FINAL JEE–MAIN EXAMINATION – JANUARY, 2024

(Held On Thursday 01 February, 2024) TIME : 9 : 00 AM to 12 : 00 NOON

MATHEMATICS

TEST PAPER WITH SOLUTION

SECTION-A

١. A bag contains ٨ balls, whose colours are either white or black. ٤ balls are drawn at random without replacement and it was found that ٢ balls are white and other ٢ balls are black. The probability that the bag contains equal number of white and black balls is:

(١) $\frac{2}{5}$ (٢) $\frac{2}{7}$

(٣) $\frac{1}{7}$ (٤) $\frac{1}{5}$

Ans. (٢)

Sol.

$P(4W4B/2W2B) =$

$$\frac{P(4W4B)\cdot P(2W2B/4W4B)}{P(2W6B)\cdot P(2W2B/2W6B)+P(3W5B)\cdot P(2W2B/3W5B)+\ldots\ldots\ldots\ldots+P(6W2B)\cdot P(2W2B/6W2B)}$$

$$=\frac{\frac{1}{5}\times\frac{{}^4C_2\times{}^4C_2}{{}^8C_4}}{\frac{1}{5}\times\frac{{}^2C_2\times{}^6C_2}{{}^8C_4}+\frac{1}{5}\times\frac{{}^3C_2\times{}^5C_2}{{}^8C_4}+\ldots+\frac{1}{5}\times\frac{{}^6C_2\times{}^2C_2}{{}^8C_4}}$$

$$=\frac{2}{7}$$

٢. The value of the integral

$\int_0^{\pi/4}\frac{x\,dx}{\sin^4(2x)+\cos^4(2x)}$ equals:

(١) $\frac{\sqrt{2}\pi^2}{8}$ (٢) $\frac{\sqrt{2}\pi^2}{16}$

(٣) $\frac{\sqrt{2}\pi^2}{32}$ (٤) $\frac{\sqrt{2}\pi^2}{64}$

Ans. (٣)

Sol. $\int_0^{\pi/4}\frac{x\,dx}{\sin^4(2x)+\cos^4(2x)}$

Let $2x=t$ then $dx=\frac{1}{2}dt$

$$I=\frac{1}{4}\int_0^{\pi/2}\frac{t\,dt}{\sin^4 t+\cos^4 t}$$

$$I=\frac{1}{4}\int_0^{\pi/2}\frac{\left(\frac{\pi}{2}-t\right)dt}{\sin^4\left(\frac{\pi}{2}-t\right)+\cos^4\left(\frac{\pi}{2}-t\right)}$$

$$I=\frac{1}{4}\int_0^{\pi/2}\frac{\frac{\pi}{2}\,dt}{\sin^4 t+\cos^4 t}-I$$

$$2I=\frac{\pi}{8}\int_0^{\pi/2}\frac{dt}{\sin^4 t+\cos^4 t}$$

$$2I=\frac{\pi}{8}\int_0^{\pi/2}\frac{\sec^4 t\,dt}{\tan^4 t+1}$$

Let $\tan t=y$ then $\sec^2 t\,dt=dy$

$$2I=\frac{\pi}{8}\int_0^{\infty}\frac{(1+y^2)dy}{1+y^4}$$

$$=\frac{\pi}{16}\int_0^{\infty}\frac{\left(1+\frac{1}{y^2}\right)}{y^2+\frac{1}{y^2}}dy$$

Put $y-\frac{1}{y}=p$

$$I=\frac{\pi}{16}\int_{-\infty}^{\infty}\frac{dp}{p^2+\left(\sqrt{2}\right)^2}$$

$$=\frac{\pi}{16\sqrt{2}}\left(\tan^{-1}\frac{p}{\sqrt{2}}\right)_{-\infty}^{\infty}$$

$$I=\frac{\pi^2}{16\sqrt{2}}$$

٣. If A = $\begin{bmatrix} \sqrt{2} & 1 \\ -1 & \sqrt{2} \end{bmatrix}$, B = $\begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$, C = ABA^T and X = A^{T}C^{2}A, then det X is equal to :

(١) ٢٤٣

(٢) ٧٢٩

(٣) ٢٧

(٤) ٨٩١

Ans. (٢)

Sol.

$A = \begin{bmatrix} \sqrt{2} & 1 \\ -1 & \sqrt{2} \end{bmatrix} \Rightarrow \det(A) = 3$

$B = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} \Rightarrow \det(B) = 1$

Now C = ABA^T ⇒ det(C) = (dct (A))^2 × det(B)

$|C| = 9$

Now |X| = |A^{T}C^{2}A|

= |A^T||C|^2|A|

= |A|^2|C|^2

= ٩ X ٨١

= ٧٢٩

٤. If tanA = $\frac{1}{\sqrt{x(x^2+x+1)}}$, $\tan B = \frac{\sqrt{x}}{\sqrt{x^2+x+1}}$

and

$\tan C = (x^{-3}+x^{-2}+x^{-1})^{\frac{1}{2}}, 0 < A,B,C < \frac{\pi}{2}$, then

A + B is equal to :

(١) C

(٢) π − C

(٣) ٢π − C

(٤) $\frac{\pi}{2} - C$

Ans. (١)

Sol.

Finding tan (A + B) we get

⇒ tan (A + B) =

$$\frac{\tan A + \tan B}{1 - \tan A \tan B} = \frac{\frac{1}{\sqrt{x(x^2+x+1)}} + \frac{\sqrt{x}}{\sqrt{x^2+x+1}}}{1 - \frac{1}{x^2+x+1}}$$

⇒ tan (A + B) = $\frac{(1+x)\sqrt{x^2+x+1}}{\sqrt{x}\,(x^2+x)}$

$$= \frac{(1+x)\sqrt{x^2+x+1}}{x^2\sqrt{x}+x\sqrt{x}}$$

$$\tan(A+B) = \frac{\sqrt{x^2+x+1}}{x\sqrt{x}} = \tan C$$

A = B = C

٥. If n is the number of ways five different employees can sit into four indistinguishable offices where any office may have any number of persons including zero, then n is equal to:

(١) ٤٧ (٢) ٥٣ (٣) ٥١ (٤) ٤٣ Ans. (٣)

Sol.

Total ways to partition ٥ into ٤ parts are :

٥، ٠، ٠، ٠ ⇒ ١ way

٤، ١، ٠، ٠ ⇒ $\frac{5!}{4!}$ = ٥ ways

٣، ٢، ٠، ٠ ⇒ $\frac{5!}{3!2!}$ = 10 ways

2, 2, 0, 1 ⇒ $\frac{5!}{2!2!2!}$ = 15 ways

2,1,1,1 ⇒ $\frac{5!}{2!(1!)^3\,3!}$ = 10 ways

3, 1, 1, 0 ⇒ $\frac{5!}{3!2!}$ = ١٠ ways

Total = ١+٥+١٠+١٥+١٠+١٠ = ٥١ ways

٦. Let $S = \{z \in \mathbb{C} : |z-1| = 1$ and $(\sqrt{2}-1)(z+\bar{z}) - i(z-\bar{z}) = 2\sqrt{2}\}$. Let $z_1, z_2 \in S$ be such that $|z_1| = \max_{z\in s}|z|$ and $|z_2| = \min_{z \in s}|z|$.

Then $\left|\sqrt{2}z_1 - z_2\right|^2$ equals :

(١) ١

(٢) ٤

(٣) ٣

(٤) ٢

Ans. (٤)

Sol. Let Z = x + iy

Then $(x-1)^2 + y^2 = 1$ ...(١)

& $(\sqrt{2}-1)(2x) - i(2iy) = 2\sqrt{2}$

$\Rightarrow (\sqrt{2}-1)x + y = \sqrt{2}$ ...(2)

Solving (١) & (٢) we get

Either x = ١ or $x = \frac{1}{2-\sqrt{2}}$ ... (3)

On solving (٣) with (٢) we get

For x = ١ ⇒ y = ١ ⇒ Z٢ = ١ + i

& for

$x = \frac{1}{2-\sqrt{2}} \Rightarrow y = \sqrt{2} - \frac{1}{\sqrt{2}} \Rightarrow Z_1 = \left(1+\frac{1}{\sqrt{2}}\right) + \frac{i}{\sqrt{2}}$

Now

$\left|\sqrt{2}z_1 - z_2\right|^2$

$= \left|\left(\frac{1}{\sqrt{2}}+1\right)\sqrt{2} + i - (1+i)\right|^2$

$= (\sqrt{2})^2$

$= 2$

٧. Let the median and the mean deviation about the median of ٧ observation ١٧٠، ١٢٥، ٢٣٠، ١٩٠، ٢١٠، a، b be ١٧٠ and $\frac{205}{7}$ respectively. Then the mean deviation about the mean of these ٧ observations is :

(١) ٣١

(٢) ٢٨

(٣) ٣٠

(٤) ٣٢

Ans. (٣)

Sol. Median = ١٧٠ ⇒ ١٢٥، a، b، ١٧٠، ١٩٠، ٢١٠، ٢٣٠

Mean deviation about

Median = $\frac{0+45+60+20+40+170-a+170-b}{7} = \frac{205}{7}$

⇒ a + b = ٣٠٠

Mean = $\frac{170+125+230+190+210+a+b}{7} = 175$

Mean deviation

About mean = $\frac{50+175-a+175-b+5+15+35+55}{7}$ = ٣٠

٨. Let $\vec{a} = 5\hat{i} - \hat{j} - 3\hat{k}$, $\vec{b} = \hat{i} + 2\hat{j} - 4\hat{k}$ and

$\vec{c} = \left(\left(\vec{a} \times \vec{b}\right) \times \hat{i}\right) \times \hat{i}$. T

equal to

(١) −١٢

(٢) −١٠

(٣) −١٣

(٤) −١٥

Ans. (١)

