A = [0, 1]$, $B = [2, 5]$ and $f : A \to B$ be function defined as $f(x) = 3x + 2$, for every $x \in A$. Then f is

(a) one-one not onto
(b) onto not one-one
(c) one-one and onto
(d) neither one-one nor onto

103. Consider the following relations in the real numbers

$R_1 = \{(x, y) \mid x^2 + y^2 \le 25\}$

$R_2 = \left\{(x, y) \mid y \ge \frac{4x^2}{9}\right\}$

Then the range of $R_1 \cup R_2$ is

(a) $(-5, 3)$ (b) $[-4, 5]$
(c) $[3, \infty)$ (d) $[-5, \infty)$

104. The function $(x^2 - 4)|x^2 - 5x + 6| + \sin(|x|)$ is not differentiable at

(a) -2 (b) 0 (c) 2 (d) 3

105. If $\alpha, \beta \ne 0$, and $f(n) = \alpha^n + \beta^n$ and

$$\begin{vmatrix} 3 & 1+f(1) & 1+f(2) \\ 1+f(1) & 1+f(2) & 1+f(3) \\ 1+f(2) & 1+f(3) & 1+f(4) \end{vmatrix}$$

$= \delta(1-\alpha)^2(1-\beta)^2(\alpha-\beta)^2$

Then δ is equal to

(a) $\alpha\beta$ (b) $\frac{1}{\alpha\beta}$ (c) 1 (d) -1

106. The value of $\begin{vmatrix} a^2+\lambda^2 & ab+c\lambda & ca-b\lambda \\ ab-c\lambda & b^2+\lambda^2 & bc+a\lambda \\ ac+b\lambda & bc-a\lambda & c^2+\lambda^2 \end{vmatrix}$

$\times \begin{vmatrix} \lambda & c & -b \\ -c & \lambda & a \\ b & -a & \lambda \end{vmatrix}$ is

(a) $\lambda^3(\lambda^2 + a^2 + b^2 + c^2)^2$
(b) $\lambda^2(\lambda^3 + a^2 + b^2 + c^2)^3$
(c) $\lambda(\lambda^2 + a^2 + b^2 + c^2)^2$
(d) $\lambda^3(\lambda^2 + a^2 + b^2 + c^2)^3$

107. If $\begin{bmatrix} 0 & 2\beta & v \\ \alpha & \beta & -v \\ \alpha & -\beta & v \end{bmatrix}$ is an orthogonal matrix, then the value of $2\alpha^2 + \beta^2 + 3v^2$ will be

(a) $\frac{13}{7}$ (b) $\frac{13}{6}$
(c) $\frac{6}{13}$ (d) $\frac{2}{13}$

108. If A is a 3×3 non-singular matrix such that : $AA^T = A^TA$ and $B = A^{-1}A^T$, then BB^T is equal to, where I is identity matrix.

(a) $I - B$ (b) I
(c) B^{-1} (d) $(B^{-1})^2$

109. Minimum area of circle which touches the parabolas $y = x^2 + 1$ and $y^2 = x - 1$ is

(a) $\frac{9\pi}{32}$ sq unit (b) $\frac{9\pi}{16}$ sq unit
(c) $\frac{9\pi}{8}$ sq unit (d) $\frac{9\pi}{4}$ sq unit

110. The length of the transverse axis of the rectangular hyperbola $xy = 18$ is

(a) 6 (b) 12
(c) 18 (d) 9

111. Locus of a point whose chord of contact with respect to the circle $x^2 + y^2 = 4$ is a tangent to the hyperbola $xy = 1$, is

(a) ellipse (b) circle
(c) hyperbola (d) parabola

112. The area bounded by the circles $x^2 + y^2 = 1$, $x^2 + y^2 = 4$ and the pair of lines $\sqrt{3}(x^2 + y^2) = 4xy$ is equal to

(a) $\frac{\pi}{2}$ (b) $\frac{5\pi}{2}$
(c) 3π (d) $\frac{\pi}{4}$

113. Let $f(x) = x^3 + x^2 + 100x + 7\sin x$, then the equation $\frac{1}{y - f(1)} + \frac{2}{y - f(2)} + \frac{3}{y - f(3)} = 0$ has

(a) both roots lying in $(f(1), f(2))$
(b) exactly one root lying in $(f(2), f(3))$
(c) exactly one root lying in $(-\infty, f(1))$
(d) exactly one root lying in $(f(3), \infty)$

114. The value of a if $f(x) = 2e^x - ae^{-x} + (2a + 1)x - 3$ increases for all x are

(a) $(-\infty, 0]$ (b) $(0, \infty)$
(c) $(-\infty, \infty)$ (d) $(1, \infty)$

115. Four tickets marked 0, 1, 10, 11 respectively are placed in a bag. A ticket is drawn at random five times being replaced each time. Then the probability that the sum of numbers on the tickets thus drawn is 23 is equal to

(a) $\frac{25}{254}$ (b) $\frac{25}{252}$
(c) $\frac{25}{256}$ (d) None of these

116. The range of the function $f(x) = \log_2\left(\frac{\sin x - \cos x + 3\sqrt{2}}{\sqrt{2}}\right)$ is

(a) $[-2, 1]$ (b) $[1, 2]$
(c) $[-1, 3]$ (d) $[2, 4]$

117. If the line $\frac{x-1}{3} = \frac{y-2}{2} = \frac{z-3}{1}$ intersects the curve $x^2 + y^2 + x - y + c = 0$, $z = 0$; then c is

(a) 98 (b) -98
(c) -106 (d) -108

118. The unit vector perpendicular to vectors $2\hat{\mathbf{i}} + \hat{\mathbf{j}} - \hat{\mathbf{k}}$ and $\hat{\mathbf{i}} - \hat{\mathbf{j}} + 2\hat{\mathbf{k}}$ is

(a) $\frac{\hat{\mathbf{i}} + 5\hat{\mathbf{j}} + 3\hat{\mathbf{k}}}{\sqrt{35}}$ (b) $\frac{5\hat{\mathbf{i}} - \hat{\mathbf{j}} - 3\hat{\mathbf{k}}}{\sqrt{35}}$
(c) $\frac{\hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}}}{\sqrt{3}}$ (d) $\frac{\hat{\mathbf{i}} - 5\hat{\mathbf{j}} - 3\hat{\mathbf{k}}}{\sqrt{35}}$

119. The values of a for which the inequality $\frac{x - 2a - 3}{x - a + 2} < 0$ is satisfied for all x belonging to $[1, 2]$, are

(a) $(-2, 3)$ (b) $(-1/3, 2)$
(c) $(-3, 2)$ (d) $(-1/2, 3)$

120. If **a** and **b** are vectors in space given by $\mathbf{a} = \frac{\hat{\mathbf{i}} + \hat{\mathbf{j}}}{\sqrt{2}}$ and $\mathbf{b} = \frac{\hat{\mathbf{i}} - \hat{\mathbf{j}} + \hat{\mathbf{k}}}{\sqrt{3}}$, then the value of $(\mathbf{a} + \mathbf{b}) \cdot [(\mathbf{a} \times \mathbf{b}) \times (\mathbf{a} - \mathbf{b})]$ is

(a) 1 (b) 2
(c) 3 (d) 4

121. If $\log_3 2$, $\log_3(2^x - 5)$ and $\log_3(2^x - 7/2)$ are in AP then the value of x is

(a) 3 (b) 2 (c) 4 (d) 5

122. For a differential equation which of the following is correct?

(a) Degree and order always exists.
(b) Degree and order both may or may not exist.
(c) Order always exists but not degree.
(d) Degree always exists but not order.

123. If two events A and B are such that $P(\bar{A}) = 0.3$, $P(B) = 0.4$ and $P(A \cap \bar{B}) = 0.5$, then $P[B \mid (A \cup \bar{B})]$ is equal to

(a) $\frac{1}{4}$ (b) $\frac{1}{3}$
(c) $\frac{1}{2}$ (d) None of these

124. The probability that a man aged x years will die in a year is p. The probability that out of n men $M_1, M_2, \dots, M_n$ each of aged x, M_1 will die and be the first to die is

(a) $\frac{1}{n^2}$ (b) $1 - (1-p)^n$
(c) $\frac{1}{n^2}\left[\frac{1}{1-(1-p)^n}\right]$ (d) $\frac{1}{n}[1-(1-p)^n]$

125. The component of vector $\mathbf{A} = 2\hat{\mathbf{i}} + 5\hat{\mathbf{j}} + 7\hat{\mathbf{k}}$ in the direction of a vector $\mathbf{B} = 3\hat{\mathbf{i}} - \hat{\mathbf{j}} + 5\hat{\mathbf{k}}$ is

(a) $\frac{36}{\sqrt{78}}$ (b) $\frac{35}{6}$
(c) $\sqrt{\frac{78}{35}}$ (d) None of these

126. In a set of four numbers the first three are in GP and the last three in AP with a common difference 6. If the first number is same as the fourth, the four numbers are

(a) 3, 9, 15, 21 (b) 1, 7, 13, 19
(c) 8, -4, 2, 8 (d) None of these

127. The solution of the differential equation $\frac{dy}{dx} = \frac{y}{x} - \frac{\phi\left(\frac{y}{x}\right)}{\phi'\left(\frac{y}{x}\right)}$, is

(a) $\phi\left(\frac{y}{x}\right) = kx$ (b) $x\phi\left(\frac{y}{x}\right) = k$
(c) $\phi\left(\frac{y}{x}\right) = ky$ (d) $y\phi\left(\frac{y}{x}\right) = k$

128. If the direction cosines of two vectors **a** and **b** are connected by the equations $l + m + n = 0$, $l^2 - m^2 + n^2 = 0$, then the angle between **a** and **b** is

(a) π (b) $\frac{2\pi}{3}$
(c) $\frac{\pi}{2}$ (d) $\frac{\pi}{3}$

129. A and B are two candidates seeking admission in AMU The probability that A is selected is 0.5 and the probability that both A and B are selected is at most 0.3. Then the probability of B getting selected is

(a) 0.9 (b) 0.8
(c) ≤ 0.8 (d) ≤ 0.3

130. The area of the region defined by $||x| - |y|| \le 2$ and $x^2 + y^2 \le 4$ in the xy-plane is

(a) 1 (b) 4π
(c) π (d) 8

131. By graphical method, the solution of linear programming problem Maximize $z = 3x_1 + 5x_2$ subject to $3x_1 + 2x_2 \le 18$, $x_1 \le 4$, $x_2 \le 6$, $x_1, x_2 \ge 0$ is

(a) $x_1 = 2, x_2 = 0, z = 6$
(b) $x_1 = 2, x_2 = 6, z = 36$
(c) $x_1 = 4, x_2 = 3, z = 27$
(d) $x_1 = 4, x_2 = 6, z = 42$

132. The greatest integer which divides the number $101^{100} - 1$ is

(a) 100 (b) 1000
(c) 10000 (d) 100000

133. Which of the following statements is incorrect? (where n is any natural number)

(a) $n(n + 1)(n + 2)$ is divisible by 12 for each $n \ge 3$.
(b) $\frac{1}{1.3} + \frac{1}{3.5} + \frac{1}{5.7} + \dots + \frac{1}{(2n-1)(2n+1)} = \frac{n}{2n+1}$, for each $n \ge 3$.
(c) $\frac{n^5}{5} + \frac{n^3}{3} + \frac{7n}{15}$ is a natural number for each $n \ge 3$.
(d) $(10^{2n-1} + 1)$ is divisible by 11 for each $n \ge 3$.

134. If a denotes the number of permutations of $x + 2$ objects taken all at a time, b the number of permutations of x objects taken 11 at a time and c the number of permutations of $x - 11$ objects taken all at a time such that $a = 182\,bc$, then the value of x is

(a) 15 (b) 12
(c) 10 (d) 18

135. The sum of real roots of the equation $x^2 - 2^{2008}x + |x - 2^{2007}| + 2(2^{4013} - 1) = 0$ is

(a) 2^{2008} (b) 2^{2007}
(c) 2^{2006} (d) 2^{2013}

136. Let a, b, c are distinct real numbers. The expression $\frac{(x-a)(x-b)}{(c-a)(c-b)} + \frac{(x-b)(x-c)}{(a-b)(a-c)} + \frac{(x-c)(x-a)}{(b-c)(b-a)} - 1$, assumes the zero value for

(a) no real value of x
(b) more than three real values of x
(c) exactly three real values of x
(d) exactly two real values of x

137. Let **a** and **c** be unit collinear vectors and $|\mathbf{b}| = 2$. If $\mathbf{b} - 2\mathbf{c} = \lambda\mathbf{a}$, then values of λ are

(a) 0, 4 (b) 0, −4
(c) 0, 2 (d) 0, −2

138. If $\lim_{x \to 0} |x|^{\sin x} = L$, then the value of L is

(a) 0 (b) 1
(c) −1 (d) e

139. The solutions of the equation $5\cos 3x + 3\cos x = 3\sin 4x$ are (where Z denotes the set of integers)

(a) $n\pi + (-1)^n \times \frac{\pi}{4}, n \in Z$
(b) $n\pi + (-1)^n \sin^{-1}\left(\frac{1}{3}\right), n \in Z$
(c) $n\pi + (-1)^n \times \frac{\pi}{6}, n \in Z$
(d) $2n\pi + \frac{\pi}{3}, n \in Z$

140. If a circle which passes through (2, 3) and cuts the circle $x^2 + y^2 = 1$ orthogonally, then locus of its centre is

(a) $2x + 3y - 7 = 0$
(b) $3x - 2y + 7 = 0$
(c) $4x + 7y - 3 = 0$
(d) $7x + 3y - 2 = 0$

141. Let A, B, C be three points represented by complex numbers z_1, z_2, z_3 respectively in a complex plane, $\angle ABC = amp\left(\frac{z_3 - z_2}{z_1 - z_2}\right)$ is equal to

(a) amp $\left(\frac{z_3 - z_2}{z_1 - z_2}\right)$ (b) amp $\left(\frac{z_3 - z_1}{z_2 - z_1}\right)$

(c) amp $\left(\frac{z_2 - z_3}{z_1 - z_3}\right)$ (d) amp $\left(\frac{z_2 - z_3}{z_3 - z_2}\right)$

142. If $\frac{ax}{\cos\theta} + \frac{by}{\sin\theta} = a^2 - b^2$ and $\frac{ax\sin\theta}{\cos^2\theta} - \frac{by\cos\theta}{\sin^2\theta} = 0$, then the value of $(ax)^{2/3} + (by)^{2/3}$ is

(a) $(a - b)^{2/3}$

(b) $(a + b)^{2/3}$

(c) $(a^2 - b^2)^{2/3}$

(d) $(a^2 + b^2)^{2/3}$

143. The solution of the inequality $\frac{1 - \sqrt{1 - 4x^2}}{x} < 3$ is

(a) $\left(-\frac{1}{2}, \frac{1}{2}\right)$ (b) $\left(-\frac{1}{2}, 0\right) \cup \left(0, \frac{1}{2}\right)$

(c) $\left[-\frac{1}{2}, 0\right) \cup \left(0, \frac{1}{2}\right]$ (d) $(-3, 3]$

144. If A and B are two disjoint sets, then $A \cup B'$, where B' is complement of B, is equal to

(a) A (b) $B - A$

(c) A' (d) B'

145. The total number of symmetric relations on a set A of cardinality 3 is

(a) 32 (b) 16

(c) 64 (d) 128

146. If a function $f : [2, \infty) \to A$ defined as $f(x) = x^2 - 4x + 5$ is bijective function, then A is equal to

(a) R (b) $[1, \infty)$

(c) $[2, \infty)$ (d) $(5, \infty)$

147. The value of $\int_{-3}^{3} \frac{\cos^2 x}{[x/\pi] + \frac{1}{2}} dx$, where $[\cdot]$ denotes the greatest integer function, is

(a) 1 (b) 0

(c) $9 - \cos 9$ (d) None of these

148. $\cos[\tan^{-1}\{\sin(\cot^{-1} x)\}]$ is equal to

(a) $\frac{1}{\sqrt{1 + x^2}}$ (b) $\tan^{-1}\frac{1}{\sqrt{1 + x^2}}$

(c) $\cot^{-1}(\sqrt{1 + x^2})$ (d) $\sqrt{\frac{x^2 + 1}{x^2 + 2}}$

149. Consider points $A(3, 4)$ and $B(7, 13)$. If P be a point on the line $y = x$ such that $PA + PB$ is minimum, then coordinates of P are

(a) $\left(\frac{12}{7}, \frac{12}{7}\right)$ (b) $\left(\frac{13}{7}, \frac{13}{7}\right)$

(c) $\left(\frac{31}{7}, \frac{31}{7}\right)$ (d) $(0, 0)$

150. Which one is true?

(a) The operation * defined by $a * b = \frac{a + b}{2}$ is a binary operation in the set I of all integers.

(b) The operation * defined by $a * b = \frac{a + b}{2}$ is a binary operation in the set Q of all rationals.

(c) The operation subtraction is a binary operation in the set of all non-zero integers.

(d) The operation subtraction is a binary operation in the set N of all natural numbers.

Answers

Chemistry

1.	(a)	2.	(c)	3.	(a)	4.	(a)	5.	(b)	6.	(a)	7.	(a)	8.	(c)	9.	(d)	10.	(c)
11.	(b)	12.	(b)	13.	(a)	14.	(d)	15.	(b)	16.	(a)	17.	(a)	18.	(d)	19.	(b)	20.	(c)
21.	(d)	22.	(c)	23.	(d)	24.	(a)	25.	(c)	26.	(a)	27.	(b)	28.	(b)	29.	(c)	30.	(a)
31.	(b)	32.	(c)	33.	(a)	34.	(d)	35.	(d)	36.	(c)	37.	(a)	38.	(c)	39.	(d)	40.	(c)
41.	(b)	42.	(d)	43.	(c)	44.	(b)	45.	(c)	46.	(a)	47.	(b)	48.	(c)	49.	(b)	50.	(c)

Physics

51.	(c)	52.	(*)	53.	(c)	54.	(b)	55.	(c)	56.	(d)	57.	(a)	58.	(a)	59.	(c)	60.	(d)
61.	(a)	62.	(c)	63.	(c)	64.	(a)	65.	(d)	66.	(b)	67.	(d)	68.	(a)	69.	(c)	70.	(a)
71.	(d)	72.	(b)	73.	(a)	74.	(c)	75.	(d)	76.	(d)	77.	(c)	78.	(c)	79.	(d)	80.	(a)
81.	(a)	82.	(b)	83.	(d)	84.	(c)	85.	(b)	86.	(a)	87.	(d)	88.	(b)	89.	(c)	90.	(d)
91.	(a)	92.	(c)	93.	(c)	94.	(c)	95.	(a)	96.	(b)	97.	(c)	98.	(c)	99.	(b)	100.	(c)

Mathematics

101.	(b)	102.	(c)	103.	(d)	104.	(b)	105.	(c)	106.	(d)	107.	(b)	108.	(b)	109.	(a)	110.	(b)
111.	(c)	112.	(d)	113.	(b)	114.	(b)	115.	(c)	116.	(b)	117.	(*)	118.	(*)	119.	(d)	120.	(b)
121.	(a)	122.	(c)	123.	(a)	124.	(d)	125.	(d)	126.	(c)	127.	(a)	128.	(b)	129.	(c)	130.	(b)
131.	(b)	132.	(c)	133.	(a)	134.	(b)	135.	(a)	136.	(b)	137.	(b)	138.	(b)	139.	(c)	140.	(a)
141.	(a)	142.	(c)	143.	(c)	144.	(d)	145.	(c)	146.	(b)	147.	(b)	148.	(d)	149.	(c)	150.	(b)

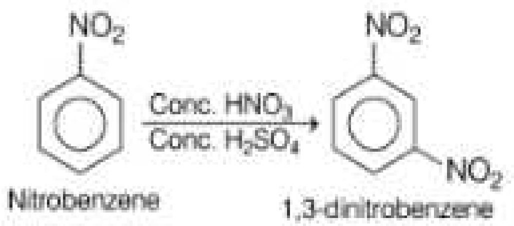
Note (*) None of the option is correct.