Sol. $\vec{a} = 5\hat{i} - \hat{j} - 3\hat{k}$

$\vec{b} = \hat{i} + 2\hat{j} - 4\hat{k}$

$(\vec{a} \times \vec{b}) \times \hat{i} = (\vec{a}\cdot\hat{i})\vec{b} - (\vec{b}\cdot\hat{i})\vec{a}$

$= 5\vec{b} - \vec{a}$

$\Rightarrow \left(\left(5\vec{b} - \vec{a}\right) \times \hat{i}\right) \times \hat{i}$

$= \left(\left(11\hat{j} - 23\hat{k}\right) \times \hat{i}\right) \times \hat{i}$

$= \left(-11\hat{k} - 23\hat{j}\right) \times \hat{i}$

$= -\left(11\hat{j} + 23\hat{k}\right)$

$\vec{c}\cdot\left(-\hat{i} + \hat{j} + \hat{k}\right) = -11 + 23 = 12$

٩. Let $S = \{x \in R : (\sqrt{3+\sqrt{2}})^x + (\sqrt{3-\sqrt{2}})^x = 10\}$.

Then the number of elements in S is :

(١) ٤ (٢) ٠

(٣) ٢ (٤) ١

Ans. (٣)

Sol. $\left(\sqrt{3}-\sqrt{2}\right)^x + \left(\sqrt{3}+\sqrt{2}\right)^x = ١٠$

Let $\left(\sqrt{3}+\sqrt{2}\right)^x = t$

$t + \frac{١}{t} = ١٠$

$t^٢ - ١٠t + ١ = ٠$

$t = \frac{10 \pm \sqrt{١٠٠ - 4}}{٢} = 5 \pm 2\sqrt{٦}$

$\left(\sqrt{3}+\sqrt{2}\right)^x = \left(\sqrt{3}+\sqrt{٢}\right)^{\pm ٢}$

x = ٢ or x = −٢

Number of solutions = ٢

١٠. The area enclosed by the curves xy + ٤y = ١٦ and x + y = ٦ is equal to :

(١) ٢٨ − ٣٠ $\log_e 2$ (٢) ٣٠ − ٢٨ $\log_e ٢$

(٣) ٣٠ − ٣٢ $\log_e 2$ (٤) ٣٢ − ٣٠ $\log_e ٢$

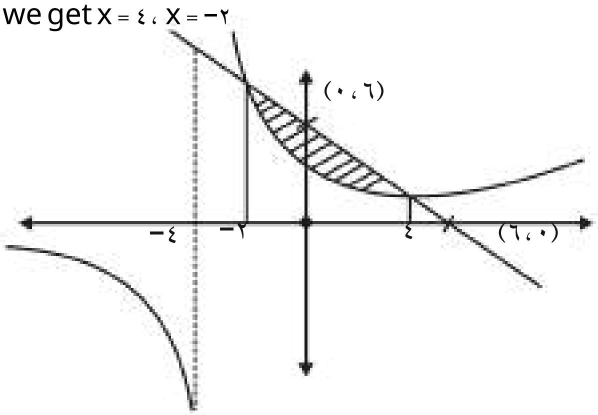
Ans. (٣)

Sol. xy + ٤y = ١٦ ، x + y = ٦

y(x + ٤) = ١٦ ______(١) ، x + y = ٦_____(٢)

on solving، (١) & (٢)

we get x = ٤ ، x = −٢



$$\text{Area} = \int_{-2}^{4} \left((6 - x) - \frac{16}{x+4} \right) dx$$

$$= 30 - 32\ln 2$$

١١. Let $f : R \to R$ and $g : R \to R$ be defined as

$f(x) = \begin{cases} \log_e x, & x > 0 \\ e^{-x}, & x \le 0 \end{cases}$ and

$g(x) = \begin{cases} x, & x \ge 0 \\ e^{x}, & x < 0 \end{cases}$. Then، $gof : R \to R$ is :

(١) one-one but not onto

(٢) neither one-one nor onto

(٣) onto but not one-one

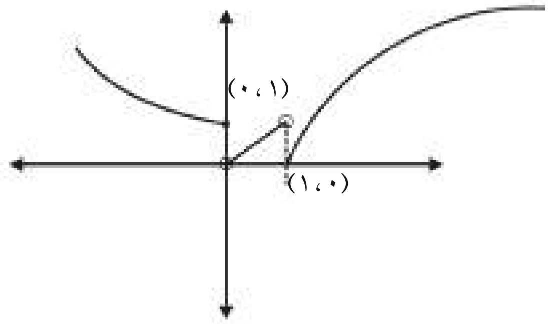
(٤) both one-one and onto

Ans. (٢)

Sol.

$g(f(x)) = \begin{cases} f(x), & f(x) \ge 0 \\ e^{f(x)}, & f(x) < 0 \end{cases}$

$g(f(x)) = \begin{cases} e^{-x}, & x \in (-\infty, 0] \\ e^{\ln x}, & (0,1) \\ \ln x, & [1, \infty) \end{cases}$



Graph of g(f(x))

g(f(x)) → Many one into

١٢. If the system of equations

٢x + ٣y − z = ٥

x + αy + ٣z = −٤

٣x − y + βz = ٧

has infinitely many solutions، then ١٣αβ is equal to

(١) ١١١٠ (٢) ١١٢٠

(٣) ١٢١٠ (٤) ١٢٢٠

Ans. (٢)

Sol. Using family of planes

$٢x + ٣y - z - ٥ = k_١ (x + \lambda y + ٣z + ٤) + k_٢ (٣x - y + \mu z - ٧)$

$٢ = k_١ + ٣k_٢,\ ٣ = k_١\lambda - k_٢,\ -١ = ٣k_١ + \mu k_٢,\ -٥ = ٤k_١ - ٧k_٢$

On solving we get

$$k_2 = \frac{13}{19},\ k_1 = \frac{-1}{19},\ \lambda = -70,\ \mu = \frac{-16}{13}$$

$$١٣\lambda\mu = ١٣(-٧٠)\left(-\frac{16}{13}\right) = 1120$$

١٣. For $٠ < \theta < \pi/٢$, if the eccentricity of the hyperbola $x^٢ - y^٢\operatorname{cosec}^٢\theta = ٥$ is $\sqrt{٧}$ times eccentricity of the ellipse $x^٢\operatorname{cosec}^٢\theta + y^٢ = ٥$, then the value of θ is :

(١) $\frac{\pi}{6}$ (٢) $\frac{5\pi}{12}$

(٣) $\frac{\pi}{3}$ (٤) $\frac{\pi}{4}$

Sol. Ans. (٣)

$$e_h = \sqrt{1 + \sin^2\theta}$$

$$e_c = \sqrt{1 - \sin^2\theta}$$

$$e_h = \sqrt{7}e_c$$

$$1 + \sin^2\theta = 7(1 - \sin^2\theta)$$

$$\sin^2\theta = \frac{6}{8} = \frac{3}{4}$$

$$\sin\theta = \frac{\sqrt{3}}{2}$$

$$\theta = \frac{\pi}{3}$$

١٤. Let $y = y(x)$ be the solution of the differential equation $\frac{dy}{dx} = ٢x(x+y)^٣ - x(x+y) - ١,\ y(٠) = ١$.

Then $\left(\frac{1}{\sqrt{2}} + y\left(\frac{1}{\sqrt{2}}\right)\right)^2$ equals :

(١) $\frac{4}{4 + \sqrt{e}}$ (٢) $\frac{3}{3 + \sqrt{e}}$

(٣) $\frac{2}{1 + \sqrt{e}}$ (٤) $\frac{1}{2 - \sqrt{e}}$

Ans. (٤)

Sol. $\frac{dy}{dx} = 2x(x+y)^3 - x(x+y) - 1$

$x + y = t$

$$\frac{dt}{dx} - 1 = 2xt^3 - xt - 1$$

$$\frac{dt}{2t^3 - t} = x\,dx$$

$$\frac{t\,dt}{٢t^٤ - t^٢} = x\,dx$$

Let $t^2 = z$

$$\frac{1}{2}\int \frac{dz}{٢z^٢ - z} = \int x\,dx$$

$$\int \frac{dz}{4z\left(z - \frac{1}{2}\right)} = \int x\,dx$$

$$\ln\left|\frac{z - \frac{1}{2}}{z}\right| = x^2 + k$$

$$z = \frac{1}{2 - \sqrt{e}}$$

١٥. Let $f : R \to R$ be defined as

$$f(x) = \begin{cases} \frac{a - b\cos 2x}{x^2} & ; \quad x < 0 \\ x^2 + cx + 2 & ; \quad 0 \le x \le 1 \\ 2x + 1 & ; \quad x > 1 \end{cases}$$

If f is continuous everywhere in R and m is the number of points where f is NOT differential then $m + a + b + c$ equals : (١) ١ (٣) ٣ Ans. (٤)

(٢) ٤

(٤) ٢

Sol. At x = ١، f(x) is continuous therefore،

$f(1^-) = f(1) = f(1^+)$

$f(١) = ٣ + c$ ....(١)

$f(1^+) = \lim_{h \to 0} ٢(١ + h) + ١$

$f(1^+) = \lim_{h \to 0} ٣ + ٢h = ٣$ ....(٢)

from (١) & (٢)

c = ٠

at x = ٠، f(x) is continuous therefore،

$f(0^-) = f(0) = f(0^+)$ ....(٣) ....(٤)

$f(٠) = f(0^+) = ٢$

$f(0^-)$ has to be equal to ٢

$$\lim_{h \to 0} \frac{a - b\cos 2h}{h^2}$$

$$\lim_{h \to 0} \frac{a - b\left(1 - \frac{4h^2}{2!} + \frac{16h^4}{4!} - \ldots\right)}{h^2}$$

$$\lim_{h \to 0} \frac{a - b + b\left(2h^2 - \frac{2}{3}h^4 \ldots\right)}{h^2}$$

for limit to exist a − b = ٠ and limit is ٢b ....(٥)

from (٣)، (٤) & (٥)

a = b = ١

checking differentiability at x = ٠

LHD: $\lim_{h \to 0} \frac{\frac{1 - \cos 2h}{h^2} - 2}{-h}$

$$\lim_{h \to 0} \frac{1 - \left(1 - \frac{4h^2}{2!} + \frac{16h^4}{4!} \ldots\right) - 2h^2}{-h^3} = 0$$

RHD: $\lim_{h \to 0} \frac{(0 \cdot h^2 + 2) - 2}{h} = 0$

Function is differentiable at every point in its domain

∴ m = ٠

m + a + b + c = ٠ + ١ + ١ + ٠ = ٢

١٦. Let $\frac{x^2}{a^2} + \frac{y^2}{b^2} = ١$، a > b be an ellipse، whose eccentricity is $\frac{1}{\sqrt{2}}$ and the length of the latus rectum is $\sqrt{١٤}$. Then the square of the eccentricity of $\frac{x^2}{a^2} - \frac{y^2}{b^2} = ١$ is:

(١) ٣ (٢) ٧/٢

(٣) ٣/٢ (٤) ٥/٢

Ans. (٣)

Sol.

$$e = \frac{1}{\sqrt{2}} = \sqrt{1 - \frac{b^2}{a^2}} \Rightarrow \frac{1}{2} = 1 - \frac{b^2}{a^2}$$

$$\frac{2b^2}{a} = \sqrt{14} \qquad \frac{b^2}{a^2} = \frac{1}{2}$$

$$e_H = \sqrt{1 + \frac{b^2}{a^2}} = \sqrt{1 + \frac{1}{2}} = \sqrt{\frac{3}{2}}$$

$$\therefore e_H^2 = \frac{3}{2}$$

١٧. Let ٣، a، b، c be in A.P. and ٣، a − ١، b + l، c + ٩ be in G.P. Then، the arithmetic mean of a، b and c is:

(١) −٤ (٢) −١

(٣) ١٣ (٤) ١١

Ans. (٤)

Sol.

٣، a، b، c ∈ A.P ⇒ ٣، ٣+d، ٣+٢d، ٣+٣d

٣، a−١، b+١، c+٩ ∈ G.P ⇒ ٣، ٢+d، ٤+٢d، ١٢+٣d

a = ٣ + d $\qquad (2+d)^2 = 3(4+2d)$

b = ٣ + ٢d $\qquad$ d = ٤، −٢

c = ٣ + ٣d

If d = ٤ $\quad$ G.P ⇒ ٣، ٦، ١٢، ٢٤

a = ٧

b = ١١

c = ١٥

$$\frac{a + b + c}{3} = 11$$

١٨. Let $C: x^2+y^2=4$ and $C': x^2+y^2-4\lambda x+9=0$ be two circles. If the set of all values of λ so that the circles C and C′ intersect at two distinct points, is $R-[a, b]$, then the point $(8a+12, 16b-20)$ lies on the curve :

(١) $x^2+2y^2-5x+6y=3$

(٢) $5x^2-y=-11$

(٣) $x^2-4y^2=7$

(٤) $6x^2+y^2=42$

Ans. (٤)

Sol. $x^2+y^2=4$

$C(0, 0)$ $r_1=2$

$C'(2\lambda, 0)$ $r_2=\sqrt{4\lambda^2-9}$

$|r_1-r_2| < CC' < |r_1+r_2|$

$\left|2-\sqrt{4\lambda^2-9}\right| < 2|\lambda| < 2+\sqrt{4\lambda^2-9}$

$4+4\lambda^2-9+4\sqrt{4\lambda^2-9} > 4\lambda^2$

True $\lambda \in R$ .... (١)

$4\lambda^2 > 4+4\lambda^2-9-4\sqrt{4\lambda^2-9}$

$5 > 4\sqrt{4\lambda^2-9}$ and $\lambda^2 \geq \frac{9}{4}$

$\frac{25}{16} > 4\lambda^2-9$ $\Rightarrow \lambda \in \left(-\infty, -\frac{3}{2}\right] \cup \left[\frac{3}{2}, \infty\right)$

$\frac{169}{64} > \lambda^2$

$\Rightarrow \lambda \in \left(-\frac{13}{8}, \frac{13}{8}\right)$ ...(٢)

from (١) and (٢)

$\Rightarrow \lambda \in \left(-\frac{13}{8}, \frac{13}{8}\right) \in R-\left[-\frac{13}{8}, \frac{13}{8}\right]$

as per question $a=-\frac{13}{8}$ and $b=\frac{13}{8}$

∴ required point is $(-1, 6)$ with satisfies option (٤)

١٩. If $5f(x)+4f\left(\frac{1}{x}\right)=x^2-2$, $\forall x \neq 0$ and $y=9x^2f(x)$, then y is strictly increasing in :

(١) $\left(0, \frac{1}{\sqrt{5}}\right) \cup \left(\frac{1}{\sqrt{5}}, \infty\right)$

(٢) $\left(-\frac{1}{\sqrt{5}}, 0\right) \cup \left(\frac{1}{\sqrt{5}}, \infty\right)$

(٣) $\left(-\frac{1}{\sqrt{5}}, 0\right) \cup \left(0, \frac{1}{\sqrt{5}}\right)$

(٤) $\left(-\infty, \frac{1}{\sqrt{5}}\right) \cup \left(0, \frac{1}{\sqrt{5}}\right)$

Ans. (٢)

Sol. $5f(x)+4f\left(\frac{1}{x}\right)=x^2-2$, $\forall x \neq 0$ ...(١)

Substitute $x \to \frac{1}{x}$

$5f\left(\frac{1}{x}\right)+4f(x)=\frac{1}{x^2}-2$ ...(٢)

On solving (١) and (٢)

$f(x)=\frac{5x^4-2x^2-4}{9x^2}$

$y=9x^2f(x)$

$y=5x^4-2x^2-4$ ...(٣)

$\frac{dy}{dx}=20x^3-4x$

for strictly increasing

$\frac{dy}{dx} > 0$

$4x(5x^2-1) > 0$

$x \in \left(-\frac{1}{\sqrt{5}}, 0\right) \cup \left(\frac{1}{\sqrt{5}}, \infty\right)$

٢٠. If the shortest distance between the lines $\frac{x-\lambda}{-2}=\frac{y-2}{1}=\frac{z-1}{1}$ and $\frac{x-\sqrt{3}}{1}=\frac{y-1}{-2}=\frac{z-2}{1}$ is 1, then the sum of all possible values of λ is :

(١) ٠ (٢) $2\sqrt{3}$

(٣) $3\sqrt{3}$ (٤) $-2\sqrt{3}$

Ans. (٢)

Sol. Passing points of lines L١ & L٢ are

$(\square,2,1) \& (3,1,2)$

$$S.D = \frac{\begin{vmatrix} \sqrt{3}\,\square & \square 1 & 1 \\ \square 2 & 1 & 1 \\ 1 & \square 2 & 1 \end{vmatrix}}{\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \square 2 & 1 & 1 \\ 1 & \square 2 & 1 \end{vmatrix}}$$

$$١ = \left| \frac{\sqrt{3}\,\square}{\sqrt{3}} \right|$$

$\square 0, \square 2\sqrt{3}$

SECTION-B

٢١. If x = x(t) is the solution of the differential equation (t + ١)dx = (٢x + (t + ١)٤) dt, x(٠) = ٢, then, x(١) equals ________________.

Ans. (١٤)

Sol. (t + ١)dx = (٢x + (t + ١)٤)dt

$$\frac{dx}{dt} \square \frac{2x \square (t \square 1)^4}{t \square 1}$$

$$\frac{dx}{dt} \square \frac{2x}{t \square 1} = (t+١)^٣$$

$$I.F. = e^{\int \frac{\square 2}{t \square 1} dt} = e^{\square 2 \ln(t \square 1)} = \frac{1}{(t \square 1)^2}$$

$$\frac{x}{(t \square 1)^2} \square \int \frac{1}{(t \square 1)^2}(t \square 1)^3\, dt \square c$$

$$\frac{x}{(t \square 1)^2} \square \frac{(t \square 1)^2}{2} \square c$$

$\square$ c = $\frac{3}{2}$

$$x = \frac{(t \square 1)^4}{2} \square \frac{3}{2}(t \square 1)^2$$

put, t = ١

x = ٢ + ٦ = ١٤

٢٢. The number of elements in the set

S = {(x, y, z) : x, y, z ∈ Z, x + ٢y + ٣z = ٤٢, x, y, z ≥ ٠} equals ________________.

Ans. (١٦٩)

Sol. x + ٢y + ٣z = ٤٢, x, y, z ≥ ٠

z = ٠	x + ٢y = ٤٢ → ٢٢
z = ١	x + ٢y = ٣٩ → ٢٠
z = ٢	x + ٢y = ٣٦ → ١٩
z = ٣	x + ٢y = ٣٣ → ١٧
z = ٤	x + ٢y = ٣٠ → ١٦
z = ٥	x + ٢y = ٢٧ → ١٤
z = ٦	x + ٢y = ٢٤ → ١٣
z = ٧	x + ٢y = ٢١ → ١١
z = ٨	x + ٢y = ١٨ → ١٠
z = ٩	x + ٢y = ١٥ → ٨
z = ١٠	x + ٢y = ١٢ → ٧
z = ١١	x + ٢y = ٩ → ٥
z = ١٢	x + ٢y = ٦ → ٤
z = ١٣	x + ٢y = ٣ → ٢
z = ١٤	x + ٢y = ٠ → ١

Total : ١٦٩

٢٣. If the Coefficient of x٣ in the expansion of $\left(1 \square \frac{1}{x}\right)^6 (١ + x)٤(٢ - x)$ ... ٣ ٨ ؛ x□□ is □, then □ equals ________________.