Answer *with* Solutions

Chemistry

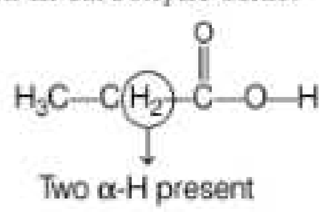
1. *(a)* Spectroscopy is not a purification technique for organic compounds. It is the measurement technique used to study matter through its interaction with different components of the electromagnetic spectrum.

2. *(c)* Nitrobenzene can undergo nitration to form 1,3-dinitrobenzene. Therefore, X is $—NO_2$.

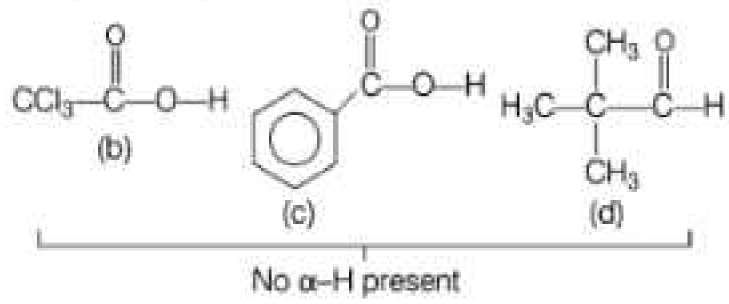


Nitro group is electron-withdrawing group. So, it will not attack at *ortho* and *para* position due to its electron-deficient in nature. So, electrophilic substitution reaction i.e., nitration takes place at *meta* position.

3. *(a)* The compounds which contain α-hydrogen will undergoes Hell-Volhard-Zelinsky (HVZ) reaction. HVZ reaction is an α-halogenation reaction in which α-C—H bond is replaced by C—X bond in carboxylic acids.



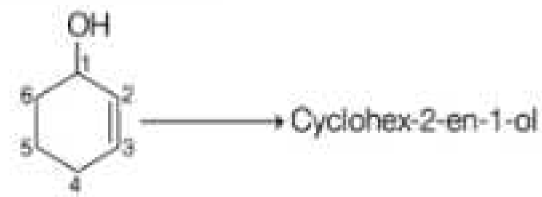
Therefore, can undergoes HVZ reaction.



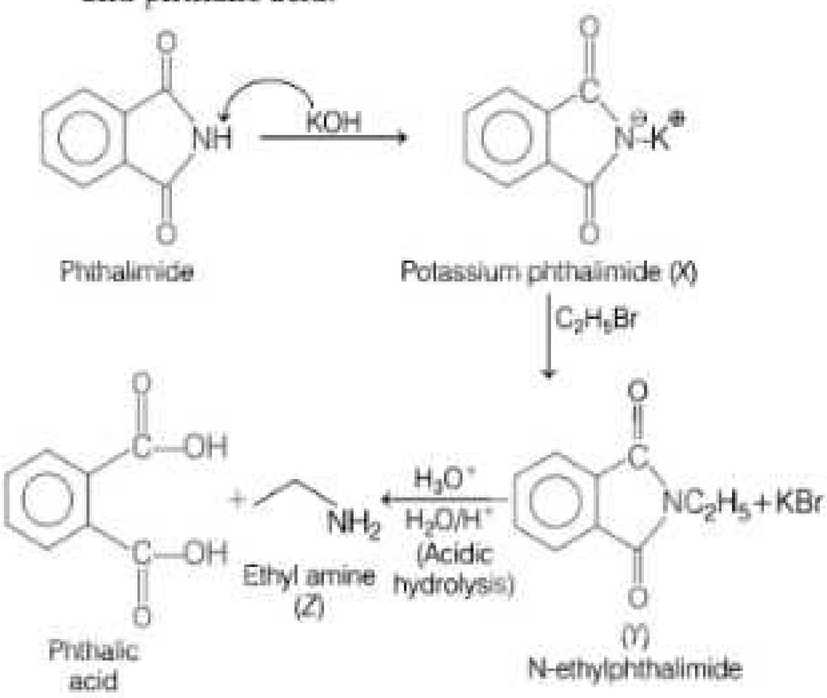
Therefore, does not undergoes HVZ reaction.

4. *(a)* Half-chair is the least stable conformation of cyclohexane due to large torsional strain. In half-chair, due to twist in bonds there is significant amount of torsional strain, particularly on C_4-atom involved in twisted bonds.

5. *(b)* In the given compound, functional group (—OH) have higher priority, than C$=$C (alkenes). So, correct IUPAC name for given compound is cyclohex-2-en-1-ol.



6. *(a)* In this sequence, phthalimide reacts with KOH to form potassium phthalimide (X) which reacts with an alkyl halide i.e. ethyl bromide to form *(Y)* i.e., N-ethylphthalimide. This N-ethylphthalimide undergoes acidic hydrolysis to give primary amine (Z) and phthalic acid.



7. *(a)* Given, initial temperature (T_i) = 300 K

Final temperature (T_f) = 300.50 K

Mass of gas burnt (m) = 10.5 g

Molecular mass of gas (C_2H_4), $M_m = 12 \times 2 + 4$

$= 28$ g/mol

Heat capacity of calorimeter = 10.5 kcal/K

Gas constant (R) = 2 cal/mol K

Change in temperature (ΔT) = $T_i - T_f$

$$= 300\text{ K} - 300.50\text{ K}$$
$$= -0.50\text{ K}$$

Energy released due to combustion of 10.5 g of gas

$$= C \times \Delta T$$
$$= 10.5\text{ kcal/K} \times -0.50\text{ K}$$
$$= -5.25\text{ kcal}$$

$\therefore$ Energy released due to combustion of 1 mole of gas i.e., 28 g of gas $= -5.25\text{ kcal} \times \frac{28\text{ g/mol}}{10.5\text{ g}}$

$$= -14\text{ kcal/mol}$$

$\therefore$ Heat of combustion of gas = −14 kcal/mol

8. *(c)* We know that,

22.4 dm^3 of a gas S.T.P. = 1 mol

$\therefore$ 5.6 dm^3 of a unknown gas $= \frac{1}{22.4} \times 5.6$

$$= 0.25\text{ mol}$$

Also, 50 J of heat is required to raise temperature by 10°C at constant volume and 0.25 mole of gas.
$\therefore$ For 1°C rise, 1 mole of gas at constant volume will required heat $= \frac{50\text{ J}}{10\text{ K} \times 0.25\text{ mol}}$

$$= 20\text{ J/K mol}$$

$\therefore \quad C_V = 20\text{ JK}^{-1}\text{ mol}^{-1}$

Now, $C_p = C_V + R$

$$= 20\text{ JK}^{-1}\text{ mol}^{-1} + 8.314\text{ JK}^{-1}\text{ mol}^{-1}$$
$$= 28.314\text{ JK}^{-1}\text{ mol}^{-1}$$

$\therefore$ Atomicity (γ) of gas $= \frac{C_p}{C_V}$

$$= \frac{28.314\text{ JK}^{-1}\text{ mol}^{-1}}{20\text{ JK}^{-1}\text{ mol}^{-1}}$$

$$\gamma = 1.4157$$

Hence, atomicity of unknown gas is 1.415.

9. *(d)* Molecular weight of naphthalene ($C_{10}H_8$) = 128 g/mol

Heat given by 1 g of naphthalene = 100 J

Heat given by 128 g of naphthalene = 128 × 100

$$= 12800\text{ J/mol}$$
$$= 12.8\text{ kJ/mol}$$

$\therefore$ Enthalpy of solidification (ΔH) = −12.8 kJ/mol.

10. *(c)* $NH_4HS(s) \rightleftharpoons NH_3(g) + H_2S(g)$

Total pressure is 3 atm. One half of this is equal to partial pressure of ammonia. It is also equal to partial pressure of H_2S.

$$p_{NH_3} = p_{H_2S} = \frac{1}{2} \times 3 = 1.5\text{ atm}$$

$$K_p = \frac{p_{NH_3} \times p_{H_2S}}{p_{NH_4HS}}$$

Since, NH_4HS is solid. So, $p_{NH_4HS} = 1$

$$K_p = p_{NH_3} \times p_{H_2S}$$
$$= 1.5 \times 1.5 = 2.25$$

11. *(b)* $2SO_2 + O_2 \rightleftharpoons 2SO_3$

Given, number of moles of SO_3 in flask is twice the number of moles of SO_2. Let number of moles of SO_2 be 'a' and O_2 be x.

$\therefore \quad 2SO_2 + O_2 \rightleftharpoons 2SO_3$

	$2SO_2$	O_2	$2SO_3$
At equilibrium :	a	x	$2a$
Concentration :	$\frac{a}{10}$	$\frac{x}{10}$	$\frac{2a}{10}$

$$\therefore \quad K_C = \frac{[SO_3]^2}{[SO_2]^2[O_2]}$$

$$= \frac{\left(\frac{2a}{10}\right)^2}{\left(\frac{a}{10}\right)^2\left(\frac{x}{10}\right)}$$

$$K_C = \frac{\frac{4a^2}{100}}{\frac{a^2}{100} \times \frac{x}{10}}$$

Given, $K_C = 100$

$$100 = \frac{4}{\frac{x}{10}}$$

$\therefore \quad x = 0.4$ moles

$\therefore$ Number of moles of O_2 = 0.4

12. *(b)* The ccp lattice is formed by the atoms of the element M. Hence, the number of tetrahedral voids generated is twice the number of atoms of element M. According to question, the atoms of element N occupy $\frac{2}{3}$rd of tetrahedral voids.

Therefore, the number of atoms of N is

$2 \times \frac{2}{3} = \frac{4}{3}$rd of atoms of M.

$\therefore$ Ratio of number of atoms of M to that of N

i.e. $M : N = 1 : \left(\frac{4}{3}\right) = 3 : 4$

$\therefore$ Formula of compound is M_3N_4.

13. *(a)* Integrated law equation for zero-order reaction is as follows :

$$[A]_0 - [A]_t = kt \quad \ldots\text{(i)}$$

where, $[A]_0$ = Initial concentration of reactant

$[A]_t$ = Concentration of reactant at time t

k = Rate constant

t = Time

Let initial concentration of reactant is 1.

So, concentration of reactant after $\frac{3}{4}$th completion of reaction $= 1 - \frac{3}{4}$

$$= \frac{1}{4}$$

From Eq. (i),

$$1 - \frac{1}{4} = kt_{3/4}$$

$$\frac{3}{4} = kt_{3/4}$$

$$\frac{3}{4t_{3/4}} = k \quad \ldots\text{(ii)}$$

Concentration of reactant after $\frac{1}{2}$ completion

$$= 1 - \frac{1}{2} = \frac{1}{2}$$

From Eq. (i),

$$1 - \frac{1}{2} = kt_{1/2}$$

$$\frac{1}{2t_{1/2}} = k \quad \ldots\text{(iii)}$$

From Eqs. (ii) and (iii),

$$\frac{3}{4t_{3/4}} = \frac{1}{2t_{1/2}}$$

$$6t_{1/2} = 4t_{3/4}$$

$$t_{3/4} = \frac{3}{2}t_{1/2}$$

$$t_{3/4} = 1.5t_{1/2}$$

14. *(d)* $Fe^{2+} + 2e^- \longrightarrow Fe; \; E^\circ_{Fe^{2+}/Fe} = -0.44\text{ V} \quad \ldots\text{(i)}$

$Fe^{3+} + e^- \longrightarrow Fe^{2+}; E^\circ_{Fe^{3+}/Fe^{2+}} = 0.77\text{ V} \ldots\text{(ii)}$

Adding Eqs. (i) and (ii),

$$Fe^{3+} + 3e^- \longrightarrow Fe$$

We know that, $\Delta G = -nFE^\circ_{cell}$

Note ΔG is an additive property.

$$\therefore \quad \Delta G^\circ_{Fe^{3+}/Fe} = \Delta G^\circ_{Fe^{2+}/Fe} + \Delta G^\circ_{Fe^{3+}/Fe^{2+}}$$

$$-nFE^\circ_{Fe^{3+}/Fe} = -nFE^\circ_{Fe^{2+}/Fe} - nFE^\circ_{Fe^{3+}/Fe^{2+}}$$

$$-3E^\circ_{Fe^{3+}/Fe} = -2 \times (-0.44) - 1\,(0.77)$$

$$-3E^\circ_{Fe^{3+}/Fe} = 0.88 - 0.77$$

$$E^\circ_{Fe^{3+}/Fe} = \frac{0.11}{3}$$

$$= 0.036\text{ V}$$

$$= 0.04\text{ V}$$

15. *(b)* Expression for net efficiency of Carnot engine is as follows :

$$\text{Net efficiency} = 1 - \frac{T_2}{T_1} \quad \ldots\text{(i)}$$

Also, net efficiency

$$= \frac{\text{Net work done }(W)}{\text{Heat with drawn from reservoir }(Q)} \quad \ldots\text{(ii)}$$

From Eqs. (i) and (ii),

$$1 - \frac{T_2}{T_1} = \frac{W}{Q}$$

Given, $T_2 = 300\text{ K}, T_1 = 500\text{ K}$

$W = 500\text{ J}$

$$\therefore \quad Q = \frac{W}{1 - \frac{T_2}{T_1}} = \frac{500}{1 - \frac{300}{500}}$$

$$= \frac{250000}{200} = 1250\text{ J}$$

16. *(a)* The conductions for non-ideal solutions showing positive deviation from Raoult's law are

(i) $\Delta H_{mix} > 0$ i.e., heat is absorbed on mixing of solute and solvent.

(ii) $\Delta V_{mix} > 0$ i.e., volume of solution is more than volume of solute and solvent.

17. *(a)* $CO_2(g) + C(s) \rightleftharpoons 2CO(g)$

$$K_p \text{ for the reaction} = \frac{p_{CO}^2}{p_{CO_2} \times p_C}$$

$$K_p = \frac{p_{CO}^2}{p_{CO_2}} \quad \ldots(i)$$

$[p_C = 1]$ Since, carbon is solid.

Let partial pressure of CO_2 initial be x.

So,	$CO_2(g) + C(s)$	$\rightleftharpoons 2CO(g)$
At $t = 0$	x	0
At equilibrium	$x - 2a$	$2a$

Given, total pressure at equilibrium $(p_T) = 2$ atm

$$p_T = p_{CO_2} + p_{CO}$$
$$2 = (x - 2a) + (2a)$$
$$2 = x$$

$\therefore$ At equilibrium $p_{CO_2} = 2 - 2a$

$$p_{CO} = 2a$$

From Eq. (i)

$$K_p = \frac{(2a)^2}{2 - 2a}$$

$$1.9 = \frac{4a^2}{2 - 2a}$$

$$2a^2 + 1.9a - 1.9 = 0$$

Solving the quadratic equation,

$a = 0.61$ and -1.55 (neglected)

So, $p_{CO} = 2a = 2 \times 0.61 = 1.22$ atm

$p_{CO_2} = (2 - 2a)$

$= (2 - 1.22) = 0.78$ atm

$\therefore$ Partial pressure of CO_2 and CO are 0.78 atm and 1.22 atm respectively.

18. *(d)* The correct statement is BaO is soluble but $BaSO_4$ is insoluble in water.

(A) $BeSO_4$ is soluble but BeO is insoluble in water.

(B) LiI is more soluble in ethanol than KI due to more covalent character of LiI, than KI. LiI has more covalent character due to smaller size of Li^+, than K^+. Therefore, Li^+ can polarise to greater extant with I^-, than K^+.

(C) KO_2 is paramagnetc due to presence of unpaired electrons.

19. *(b)* Molality of solution

$$= \frac{\text{Mass of solute } (m)}{\text{Molecular mass of solute } (M) \times \text{Mass of solvent } (m_s) \text{ (in kg)}}$$

$$= \frac{m}{M} \times \frac{1}{m_s}$$

$$= \frac{30}{M} \times \frac{1}{1}$$

We know that,

$$\Delta T_f = K_f \times m$$

where, ΔT_f = change in freezing point

m = molality

K_f = cryoscopic constant

$$0.93 = 1.86 \times \frac{30}{M} \times 1$$

$$= \frac{1.86 \times 30}{0.93}$$

$$= 60 \text{ g/mol}$$

General molecular formula = $C_nH_{2n}O_n$

As molecular mass = 60 g/mol

$\therefore \quad n(12) + 2n(1) + n(16) = 60$

$30n = 60$

$\therefore \quad n = 2$

20. *(c)* Integrated rate-equation for 1st order reaction is as follows :

$$kt = \ln \frac{a}{a - x} \quad \ldots(i)$$

$$kt = 2.303 \log \frac{a}{a - x}$$

where, k = rate constant

t = time

a = initial concentration of CH_3CN

$a - x$ = concentration of CH_3CN at time t

From Eq. (i),

$$5.11 \times 10^{-5} \text{ s}^{-1} \times 2 \times 3600 \text{ s} = \ln\left[\frac{(0.0340\ M)}{(a - x)}\right]$$

$$0.36792 = \ln(0.0340) - \ln(a - x)$$

$$0.36792 = -3.3814 - \ln(a-x)$$
$$3.7493 = -\ln(a-x)$$
$$\ln(a-x) = -3.7493$$
$$(a-x) = e^{-3.7493}$$

Hence, concentration (molarity) of CH_3NC after 2 hrs is $e^{-3.7493}$ M.