Ans. (٦٧٨)

Sol. coeff of $x^{٠}$ in $\dfrac{(x-١)(١-x^{٢})(1-x^{٣})}{x^{٦}}$

coeff. of $x^{٦}$ in $(x-١)(١-x^{٢})(1-x^{٣})$

General term

$^{٦}C_{r_1}\,{}^{٧}C_{r_2}\,{}^{٨}C_{r_3}\,(-١)^{r_1}\,x^{r_1+٢r_2+٣r_3}$

$r_1+٢r_2+٣r_3=٣٦$

Case-I :

r_1	r_2	r_3
٠	٦	٨
٢	٥	٨
٤	٤	٨
٦	٣	٨

$r_1+٢r_2=١٢$ (Taking $r_3=٨$)

Case-II :

r_1	r_2	r_3
١	٧	٧
٣	٦	٧
٥	٥	٧

$r_1+٢r_2=١٥$ (Taking $r_3=٧$)

Case-III :

r_1	r_2	r_3
٤	٧	٦
٦	٦	٦

$r_1+٢r_2=١٨$ (Taking $r_3=٦$)

Coeff. = ٧ + (١٥ × ٢١) + (١٥ × ٣٥) + (٣٥) −(٦ × ٨) − (٢٠ × ٧ × ٨) − (٦ × ٢١ × ٨) + (١٥ × ٢٨) + (٧ × ٢٨) = −٦٧٨ = [illegible]

٢٤. Let ٣، ٧، ١١، ١٥، . . . . ، ٤٠٣ and ٢، ٥، ٨، ١١، . . . . ، ٤٠٤ be two arithmetic progressions. Then the sum، of the common terms in them، is equal to ____________.

Ans. (٦٦٩٩)

Sol. ٣، ٧، ١١، ١٥، ..... ، ٤٠٣

٢، ٥، ٨، ١١، ..... ، ٤٠٤

LCM (٤ ، ٣) = ١٢

١١، ٢٣، ٣٥، ..... let (٤٠٣)

٤٠٣ = ١١ + (n − ١) x ١٢

$\frac{392}{12} = n-1$

$33 = 66\ n$ [illegible]

$n = 33$

Sum $\frac{33}{2}\left[22+32\times12\right]$

= ٦٦٩٩

٢٥. Let {x} denote the fractional part of x and

$f(x) = \dfrac{\cos^{-1}(1-\{x\}^2)\sin^{-1}(1-\{x\})}{\{x\}-\{x\}^3}$ ، x ≠ ٠. If L and R respectively denotes the left hand limit and the right hand limit of f(x) at x = ٠، then $\frac{32}{\pi^2}(L^2+R^2)$ is equal to ____________.

Ans. (١٨)

Sol. Finding right hand limit

$\lim_{x\to0^+} f(x) = \lim_{h\to0} f(٠+h)$

$= \lim_{h\to0} f(h)$

$= \lim_{h\to0}\dfrac{\cos^{-1}(1-h^2)\sin^{-1}(1-h)}{h(1-h^2)}$

$= \lim_{h\to0}\dfrac{\cos^{-1}(1-h^2)}{h}\cdot\dfrac{\sin^{-1}(1)}{1}$

Let $\cos^{-1}(1-h^2)=\theta \Rightarrow \cos\theta = 1-h^2$

$= \frac{\pi}{2}\lim_{\theta\to0}\dfrac{\theta}{\sqrt{1-\cos\theta}}$

$= \frac{\pi}{2}\lim_{\theta\to0}\dfrac{1}{\sqrt{\dfrac{1-\cos\theta}{\theta^2}}}$

$= \frac{\pi}{2}\cdot\dfrac{1}{\sqrt{1/2}}$

$R = \dfrac{\pi}{\sqrt{2}}$

Now finding left hand limit

$L = \lim_{x \to 0^-} f(x)$

$= \lim_{h \to 0} f(-h)$

$= \lim_{h \to 0} \frac{\cos^{-1}(1-\{-h\}^2)\sin^{-1}(1-\{-h\})}{\{-h\} - \{-h\}^3}$

$= \lim_{h \to 0} \frac{\cos^{-1}(1-(-h+1)^2)\sin^{-1}(1-(-h+1))}{(-h+1)-(-h+1)^3}$

$= \lim_{h \to 0} \frac{\cos^{-1}(2h-h^2)\sin^{-1}(h)}{(1-h)(1-(1-h)^2)}$

$= \lim_{h \to 0} \frac{\frac{\pi}{2}\sin^{-1}h}{(1-h)(2h-h^2)}$

$= \lim_{h \to 0} \frac{\pi}{2} \cdot \frac{\sin^{-1}h}{h(2-h)(1-h)}$

$= \frac{\pi}{2} \lim_{h \to 0} \frac{\sin^{-1}h}{h} \cdot \frac{1}{2} = \frac{\pi}{4}$

$L = \frac{\pi}{4}$

$32\left(L^2 + R^2\right) = 32\left(\frac{\pi^2}{2} \cdot \frac{1}{16} + \frac{\pi^2}{2}\cdot\frac{1}{16}\right)$

$= 16 + 2 = 18$

26. Let the line $L : \sqrt{2}x + y = \alpha$ pass through the point of the intersection P (in the first quadrant) of the circle $x^2 + y^2 = 3$ and the parabola $x^2 = 2y$. Let the line L touch two circles C_1 and C_2 of equal radius $2\sqrt{3}$. If the centres Q_1 and Q_2 of the circles C_1 and C_2 lie on the y-axis, then the square of the area of the triangle PQ_1Q_2 is equal to ______________.

Ans. (72)

Sol. $x^2 + y^2 = 3$ and $x^2 = 2y$

$y^2 + 2y - 3 = 0 \Rightarrow (y+3)(y-1) = 0$

$y = -3$ or $y = 1$

$y = 1 \; x = \sqrt{2} \Rightarrow P(\sqrt{2}, 1)$

p lies on the line

$\sqrt{2}x + y = \alpha$

$\sqrt{2}(\sqrt{2}) + 1 = \alpha$

$\alpha = 3$

For circle C_1

Q_1 lies on y axis

Let $Q_1(0, \alpha)$ coordinates

$R_1 = 2\sqrt{3}$ (Given)

Line L act as tangent

Apply $P = r$ (condition of tangency)

$\Rightarrow \left|\frac{\alpha - 3}{\sqrt{3}}\right| = 2\sqrt{3}$

$\Rightarrow |\alpha - 3| = 6$

$\Rightarrow \alpha - 3 = 6$ or $\alpha - 3 = -6$

$\Rightarrow \alpha = 9$ $\quad \alpha = -3$

$\Delta PQ_1Q_2 = \frac{1}{2}\begin{vmatrix} \sqrt{2} & 1 & 1 \\ 0 & 9 & 1 \\ 0 & -3 & 1 \end{vmatrix}$

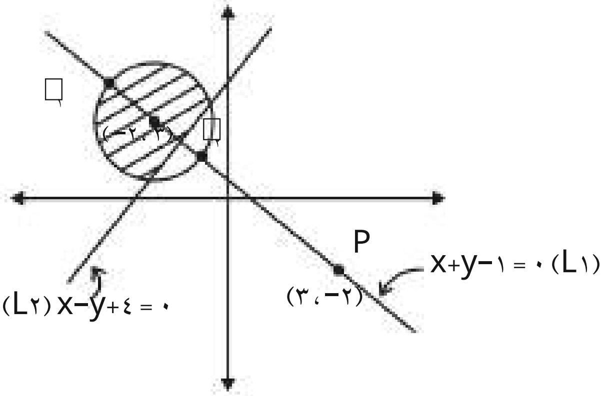
$= \frac{1}{2} \times \sqrt{2}(12) = 6\sqrt{2}$

$\Rightarrow (\Delta PQ_1Q_2)^2 = 72$

٢٧. Let $P = \{z \in \mathbb{C} : |z + 2 - 3i| \le 1\}$ and $Q = \{z \in \mathbb{C} : z(1 + i) + \bar{z}(1 - i) \le -8\}$. Let in $P \cap Q$, $|z - 3 + 2i|$ be maximum and minimum at z_1 and z_2 respectively. If $|z_1|^2 + 2|z_2|^2 = \alpha + \beta\sqrt{2}$ where α, β are integers, then $\alpha + \beta$ equals ____________.

Ans. (٣٦)

Sol.



Clearly for the shaded region z_1 is the intersection of the circle and the line passing through P (L_1) and z_2 is intersection of line L_1 & Circle

Circle : $(x + 2)^2 + (y - 3)^2 = 1$

$L_1 : x + y - 1 = 0$

$L_2 : x - y + 4 = 0$

On solving circle & L_1 we get

$$z_1 : \left(-2 - \frac{1}{\sqrt{2}}, 3 + \frac{1}{\sqrt{2}}\right)$$

On solving L_1 and z_2 is intersection of line L_1 & L_2

we get $z_2 : \left(-\frac{3}{2}, \frac{5}{2}\right)$

$$|z_1|^2 + 2|z_2|^2 = 14 + 5\sqrt{2} + 17$$

$$= 31 + 5\sqrt{2}$$

So $\alpha = 31$

$\beta = 5$

$\alpha + \beta = 36$

٢٨. If $\int_{-\pi/2}^{\pi/2} \frac{8\sqrt{2}\cos x\, dx}{(1 + e^{\sin x})(1 + \sin^4 x)} = \alpha\pi + \beta \log_e(3 + 2\sqrt{2})$, where α, β are integers, then $\alpha^2 + \beta^2$ equals ________________.

Ans. (٨)

Sol. $$I = \int_{-\pi/2}^{\pi/2} \frac{8\sqrt{2}\cos x}{(1 + e^{\sin x})(1 + \sin^4 x)} dx$$

Apply king

$$I = \int_{-\pi/2}^{\pi/2} \frac{8\sqrt{2}\cos x\, e^{\sin x}}{(1 + e^{\sin x})(1 + \sin^4 x)} dx \quad \ldots(2)$$

adding (1) & (2)

$$2I = \int_{-\pi/2}^{\pi/2} \frac{8\sqrt{2}\cos x}{1 + \sin^4 x} dx$$

$$I = \int_0^{\pi/2} \frac{8\sqrt{2}\cos x}{1 + \sin^4 x} dx, \quad \sin x = t$$

$$I = \int_0^1 \frac{8\sqrt{2}}{1 + t^4} dt$$

$$I = 4\sqrt{2}\int_0^1 \left(\frac{1 + \frac{1}{t^2}}{t^2 + \frac{1}{t^2}} - \frac{1 - \frac{1}{t^2}}{t^2 + \frac{1}{t^2}}\right) dt$$

$$I = 4\sqrt{2}\int_0^1 \left(\frac{1 + \frac{1}{t^2}}{\left(t - \frac{1}{t}\right)^2 + 2} - \frac{1 - \frac{1}{t^2}}{\left(t + \frac{1}{t}\right)^2 - 2}\right) dt$$

Let $t - \frac{1}{t} = z$ & $t + \frac{1}{t} = k$

$= 4\sqrt{2}\left[\int_{-\infty}^{0}\frac{dz}{z^2+2} + \int_{0}^{2}\frac{dk}{k^2-2}\right]$

$= 4\sqrt{2}\left[\frac{1}{\sqrt{2}}\tan^{-1}\frac{z}{\sqrt{2}}\Big|_{-\infty}^{0} + \frac{1}{2\sqrt{2}}\ln\left|\frac{k-\sqrt{2}}{k+\sqrt{2}}\right|_{0}^{2}\right]$

$= 4\sqrt{2}\left[\frac{\pi}{2\sqrt{2}} + \frac{1}{2\sqrt{2}}\ln\left(\frac{2-\sqrt{2}}{2+\sqrt{2}}\right)\right]$

$= 2\pi + 2\ln(3-2\sqrt{2})$

$\alpha = 2$

$\beta = 2$

29. Let the line of the shortest distance between the lines

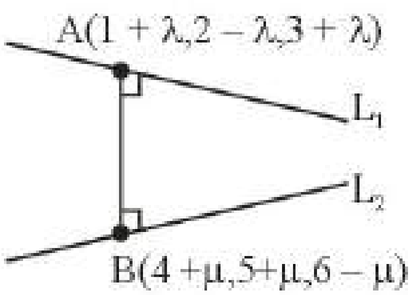
$L_1 : \vec{r} = (\hat{i} + 2\hat{j} + 3\hat{k}) + \lambda(\hat{i} - \hat{j} + \hat{k})$ and

$L_2 : \vec{r} = (4\hat{i} + 5\hat{j} + 6\hat{k}) + \mu(\hat{i} + \hat{j} - \hat{k})$

intersect L_1 and L_2 at P and Q respectively. If (α, β, γ) is the midpoint of the line segment PQ, then $2(\alpha + \beta + \gamma)$ is equal to ____________.

Ans. (21)

Sol.



$\vec{b} = \hat{i} - \hat{j} + \hat{k}$ (DR's of L_1)

$\vec{d} = \hat{i} + \hat{j} - \hat{k}$ (DR's of L_2)

$\vec{b} \times \vec{d} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -1 & 1 \\ 1 & 1 & -1 \end{vmatrix}$

$= 0\hat{i} + 2\hat{j} + 2\hat{k}$ (DR's of Line perpendicular to L_1 and L_2)

DR of AB line

$= (0,2,2) = (3+\mu-\lambda, 3+\mu+\lambda, 3-\mu-\lambda)$

$\frac{3+\mu-\lambda}{0} = \frac{3+\mu+\lambda}{2} = \frac{3-\mu-\lambda}{2}$

Solving above equation we get $\mu = -\frac{3}{2}$ and $\lambda = \frac{3}{2}$

point A $\left(\frac{5}{2}, \frac{1}{2}, \frac{9}{2}\right)$

B $\left(\frac{5}{2}, \frac{7}{2}, \frac{15}{2}\right)$

Point of AB = $\left(\frac{5}{2}, 2, 6\right) = (\alpha, \beta, \gamma)$

$2(\alpha + \beta + \gamma) = 5 + 4 + 12 = 21$

30. Let A = {1, 2, 3, ... 20}. Let R_1 and R_2 two relation on A such that

R_1 = {(a, b) : b is divisible by a}

R_2 = {(a, b) : a is an integral multiple of b}.

Then, number of elements in $R_1 - R_2$ is equal to ____________.

Ans. (46)

Sol. $n(R_1) = 20 + 10 + 6 + 5 + 4 + 3 + 2 + 2 + 2$

$+ 2 + \underbrace{1 + \ldots + 1}_{10\text{ times}}$

n(R١) = ٦٦

R١ □ R٢ □□□١،١□□ ٢،٢□، . . . .□□٢٠،٢٠□

n□R١□ R٢ □□٢٠ n□R

n□R١□R٢ □□ 1□n□R١R٢ □

□n□R١□□٢٠

= ٦٦ − ٢٠

R١ − R٢ = ٤٦ Pair

SECTION-A

٣١. With rise in temperature، the Young's modulus of elasticity

(١) changes erratically

(٢) decreases

(٣) increases

(٤) remains unchanged

Ans. (٢)

Sol. Conceptual questions

٣٢. If R is the radius of the earth and the acceleration due to gravity on the surface of earth is $g = \pi^2$ m/s²، then the length of the second's pendulum at a height h = ٢R from the surface of earth will be ::

(١) $\frac{2}{9}$ m

(٢) $\frac{4}{9}$ m

(٣) $\frac{8}{9}$ m

(٤) $\frac{1}{9}$ m

Ans. (٢)

Sol. $g' = \frac{GMe}{(3R)^2} = \frac{1}{9} g$

$T = ٢\pi \sqrt{\frac{\ell}{g'}}$

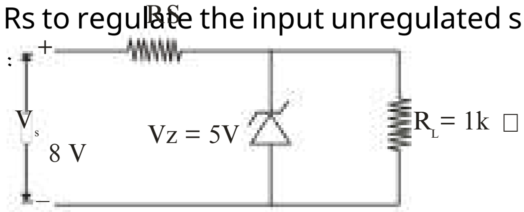
Since the time period of second pendulum is ٢ sec.

T = ٢ sec

$٢ = ٢\pi \sqrt{\frac{\ell 9}{g}}$

$\boxed{\ell = \frac{1}{9} m}$

٣٣. In the given circuit if the power rating of Zener diode is ١٠ mW، the value of series resistance Rs to regulate the input unregulated supply is :

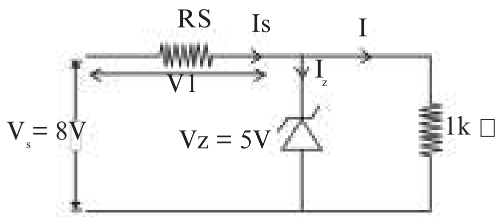


(١) ٥kΩ (٢) ١٠Ω

(٣) ١kΩ (٤) ١٠kΩ

Ans. (BONUS)

Sol.



Pd across Rs

V١ = ٨ – ٥ = ٣V

Current through the load resistor

$I = \frac{5}{1 \times 10^3}$ = ٥mA

Maximum current through Zener diode

Iz max. $= \frac{10}{5}$ = ٢mA

And minimum current through Zener diode

Iz min. = ٠

∴ Is max. = ٥ + ٢ = ٧mA

And Rs min $= \frac{V_1}{Is max.} = \frac{3}{7}$ kΩ

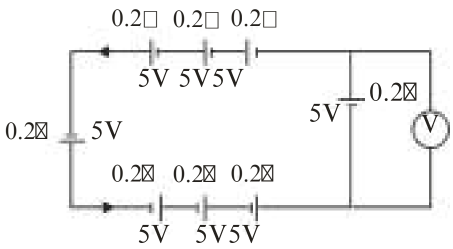
Similarly

Is min. = ٥mA

And Rs max. $= \frac{V_1}{I_{s\ min.}}$ = $\frac{٣}{٥}$ kΩ

∴ $\frac{3}{7}$ kΩ < Rs < $\frac{٣}{٥}$ kΩ

٣٤. The reading in the ideal voltmeter (V) shown in the given circuit diagram is :



(١) ٥V (٢) ١٠V

(٣) ٠V (٤) ٣V

Ans. (٣)

Sol. $i = \frac{E_{eq}}{r_{eq}} = \frac{8 \times 5}{8 \times 0.2}$

I = ٢٥A

V = E – ir

= ٥ – ٠.٢ × ٢٥

= ٠

٣٥. Two identical capacitors have same capacitance C. One of them is charged to the potential V and other to the potential ٢V. The negative ends of both are connected together. When the positive ends are also joined together، the decrease in energy of the combined system is :

(١) $\frac{1}{4}CV^2$

(٢) ٢ CV ٢

(٣) $\frac{1}{2}CV^2$

(٤) $\frac{3}{4}CV^2$

Ans. (١)

Sol. $V_C = \frac{q_{net}}{C_{net}} = \frac{CV + 2CV}{2C}$

$V_C = \frac{3V}{2}$

Loss of energy

$= \frac{1}{2}CV^2 + \frac{1}{2}C(2V)^2 - \frac{1}{2}2C\left(\frac{3V}{2}\right)^2$

$= \left(\frac{CV^2}{4}\right)$

٣٦. Two moles a monoatomic gas is mixed with six moles of a diatomic gas. The molar specific heat of the mixture at constant volume is :

(١) $\frac{9}{4}R$ (٢) $\frac{7}{4}R$

(٣) $\frac{3}{2}R$ (٤) $\frac{5}{2}R$

Ans. (١)

Sol. $C_V = \frac{n_1 C_{v_1} + n_2 C_{v_2}}{n_1 + n_2}$

$= \frac{2 \times \frac{3}{2}R + 6 \times \frac{5}{2}R}{2 + 6}$

$= \frac{9}{4}R$

٣٧. A ball of mass ٠.٥ kg is attached to a string of length ٥٠ cm. The ball is rotated on a horizontal circular path about its vertical axis. The maximum tension that the string can bear is ٤٠٠ N. The maximum possible value of angular velocity of the ball in rad/s is، :

(١) ١٦٠٠ (٢) ٤٠

(٣) ١٠٠٠ (٤) ٢٠

Ans. (٢)

Sol. $T = m\omega^2 \ell$

٤٠٠ = ٠.٥ω × ٢ ٠.٥

ω = ٤٠ rad/s.