21. *(d)* Depression in freezing point is given as

$$\Delta T_f = K_f \times m$$

where, ΔT_f = change in freezing point

K_f = cryoscopic constant

m = molality of solution

$$0.112 = 4.90 \times \frac{3.26}{\text{Molecular mass of selenium}} \times \frac{1}{\frac{226}{1000}\text{ kg}}$$

$$\left\{\text{Molality} = \frac{\text{Mass of solute}}{\text{Molecular mass of solute}} \times \frac{1}{\text{Mass of solvent (in kg)}}\right\}$$

$$\text{Molecular mass of selenium} = \frac{4.90 \times 3.26}{226 \times 0.112} \times 1000$$

$$= 631.08 \text{ g/mol}$$

∴ In polymer Se_x, x is equal to 8 and molecular formula of selenium is Se_8.

22. *(c)* This question deals with selective precipitation of copper (II) and zinc (II).

Molarity of Cu^{2+} = 0.010 M

Molarity of Zn^{2+} = 0.10 M

Given, sulphide anion molarity is 8.1×10^{-19} M.

Comparing solubility product Q with K_{sp} values as follows.

$$Q_1 = [Cu^{2+}][S^{2-}] = (0.010 \text{ M})(8.1 \times 10^{-19} \text{ M})$$
$$Q_1 = 8.1 \times 10^{-21} \text{ M} > 8.0 \times 10^{-36} \text{ i.e., } K_{sp}$$

∴ CuS will precipitate.

Similarly, $Q_2 = [Zn^{2+}][S^{2-}]$

$$= (0.10 \text{ M})[8.1 \times 10^{-19} \text{ M}]$$
$$Q_2 = 8.1 \times 10^{-20} > K_{sp} \times 3 \times 10^{-22}$$

∴ ZnS will precipitates.

23. *(d)* IUPAC name of $[PdI_2(NOO)_2(H_2O)_2]$ is diaquadiiodonitrito-N- palladium (IV).

Oxidaiton number of Pd$(x) \Rightarrow x + (-2) + (-2) = 0$

∴ $x = 4$

Note Follow alphabetical order and linking atom in NOO is N.

24. *(a)* Copper reacts with dil HNO_3 to form copper nitrate $[Cu(NO_3)_2]$, nitric oxide (NO) and water as products. In this copper is oxidised while nitric acid is reduced to nitric oxide.

$$\overset{0}{Cu} + \text{dil } HNO_3 \longrightarrow \overset{+2}{Cu}(NO_3)_2 + H_2O + \underset{(X)}{NO}$$

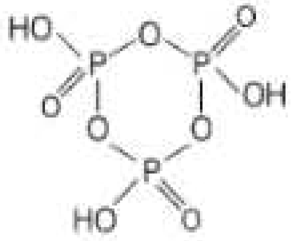
Copper reacts with conc. HNO_3 to form $Cu(NO_3)_2$, H_2O and nitrogen dioxide (NO_2) as products. In this, nitric acid is reduced to NO_2.

$$Cu + \text{conc. } HNO_3 \longrightarrow Cu(NO_3)_2 + H_2O + \underset{(Y)}{NO_2}$$

∴ X is NO and Y is NO_2.

25. *(c)* $[Co(NH_3)_3Cl_3]$ has similar electrical conductance as that of glucose. Both have zero conductance as both do not dissociate into ions in water as they are non-electrolyte.

26. *(a)* There are 12 phosphorus oxygen bonds present in cyclotrimetaphosphoric acid.



Cyclotrimetaphosphoric acid

27. *(b)* Mond's process is method used for refining of nickel metal. In this process, nickel is heated in steam of CO to form volatile nickel carbonyl. $[Ni(CO)_4]$ which on further heating decomposes giving pure nickel.

28. *(b)* Bismuth (Bi) has least tendency to exhibit –3 oxidation state. Bi hardly forms any compound in –3 oxidation state. The tendency to exhibit –3 oxidation state decreases as we move down the group due to increase in size of atom and decrease in nuclear charge or holding capacity of nucleus.

29. *(c)* Nitrogen does not show inert-pair effect. It is exhibited by some heavier nucleus *p*-block elements such as Ti, Sn, Pb, etc.

30. *(a)* In 15th group as we move down the group acidic character of oxides decreases. Oxides of nitrogen and phosphorus are purely acidic. Therefore, N_2O_3 is purely acidic character.

31. *(b)* Various zinc antimonides (ZnSb), (Zn_3Sb_2), (Zn_4Sb_3) are possible.

Oxidation states of antiomony in all the above compounds are as follows :

In ZnSb $\Rightarrow +2 + x = 0$

$$x = -2$$

In $Zn_3Sb_2 \Rightarrow +2 \times (3) + 2x = 0$

$$2x = -6$$

$$x = -3$$

In $Zn_4Sb_3 \Rightarrow 4 \times (+2) + 3x = 0$

$$3x = -8$$

$$x = \frac{-8}{3}$$

$\therefore$ Oxidation states of antimony in zinc antimonide are −2, −3, and $\frac{-8}{3}$.

32. *(c)* As we move down in 16th group, electronegativitiy (to attract shared pair of electrons) decreases down the group due to increase in size of atom. Hence, an increase in metallic character occur i.e. tendency to loose electrons increases which indicates decrease in ionisation enthalpies.

33. *(a)* In PCl_5, the *d*-orbital involved in d_{z^2} but when it is changed to PCl_6^-, then in sp^3d^2 hybridisation *d*-orbital involved are d_{z^2}, and $d_{x^2-y^2}$. So, *d*-orbital involved when PCl_5 is changed to PCl_6^- ion is $d_{x^2-y^2}$.

34. *(d)* Electronic configuration of O_2 is
$\sigma 1s^2, \sigma^* 1s^2, \sigma 2s^2, \sigma^* 2s^2, \sigma 2p_z^2, \pi 2p_x^2 = \pi 2p_y^2,$
$\pi^* 2p_x^1 = \pi^* 2p_y^1$

Since, π-anti-bonding orbital has unpaired electrons. So, O_2 is paramagnetic in nature.

Electronic configuration of S_2 is
$\sigma 1s^2, \sigma^* 1s^2, \sigma 2s^2, \sigma^* 2s^2, \sigma 2p_z^2, \pi 2p_x^2 = \pi 2p_y^2,$
$\pi^* 2p_x^2 = \pi^* 2p_y^2, \sigma^* 2p_z^2, \sigma 3s^2, \sigma^* 3s^2, \sigma 3p_z^2,$
$\pi 3p_x^2 = \pi 3p_y^2, \pi^* 3p_x^1 = \pi^* 3p_y^1$

Since, π-anti-bonding orbital of S_2 has unpaired electrons. So, S_2 is paramagnetic in nature.

Electronic configuration of S_2^+ is
$\sigma 1s^2, \sigma^* 1s^2, \sigma 2s^2, \sigma^* 2s^2, \sigma 2p_z^2, \pi 2p_x^2 = \pi 2p_y^2,$
$\pi^* 2p_x^2 = \pi^* 2p_y^2, \sigma^* 2p_z^2, \sigma 3s^2, \sigma^* 3s^2, \sigma 3p_z^2,$
$\pi 3p_x^2 = \pi 3p_y^2, \pi^* 3p_x^1 = \pi^* 3p_y^0$

Since, $\pi^* 3p_x$ has one unpaired electron. So, S_2^+ is also paramagnetic in nature.

Electronic configuration of O_2^{2+} is
$\sigma 1s^2, \sigma^* 1s^2, \sigma 2s^2, \sigma^* 2s^2, \sigma 2p_z^2, \pi 2p_x^2 = \pi 2p_y^2,$
$\pi^* 2p_x^0 = \pi 2p_y^0$

Since, all electrons are paired. Hence, O_2^{2+} is diamagnetic in nature.

35. *(d)* Low temperature is incorrect condition for Haber's process. It is the process used for production of ammonia. This process takes place at high pressure and high temperature conditions. It takes place in presence of iron oxide as catalyst with use of small amount of K_2O and Al_2O_3.

Complete reaction is as follows :

$$N_2(g) + 3H_2(g) \xrightarrow[250\text{ atm}]{\text{Iron oxide } 450°C} 2NH_3(g)$$

36. *(c)* Aqueous SO_2 decolourises acidified $KMnO_4$. SO_2 behaves as reducing agent. It reduces MnO_4^- to Mn^{2+}.

$$\underset{\text{(Purple)}}{2MnO_4^-} + 5SO_2 + 2H_2O \longrightarrow 5SO_4^{2-} + 4H^+ + \underset{\text{(Colourless)}}{2Mn^{2+}}$$

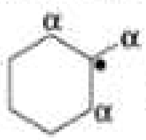
37. *(a)* On moving from top to bottom in group 15, the atomic size increases. With increase in size, the extent of van der Waals' force also increases. Hence, the boiling point increases. But an exception is seen with ammonia (NH_3) having higher boiling point than PH_3 and AsH_3 due to presence of intermolecular H-bonding in ammonia molecules which leads to molecular association and increase in boiling point. Hence, correct order of boiling point of hydrides of group 15 is

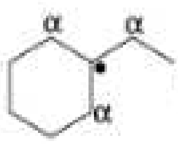
$$BiH_3 > SbH_3 > NH_3 > AsH_3 > PH_3$$

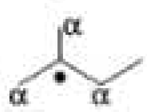
38. *(c)* Sphalerite is a zinc sulphide mineral with chemical composition of (ZnS or FeS). It is a zinc mineral and an ore of zinc.

39. *(d)* Aluminium is not refined by zone refining process. Hoope's method is used to obtain aluminium with very high purity. This is an electrolytic process.

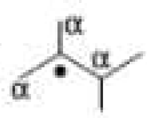
40. *(c)* The maximum number of hyperconjugating structure is seen in compound having highest number of α-hydrogen atoms.

(a)  It contain 7 α-hydrogen atoms.

(b)  It has 6 α-hydrogen atoms.

(c)  It has 8 α-hydrogen atoms.

Therefore, maximum number of hyperconjugating structure is seen.

(d)  It has 6 α-hydrogen atoms.

41. *(b)* H_3C^- has highest nucleophilicity. This is due to less electronegativity of C-atom among N, O, F and C. As, nucleophilicity depends on availability of electron to donate. So, atom with less electronegativity is capable of donating electrons.

42. *(d)* Ethyne reacts with base $NaNH_2$ to give an anion (*A*), $NaNH_2$ is a base that will abstract proton from ethyne.

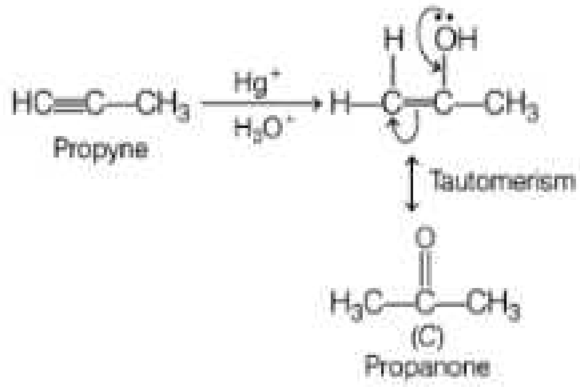
$$\underset{\text{Ethyne}}{H-C\equiv C-H} \xrightarrow{NaNH_2} \underset{\text{Sodium ethynide}}{\underset{(A)}{H-C\equiv C^{\ominus}Na^+}} + NH_3$$

Anion reacts with methyl iodide to give propyne (*B*).

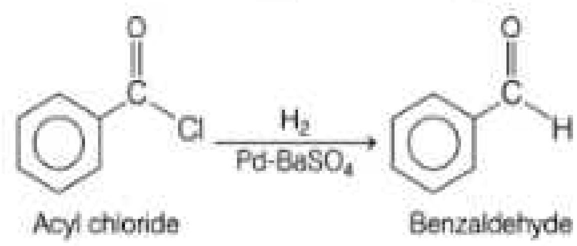
$$H-C\equiv C^{\ominus}Na^+ + CH_3I \longrightarrow \underset{\text{Propyne}}{\underset{(B)}{HC\equiv C-CH_3}} + NaI$$

At last, unsymmetrical alkyne (propyne) reacts with Hg^{2+}/H_3O^+ to give a hydrated product propanone (*C*), addition of water to alkyne is according to Markownikoff's rule.

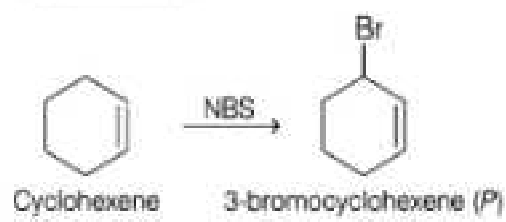
Note Negative part of water i.e. OH^- is added to carbon with lesser number of hydrogen atoms.



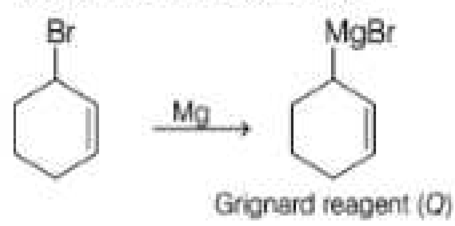
43. *(c)* The given reaction is Rosenmund reaction. This reaction is used to reduce acyl chlorides to their corresponding aldehydes in presence of rosenmund catalyst (H_2/Pd-$BaSO_4$)



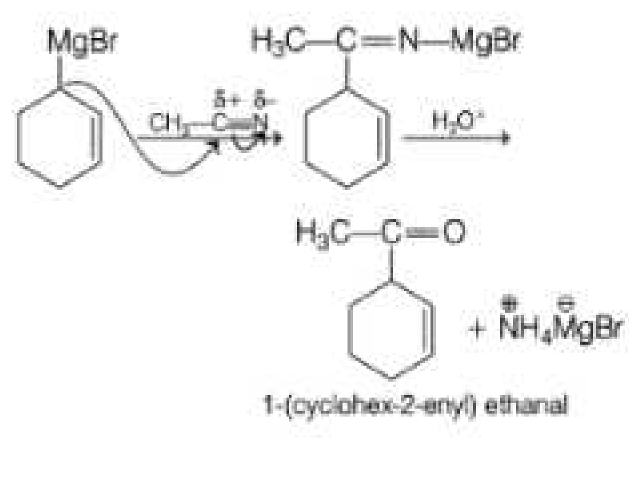
44. *(b)* Cyclohexene reacts with NBS to form 3-bromo-cyclohexene. NBS is used for allylic bromination.



The compound *P* reacts with magnesium to form a Grignard's reagent (*Q*).



The compound *Q* reacts with acetonitrile (CH_3CN) followed by acidic hydrolysis to form 1-(cyclohex-2-enyl) ethanone.



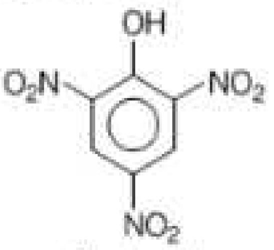
45. ***(c)*** Zymase is the enzyme that converts glucose and fructose into ethanol through the process of fermentation.

$$\underset{\text{Glucose/Fructose}}{C_6H_{12}O_6} \xrightarrow{\text{Zymase}} \underset{\text{Ethanol}}{2C_2H_5OH} + 2CO_2$$

46. ***(a)*** Primary alkyl halides are most reactive towards S_N2 displacement reaction because primary alkyl halides have least steric hinderance. 1-bromobutane is primary alkyl halide.

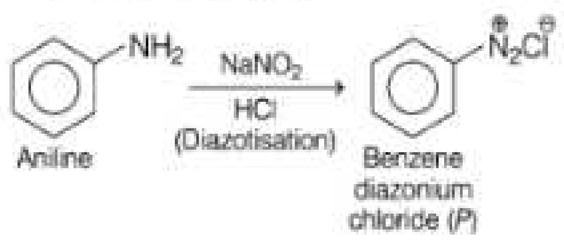
47. ***(b)*** Alcohols and phenols do not react with sodium bicarbonate ($NaHCO_3$) to liberate CO_2 gas while carboxylic acids reacts with $NaHCO_3$ to liberate CO_2 gas.

∴ Picric acid will not liberate CO_2 gas as it contain —OH group only.

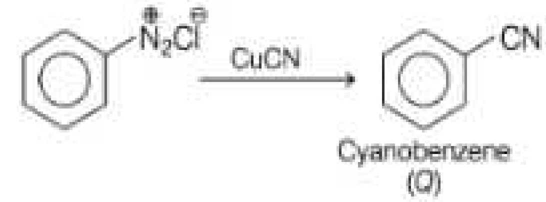


Picric acid

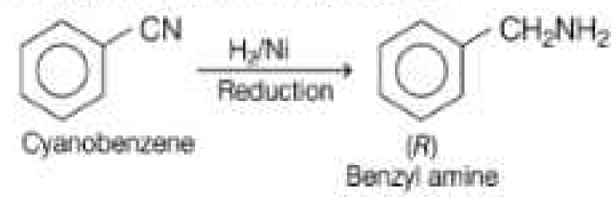
48. ***(c)*** Aniline reacts with $NaNO_2$ and HCl to form benzene diazonium chloride.



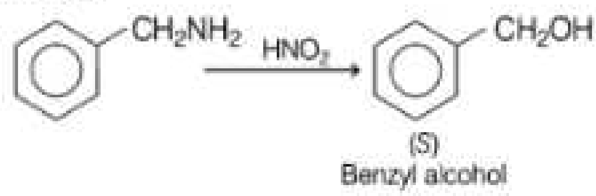
This benzene diazonium chloride reacts with CuCN to form cyanobenzene (Q).



Cyanobenzene undergoes reduction in presence of H_2/Ni to form benzyl amine.



Benzyl amine reacts with HNO_2 to form benzyl alcohol(*S*).



49. ***(b)*** Heroin is an opioid drug made from morphine, a natural substance taken from seed pod of various opium poppy plants grown in south-east. It is a white or brown powder.

50. ***(c)*** DNA consists of 4 bases namely adenine, thymine, guanine, cytosine, whereas RNA has adenine, guanine, uracil and cytosine. Therefore, thymine is present in DNA but not in RNA.

Physics

51. *(c)* Given, peak voltage of carrier signal, $V_c = 15$ V

Modulation index = 65%

$$= \frac{65}{100}$$

Since, modulation index $= \frac{V_m}{V_c}$

where, V_m is peak of message signal.

$$\therefore \quad \frac{65}{100} = \frac{V_m}{15}$$

$$\Rightarrow \quad V_m = \frac{65 \times 15}{100}$$

$$= 9.75 \text{ V} \simeq 9.7 \text{ V}$$

52. (*) Given, first wavelength, $\lambda_1 = 650$ nm

Second wavelength, $\lambda_2 = 520$ nm

Let nth fringe of λ_1 coincides with $(n+1)$th fringe of λ_2.

Since, $y = \frac{n\lambda D}{d}$

where, y is least distance of bright fringe from central bright,

n is number of bright fringe,

λ is wavelength,

d is separation between slits

and D is separation of slits from screen.

$$\therefore \quad \frac{n\lambda_1 D}{d} = \frac{(n+1)\lambda_2 D}{d}$$

$$\Rightarrow n \times 650 \times 10^{-9} = (n+1)\, 520 \times 10^{-9}$$

$$\Rightarrow \quad \frac{n+1}{n} = \frac{650}{520} = 1.25$$

$$\Rightarrow \quad n + 1 = 1.25\, n$$

$$\Rightarrow \quad n = \frac{1}{0.25}$$

$$= \frac{100}{25} = 4$$

i.e. at 4th bright fringe both wavelength will coincide.