٣٨. A parallel plate capacitor has a capacitance C = ٢٠٠ pF. It is connected to ٢٣٠ V ac supply with an angular frequency ٣٠٠ rad/s. The rms value of conduction current in the circuit and displacement current in the capacitor respectively are :

(١) ١.٣٨ μA and ١.٣٨ μA

(٢) ١٤.٣ μA and ١٤٣ μA

(٣) ١٣.٨ μA and ١٣٨ μA

(٤) ١٣.٨ μA and ١٣.٨ μA

Ans. (٤)

Sol. $I = \frac{V}{X_C}$ = ٢٣٠ × ٣٠٠ × ٢٠٠ × ١٠ ⁻¹² = ١٣.٨ μA

٣٩. The pressure and volume of an ideal gas are related as $PV^{٣/٢}$ = K (Constant). The work done when the gas is taken from state A (P١, V١, T١) to state B (P٢, V٢, T٢) is :

(١) ٢(P١V١ – P٢V٢)

(٢) ٢(P٢V٢ – P١V١)

(3) 2(P1 √V1 □ P2 √V2)

(4) 2(P2 √V2 □ P1 √V1)

Ans. (١ or ٢)

Sol. For PV^x = constant

If work done by gas is asked then

$$W = \frac{nR□T}{1□x}$$

Here x = $\frac{3}{2}$

$$□W = \frac{P2\ V2\ □\ P1V1}{□\frac{1}{2}}$$

= ٢(P١V١ – P٢V٢) ..... Option (١) is correct

If work done by external is asked then

W = –٢(P١V١ – P٢V٢) ..... Option (٢) is correct

٤٠. A galvanometer has a resistance of ٥٠ □ and it allows maximum current of ٥ mA. It can be converted into voltmeter to measure upto ١٠٠ V by connecting in series a resistor of resistance

(١) ٥٩٧٥□□□

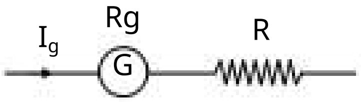
(٢) ٢٠٠٥٠□□

(٣) ١٩٩٥٠ □□□

(٤) ١٩٥٠٠□□

Ans. (٣)

Sol.



$$R = \frac{V}{I_g} □ Rg□ \frac{100}{5□10^{□3}} □ 50$$

= ٢٠٠٠٠ – ٥٠

= ١٩٩٥٠□

٤١. The de Broglie wavelengths of a proton and an □ particle are □ and ٢□□ respectively. The ratio of the velocities of proton and □ particle will be : (١) ١ : ٨ (٢) ١ : ٢ (٣) ٤ : ١ (٤) ٨ : ١

Ans. (٤)

Sol. □ = $\frac{h}{p} □ \frac{h}{mv} □ v = \frac{h}{m□}$

$$\frac{v_p}{v_□} □ \frac{m_□}{m_p} □ \frac{□_□}{□_p}$$

= ٤ × ٢ = ٨

٤٢. ١٠ divisions on the main scale of a Vernier calliper coincide with ١١ divisions on the Vernier scale. If each division on the main scale is of ٥ units, the least count of the instrument is :

(١) $\frac{1}{2}$

(٢) $\frac{10}{11}$ $\frac{11}{50}$

(٣) $\frac{11}{5}$

(٤) $\frac{11}{}$

Ans. (٤)

Sol. ١٠ MSD = ١١ VSD

١ VSD = $\frac{10}{11}$ MSD

LC = ١MSD – ١VSD

= ١ MSD □ $\frac{10}{11}$ MSD

= $\frac{1MSD}{11}$

= $\frac{5}{11}$ units

٤٣. In series LCR circuit، the capacitance is changed from C to ٤C. To keep the resonance frequency unchanged، the new inductance should be :

(١) reduced by $\frac{1}{4}$ L

(٢) increased by ٢L

(٣) reduced by $\frac{3}{4}$ L

(٤) increased to ٤L

Ans. (٣)

Sol. □' = □

$\frac{١}{\sqrt{L'C'}} □ \frac{١}{\sqrt{LC}}$

□ L'C' = LC

L'(٤C) = LC

L' = $\frac{L}{4}$

□ Inductance must be decreased by $\frac{3L}{4}$

٤٤. The radius (r)، length (l) and resistance (R) of a metal wire was measured in the laboratory as

r = (٠.٣٥ ± ٠.٠٥) cm

R = (١٠٠ ± ١٠) ohm

l = (١٥ ± ٠.٢) cm

The percentage error in resistivity of the material of the wire is :

(١) ٢٥.٦٪ (٢) ٣٩.٩٪

(٣) ٣٧.٣٪ (٤) ٣٥.٦٪

Ans. (٢)

Sol. □ = R $\frac{□}{\ell}$

$\frac{□□}{□} □ \frac{□R}{R} □ 2\frac{□r}{r} □ \frac{□\ell}{\ell}$

= $\frac{10}{100} □ 2 □ \frac{0.05}{0.35} □ \frac{0.2}{15}$

= $\frac{1}{10} □ \frac{2}{7} □ \frac{1}{75}$

$\frac{□□}{□}$ = ٣٩.٩٪

٤٥. The dimensional formula of angular impulse is :

(١) [M L²T] (٢) [M LT]

(٣) [M L T] (٤) [M LT]

Ans. (٤)

Sol. Angular impulse = change in angular momentum.

[Angular impulse] = [Angular momentum] =

= [M LT]

٤٦. A simple pendulum of length ١ m has a wooden bob of mass ١ kg. It is struck by a bullet of mass ١٠ kg moving with a speed of ٢ × ١٠ ms. ٢ -١

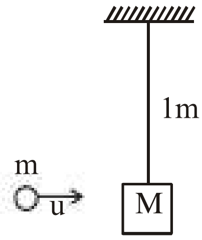
The bullet gets embedded into the bob. The height to which the bob rises before swinging back is.

(use g = ١٠ m/s²)

(١) ٠.٣٠ m (٢) ٠.٢٠ m

(٣) ٠.٣٥ m (٤) ٠.٤٠ m

Ans. (٢)



Sol.

mu = (M + m)V

١٠ × ٢ × ١٠ □ ١ × V

V □ ٢m/s

h = $\frac{V2}{2g}$ = ٠.٢ m

٤٧. A particle moving in a circle of radius R with uniform speed takes time T to complete one revolution. If this particle is projected with the same speed at an angle □ to the horizontal، the maximum height attained by it is equal to ٤R. The angle of projection □ is then given by :

(١) $\sin^{-1}\left[\frac{2gT^2}{\pi^2 R}\right]^{\frac{1}{2}}$ (٢) $\sin^{-1}\left[\frac{\pi^2 R}{2gT^2}\right]^{\frac{1}{2}}$

(٣) $\cos^{-1}\left[\frac{2gT^2}{\pi^2 R}\right]^{\frac{1}{2}}$ (٤) $\cos^{-1}\left[\frac{\pi R}{2gT^2}\right]^{\frac{1}{2}}$

Ans. (١)

Sol. $\frac{2\pi R}{T} = V$

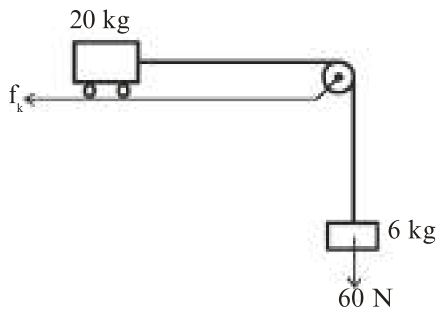
Maximum height $H = \frac{v^2 \sin^2\theta}{2g}$

٤R = $\frac{4\pi^2 R^2}{T^2 2g}\sin^2\theta$

$\sin\theta = \sqrt{\frac{2gT^2}{\pi^2 R}}$

$\theta = \sin^{-1}\left(\frac{2gT^2}{\pi^2 R}\right)^{\frac{1}{2}}$

٤٨. Consider a block and trolley system as shown in figure. If the coefficient of kinetic friction between the trolley and the surface is ٠.٠٤, the acceleration of the system in m/s² is :
(Consider that the string is massless and unstretchable and the pulley is also massless and frictionless) :



(١) ٣ (٢) ٤
(٣) ٢ (٤) ١.٢

Ans. (٣)

Sol. $f_k = \mu N$ = ٠.٠٤ × ٢٠g = ٨ Newton

$a = \frac{60 - 8}{26}$ = ٢ m/s²

٤٩. The minimum energy required by a hydrogen atom in ground state to emit radiation in Balmer series is nearly :

(١) ١.٥ eV (٢) ١٣.٦ eV
(٣) ١.٩ eV (٤) ١٢.١ eV

Ans. (٤)

Sol. Transition from n = ١ to n = ٣

ΔE = ١٢.١ eV

٥٠. A monochromatic light of wavelength ٦٠٠٠Å is incident on the single slit of width ٠.٠١ mm. If the diffraction pattern is formed at the focus of the convex lens of focal length ٢٠ cm, the linear width of the central maximum is : (١) ٦٠ mm (٢) ٢٤ mm (٣) ١٢٠ mm (٤) ١٢ mm

Ans. (٢)

Sol. Linear width

$W = \frac{2\lambda d}{a} = \frac{2 \times 6 \times 10^{-7} \times 0.2}{1 \times 10^{-5}}$

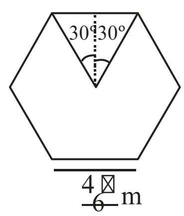
= ٢.٤ × ١٠$^{-٢}$ = ٢٤ mm

SECTION-B

٥١. A regular polygon of ٦ sides is formed by bending a wire of length ٤π meter. If an electric current of ٤π$\sqrt{٣}$ A is flowing through the sides of the polygon, the magnetic field at the centre of the polygon would be x × ١٠$^{-٧}$ T. The value of x is ______.

Ans. (٧٢)

Sol.



$B = 6\left(\frac{\mu_0 I}{4\pi r}\right)$(sin ٣٠° + sin ٣٠°)

= ٦ $\frac{10^{-7} \times 4\pi\sqrt{3}}{\left(\frac{\sqrt{3}}{2}\right)\left(\frac{4\pi}{6}\right)}$

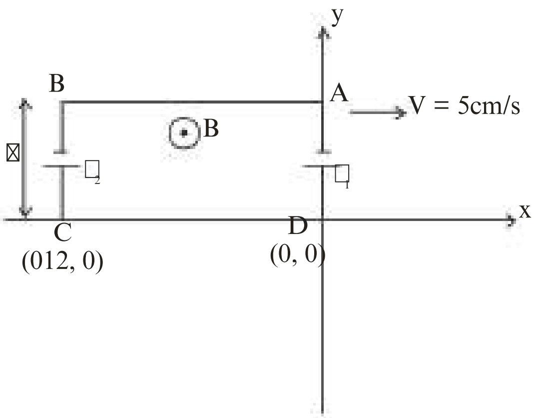
= ٧٢ × ١٠$^{-٧}$ T

٥٢. A rectangular loop of sides ١٢ cm and ٥ cm، with its sides parallel to the x-axis and y-axis respectively moves with a velocity of ٥ cm /s in the positive x axis direction، in a space containing a variable magnetic field in the positive z direction.

The field has a gradient of ١٠T/cm along the negative x direction and it is decreasing with time at the rate of ١٠ T /s. If the resistance of the loop is ٦ mΩ، the power dissipated by the loop as heat is ________ × ١٠⁻٩ W.

Ans. (٢١٦)

Sol.



B٠ is the magnetic field at origin

$$\frac{dB}{dx} = -\frac{10^{-3}}{10^{-2}}$$

$$\int_{B_0}^{B} dB = -\int_0^x 10^{-1} dx$$

B − B٠ = −١٠x⁻¹

$$B = \left(B_0 - \frac{x}{10}\right)$$

Motional emf in AB = ٠

Motional emf in CD = ٠

Motional emf in AD = ε١ = $B_0 \ell v$

Magnetic field on rod BC B

$$= \left(B_0 - \frac{(-12 \times 10^{-2})}{10}\right)$$

Motional emf in BC = ε٢ = $\left(B_0 + \frac{12 \times 10^{-2}}{10}\right)\ell v$

εeq = ε٢ − ε١ = ٣٠٠ × ١٠⁻٧ V

For time variation

$(\varepsilon_{eq})' = A\frac{dB}{dt}$ = ٦٠ × ١٠⁻٧ V

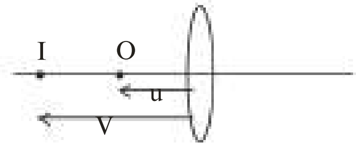
(εeq)net = εeq + (εeq)′ = ٣٦٠ × ١٠⁻٧ V

Power = $\frac{(\varepsilon_{eq})^2_{net}}{R}$ = ٢١٦ × ١٠⁻٩ W

٥٣. The distance between object and its ٣ times magnified virtual image as produced by a convex lens is ٢٠ cm. The focal length of the lens used is ____________ cm.

Ans. (١٥)

Sol.



v = ٣u

v − u = ٢٠ cm

٢u = ٢٠ cm

u = ١٠ cm

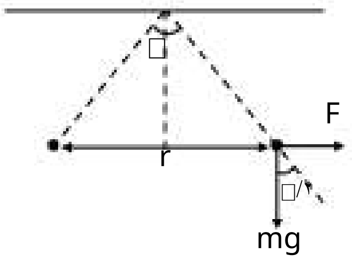
$$\frac{1}{(-30)} - \frac{1}{(-10)} = \frac{1}{f}$$

f = ١٥ cm

٥٤. Two identical charged spheres are suspended by strings of equal lengths. The strings make an angle θ with each other. When suspended in water the angle remains the same. If density of the material of the sphere is ١.٥ g /cc، the dielectric constant of water will be __________

(Take density of water = ١ g /cc)

Ans. (٣)

Sol. 

In air $\tan\frac{\theta}{2} = \frac{F}{mg} = \frac{q^2}{4\pi\varepsilon_0 r^2 mg}$

In water $\tan\frac{\theta}{2} = \frac{F'}{mg'} = \frac{q^2}{4\pi\varepsilon_0 \varepsilon_r r^2 mg_{eff}}$

Equate both equations

$\rho_0 g = \rho_0 \varepsilon_r g\left(1 - \frac{1}{1.5}\right)$

$\varepsilon_r = ٣$

٥٥. The radius of a nucleus of mass number ٦٤ is ٤.٨ fermi. Then the mass number of another nucleus having radius of ٤ fermi is $\frac{1000}{x}$, where x is ________.

Ans. (٢٧)

Sol. $R = R_0 A^{١/٣}$

$R^3 \propto A$

$\left(\frac{4.8}{4}\right)^3 = \frac{64}{A}$

$= \frac{٦٤}{A} = (١.٢)^3$

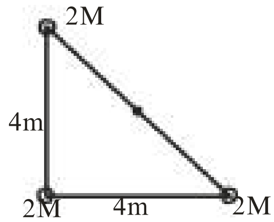
$\frac{٦٤}{A} = ١.٤٤ \times ١.٢$

$A = \frac{٦٤}{١.٤٤ \times ١.٢} = \frac{١٠٠٠}{x}$

$x = \frac{١٤٤ \times ١٢}{٦٤} = ٢٧$

٥٦. The identical spheres each of mass ٢M are placed at the corners of a right angled triangle with mutually perpendicular sides equal to ٤ m each. Taking point of intersection of these two sides as origin, the magnitude of position vector of the centre of mass of the system is $\frac{4\sqrt{2}}{x}$, where the value of x is ________________

Ans. (٣)

Sol. 

Position vector $\vec{r}_{COM} = \frac{m_١\vec{r}_١ + m_٢\vec{r}_٢ + m_٣\vec{r}_٣}{m_١ + m_٢ + m_٣}$

$\vec{r}_{COM} = \frac{٢M(0) + ٢M(٤\hat{i}) + ٢M(٤\hat{j})}{٦M}$

$\vec{r} = \frac{4}{3}\hat{i} + \frac{4}{3}\hat{j}$

$|\vec{r}| = \frac{4\sqrt{2}}{3}$

$x = ٣$

٥٧. A tuning fork resonates with a sonometer wire of length ١ m stretched with a tension of ٦ N. When the tension in the wire is changed to ٥٤ N, the same tuning fork produces ١٢ beats per second with it. The frequency of the tuning fork is __________ Hz.

Ans. (٦)

Sol. $f = \frac{1}{2\ell}\sqrt{\frac{T}{\mu}}$

$f_١ = \frac{1}{2}\sqrt{\frac{6}{\mu}}$ $\quad$ $f_٢ = \frac{1}{2}\sqrt{\frac{54}{\mu}}$

$\frac{f_1}{f_2} = \frac{1}{3}$ $\quad$ $f_٢ - f_١ = ١٢$

$f_١ = ٦HZ$

٥٨. A plane is in level flight at constant speed and each of its two wings has an area of ٤٠ $m^{٢}$. If the speed of the air is ١٨٠ km/h over the lower wing surface and ٢٥٢ km/h over the upper wing surface، the mass of the plane is ____________kg. (Take air density to be ١ kg $m^{-٣}$ and g = ١٠ $ms^{-٢}$)

Ans. (٩٦٠٠)

Sol. A = ٨٠ $m^{٢}$

Using Bernonlli equation

$A(P_٢ - P_١) = \frac{1}{2}\rho(V_2^2 - V_1^2)A$

$mg = \frac{1}{2} \times ١ (٧٠^{٢} - ٥٠^{٢}) \times ٨٠$

mg = ٤٠ × ٢٤٠٠

m = ٩٦٠٠ kg

٥٩. The current in a conductor is expressed as $I = ٣t^{٢} + ٤t^{٣}$، where I is in Ampere and t is in second. The amount of electric charge that flows through a section of the conductor during t = ١s to t = ٢s is _________________ C.

Ans. (٢٢)

Sol. $q = \int_1^2 i\,dt = \int_1^2 (3t^2 + 4t^3)\,dt$

$q = \left[t^3 + t^4\right]_1^2$

q = ٢٢C

٦٠. A particle is moving in one dimension (along x axis) under the action of a variable force. It's initial position was ١٦ m right of origin. The variation of its position (x) with time (t) is given as $x = -٣t^{٣} + ١٨t^{٢} + ١٦t$، where x is in m and t is in s. The velocity of the particle when its acceleration becomes zero is ______________ m/s.

Ans. (٥٢)

Sol. $x = ٣t^{٣} + ١٨t^{٢} + ١٦t$

$v = -٩t^{٢} + ٣٦t + ١٦$

a = − ١٨t + ٣٦

a = ٠ at t = ٢s

$v = -٩(٢)^{٢} + ٣٦ \times ٢ + ١٦$

v = ٥٢ m/s

CHEMISTRY   TEST PAPER WITH SOLUTION

SECTION-A

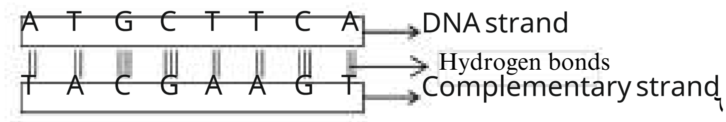
٦١. If one strand of a DNA has the sequence ATGCTTCA, sequence of the bases in complementary strand is:

(١) CATTAGCT (٢) TACGAAGT

(٣) GTACTTAC (٤) ATGCGACT

Ans. (٢)

Sol. Adenine base pairs with thymine with ٢ hydrogen bonds and cytosine base pairs with guanine with ٣ hydrogen bonds.