Now, $y = \frac{n\lambda D}{d}$

But, since value of D and d are not given in question, so cannot be solved further.

53. *(c)* Given, mass of object, $m = 10$ kg

Change in force, $F = 50$ N

Time taken, $t = 4$ s

Since, $F = ma$

where, a is acceleration

$$\Rightarrow \quad a = \frac{v}{t} = \frac{F}{m}$$

where, v is velocity

$$\Rightarrow \quad v = \frac{F}{m} t$$

$$\therefore \quad v = \frac{50 - 0}{10} \times 4$$

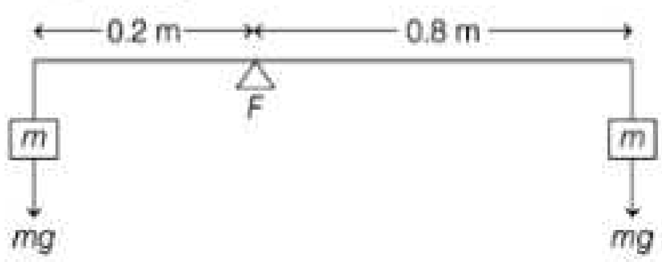
$$= 5 \times 4 = 20 \text{ ms}^{-1}$$

54. *(b)* Given, mass of two blocks $= m$,

Let l_1, l_2 be the distance of blocks from fulcrum, then $l_1 = 20$ cm $= 0.2$ m

$l_2 = 80$ cm $= 0.8$ m

As the blocks and rod will rotate together and torque will produce.



$$\therefore \quad \text{Torque, } \tau = mgl_2 - mgl_1$$

$$I\alpha = mg(l_2 - l_1)$$

where, I is inertia of blocks and α is angular acceleration.

$$\therefore \quad (ml_1^2 + ml_2^2)\alpha = mg(l_2 - l_1)$$

$$\Rightarrow \quad \alpha = \frac{g(l_2 - l_1)}{l_1^2 + l_2^2}$$

Hence, acceleration of blocks closer to fulcrum,

$$a = \alpha l_1 = g\left(\frac{l_2 - l_1}{l_2^2 + l_1^2}\right) l_1$$

$$= 9.8\left[\frac{0.8 - 0.2}{(0.8)^2 + (0.2)^2}\right] 0.2$$

$$= 1.73 \simeq 1.7 \text{ ms}^{-2}$$

55. ***(c)*** Given, inner radius of hollow sphere,

$$r_1 = 8 \text{ cm} = 8 \times 10^{-2} \text{ m}$$

Outer radius of hollow sphere,

$$r_2 = 9 \text{ cm} = 9 \times 10^{-2} \text{ m}$$

Density of liquid, $\rho_l = 800 \text{ kg m}^{-3}$

Let density, mass and volume of sphere and volume of sphere submerged be ρ, m, V and V_l

As, sphere is floating.

$\therefore$ Weight of sphere = Upthrust force on body

$$mg = \rho_l g V_l$$

$$\Rightarrow \quad m = \frac{\rho_l V}{2} \quad \left[\text{Since, } V_l = \frac{V}{2}\right]$$

$$= 800 \times \frac{\frac{4}{3}\pi r_2^3}{2}$$

$$= 800 \times \frac{4}{6}\pi (9 \times 10^{-2})^3$$

$$= 1.2 \text{ kg}$$

$\therefore$ Density of sphere, $\rho_l = \frac{m}{V}$

$$= \frac{1.2}{\frac{4}{3}\pi (r_2^3 - r_1^3)}$$

$$= \frac{1.2}{\frac{4}{3}\pi (9^3 - 8^3) \times 10^{-6}}$$

$$= 1300 \text{ kg m}^{-3}$$

56. ***(d)*** Given, diameter of circular path of proton,

$d_p = 23 \text{ m}$

Temperature, $T = 295 \text{K}$

Pressure, $p = 1 \times 10^{-6}$ torr

$$= 133 \times 10^{-6} \text{ Pa}$$

Diameter of molecule, $d_m = 2 \times 10^{-8}$ cm

$$= 2 \times 10^{-10} \text{ m}$$

Since, mean free path, $\lambda = \frac{RT}{\sqrt{2}\pi d^2 N_A p}$ ...(i)

where, R = universal gas constant

$$= 8.314 \text{ J mol}^{-1} \text{ K}^{-1}$$

$$\therefore \quad \lambda = \frac{8.314 \times 295}{\sqrt{2} \times 3.14 (2 \times 10^{-8} \times 10^{-2})^2 \times 6.022 \times 10^{23} \times 133 \times 10^{-6}}$$

$$= \frac{1}{\sqrt{2} \times 3.14 \times (2 \times 10^{-10})^2 \times 3.27 \times 10^{16}}$$

$$= 172 \text{ m}$$

57. ***(a)*** Given, surface charge density,

$$\sigma = 0.1 \,\mu\text{C/m}^2$$

$$= 0.1 \times 10^{-6} \text{ C/m}^2$$

Potential difference, $V = 50$ V

Since,

$$V = \frac{\sigma}{2\varepsilon_0} d$$

where,

ε_0 = free space permittivity

$$= 8.854 \times 10^{-12} \text{ C}^2 \text{ N}^{-1} \text{m}^{-2}$$

and d = separation from sheet.

$$\therefore \quad d = \frac{2V\varepsilon_0}{\sigma}$$

$$= \frac{2 \times 50 \times 8.854 \times 10^{-12}}{0.1 \times 10^{-6}}$$

$$= 885.4 \times 10^{-12+7}$$

$$= 8.854 \times 10^{-3}$$

$$= 8.854 \text{ mm} \simeq 8.8 \text{ mm}$$

58. ***(a)*** Given, radius of wire, $R = 8$ cm

$$= 8 \times 10^{-2} \text{ m}$$

Current, $I = 0.2$ A

Dipole moment, $\mu = 0.6\,\hat{\mathbf{i}} - 0.8\,\hat{\mathbf{j}}$

Magnetic field, $\mathbf{B} = (0.25\text{T})\hat{\mathbf{i}} + (0.30\text{T})\hat{\mathbf{k}}$

Since, potential energy, $U = -\mu \cdot \mathbf{B}$

and $\quad \mu = IA$

$$= 0.2 \times \pi (8 \times 10^{-2})^2$$

$$= 4 \times 10^{-3}$$

$$\therefore \quad U = -(4 \times 10^{-3})(0.6\hat{\mathbf{i}} - 0.8\hat{\mathbf{j}}) \cdot (0.25\hat{\mathbf{i}} + 0.30\hat{\mathbf{k}})$$

$$= -(4 \times 10^{-3})(0.6)(0.25)$$

$$= -6 \times 10^{-4} \text{ J}$$

59. *(c)* Given, atomic number of lithium, $Z = 3$

Since, electronic configuration of lithium $= 1s^2 2s^1$

For $2s$ orbital, $n = 2, l = 0$

Magnetic quantum number, $m_l = 0$

Spin quantum number, $m_S = +\frac{1}{2}, -\frac{1}{2}$

Hence, option (c) is correct.

60. *(d)* Since, fusion reaction is only performed by hydrogen atom or hydrogen like atom but as atomic number of chlorine is 35 and addition of electron will make it inert due to its large binding energy. Hence, reaction $^{35}Cl + {}^{35}Cl$ will not result in the net release of energy.

61. *(a)* Given, thermal energy of hydrogen

$$E = 26.2 \text{ MeV}$$
$$= 26.2 \times 10^6 \times 1.6 \times 10^{-19} \text{ J}$$

Number of proton, $n = 4$

Mass of proton, $m = 1.6 \times 10^{-27}$ kg

Power, $P = 3.9 \times 10^{26}$ W

Since,

$$nP = \frac{E}{t}$$

$$\frac{1}{t} = \frac{nP}{E}$$

$$\Rightarrow \quad \frac{m}{t} = \frac{nmP}{E}$$

$$= \frac{4 \times 1.6 \times 10^{-27} \times 3.9 \times 10^{26}}{26.2 \times 10^6 \times 1.6 \times 10^{-19}}$$

$$= 0.6 \times 10^{12}$$

$$= 6 \times 10^{11} \approx 6.2 \times 10^{11} \text{ kg/s}$$

62. *(c)* Given, energy of quantum state above Fermi energy level $(E - E_F) = 0.1$ eV

Temperature $(T) = 800$ K

According to Fermi-Dirac distribution, probability

$$P(E) = \frac{1}{e^{(E - E_F)/kT} + 1} \quad \ldots \text{(i)}$$

where, k is Boltzmann constant $= 1.38 \times 10^{-23}$ J

$$= 8.625 \times 10^{-5} \text{ eV K}^{-1}$$

$$\therefore \quad \frac{E - E_F}{kT} = \frac{0.1}{8.625 \times 10^{-5} \times 800} = 1.45$$

Hence, from Eq. (i), we get

$$P(E) = \frac{1}{e^{1.45} + 1} = 0.19$$

$$= 19\% \approx 20\%$$

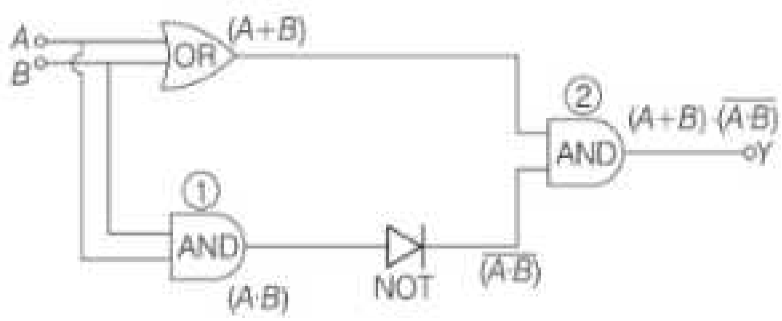
63. *(c)* The given circuit diagram is of AND gate, i.e. $A \cdot B =$ output.

If any of the input will be off (0), the output is 0.

Truth Table

A	B	Y
0	0	0
0	1	0
1	0	0
1	1	1

64. *(a)* According to given logic diagram,



Final output, $Y = (A + B) \cdot (\overline{A \cdot B})$

65. *(d)*

As we know that,

(a) Surface tension $= \frac{\text{Force}}{\text{Length}}$

Dimensionally,

$$[\text{Surface tension}] = \frac{[\text{Force}]}{[\text{Length}]} = \frac{[MLT^{-2}]}{[L]} = [MT^{-2}]$$

$$[\text{Stress}] = \frac{[\text{Force}]}{[\text{Area}]}$$

$$= \left[\frac{MLT^{-2}}{L^2}\right] = [ML^{-1}T^{-2}]$$

(b) Thermal conductivity

$$= \frac{\text{Heat} \times \text{Distance}}{\text{Area} \times \text{Temperature}}$$

$\therefore \left[\dfrac{\text{Heat} \times \text{Distance}}{\text{Area} \times \text{Temperature}}\right] = \dfrac{[ML^2T^{-2}][L]}{[L^2][K]}$

$= [MLT^{-2}K^{-1}]$

(c) Gravitational constant $(G) = \dfrac{(\text{Force})\,(\text{Distance})^2}{(\text{Mass})^2}$

$\therefore \left[\dfrac{(\text{Force})^2 \times (\text{Distance})^2}{(\text{Mass})^2}\right] = \dfrac{[MLT^{-2}][L^2]}{[M^2]}$

$= [M^{-1}L^3T^{-2}]$

(d) Viscosity $(\eta) = \dfrac{1}{6\pi} \cdot \dfrac{(\text{Force})}{(\text{Radius})\,(\text{Velocity})}$

$= \dfrac{[MLT^{-2}]}{[L]\,[LT^{-1}]}$

$= [ML^{-1}T^{-1}]$

Hence, option (d) is the correct.

66. *(b)* Given, initial speed, $u = 0\ \text{ms}^{-1}$

Time, $t_1 = 10\ \text{s}$

Velocity after 10 s, $v_1 = 100\ \text{ms}^{-1}$

Velocity after 11s, $v_2 = 150\ \text{ms}^{-1}$

Since, $v = u + at$

where, v is final velocity and a is acceleration.

$\therefore \quad v_1 = u + at_1$

$100 = at_1 \quad \ldots\text{(i)}$

$150 = a(t_1 + 1) \quad \ldots\text{(ii)}$

On subtracting Eq. (i) from Eq. (ii), we get

$150 - 100 = at_1 + a - at_1$

$a = 50\ \text{ms}^{-2}$

Hence, distance travelled in 11th sec,

$s = \dfrac{v_2^2 - v_1^2}{2a}$

$= \dfrac{(150)^2 - (100)^2}{2 \times 50}$

$= \dfrac{22500 - 10000}{100}$

$= 125\ \text{m}$

67. *(d)* Given,

Case I

Angle of projection, $\theta_1 = 15°$

Range, $R_1 = 50$ m

Case II

Angle of projection, $\theta_2 = 45°$

Range, $R_2 = ?$

Since, range $= \dfrac{u^2 \sin 2\theta}{g}$

$\Rightarrow \quad \dfrac{u^2}{g} = \dfrac{\text{Range}}{\sin 2\theta}$

$= \dfrac{50}{\sin (2 \times 15°)}$

$= \dfrac{50}{\sin 30°} = 100 \quad \ldots\text{(i)}$

Again, for case II,

range $= \dfrac{u^2}{g} \sin (2 \times 45°)$

$= \dfrac{u^2}{g} = 100$ m

68. *(a)* Given, mass of particle, $m = 10$ g

Force, $F = 5\hat{\mathbf{i}} + 2.5\hat{\mathbf{j}}$ N

Time, $t = 5$ s

Since, $F = ma$

$\Rightarrow \quad a = \dfrac{F}{m}$

where, a is acceleration.

$\therefore \quad a = \left(\dfrac{5\hat{\mathbf{i}} + 2.5\hat{\mathbf{j}}}{10 \times 10^{-3}}\right) \text{ms}^{-2}$

$= (5\hat{\mathbf{i}} + 2.5\hat{\mathbf{j}}) \times 10^2\ \text{ms}^{-2}$

$= 500\,\hat{\mathbf{i}} + 250\,\hat{\mathbf{j}}$

$\because \quad a = \dfrac{dv}{dt}$

$\therefore \quad \int dv = \int a\, dt$

$\int dv = \int (500\,\hat{\mathbf{i}} + 250\hat{\mathbf{j}})\, dt$

$v = 500t\,\hat{\mathbf{i}} + 250t\hat{\mathbf{j}} = \dfrac{dx}{dt}$

$\Rightarrow \quad \int dx = \int 500\, t\, dt \hat{\mathbf{i}} + \int 250t\, dt\, \hat{\mathbf{j}}$

$\Rightarrow \quad x = 500 \dfrac{t^2}{2}\,\hat{\mathbf{i}} + 250 \dfrac{t^2}{2}\,\hat{\mathbf{j}}$

$\Rightarrow \quad x = 250t^2\hat{\mathbf{i}} + 125t^2\hat{\mathbf{j}}$

At $t = 5$ s,

$$x = 250 \times 5^2\,\hat{\mathbf{i}} + 125 \times 5^2\,\hat{\mathbf{j}}$$

$$= (6250\,\hat{\mathbf{i}} + 3125\hat{\mathbf{j}})\text{ m}$$

69. *(c)* Given, mass of rain drop, $m = 3 \times 10^{-5}$ kg

Terminal velocity, $v = 9\text{ ms}^{-1}$

Amount of rain, $h = 100$ cm/year

$= 1$ m/year

$\therefore$ Volume, $V = 1\text{ m}^3$/year

Since, energy $E = \frac{1}{2}mv^2$

and $m =$ density (ρ) $\times$ volume (V)

$= 10^3 \times 1 = 1000$ kg

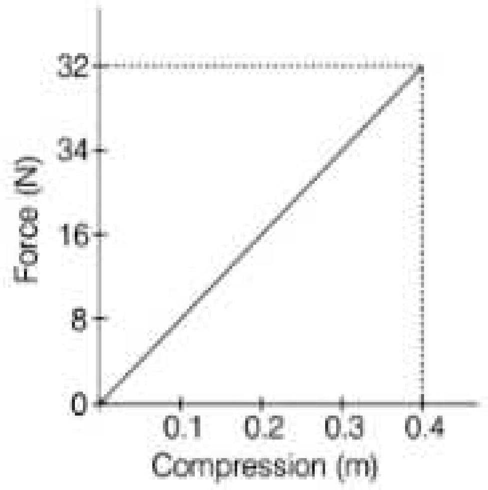
$$\therefore \quad E = \frac{1}{2} \times 1000 \times 9^2$$

$$= \frac{81000}{2} = 40500 = 4.1 \times 10^4\text{ J}$$

70. *(a)* Given, mass, $m = 5$ kg and

velocity, $v = 8\text{ ms}^{-1}$

According to given force *versus* compression graph,



As we know that,

force (F) = spring constant (k) $\times$ compression (x)

$$\Rightarrow \quad k = \frac{F}{x} = \frac{32}{0.4}$$

$$= \frac{320}{4} = 80\text{ Nm}^{-1}$$

and

since, kinetic energy of body = energy stored in spring

$$\Rightarrow \quad \frac{1}{2}mv^2 = \frac{1}{2}kx^2$$

$$\Rightarrow \quad x = \sqrt{\frac{mv^2}{k}}$$

$$= \sqrt{\frac{5 \times 8^2}{80}}$$

$$= \sqrt{\frac{5 \times 64}{80}} = \sqrt{4} = 2\text{ m}$$

71. *(d)* Given, mass of body, $m = 0.5$ kg

Velocity, $v = \alpha x^{3/2}$

$\alpha = 5\text{ m}^{-1/2}\text{s}^{-1}$

Position $x_1 = 0$ to $x_2 = 2$ m

Since,

work (W) = force (F) $\times$ displacement (dx)

and $F = ma$

where, a is acceleration $= \frac{dv}{dt}$

$$\therefore \quad a = \frac{d}{dt}(\alpha x^{3/2})$$

$$= \alpha\frac{3}{2}x^{\frac{3}{2}-1}\frac{dx}{dt}$$

$$= \frac{3}{2}\alpha x^{1/2}v$$

$$= \frac{3}{2}\alpha^2 x^{\frac{1}{2}+\frac{3}{2}}$$

$$= \frac{3}{2}\alpha^2 x^{4/2}$$

$$= \frac{3}{2}\alpha^2 x^2$$

$$\therefore \quad F = \frac{1}{2} \times \frac{3}{2}\alpha^2 x^2$$

$$= \frac{3}{4}\alpha^2 x^2$$

Now, since, work, $W = \int_{x=0}^{x=2} F \cdot dx$

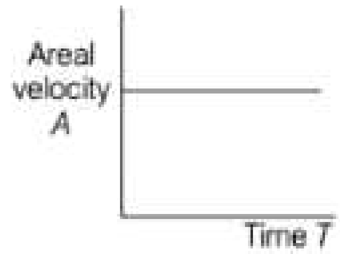
$$\therefore \quad W = \frac{3}{4}\alpha^2\int_0^2 x^2\,dx$$

$$= \frac{3}{4}\alpha^2\frac{x^3}{3}\Big|_0^2$$

$$= \frac{\alpha^2}{4} \times 2^3$$

$$= \frac{8 \times 5^2}{4} = 50\text{ J}$$

72. *(b)* As we know that, from Kepler's second law, equal area swept by planet in equal interval of time, i.e. areal velocity of planets $\left(\frac{dA}{dt} = \frac{L}{2m} = \text{constant}\right)$

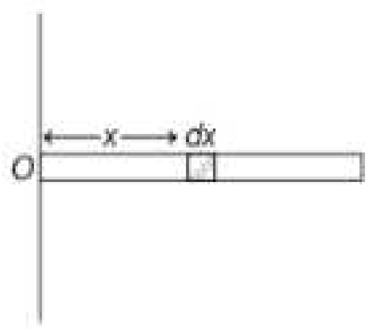


where, $\frac{dA}{dt}$ is areal velocity, L is angular momentum and m is mass.

73. *(a)* Given, length of rod, $l = 1$ m

Density of rod, $\rho = a(1 + bx^2)$

and $0 \le x \le 1$



Let dx be the element having mass dm from O at distance x

and $dm = \rho\, dx = a(1 + bx^2)\, dx$

$\because \quad X_{CM} = \frac{\int x\, dm}{\int dm}$

$\therefore \quad X_{CM} = \frac{\int_0^1 a(1 + bx^2)x\, dx}{\int a(1 + bx^2)\, dx}$

$$= \frac{\int_0^1 (x + bx^3)\, dx}{\int_0^1 (1 + bx^2)\, dx}$$

$$= \left.\frac{\frac{x^2}{2} + \frac{bx^4}{4}}{x + \frac{bx^3}{3}}\right|_0^1$$

$$= \left.\frac{\frac{x}{2} + \frac{bx^3}{4}}{1 + \frac{bx^2}{3}}\right|_0^1$$

$$= \frac{\frac{1}{2} + \frac{b}{4}}{1 + \frac{b}{3}}$$

$$= \frac{\frac{(2+b)}{4}}{\frac{(3+b)}{3}}$$

$$= \frac{3}{4}\frac{(2+b)}{(3+b)}$$

74. *(c)* (a) As we know that for perfect rolling friction force (f) will support rolling.

So, option (a) is correct.

(b) Since, rolling body can also be assumed as rotating about its own axis.

$\therefore$ Instantaneous speed of the point of contact is zero.

So, option (b) is correct

(c) But as the body is rotating (changing direction), so instantaneous acceleration is not zero.

So, option (c) is not correct.

(d) Since, during pure rotation (or rolling), friction force, $f = 0$ N

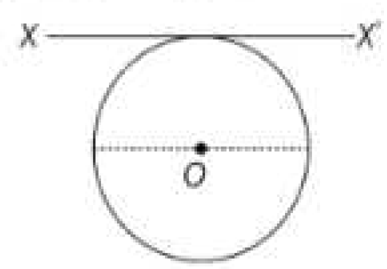
$\therefore$ Work done against friction = 0

So, option (d) is correct.

75. *(d)* Given, length of wire = L

Mass density of loop = ρ

According to given figure,



Moment of inertia about, XX', $I_{xx'} = I' + MR^2$

where, I' is moment of inertia about diameter, M is mass of ring and R is radius of ring.

$$I_{XX'} = \frac{MR^2}{2} + MR^2$$

$$= \frac{3MR^2}{2} \quad \text{...(i)}$$

Since, $L = 2\pi R$

$\therefore \quad R = \frac{L}{2\pi}$

and $\quad \rho = \frac{\text{mas } (M)}{\text{length } (L)}$

$\Rightarrow \quad M = \rho L$

Putting in Eq. (i), we get

$$I_{XX'} = \frac{3}{2}(\rho L)\left(\frac{L}{2\pi}\right)^2$$

$$= \frac{3}{2}(\rho L)\frac{L^2}{4\pi^2}$$

$$= \frac{3}{8}\frac{\rho L^3}{\pi^2}$$

76. *(d)* Given, mass of body $= m$

Angle of projection $= \theta$

At time $t = \frac{v_0 \sin\theta}{g}$ (body will be at peak because it is the half the value of time of flight.)

Angular momentum, $L = m(\mathbf{r} \times \mathbf{v}_0)$

$$= m\left[v_0 \cos\theta\, t\hat{\mathbf{i}} + \left(v_0 \sin\theta t - \frac{1}{2}gt^2\right)\hat{\mathbf{j}}\right] \times [v_0 \cos\theta\,\hat{\mathbf{i}} + (v_0 \sin\theta - gt)\,\hat{\mathbf{j}}]$$

$$= m\, v_0 \cos\theta\, t\left[\frac{-gt}{2}\right]\hat{\mathbf{k}}$$

$$= -\frac{mv_0}{2}\, g\, t^2 \cos\theta\,\hat{\mathbf{k}}$$

77. *(c)* As we know that, artificial satellites are used to study atmosphere, weather forcast, shape and environment of planet, radiations coming from sun but not used to study the shape of the planets.

78. *(c)* Given, depth of lake, $h = 200$ m

Decrease in volume, $\frac{\Delta V}{V} = 0.1\%$

$$= \frac{0.1}{100} = \frac{1}{1000}$$

Since, $p = \rho g h$

where, p is pressure

ρ is density of water $= 10^3$ kg m^{-3}

and g is acceleration due to gravity $= 10$ ms^{-2}

$\therefore \quad p = 10^3 \times 10 \times 200$

$\quad = 2 \times 10^6$ Pa

$\because$ Bulk modulus, $K = \frac{p}{\frac{\Delta V}{V}}$

$$= \frac{2 \times 10^6}{\frac{1}{1000}} = 2 \times 10^9 \text{ Nm}^{-2}$$

79. *(d)* Given, $A_1 = A_2 = A$

$L_1 = L, L_2 = 2L$

Since, we know that,

Thermal resistance, $R = \frac{L}{KA}$

$\therefore$ For plate 1, $R_1 = \frac{L_1}{K_1 A_1} = \frac{L}{KA}$

Plate 2, $R_2 = \frac{L_2}{K_2 A_2} = \frac{2L}{2KA} = \frac{L}{KA}$

Now, series equivalent resistance $= R_1 + R_2$

$$\Rightarrow \quad \frac{3L}{K_{eq} \cdot A} = \frac{L}{KA} + \frac{L}{KA}$$

$$= \frac{2L}{KA}$$

$$\Rightarrow \quad \frac{3}{K_{eq}} = \frac{2}{K}$$

$$\Rightarrow \quad K_{eq} = \frac{3K}{2}$$

Equivalent thermal conductivity, $K_{eq} = \frac{3K}{2}$

80. *(a)* Given, work done, $W = 146$ kJ $= 146 \times 10^3$ J

Number of moles, $n = 1$ kilomole $= 10^3$ mol

Change in temperature, $\Delta T = 7°$C

Universal gas constant, $R = 8.314$ J mol^{-1} K^{-1}

Since, work done in adiabatic condition,

$$W = \frac{nR\Delta T}{\gamma - 1}$$

$$\therefore \quad \gamma - 1 = \frac{nR\Delta T}{W}$$

$$= \frac{1000 \times 8.314 \times 7}{146 \times 10^3}$$

$$= 0.398$$

$$\Rightarrow \quad \gamma = 1 + 0.398$$

$$= 1.398 \simeq 1.4$$

Hence, given gas is diatomic.

81. *(a)* Given, potential energy, $U(x) = K[1 - e^{-x^2}]$

$$-\infty < x < \infty$$

Since,

$$F = -\frac{dU(x)}{dx}$$

and for equilibrium, $F = -\frac{dU(x)}{dx} = 0$

$$= -K\frac{d}{dx}[1 - e^{-x^2}]$$

$$= -K(+2x)e^{-x^2}$$

$$= +K(-2x)e^{-x^2}$$

$$F = -K2xe^{-x^2}$$

$$F \propto -x$$

∴ It is SHM only.

82. *(b)* Given, length of wire, $l = 1.5$ m

Frequency, $f = 175$ Hz

Change in tension, $\Delta T = 3\%$

$$= \frac{3}{100}$$

∵ Frequency ∝ $\sqrt{\text{Tension}}$

$\Rightarrow \quad f \propto \sqrt{T}$

$\Rightarrow \quad \frac{f_2}{f_1} = \sqrt{\frac{T_2}{T_1}} = \sqrt{\frac{100\,T_1 + 3T_1}{100\,T_1}}$

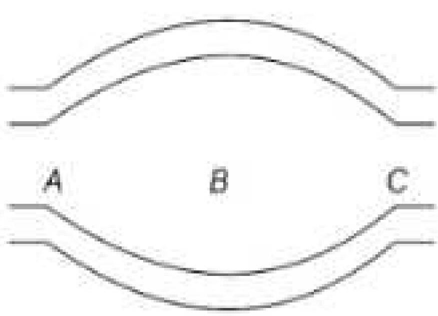
$$= \sqrt{\frac{103}{100}} = \sqrt{1.03} = 1.015$$

$$f_2 = 1.015 f_1$$

$$= \frac{f_2 - f_1}{f_1} = \frac{1.015 f_1 - f_1}{f_1}$$

$$= 0.015 = 1.5\%$$

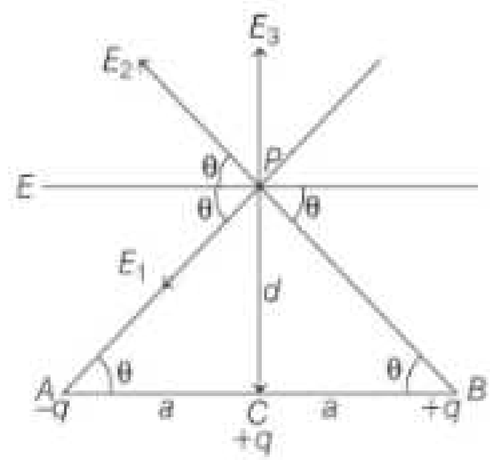
83. *(d)* According to given figure.



As the number of field lines at A and C is same but at B is less.

∴ Field strength at $E_A = E_C < E_B$

84. *(c)* According to given diagram,



Since, dipole moment, $p = q(2a)$

Electric field along X-axis is given as

$\therefore \quad E = E_1 \cos\theta + E_2 \cos\theta$

$$= \frac{2Kqp}{a^2 + d^2}\cos\theta$$

$$= \frac{2Kqp}{a^2 + d^2} \cdot \frac{a}{(a^2 + d^2)^{1/2}}$$

$$= \frac{2Kqa}{(a^2 + d^2)^{3/2}}$$

$$= \frac{2Kqa}{d^3} \qquad [\text{For } d >> a]$$

Electric field at point P due to charge q present at middle point C, $E_3 = K \cdot \frac{q}{d^2}$

∴ Net electric field at P,

$$E_{net} = \sqrt{E^2 + E_3^2}$$

$$= \sqrt{\left(\frac{2Kqa}{d^3}\right) + \left(\frac{Kq}{d^2}\right)^2}$$

$$= \frac{Kq}{d^3}\sqrt{d^2 + 4a^2}$$

$$= \frac{q}{4\pi\varepsilon_0 d^3}\sqrt{d^2 + 4a^2}$$

85. *(b)* Given, linear charge density, $\lambda = 4 \times 10^{-4}$ C/m

Charge of dipole, $q = \pm\, 2 \times 10^{-8}$ C

Separation distance, $d = 2 \times 10^{-3}$ m

Since, force, $F = qE$

where, E is electric field.

$$E_1 = \frac{\lambda}{2\pi\varepsilon_0 r_1} \times \frac{2}{2}$$

due to charge at A

$\therefore \quad E_1 = \dfrac{9 \times 10^9 \times 4 \times 10^{-4} \times 2}{0.02}$

$(\because r = 2\,\text{cm} = 0.02\,\text{m})$

$= 3.6 \times 10^8\ \text{NC}^{-1}$

and force, $F_1 = (3.6 \times 10^8 \times 2 \times 10^{-8})$

$= 7.2\,\text{N}$ towards L

Similarly, electric field at $B(E_2) = \dfrac{2\lambda}{4\pi\varepsilon_0 r_2}$

$= \dfrac{2 \times 9 \times 10^9 \times 4 \times 10^{-4}}{(0.02 + 0.002)}$

$= 3.3 \times 10^8\ \text{NC}^{-1}$

$\therefore \quad F_2 = 3.3 \times 10^8 \times 2 \times 10^{-8} = 6.6\,\text{N}$

Hence, net force on dipole, $F_{\text{net}} = F_1 - F_2$

$= 7.2 - 6.6 = 0.6\,\text{N}$ away from L

86. *(a)* Given, coloured bands given on resistor are brown, yellow, green and gold.

As we know that,

colour code of resistor

Brown = 1

Yellow = 4

Green = 10^5

Gold = 5%

$\therefore$ Resistor $= 14 \times 10^5 \pm 5\%$

$= 1.4 \times 10^6 \pm 5\%$

$= (1.4 \pm 0.07)\,\text{M}\Omega$

87. *(d)* Given, energy developed by resistor,

$U_1 = 200\,\text{J}$

Time, $t_1 = 5\,\text{s}$

Current, $I_1 = 1\,\text{A}$

If increased current, $I_2 = 3\,\text{A}$, then

developed energy be (U_2)

Since,

$U = I^2RT$

$\therefore \quad U_1 = I_1^2Rt_1$

$\Rightarrow \quad R = \dfrac{U_1}{I_1^2 t_1} = \dfrac{200}{1^2 \times 5} = 40\,\Omega$

and $\quad U_2 = I_2^2Rt_2$

$= 3^2 \times 40 \times 5$

$= 9 \times 200 = 1800\,\text{J}$

88. *(b)* Given, capacitance of capacitor,

$C = 10\,\mu\text{F} = 10 \times 10^{-6}\,\text{F}$

Charge, $Q = 40\,\mu\text{C}$

$= 40 \times 10^{-6}\,\text{C}$

Resistance, $R = 2.5\,\Omega$

$t_1 = 25\mu\text{-s}$, $t_2 = 50\mu\text{-s}$

Let energy dissipated be U

$\therefore \quad dU = I^2Rdt$

where, I = current.

$= \dfrac{Q}{RC} e^{-t/\tau}$

where, τ = time constant.

$= RC$

$= 2.5 \times 10 \times 10^{-6}$

$= 25 \times 10^{-6}$

$\therefore \quad \int dU = \int_{25}^{50} \left(\dfrac{Q}{RC} e^{-t/\tau} \right)^2 R\,dt$

$= \int_{25}^{50} \dfrac{Q^2}{R^2C^2} e^{-2t/\tau} \cdot R dt$

$= \int_{25}^{50} \dfrac{Q^2}{RC^2} \cdot e^{-2t/\tau}\,dt$

$= \dfrac{Q^2}{RC^2} \int_{25}^{50} e^{-2t/\tau}\,dt$

$= \dfrac{Q^2}{RC^2} \left(\dfrac{e^{-2t/\tau}}{-2/\tau} \right)_{25}^{50}$

$= -\dfrac{Q^2}{RC^2} \times \dfrac{RC}{2} e^{-2t/\tau} \Big|_{25}^{50}$

$= -\dfrac{Q^2}{2C} \left(e^{-\frac{2 \times 50}{25}} - e^{-2 \times \frac{25}{25}} \right)$

$= \dfrac{(40 \times 10^{-6})^2}{2 \times (10 \times 10^{-6})} (e^{-4} - e^{-2})$

$= \dfrac{40 \times 10^{-6} \times 40 \times 10^{-6}}{20 \times 10^{-6}}$

$= 80 \times (e^{-4} - e^{-2}) \times 10^{-6}$

$= 9.36 \times 10^{-6}\,\text{J}$

$= 9.36\mu\,\text{J} = 9.4\,\mu\text{J}$

89. *(c)* Magnetic susceptibility of diamagnetic substance is small and negative. Its relative permeability is slightly less than 1.

90. *(d)* Given, initial orbit, $n_1 = 4$

Final orbit, $n_2 = 2$

Work-function, $W_0 = 1.9$ eV

According to Einstein's photoelectric equation,

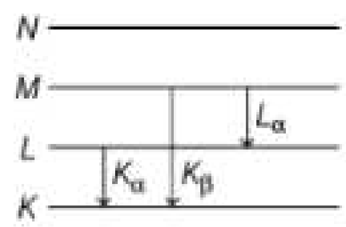
$$KE = E - W_0$$
$$= 13.6\left(\frac{1}{n_2^2} - \frac{1}{n_1^2}\right) - W_0$$
$$\therefore \quad KE = 13.6\left(\frac{1}{2^2} - \frac{1}{4^2}\right) - 1.9$$
$$= 13.6\left(\frac{1}{4} - \frac{1}{16}\right) - 1.9$$
$$= \frac{40.8}{16} - 1.9$$
$$= 2.55 - 1.9$$
$$= 0.65 \text{ eV}$$

91. *(a)* Given, frequencies are ν_{K_β}, ν_{K_α} and ν_{L_α}

Kinetic energies be $K_\beta, K_\alpha, L_\alpha$

As we know that,

acccording to energy diagram of K, L, M and N shell,

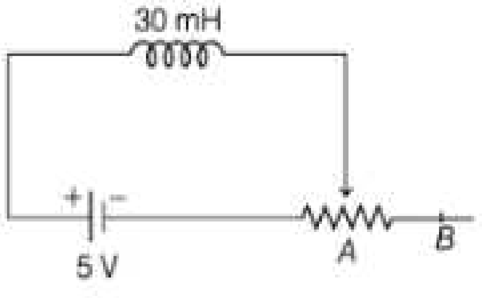


$$\therefore \quad K_\alpha = K_\beta - L_\alpha$$
$$\Rightarrow \quad h\nu_{K_\alpha} = h\nu_{K_\beta} - h\nu_{L_\alpha}$$
$$\Rightarrow \quad \nu_{K_\alpha} = \nu_{K_\beta} - \nu_{L_\alpha}$$
$$\Rightarrow \quad \nu_{K_\beta} = \nu_{K_\alpha} + \nu_{L_\alpha}$$

92. *(c)* Given, resistance of circuit, $R = 10\ \Omega$

Inductance, $L = 30 \text{ mH} = 30 \times 10^{-3}$ H

Supply voltage = 5 V



By using Ohm's law,

current, $i = \frac{V}{R} = \frac{5}{10} = 0.5$ A

and by using Kirchhoff's voltage law,

$$5 - L\frac{di}{dt} - iR = 0$$
$$\Rightarrow \quad \text{Current } (i) = \frac{5 - L\frac{di}{dt}}{R}$$
$$= \frac{5 - 30 \times 10^{-3}\frac{di}{dt}}{10}$$
$$= 0.5 - 3 \times 10^{-3}\frac{di}{dt}$$

Now, if R increases, –ve part in RHS decreases.

$\therefore$ i increases, so $i > 0.5$ A.

93. *(c)* Given, stored energy, $E = 20$ J

Power, $P = 200$ W

Current, $I = 2$ A

Let time constant be τ

and inductance be L.

$$\because \quad E = \frac{1}{2}LI^2 \quad \text{...(i)}$$
$$\therefore \quad L = \frac{2E}{I^2}$$
$$= \frac{2 \times 20}{2^2}$$
$$= 10 \text{ H} \quad \text{...(i)}$$
$$\because \quad \tau = \frac{L}{R} \quad \text{...(ii)}$$

where, R = resistance

and $\quad P = I^2R$

$$\therefore \quad R = \frac{P}{I^2}$$
$$= \frac{200}{2^2}$$
$$= \frac{200}{4} = 50\ \Omega \quad \text{...(iii)}$$

From Eqs. (i), (ii) and (iii), we get

$$\tau = \frac{10}{50} = \frac{1}{5} = 0.2 \text{ s}$$

94. *(c)* Given, length of laser beam,
$l = 90 \text{ cm} = 90 \times 10^{-2} \text{ m}$

Operating power, $P = 3\text{m W}$

$$= 3 \times 10^{-3} \text{ W}$$

Let stored energy be E.

Since, energy = power (P) × time (t)

and speed of light (c) = $\frac{\text{wavelength } (\lambda)}{\text{time } (t)}$

$$\Rightarrow \text{Time } (t) = \frac{90 \times 10^{-2}}{3 \times 10^{8}}$$

$$= 30 \times 10^{-10} \text{ s}$$

$$= 3 \times 10^{-9} \text{ s}$$

$$\therefore \text{Energy} = 3 \times 10^{-3} \times 3 \times 10^{-9}$$

$$= 9 \times 10^{-12} \text{ J}$$

95. *(a)* Given, wavelength, $\lambda = 242$ nm

$= 242 \times 10^{-9}$ m

Since, linear momentum, $p = mv = \frac{h}{\lambda}$

where, m is mass of hydrogen,

v is speed of hydrogen atom,

$h = 6.63 \times 10^{-34}$ J-s

$$\therefore \qquad v = \frac{h}{m\lambda}$$

$$= \frac{6.63 \times 10^{-34}}{1.67 \times 10^{-27} \times 242 \times 10^{-9}}$$

$$= 0.0164 \times 10^{2}$$

$$= 1.64 \text{ ms}^{-1}$$

$$\simeq 1.6 \text{ m/s}$$

96. *(b)* Given, energy of hydrogen in 1st excited state
$= 41$ eV

Since, energy for excitation in hydrogen atom
$$= \frac{13.6}{n^2} Z^2$$

$\therefore$ Energy in $n = 1$

$$\Rightarrow \qquad E_1 = \frac{13.6}{1^2} Z^2$$

and energy in $n = 2$

$$\Rightarrow \qquad E_2 = \frac{13.6}{2^2} Z^2$$

$\therefore$ Energy remain in excited state = 41 eV

$$\therefore \qquad 41 = 13.6\left(\frac{1}{1} - \frac{1}{4}\right) Z^2$$

$$\Rightarrow \qquad 3 = \left(\frac{3}{4}\right) Z^2$$

$$\Rightarrow \qquad Z = 2$$

$$\therefore \qquad E_1 = 13.6 \times 2^2$$

$$= 54.4 \text{ eV} \simeq 54 \text{ eV}$$

97. *(c)* As we know that, iron (Fe^{56}) is most stable mass number because of lowest mass per nucleon.

$\therefore$ $50 < A < 80$ will be most stable.

98. *(c)* When α-rays strike on fluorescent material, then it produce scientillation, hence option (c) is not correct.

(a) α-particles have low penetrating power.

(b) Also have speed 5000 km s^{-1}

$$= 5 \times 10^{6} \text{ ms}^{-1}$$

(d) It can also ionise gases.

$\therefore$ Option c is incorrect.

99. *(b)* Given, wavelength, $\lambda = 6000$ Å

$$= 6000 \times 10^{-10} \text{ m}$$

$$= 6 \times 10^{-7} \text{ m}$$

Distance, $a = \frac{200}{2}$ inch

$$= 100 \times 0.0254 \text{ m}$$

$$= 2.54 \text{ m}$$

$\because$ Limit of resolution of telescope, $\Delta\theta = 1.22 \frac{\lambda}{a}$

$$= 1.22 \times \frac{6 \times 10^{-7}}{2.54}$$

$$= 2.88 \times 10^{-7} \text{ rad}$$

$$= 2.9 \times 10^{-7} \text{ rad}$$

100. *(c)* As we know that, Zener diode can work in reverse bias condition and Zener voltage is steady and constant.

Hence, zener diode () is used as voltage regulator.

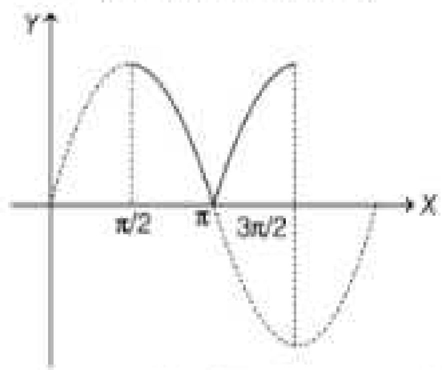
Zener diode also called breakdown diode.

Mathematics

101. *(b)* $|y| = |\sin x|$ between $x = \frac{\pi}{2}$ to $x = \frac{3\pi}{2}$

$\therefore$ Area $= \int_{\pi/2}^{\pi} \sin x \, dx + \int_{\pi}^{3\pi/2} -(\sin x)\, dx$

$$= -[\cos x]_{\pi/2}^{\pi} + [\cos x]_{\pi}^{3\pi/2}$$



$$= -\left[\cos \pi - \cos\frac{\pi}{2}\right] + \left[\cos\frac{3\pi}{2} - \cos \pi\right] = 1 + 1 = 2$$

102. *(c)* $A = [0, 1]$, $B = [2, 5]$

$f : A \to B$ and $f(x) = 3x + 2, \forall x \in A$

One-One

Let $f(x_1) = f(x_2)$

$$3x_1 + 2 = 3x_2 + 2 \Rightarrow x_1 = x_2$$

Hence, f is one-one.

Onto : Let $f(x) = y$

$$3x + 2 = y \Rightarrow x = \frac{y - 2}{3}$$

which is defined for all value of $y \in [2, 5]$

$\therefore f$ is onto.

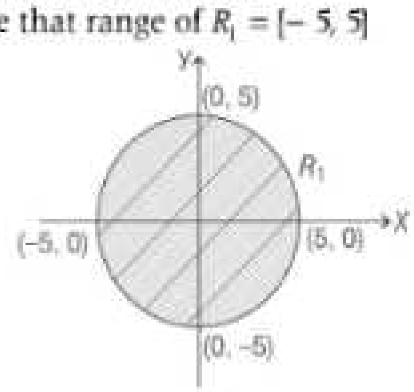
Thus function is one-one and onto.

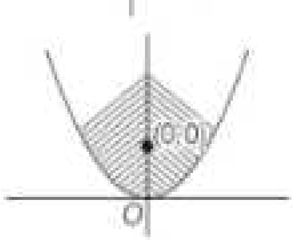
103. *(d)* $R_1 = \{(x, y) : x^2 + y^2 \le 25\}$

$$R_2 = \left\{(x, y) : y \ge \frac{4x^2}{9}\right\}$$

Shaded region represents R_1.

We can see that range of $R_1 = [-5, 5]$





$$y \ge \frac{4x^2}{9} \Rightarrow x^2 \le \frac{9}{4} y$$

For shaded region; at (0, 1)

$$\Rightarrow 0^2 \le \frac{9}{4} \times 1 \Rightarrow 0 \le \frac{9}{4} \text{ (true)}$$

Range of R_2 from the figure $= [0, \infty)$

$\therefore R_1 \cup R_2 = [-5, 5] \cup [0, \infty) = [-5, \infty)$

104. *(b)*

$$f(x) = \begin{cases} (x^2 - 4)(x^2 - 5x + 6) - \sin x; & x \in (-\infty, 0) \\ (x^2 - 4)(x^2 - 5x + 6) + \sin x; & x \in [0, 2] \cup [3, \infty) \\ -(x^2 - 4)(x^2 - 5x + 6) + \sin x; & x \in (2, 3) \end{cases}$$

$$\therefore f'(x) = \begin{cases} (2x - 0)(x^2 - 5x + 6) + (2x - 5)(x^2 - 4) - \cos x; & x \in (-\infty, 0) \\ 2x(x^2 - 5x + 6) + (2x - 5)(x^2 - 4) + \cos x; & x \in [0, 2] \cup [3, \infty) \\ -2x(x^2 - 5x + 6) - (2x - 5)(x^2 - 4) + \cos x; & x \in (2, 3) \end{cases}$$

At $x = 0$

LHD $f'(0^-) = 0 + (-5)(-4) - 1 = 19$

RHS $f'(0^+) = 0 + (-5)(-4) + 1 = 21$

$\because$ LHD $\ne$ RHD at $x = 0$.

$\therefore f(x)$ is not differentiable at $x = 0$.

105. *(c)* $\alpha, \beta \ne 0$ and $f(x) = \alpha^n + \beta^n$

$$\text{Let } \Delta = \begin{vmatrix} 3 & 1 + f(1) & 1 + f(2) \\ 1 + f(1) & 1 + f(2) & 1 + f(3) \\ 1 + f(2) & 1 + f(3) & 1 + f(4) \end{vmatrix}$$

$$= \delta(1 - \alpha)^2 (1 - \beta)^2 (\alpha - \beta)^2$$

$$\Delta = \begin{vmatrix} 3 & 1 + \alpha + \beta & 1 + \alpha^2 + \beta^2 \\ 1 + \alpha + \beta & 1 + \alpha^2 + \beta^2 & 1 + \alpha^3 + \beta^3 \\ 1 + \alpha^2 + \beta^2 & 1 + \alpha^3 + \beta^3 & 1 + \alpha^4 + \beta^4 \end{vmatrix}$$

$$\Delta = \begin{vmatrix} 1\cdot 1 + 1\cdot 1 + 1\cdot 1 & 1\cdot 1 + 1\cdot\alpha + 1\cdot\beta & 1\cdot 1 + 1\cdot\alpha^2 + 1\cdot\beta^2 \\ 1\cdot 1 + 1\cdot\alpha + 1\cdot\beta & 1\cdot 1 + \alpha\cdot\alpha + \beta\cdot\beta & 1\cdot 1 + \alpha\cdot\alpha^2 + \beta\cdot\beta^2 \\ 1\cdot 1 + 1\cdot\alpha^2 + 1\cdot\beta^2 & 1\cdot 1 + \alpha^2\cdot\alpha + \beta^2\cdot\beta & 1\cdot 1 + \alpha^2\cdot\alpha^2 + \beta^2\cdot\beta^2 \end{vmatrix}$$

$$= \begin{vmatrix} 1 & 1 & 1 \\ 1 & \alpha & \beta \\ 1 & \alpha^2 & \beta^2 \end{vmatrix} \cdot \begin{vmatrix} 1 & 1 & 1 \\ 1 & \alpha & \alpha^2 \\ 1 & \beta & \beta^2 \end{vmatrix}$$

$$= [(1 - \alpha)(\alpha - \beta)(\beta - 1)]^2 = (1 - \alpha)^2 (1 - \beta)^2 (\alpha - \beta)^2$$

$\therefore \delta = 1$

106. *(d)* $\begin{vmatrix} a^2+\lambda^2 & ab+c\lambda & ca-b\lambda \\ ab-c\lambda & b^2+\lambda^2 & bc+a\lambda \\ ac+b\lambda & bc-a\lambda & c^2+\lambda^2 \end{vmatrix} \cdot \begin{vmatrix} \lambda & c & -b \\ -c & \lambda & a \\ b & -a & \lambda \end{vmatrix}$

Let $\Delta = \begin{vmatrix} \lambda & c & -b \\ -c & \lambda & a \\ b & -a & \lambda \end{vmatrix}$

Let's find out the cofactors of all the elements of determinant Δ.

$$A_{11} = \begin{vmatrix} \lambda & a \\ -a & \lambda \end{vmatrix} = \lambda^2 + a^2$$

$$A_{12} = -\begin{vmatrix} -c & a \\ b & \lambda \end{vmatrix} = ab + c\lambda$$

$$A_{13} = \begin{vmatrix} -c & \lambda \\ b & -a \end{vmatrix} = ca - b\lambda$$

$$A_{21} = -\begin{vmatrix} c & -b \\ -a & \lambda \end{vmatrix} = ab - c\lambda$$

$$A_{22} = \begin{vmatrix} \lambda & -b \\ b & \lambda \end{vmatrix} = \lambda^2 + b^2$$

$$A_{23} = -\begin{vmatrix} \lambda & c \\ b & -a \end{vmatrix} = bc + a\lambda$$

$$A_{31} = \begin{vmatrix} c & -b \\ \lambda & a \end{vmatrix} = ac + b\lambda$$

$$A_{32} = -\begin{vmatrix} \lambda & -b \\ -c & a \end{vmatrix} = bc - a\lambda$$

$$A_{33} = \begin{vmatrix} \lambda & c \\ -c & \lambda \end{vmatrix} = c^2 + \lambda^2$$

We can see that,
Given determinant

$\Delta' = \begin{vmatrix} a^2+\lambda^2 & ab+c\lambda & ca-b\lambda \\ ab-c\lambda & b^2+\lambda^2 & bc+a\lambda \\ ac+b\lambda & bc-a\lambda & c^2+\lambda^2 \end{vmatrix}$ is the determinant of cofactors of element of Δ.

Since, we know that $\Delta' = \Delta^2$

Hence,

$$\begin{vmatrix} a^2+\lambda^2 & ab+c\lambda & ca-b\lambda \\ ab-c\lambda & b^2+\lambda^2 & bc+a\lambda \\ ac+b\lambda & bc-a\lambda & c^2+\lambda^2 \end{vmatrix} \times \begin{vmatrix} \lambda & c & -b \\ -c & \lambda & a \\ b & -a & \lambda \end{vmatrix}$$

$$= \begin{vmatrix} \lambda & c & -b \\ -c & \lambda & a \\ b & -a & \lambda \end{vmatrix}^2 \times \begin{vmatrix} \lambda & c & -b \\ -c & \lambda & a \\ b & -a & \lambda \end{vmatrix}$$

$$= \begin{vmatrix} \lambda & c & -b \\ -c & \lambda & a \\ b & -a & \lambda \end{vmatrix}^3 = \lambda^3(\lambda^2 + a^2 + b^2 + c^2)^3$$

107. *(b)* Let $A = \begin{bmatrix} 0 & 2\beta & v \\ \alpha & \beta & -v \\ \alpha & -\beta & v \end{bmatrix}$

$\because A$ is orthogonal $\Rightarrow AA^T = I$, where A^T is transpose of A.

$$\Rightarrow \begin{bmatrix} 0 & 2\beta & v \\ \alpha & \beta & -v \\ \alpha & -\beta & v \end{bmatrix} \begin{bmatrix} 0 & \alpha & \alpha \\ 2\beta & \beta & -\beta \\ v & -v & v \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 4\beta^2 + v^2 & 2\beta^2 - v^2 & -2\beta^2 + v^2 \\ 2\beta^2 - v^2 & \alpha^2 + \beta^2 + v^2 & \alpha^2 - \beta^2 - v^2 \\ -2\beta^2 + v^2 & \alpha^2 - \beta^2 - v^2 & \alpha^2 + \beta^2 + v^2 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\Rightarrow \quad 4\beta^2 + v^2 = 1,\ 2\beta^2 - v^2 = 0,\ -2\beta^2 + v^2 = 0$$

$$\alpha^2 - \beta^2 - v^2 = 0,\ \alpha^2 + \beta^2 + v^2 = 1$$

$$\therefore \quad \alpha = \pm\frac{1}{\sqrt{2}},\ \beta = \pm\frac{1}{\sqrt{6}},\ v = \pm\frac{1}{\sqrt{3}}$$

$$\therefore \quad 2\alpha^2 + \beta^2 + 3v^2 = 2\left(\frac{1}{2}\right) + \frac{1}{6} + 3\left(\frac{1}{3}\right)$$

$$= 1 + \frac{1}{6} + 1 = 13/6$$

108. *(b)* Given, $|A|_{3\times 3} \neq 0$

$$AA^T = A^TA \text{ and } B = A^{-1}A^T$$

$$\therefore \quad BB^T = (A^{-1}A^T)(A^{-1}A^T)^T$$

$$= (A^{-1}A^T)[(A^T)^T(A^{-1})^T]$$

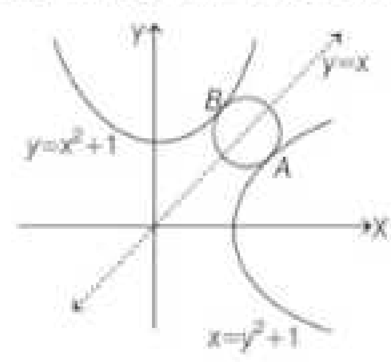
$$= (A^{-1}A^T)[A(A^T)^{-1}]$$

$$= A^{-1}(A^TA)(A^T)^{-1}$$

$$= A^{-1}A\,A^T(A^T)^{-1} = I \cdot I = I$$

109. *(a)* $\because$ Parabola $y = x^2 + 1$ and $x = y^2 + 1$ are symmetrical about $y = x$.

Therefore, tangent at point A is parallel to $y = x$



$$\therefore \quad \frac{dy}{dx} = \frac{d}{dx}(x^2 + 1) = 2x$$

$\because$ Slope of $y = x$ is 1.

$\Rightarrow \quad 2x = 1$

$\Rightarrow \quad x = \frac{1}{2}$ and $y = \left(\frac{1}{2}\right)^2 + 1 = \frac{5}{4}$

$\therefore \quad A = \left(\frac{1}{2}, \frac{5}{4}\right)$ and $B = \left(\frac{5}{4}, \frac{1}{2}\right)$

$\therefore$ Radius of the circle $= \frac{1}{2}\sqrt{\left(\frac{1}{2} - \frac{5}{4}\right)^2 + \left(\frac{5}{4} - \frac{1}{2}\right)^2}$

$= \frac{3}{8}\sqrt{2}$

$\therefore$ Area of circle $= \pi \cdot \left(\frac{3}{8}\sqrt{2}\right)^2 = \frac{9\pi}{32}$ sq unit

110. *(b)* Given equation of rectangular hyperbola is

$$xy = 18 \quad \ldots \text{(i)}$$

On comparing Eq. (i) with general equation of rectangular Hyperbola

$xy = \frac{a^2}{2} \Rightarrow \frac{a^2}{2} = 18 \Rightarrow a^2 = 36 \Rightarrow a = 6$

$\therefore$ Length of the transverse axis $= 2a = 12$

111. *(c)* Let the point be (h, k).

Then, equation of the chord of contact is

$$hx + ky = 4 \quad \ldots \text{(i)}$$

Since, Eq. (i) is tangent to $xy = 1$

$\therefore \quad x\left(\frac{4 - hx}{k}\right) = 1$ has two equal roots

$\Rightarrow \quad hx^2 - 4x + k = 0$

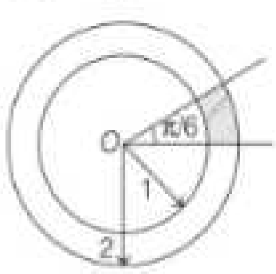
Discriminant = 0

$\Rightarrow \quad 16 - 4hk = 0 \Rightarrow hk = 4$

Thus, the locus of (h, k) is $xy = 4$

Hence, it is a hyperbola.

112. *(d)* Shaded region represents the Area bounded by the circles $x^2 + y^2 = 1$, $x^2 + y^2 = 4$ and the pair of lines $\sqrt{3}(x^2 + y^2) = 4xy$



Angle between the lines $\tan\theta = \frac{2\sqrt{2^2 - 3}}{\sqrt{3} + \sqrt{3}} = \frac{1}{\sqrt{3}}$

$\therefore \quad \theta = \frac{\pi}{6}$

$\therefore$ Shaded Area $= \frac{\pi/6}{2\pi}(2^2 - 1^2) = \frac{\pi}{4}$

113. *(b)* Given, $f(x) = x^3 + x^2 + 100x + 7\sin x$

$\therefore \quad f'(x) = 3x^2 + 2x + 100 + 7\cos x > 0, \forall x \in R$

$\therefore \quad f(x)$ is increasing function.

$\Rightarrow \quad f(1) < f(2) < f(3)$

Let $\quad \alpha = f(1), \beta = f(2), \gamma = f(3)$

Then, $\alpha < \beta < \gamma \quad \ldots \text{(i)}$

Given equation $\frac{1}{y - \alpha} + \frac{2}{y - \beta} + \frac{3}{y - \gamma} = 0$

$\Rightarrow \quad (y - \beta)(y - \gamma) + 2(y - \alpha)(y - \gamma) + 3(y - \alpha)(y - \beta) = 0 \quad \ldots \text{(ii)}$

Let $g(y) = (y - \beta)(y - \gamma) + 2(y - \alpha)(y - \gamma) + 3(y - \alpha)(y - \beta)$

Then, $g(\alpha) = (\alpha - \beta)(\alpha - \gamma) > 0 \quad \ldots \text{(iii)}$

$g(\beta) = 2(\beta - \alpha)(\beta - \gamma) < 0$

and $\quad g(\gamma) = 3(\gamma - \alpha)(\gamma - \beta) > 0 \quad \ldots \text{(iv)}$

{from Eq. (i)}

From Eqs. (iii) and (iv), we get

$g(y) = 0$ has one real root between α and β and other between β and γ.

$\Rightarrow$ Given equation has exactly one root lying in $(f(2), f(3))$.

114. *(b)* Given, $f(x) = 2e^x - ae^{-x} + (2a + 1)x - 3$ is increasing for all x.

$\Rightarrow \quad f'(x) > 0 \Rightarrow 2e^x + ae^{-x} + 2a + 1 > 0$

$\Rightarrow \quad 2e^{2x} + a + (2a + 1)e^x > 0$

Let $\quad e^x = y$

$\Rightarrow \quad 2y^2 + (2a + 1)y + a > 0$

$\Rightarrow \quad 2\left[y^2 + \left(a + \frac{1}{2}\right)y + \frac{a}{2}\right] > 0$

$\Rightarrow \quad (y + a)\left(y + \frac{1}{2}\right) > 0$

$\Rightarrow \quad (e^x + a)\left(e^x + \frac{1}{2}\right) > 0$

↓

Positive

For $f(x)$ to be increasing

$$e^x + a > 0, \forall x \in R \Rightarrow a \in (0, \infty)$$

115. *(c)* Tickets marked in a bag are 0, 1, 10, 11.

Let A be the event that the sum of numbers on the tickets drawn is 23.

If S is the sample space, then $n(S) = 4^5 = 1024$

$n(A) =$ Coefficient of x^{22} in $(1 + x + x^{10} + x^{11})^5$

$=$ Coefficient of x^{22} in $[(1 + x)(1 + x^{10})]^5$

= Coefficient of x^{22} in $(1 + 5x^{10} + 10x^{20} + 10x^{30} + 5x^{40} + x^{50})$

$(1 + 5x + 10x^2 + 10x^3 + 5x^4 + x^5) = 100$

$\therefore \quad P(A) = \frac{n(A)}{n(S)} = \frac{100}{1024} = \frac{25}{256}$

116. *(b)* $f(x) = \log_2\left(\frac{\sin x - \cos x + 3\sqrt{2}}{\sqrt{2}}\right)$

$= \log_2\left(\frac{\sin x - \cos x}{\sqrt{2}} + 3\right)$

$f(x) = \log_2\left(\sin\left(x - \frac{\pi}{4}\right) + 3\right)$

$\because \quad -1 \le \sin\left(x - \frac{\pi}{4}\right) \le 1$

$-1 + 3 \le \sin\left(x - \frac{\pi}{4}\right) + 3 \le 1 + 3$

$2 \le \sin\left(x - \frac{\pi}{4}\right) + 3 \le 4$

$\log_2 2 \le \log_2\left(\sin\left(x - \frac{\pi}{4}\right) + 3\right) \le \log_2 2^2$

$1 \le \log_2\left(\sin\left(x - \frac{\pi}{4}\right) + 3\right) \le 2$

$\therefore$ Range = [1, 2]

117. *(*)* Let $\frac{x-1}{3} = \frac{y-2}{2} = \frac{z-3}{1} = k$

$\Rightarrow \quad x = 3k + 1,\ y = 2k + 2,\ z = k + 3$

$\therefore$ Point on the line $A = (3k + 1, 2k + 2, k + 3)$

$\because$ Line intersects the curve $\Rightarrow$ point A will satisfy the equation of curve

$x^2 + y^2 + x - y + C = 0$

$(3k + 1)^2 + (2k + 2)^2 + (3k + 1) - (2k + 2) + C = 0$

$\Rightarrow \quad 9k^2 + 1 + 6k + 4k^2 + 4 + 8k + 3k + 1 - 2k - 2 + C = 0$

$\Rightarrow \quad 13k^2 + 15k + 4 + C = 0 \quad \ldots(i)$

Intersection of line $\frac{x-1}{3} = \frac{y-2}{2} + \frac{z-3}{1}$ and $z = 0$

$\therefore \quad k + 3 = 0 \ \Rightarrow\ k = -3$

From Eq. (i), we get

$13(-3)^2 + 15(-3) + 4 + c = 0$

$\Rightarrow \quad 117 - 45 + 4 + c = 0$

$\therefore \quad c = -76$

118. *(*)* Unit vector perpendicular to $(2\hat{i} + \hat{j} - \hat{k})$ and $(\hat{i} - \hat{j} + 2\hat{k})$ is $\frac{(2\hat{i} + \hat{j} - \hat{k}) \times (\hat{i} - \hat{j} + 2\hat{k})}{|2\hat{i} + \hat{j} - \hat{k}||\hat{i} - \hat{j} + 2\hat{k}|} \quad \ldots(i)$

Consider,

$(2\hat{i} + \hat{j} - \hat{k}) \times (\hat{i} - \hat{j} + 2\hat{k}) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 1 & -1 \\ 1 & -1 & 2 \end{vmatrix}$

$= \hat{i} - 5\hat{j} - 3\hat{k}$

$\therefore$ Unit vector $= \frac{\hat{i} - 5\hat{j} - 3\hat{k}}{\sqrt{6}\sqrt{6}} = \frac{1}{6}(\hat{i} - 5\hat{j} - 3\hat{k})$

119. *(d)* $\frac{x - 2a - 3}{x - a + 2} < 0\ x \in [1, 2]$

$\Rightarrow \quad \frac{x - (2a + 3)}{x - (a - 2)} < 0$

When $2a + 3 > a - 2$

$\because \quad x \in [1, 2]$

$2a + 3 > 2$ and $a - 2 < 1$

$\Rightarrow \quad 2a > -1;\ a < 3 \ \Rightarrow a > \frac{-1}{2}, a < 3$

When $2a + 3 < a - 2$

$\Rightarrow \quad a - 2 \ge 2$ and $2a + 3 \le 1$

$\Rightarrow \quad a \ge 4$ and $a \le -1$ not possible

Hence, $a > \frac{-1}{2}$ and $a < 3$ is the solution.

$a \in \left(\frac{-1}{2}, 3\right)$

120. *(b)* Given, $a = \frac{\hat{i} + \hat{j}}{\sqrt{2}}$ and $b = \frac{\hat{i} - \hat{j} + \hat{k}}{\sqrt{3}}$

$\therefore \quad a \times b = \frac{1}{\sqrt{6}}\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & 0 \\ 1 & -1 & 1 \end{vmatrix}$

$= \frac{1}{\sqrt{6}}(\hat{i} - \hat{j} - 2\hat{k})$

$a + b = \frac{(\sqrt{3} + \sqrt{2})\hat{i} + (\sqrt{3} - \sqrt{2})\hat{j} + \sqrt{2}\hat{k}}{\sqrt{6}}$

$a - b = \frac{(\sqrt{3} - \sqrt{2})\hat{i} + (\sqrt{3} + \sqrt{2})\hat{j} - \sqrt{2}\hat{k}}{\sqrt{6}}$

$\therefore \quad [(a \times b) \times (a - b)]$

$= \frac{1}{6}\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -1 & -2 \\ \sqrt{3} - \sqrt{2} & \sqrt{3} + \sqrt{2} & -\sqrt{2} \end{vmatrix}$

$= \frac{1}{6}[\hat{i}(\sqrt{2} + 2\sqrt{3} + 2\sqrt{2}) - \hat{j}(-\sqrt{2} + 2\sqrt{3} - 2\sqrt{2}) + \hat{k}(\sqrt{3} + \sqrt{2} + \sqrt{3} - \sqrt{2})]$

$[(a \times b) \times (a - b)] = \frac{1}{6}[(3\sqrt{2} + 2\sqrt{3})\hat{i} - \hat{j}(2\sqrt{3} - 3\sqrt{2}) + 2\sqrt{3}\hat{k}]$

$\therefore \quad (\mathbf{a}+\mathbf{b})\cdot[(\mathbf{a}\times\mathbf{b})\times(\mathbf{a}-\mathbf{b})]$

$$= \frac{1}{6\sqrt{6}}[3\sqrt{2}+2\sqrt{3})(\sqrt{3}+\sqrt{2}) - (2\sqrt{3}-3\sqrt{2})(\sqrt{3}-\sqrt{2})+2\sqrt{6}]$$

$$= \frac{1}{6\sqrt{6}}[3\sqrt{6}+6+6+2\sqrt{6}-6+3\sqrt{6} + 2\sqrt{6}-6+2\sqrt{6}]$$

$$= \frac{12\sqrt{6}}{6\sqrt{6}} = 2$$

121. *(a)* $\log_3 2, \log_3(2^x-5)$ and $\log_3(2^x-7/2)$ are in AP.

$\therefore \quad 2\log_3(2^x-5) = \log_3 2 + \log_3(2^x-7/2)$

$\log_3(2^x-5)^2 = \log_3 2(2^x-7/2)$

$2^{2x}+25-10\cdot 2^x = 2\cdot 2^x - 7$

Let $\quad 2^x = y$

$\therefore \quad y^2-10y-2y+25+7=0$

$\Rightarrow \quad y^2-12y+32=0$

$\Rightarrow \quad (y-4)(y-8)=0$

$\Rightarrow \quad y=4$ or $y=8 \Rightarrow 2^x=2^2$ or $2^x=2^3$

$\Rightarrow \quad x=2 \quad x=3$

$x=2$ is not possible because 2^x-5 will be negative.

Hence, $x=3$.

122. *(c)* For a differential equation, order always exist but degree may not exist in many cases.

123. *(a)* Given, $P(\bar{A})=0.3, P(B)=0.4, P(A\cap\bar{B})=0.5$

$\because \quad P(A\cup\bar{B}) = P(A)+P(\bar{B})-P(A\cap\bar{B})$

$= \{1-0.3\}+\{1-0.4\}-0.5$

$= 0.7+0.6-0.5 = 0.8$

$\because \quad P(A\cap\bar{B}) = P(A)-P(A\cap B)$

$0.5 = 0.7 - P(A\cap B)$

$\Rightarrow \quad P(A\cap B) = 0.2$

$$\therefore \quad P[B\,|\,(A\cup\bar{B})] = \frac{P[B\cap(A\cup\bar{B})]}{P(A\cup\bar{B})} = \frac{P(A\cap B)}{P(A\cup\bar{B})} = \frac{0.2}{0.8} = \frac{1}{4}$$

124. *(d)* Given that, the probability that a man aged x dies in a year $= p$

$\Rightarrow$ The probability that at least one man dies in a years $= 1-(1-p)^n$

Probability that out of n men, M_1 is first to die $= \frac{1}{n}$

Since, this event is independent.

Hence, the probability that M_1 dies in the year and he is first to die $= \frac{1-(1-p)^n}{n}$

125. *(d)* Unit vector in the direction of

$$B = \frac{3\hat{i}-\hat{j}+5\hat{k}}{\sqrt{3^2+(-1)^2+5^2}} = \frac{3\hat{i}-\hat{j}+5\hat{k}}{\sqrt{35}}$$

Now, component of vector A in the direction of $B = A\cdot B$

$$= (2\hat{i}+5\hat{j}+7\hat{k})\cdot\frac{3\hat{i}-\hat{j}+5\hat{k}}{\sqrt{35}} = \frac{36}{\sqrt{35}}$$

126. *(c)* Let the last three numbers in AP be $b, b+6, b+12$ and the first number be a.

Hence, four numbers are : $a, b, b+6, b+12$

$\because$ First number = fourth number

$$a = b+12 \qquad \ldots (i)$$

and $a, b, b+6$ are in G.P.

$\Rightarrow \quad b^2 = a(b+6)$

$b^2 = (b+12)(b+6)$

$\Rightarrow \quad 18b = -72 \Rightarrow b = -4$

From Eq. (i), we get

$a = -4+12 \Rightarrow a = 8$

Hence, four numbers are $8, -4, -4+6, -4+12$

$\Rightarrow 8, -4, 2, 8$

127. *(a)* $$\frac{dy}{dx} = \frac{y}{x} - \frac{\phi(y/x)}{\phi'(y/x)} \qquad \ldots (i)$$

Let $\quad y = vx$

$\Rightarrow \quad \frac{dy}{dx} = v + x\frac{dv}{dx}$

From Eq. (i),

$$v + x\frac{dv}{dx} = \frac{vx}{x} - \frac{\phi(v)}{\phi'(v)}$$

$\Rightarrow \quad \frac{\phi'(v)}{\phi(v)}dv = \frac{1}{x}dx \Rightarrow \int\frac{\phi'(v)}{\phi(v)}dv = \int\frac{1}{x}dx$

$\Rightarrow \quad \log\phi(v) = \log x + \log k \Rightarrow \phi(v) = kx$

$\Rightarrow \quad \phi\left(\frac{y}{x}\right) = kx$

128. *(b)* Given, $l+m+n=0, l^2-m^2+n^2=0$

$\Rightarrow \quad m = -(l+n)$

$\therefore \quad l^2+n^2-\{-(l+n)\}^2 = 0$

$l^2+n^2-l^2-n^2-2ln = 0 \Rightarrow 2ln = 0$

Either $l=0$ or $n=0$

When, $l=0$

$m+n=0$ or $m=-n$

$\therefore \quad \frac{l}{0} = \frac{m}{1} = \frac{n}{-1}$

When, $n=0$

$l+m=0 \Rightarrow l=-m$

$\therefore \qquad \frac{l}{1} = \frac{m}{-1} = \frac{n}{0}$

Hence, direction ratios of the vector a and b are $(0, 1, -1)$ and $(1, -1, 0)$.

$\therefore \qquad \cos\theta = \frac{0 - 1 + 0}{\sqrt{2}\sqrt{2}} = \frac{-1}{2} = \cos\frac{2\pi}{3}$

$\therefore \qquad \theta = \frac{2\pi}{3}$

129. ***(c)*** Given, $P(A) = 0.5, P(B) = ?$

$P(A \cap B) \leq 0.3$

$\because \quad P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$\therefore \quad P(B) = P(A \cap B) + P(A \cup B) - P(A)$

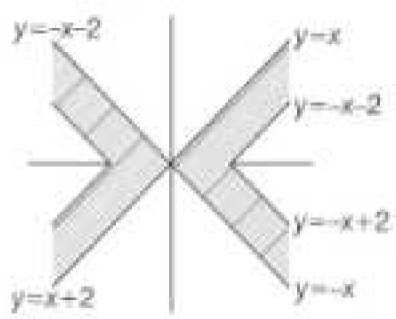
$\because \quad P(A \cup B) \leq 1, P(A \cap B) \leq 0.3$

$\therefore \quad P(B) = P(A \cap B) + P(A \cup B) - P(A)$

$\leq 1 + 0.3 - 0.5 \Rightarrow P(B) \leq 0.8$

130. ***(b)*** $\because || x | - | y || \leq 2 \qquad \ldots \text{(i)}$

$x^2 + y^2 \leq 4 \qquad \ldots \text{(ii)}$



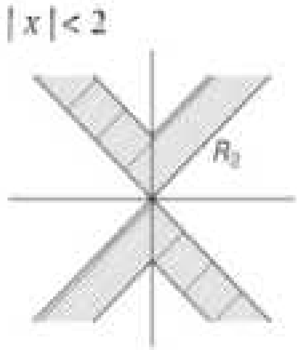
Case I For $| x | \geq | y |$

Then from Eq. (i), $| x | - | y | \leq 2$ which is symmetric about X and Y-axes.

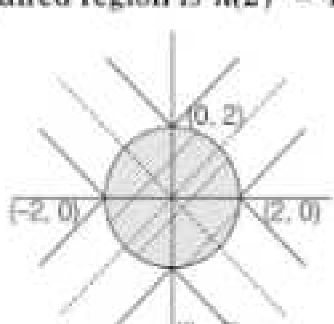
Case II For $| x | < | y |$

From Eq. (i), we get

$| y | - | x | < 2$



Thus, the required region is $\pi(2)^2 = 4\pi$ sq units

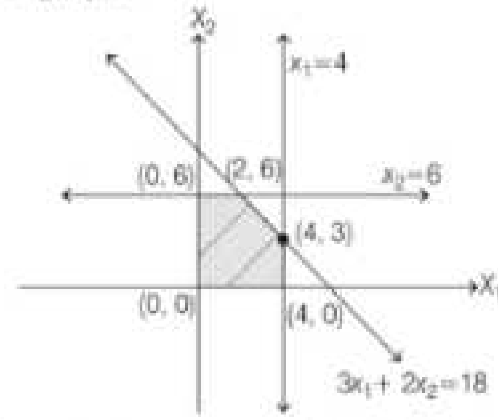


131. ***(b)*** Maximise : $Z = 3x_1 + 5x_2$

Subject to contraints $3x_1 + 2x_2 \leq 18 \Rightarrow x_1 \leq 4$

$x_2 \leq 6 \Rightarrow x_1, x_2 \geq 0$

From the graph,



Feasible region is represented by shaded region and corner points are (0, 0), (0, 6) (2, 6), (4, 3), (4, 0).

$\therefore$ Value of Z at (0, 0), $Z = 0$ at (4, 3), $Z = 27$

at (0, 6), $Z = 30$ at (4, 0), $Z = 12$

at (2, 6), $Z = 36$

As the feasible region is bounded so $Z = 36$ is the maximum value at (2, 6).

$\therefore \qquad x_1 = 2, x_2 = 6, Z = 36$

132. ***(c)*** Given, number is $(101)^{100} - 1$

$\because \quad (101)^{100} = (1 + 100)^{100}$

$= 1 + 100 \cdot 100 + \frac{100 \cdot 99}{2!}(100)^2 + \ldots$

$\therefore \quad (101)^{100} - 1 = 10000\left[1 + \frac{100 \cdot 99}{2!} + \ldots\right]$

$\therefore$ The greatest integer is 10000 which divides the given number.

133. ***(a)*** $n(n + 1)(n + 2)$

Let $\quad n = 5$

$\therefore n(n + 1)(n + 2) = 5 \cdot 6 \cdot 7$ which is not divisible by 12.

134. ***(b)*** $\because a$ = permutation of $x + 2$ objects taken all at a time.

$\therefore \quad a = {}^{x+2}P_{x+2} = (x + 2)!$

$b = {}^{x}P_{11} = \frac{x!}{(x - 11)!}$

$c = {}^{x-11}P_{x-11} = (x - 11)!$

$\because \quad a = 182bc$

$\therefore \quad (x + 2)! = 182\frac{x!}{(x - 11)!}(x - 11)!$

$\Rightarrow \quad (x + 2)(x + 1) = 182$

$$x^2 + 3x - 180 = 0$$
$$\Rightarrow \quad (x+15)(x-12) = 0$$
$$\Rightarrow \quad x = 12 \text{ or } -15 \text{ (neglect)}$$
$$\therefore \quad x = 12$$

135. *(a)* Given equation,
$$x^2 - 2^{2008}x + |x - 2^{2007}| + 2(2^{4013} - 1) = 0$$
$$\Rightarrow \quad x^2 - 2\cdot 2^{2007}x + 2^{4014} + |x - 2^{2007}| - 2 = 0$$
$$(x - 2^{2007})^2 + |x - 2^{2007}| - 2 = 0$$
Let $|x - 2^{2007}| = t \geq 0$
$$\therefore \quad t^2 + t - 2 = 0 \Rightarrow (t-1)(t+2) = 0$$
$t = 1$ or $t = -2$ which is not possible
$$|x - 2^{2007}| = 1 \Rightarrow x - 2^{2007} = \pm 1$$
$$x = 2^{2007} + 1 \text{ or } x = 2^{2007} - 1$$
$\therefore$ Sum of real roots $= 2^{2007} + 1 + 2^{2007} - 1$
$$= 2\cdot 2^{2007} = 2^{2008}$$

136. *(b)* Given, expression
$$f(x) = \frac{(x-a)(x-b)}{(c-a)(c-b)} + \frac{(x-b)(x-c)}{(a-b)(a-c)} + \frac{(x-c)(x-a)}{(b-c)(b-a)} - 1$$
At $x = a$
$$f(a) = 0 + 1 + 0 - 1 = 0$$
At $x = b$
$$f(b) = 0 + 0 + 1 - 1 = 0$$
At $x = c$
$$f(c) = 1 + 0 + 0 - 1 = 0$$
Hence, $f(x)$ is zero for all value of x.

137. *(b)* Given, $|\mathbf{a}| = 1, |\mathbf{c}| = 1, |\mathbf{b}| = 2$

$\because$ a and c are collinear vector.

Hence, angle between a and c is 0°.
$$\because \quad \mathbf{b} - 2\mathbf{c} = \lambda\mathbf{a} \quad \ldots\text{(i)}$$
$$\Rightarrow \quad (\mathbf{b} - 2\mathbf{c})\cdot\mathbf{c} = \lambda\mathbf{a}\cdot\mathbf{c}$$
$$\Rightarrow \quad \mathbf{b}\cdot\mathbf{c} - 2\mathbf{c}\cdot\mathbf{c} = \lambda\mathbf{a}\cdot\mathbf{c}$$
$$\Rightarrow \quad \mathbf{b}\cdot\mathbf{c} - 2(1) = \lambda|\mathbf{a}||\mathbf{c}|\cos 0^\circ$$
$$\Rightarrow \quad \mathbf{b}\cdot\mathbf{c} = 2 + \lambda \quad \ldots\text{(iii)}$$
Again from Eq. (i),
$$|\mathbf{b} - 2\mathbf{c}|^2 = \lambda^2|\mathbf{a}|^2$$
$$\Rightarrow \quad |\mathbf{b}|^2 + 4|\mathbf{c}|^2 - 4\mathbf{b}\cdot\mathbf{c} = \lambda^2|\mathbf{a}|^2$$
$$\Rightarrow \quad 4 + 4 - 4(\lambda + 2) = \lambda^2 \Rightarrow -4\lambda = \lambda^2$$
$$\Rightarrow \quad \lambda^2 + 4\lambda = 0$$
$$\Rightarrow \quad \lambda(\lambda + 4) = 0 \Rightarrow \lambda = 0, -4$$

138. *(b)* $\lim_{x\to 0} |x|^{\sin x} = L$

RHL At $x \to 0^+$

Let $\log \lim_{x\to 0} x^{\sin x} = \log l_1$
$$\lim_{x\to 0} \log x^{\sin x} = \log l_1$$
$$\lim_{x\to 0} \frac{\log x}{1/\sin x} = \log l_1$$
Using L-H rule
$$\lim_{x\to 0} \frac{1/x}{-\operatorname{cosec} x \cdot \cot x} = -\lim_{x\to 0} \frac{\sin x \cdot \tan x}{x} = -(1)\cdot(0)$$
$$\log l_1 = 0$$
$$\therefore \quad l_1 = e^0 = 1$$
LHL At $x \to 0^-$ Let $\lim_{x\to 0^-} (-x)^{\sin x} = l_2$

Similarly,
$$\log l_2 = 0 \Rightarrow l_2 = e^0 = 1$$
$$\therefore \quad l_1 = l_2 = 1$$
$$\therefore \quad L = 1$$

139. *(c)* $5\cos 3x + 3\cos x = 3\sin 4x$
$$\Rightarrow 5(4\cos^3 x - 3\cos x) + 3\cos x = 3\cdot 2\sin 2x \cdot \cos 2x$$
$$\Rightarrow 20\cos^3 x - 15\cos x + 3\cos x = 6\cdot(2\sin x\cos x)(2\cos^2 x - 1)$$
$$\Rightarrow 20\cos^3 x - 12\cos x = 24\sin x\cdot\cos^3 x - 12\sin x\cdot\cos x$$
$$\Rightarrow 20\cos^3 x - 24\sin x\cdot\cos^3 x - 12\cos x + 12\sin x\cos x = 0$$
$$\Rightarrow 4\cos x\,[5\cos^2 x - 6\sin x\cdot\cos^2 x + 3\sin x - 3] = 0$$
$$\Rightarrow 4\cos x\,[5(1 - \sin^2 x) - 6\sin x(1 - \sin^2 x) + 3\sin x - 3] = 0$$
$$\Rightarrow 4\cos x\,[5 - \sin^2 x - 6\sin x + 6\sin^3 x + 3\sin x - 3] = 0$$
$$\Rightarrow 4\cos x\,[6\sin^3 x - 5\sin^2 x - 3\sin x + 2] = 0$$
$$\Rightarrow 4\cos x\,[(\sin x - 1)(6\sin^2 x + \sin x - 2)] = 0$$
$$\Rightarrow 4\cos x\,(\sin x - 1)\left(\sin x - \frac{1}{2}\right)\left(\sin x + \frac{2}{3}\right) = 0$$
I. $\cos x = 0 \Rightarrow x = 2n\pi \pm \dfrac{\pi}{2}$

II. $\sin x = 1 \Rightarrow x = n\pi + (-1)^n \cdot \dfrac{\pi}{2}$

III. $\sin x = \dfrac{1}{2} \Rightarrow x = n\pi + (-1)^n \cdot \dfrac{\pi}{6}$

IV. $\sin x = -\frac{2}{3} \Rightarrow x = n\pi + (-1)^n \cdot \sin^{-1}\left(-\frac{2}{3}\right)$

$$= n\pi - (-1)^n \cdot \sin^{-1}\left(-\frac{2}{3}\right)$$

Hence, $C(x) = n\pi + (-1)^n, n \in Z$ is the correct option.

140. *(a)* Let the equation of circle which passes through (2, 3) be

$$x^2 + y^2 + 2gx + 2fy + C = 0 \quad \dots \text{(i)}$$

Circle (i) cuts $x^2 + y^2 - 1 = 0$ orthogonally

$\Rightarrow \quad 2g \times 0 + 2f \times 0 = C - 1 \Rightarrow C = 1$

Since, circle (i) passes through (2, 3)

$\therefore \quad 2^2 + 3^2 + 2g(2) + 2f(3) + 1 = 0$

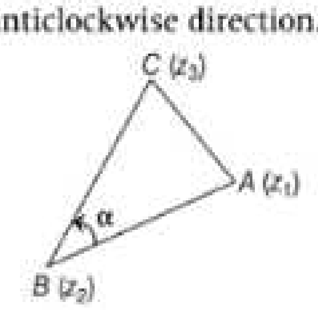
$$4g + 6f + 14 = 0$$

$\therefore$ Locus of centre $(-g, -f)$ is

$$-4g - 6f + 14 = 0 \Rightarrow 4g + 6f - 14 = 0$$

$\Rightarrow \quad 4x + 6y - 14 = 0 \Rightarrow 2x + 3y - 7 = 0$

141. *(a)* Let z_1, z_2 and z_3 be the vertices of ΔABC described in anticlockwise direction.



Let $\angle ABC = \alpha$

Then, $\frac{z_3 - z_2}{z_1 - z_2} = \frac{CB}{AB} e^{i\alpha} = \frac{|z_3 - z_2|}{|z_1 - z_2|} e^{i\alpha}$

$\Rightarrow \quad \text{amp}\left(\frac{z_3 - z_2}{z_1 - z_2}\right) = \alpha = \angle ABC$

142. *(c)* $\frac{ax}{\cos\theta} + \frac{by}{\sin\theta} = a^2 - b^2$

and $\frac{ax\sin\theta}{\cos^2\theta} - \frac{by\cos\theta}{\sin^2\theta} = 0$

To find, $(ax)^{2/3} + (by)^{2/3} = ?$

Let $\quad \frac{1}{\cos\theta} = l, \frac{1}{\sin\theta} = m$

$\therefore$ Above expression can be re-written as

$$axl + bym = a^2 - b^2 \quad \dots \text{(i)}$$

$$ax\left(\frac{l^2}{m}\right) - by\left(\frac{m^2}{l}\right) = 0 \quad \dots \text{(ii)}$$

$\Rightarrow \quad axl = \frac{by(m^3)}{l^2}$

From Eq. (i),

$$bym\left(\frac{m^2}{l^2}\right) + bym = a^2 - b^2$$

$$bym\left(\frac{m^2 + l^2}{l^2}\right) = a^2 - b^2 \quad \dots \text{(iii)}$$

$\because \quad \frac{1}{\cos\theta} = l$ and $\frac{1}{\sin\theta} = m$

$\Rightarrow \quad \frac{1}{l^2} + \frac{1}{m^2} = 1 \Rightarrow l^2 + m^2 = l^2m^2$

From Eq. (iii), we get

$$bym\left(\frac{l^2m^2}{l^2}\right) = a^2 - b^2$$

$\Rightarrow \quad bym^3 = a^2 - b^2 \Rightarrow \frac{1}{m^3} = \frac{by}{a^2 - b^2}$

$\therefore \quad \sin\theta = \left\{\frac{by}{(a^2 - b^2)}\right\}^{1/3} \quad \dots \text{(iv)}$

Similarly, we can get

$$\cos\theta = \left(\frac{ax}{a^2 - b^2}\right)^{1/3} \quad \dots \text{(v)}$$

Squaring and adding Eqs. (iv) and (v)

$$\left(\frac{by}{a^2 - b^2}\right)^{2/3} + \left(\frac{ax}{a^2 - b^2}\right)^{2/3} = 1$$

$$(by)^{2/3} + (ax)^{2/3} = (a^2 - b^2)^{2/3}$$

143. *(c)* $\frac{1 - \sqrt{1 - 4x^2}}{x} < 3$

Which will be defined if $1 - 4x^2 \geq 0$ and $x \neq 0$

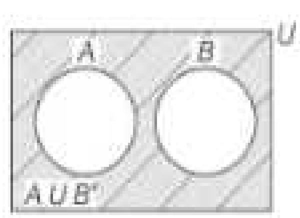
$$1 - 4x^2 \geq 0 \Rightarrow 4x^2 \leq 1 \Rightarrow x^2 \leq \frac{1}{4}$$

$\Rightarrow \quad |x| \leq \frac{1}{2} \Rightarrow x \in \left[\frac{-1}{2}, \frac{1}{2}\right]$ but $x \neq 0$

$\therefore \quad x \in \left[\frac{-1}{2}, 0\right) \cup \left(0, \frac{1}{2}\right]$

144. *(d)* Given that, A and B are disjoint sets.

$\therefore A \cap B = \phi$



$\therefore A \cup B' = B'$ from the Venn diagram.

145. *(c)* Since, we know that, if set has n elements, then the number of symmetric relations $= 2^{\frac{n(n+1)}{2}}$

Here, $n = 3$

$\therefore$ Number of symmetric relations $= 2^{\frac{3 \times 4}{2}} = 2^6 = 64$

146. *(b)* $f:[2,\infty)\to A$

$f(x) = x^2 - 4x + 5$ is bijective

$= (x-2)^2 + 1$

$\because \quad (x-2)^2 \ge 0$

$\therefore \quad 1 + (x-2)^2 \ge 1$

$\therefore$ Range $= [1, \infty)$

$\because f(x)$ is bijection $\Rightarrow$ range = codomain

$\therefore \quad A = [1, \infty)$

147. *(b)* Let $I = \int_{-3}^{3} \frac{\cos^2 x}{\left[\frac{x}{\pi}\right] + \frac{1}{2}} dx$

$$= \int_{-3}^{0} \frac{\cos^2 x}{\left[\frac{x}{\pi}\right] + \frac{1}{2}} dx + \int_{0}^{3} \frac{\cos^2 x}{\left[\frac{x}{\pi}\right] + \frac{1}{2}} dx$$

$$= \int_{-3}^{0} \frac{\cos^2 x}{-1 + \frac{1}{2}} dx + \int_{0}^{3} \frac{\cos^2 x}{0 + \frac{1}{2}} dx$$

$$I = -2\int_{-3}^{0} \cos^2 x\, dx + 2\int_{0}^{3} \cos^2 x\, dx$$

Let $\quad x = -t$

$dx = -dt$

$$I = 2\int_{3}^{0} \cos^2 t\, dt + 2\int_{0}^{3} \cos^2 x\, dx$$

$$= -2\int_{0}^{3} \cos^2 x\, dx + 2\int_{0}^{3} \cos^2 x\, dx$$

$I = 0$

148. *(d)* Let $\cot^{-1} x = \theta$

$\Rightarrow \quad x = \cot\theta \Rightarrow \operatorname{cosec}\theta = \sqrt{1+x^2}$

$\therefore \quad \cos[\tan^{-1}\{\sin(\cot^{-1} x)\}] = \cos[\tan^{-1}(\sin\theta)]$

$$= \cos\left[\tan^{-1}\left(\frac{1}{\operatorname{cosec}\theta}\right)\right]$$

$$= \cos\left[\tan^{-1}\left(\frac{1}{\sqrt{1+x^2}}\right)\right] \quad \ldots \text{(i)}$$

Let $\tan^{-1}\left(\frac{1}{\sqrt{1+x^2}}\right) = \phi$

$\therefore \quad \frac{1}{\sqrt{1+x^2}} = \tan\phi$

From Eq. (i),

$$\cos\phi = \frac{1}{\sec\phi} = \frac{1}{\sqrt{1+\tan^2\phi}}$$

$$= \frac{1}{\sqrt{1 + \frac{1}{1+x^2}}} = \sqrt{\frac{1+x^2}{2+x^2}}$$

149. *(c)* $\because A = (3, 4)$ and $B = (7, 13)$

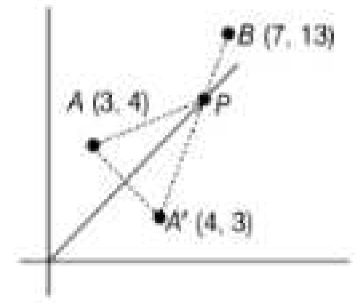
Consider a point A', then image of A through $y = x$.

$\therefore \quad A' = (4, 3) \Rightarrow PA = PA'$

Thus, $PA + PB$ is minimum.

If $PA' + PB$ is minimum.

or If A', P, B are collinear.



Now, $A'B$ is

$$y - 3 = \frac{13-3}{7-4}(x-4) \Rightarrow 3y - 10x + 31 = 0$$

It intersects $y = x$

$\therefore \quad 3(x) - 10x + 31 = 0$

$\Rightarrow \quad x = \frac{31}{7}$

$\therefore \quad y = \frac{31}{7}$

$\therefore \quad P = \left(\frac{31}{7}, \frac{31}{7}\right)$

150. *(b)* If $a * b = \frac{a+b}{2}$

If $\quad a = 3, b = 4$

Then $\quad a * b = \frac{3+4}{2} = \frac{7}{2} \notin I$

But $\quad \frac{7}{2} \in Q$

Hence, $a * b = \frac{a+b}{2}$ is a binary operation in the set Q but not in I.