٦٢. Given below are two statements : one is labelled as Assertion (A) and the other is labelled as Reason (R).

Assertion (A) : Haloalkanes react with KCN to form alkyl cyanides as a main product while with AgCN form isocyanide as the main product.

Reason (R) : KCN and AgCN both are highly ionic compounds.

In the light of the above statement, choose the most appropriate answer from the options given below:

(١) (A) is correct but (R) is not correct

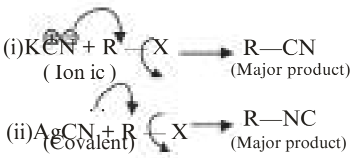
(٢) Both (A) and (R) are correct but (R) is not the correct explanation of (A)

(٣) (A) is not correct but (R) is correct

(٤) Both (A) and (R) are correct and (R) is the correct explanation of (A)

Ans. (١)

Sol.



AgCN is mainly covalent in nature and nitrogen is available for attack, so alkyl isocyanide is formed as main product.

٦٣. In acidic medium, $K_2Cr_2O_7$ shows oxidising action as represented in the half reaction

$$Cr_2O_7^{2-} + XH^+ + Ye^- \rightarrow 2A + ZH_2O$$

X, Y, Z and A are respectively are:

(١) ٨, ٦, ٤ and Cr_2O_3 (٢) ١٤, ٧, ٦ and Cr^{3+}

(٣) ٨, ٤, ٦ and Cr_2O_3 (٤) ١٤, ٦, ٧ and Cr^{3+}

Ans. (٤)

Sol. The balanced reaction is,

$$Cr_2O_7^{2-} + 14H^+ + 6e^- \rightarrow 2Cr^{3+} + 7H_2O$$

X = ١٤

Y = ٦

A = ٧

٦٤. Which of the following reactions are disproportionation reactions?

(A) $Cu^+ \rightarrow Cu^{2+} + Cu$

(B) $3MnO_4^{2-} + 4H^+ \rightarrow 2MnO_4^- + MnO_2 + 2H_2O$

(C) $2KMnO_4 \rightarrow K_2MnO_4 + MnO_2 + O_2$

(D) $2MnO_4^- + 3Mn^{2+} + 2H_2O \rightarrow 5MnO_2 + 4H^+$

Choose the correct answer from the options given below:

(١) (A), (B) (٢) (B), (C), (D)

(٣) (A), (B), (C) (٤) (A), (D)

Ans. (١)

Sol. When a particular oxidation state becomes less stable relative to other oxidation state, one lower, one higher, it is said to undergo disproportionation.

$Cu^+ \rightarrow Cu + Cu^{2+}$

$3MnO_4^{2-} + 4H^+ \rightarrow 2MnO_4^- + MnO_2 + 2H_2O$

٦٥. In case of isoelectronic species the size of F, Ne and Na is affected by:

(١) Principal quantum number (n)

(٢) None of the factors because their size is the same

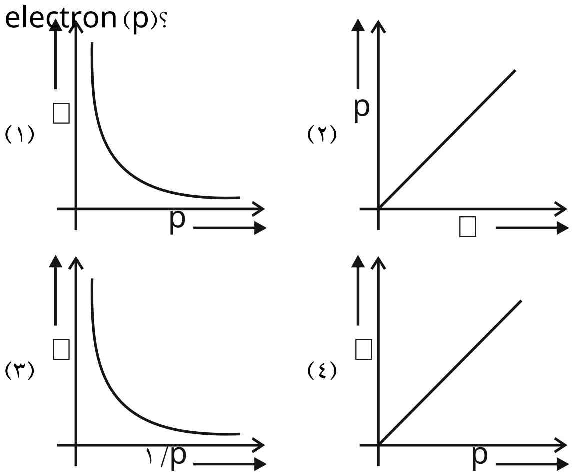
(٣) Electron-electron interaction in the outer orbitals

(٤) Nuclear charge (z)

Ans. (٤)

Sol. In F^-, Ne, Na^+ all have $1s^2, 2s^2, 2p^6$ configuration. They have different size due to the difference in nuclear charge.

٦٦. According to the wave-particle duality of matter by de-Broglie, which of the following graph plot presents most appropriate relationship between wavelength of electron (λ) and momentum of electron (p)?

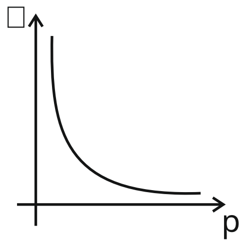


Ans. (١)

Sol. $\lambda = \frac{h}{p} \Rightarrow \lambda \propto \frac{1}{p}$

$\lambda p = h$ (constant)

So, the plot is a rectangular hyperbola.



٦٧. Given below are two statements:

Statement (I): A solution of $[Ni(H_2O)_6]^{2+}$ is green in colour.

Statement (II): A solution of $[Ni(CN)_4]^{2-}$ is colourless.

In the light of the above statements, choose the most appropriate answer from the options given below:

(١) Both Statement I and Statement II are incorrect
(٢) Both Statement I and Statement II are correct
(٣) Statement I is incorrect but Statement II is correct
(٤) Statement I is correct but Statement II is incorrect

Ans. (٢)

Sol. $[Ni(H_2O)_6]^{2+}$ → Green colour solution due to d-d transition.

$[Ni(CN)_4]^{2-}$ → is diamagnetic and it is colourless.

٦٨. Given below are two statements: one is labelled as Assertion (A) and the other is labelled as Reason (R).

Assertion (A): PH_3 has lower boiling point than NH_3.

Reason (R): In liquid state NH_3 molecules are associated through vander waal's forces, but PH_3 molecules are associated through hydrogen bonding.

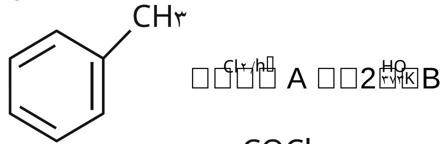
In the light of the above statements, choose the most appropriate answer from the options given below:

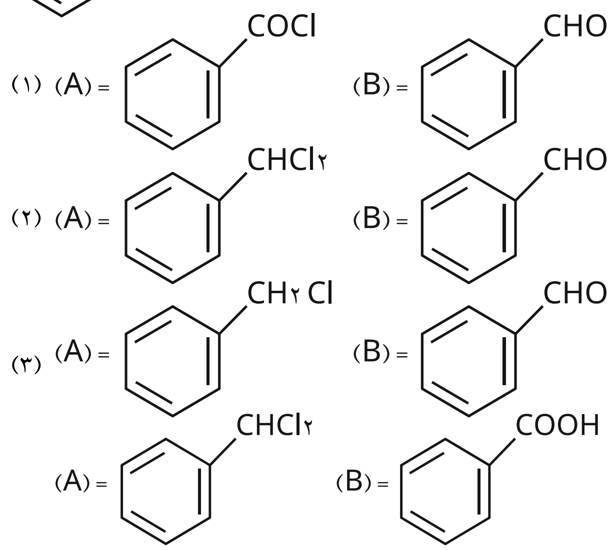
(١) Both (A) and (R) are correct and (R) is not the correct explanation of (A)
(٢) (A) is not correct but (R) is correct
(٣) Both (A) and (R) are correct but (R) is the correct explanation of (A)
(٤) (A) is correct but (R) is not correct

Ans. (٤)

Sol. Unlike NH_3, PH_3 molecules are not associated through hydrogen bonding in liquid state. That is why the boiling point of PH_3 is lower than NH_3.

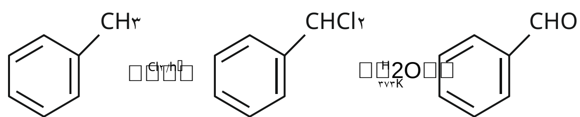
٦٩. Identify A and B in the following sequence of reaction





Ans. (٢)

Sol.



Toluene Benzal chloride Benzaldehyde

٧٠. Given below are two statements: Statement (I) : Aminobenzene and aniline are same organic compounds. Statement (II) : Aminobenzene and aniline are different organic compounds. In the light of the above statements، choose the most appropriate answer from the options given below:

(١) Both Statement I and Statement II are correct (٢) Statement I is correct but Statement II is incorrect (٣) Statement I is incorrect but Statement II is correct (٤) Both Statement I and Statement II are incorrect

Ans. (٣)

Sol. Aniline is also known as amino benzene.

٧١. Which of the following complex is homoleptic؟

(١) [Ni(CN)٤]

(٢) [Ni(NH٣)٢Cl٢]

(٣) [Fe(NH٣)٤Cl٢]

(٤) [Co(NH٣)٤Cl٢]

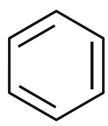
+

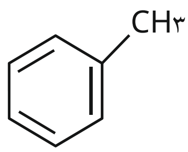
Ans. (١)

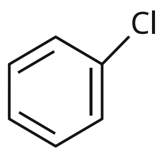
Sol. In Homoleptic complex all the ligand attached with the central atom should be the same. Hence

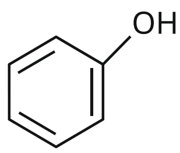
[Ni(CN)٤] is a homoleptic complex.

٧٢. Which of the following compound will most easily be attacked by an electrophile؟

(١) 

(٢) 

(٣) 

(٤) 

Ans. (٤)

Sol. Higher the electron density in the benzene ring more easily it will be attacked by an electrophile. Phenol has the highest electron density amongst all the given compound.

٧٣. Ionic reactions with organic compounds proceed through: (A) Homolytic bond cleavage (B) Heterolytic bond cleavage (C) Free radical formation (D) Primary free radical (E) Secondary free radical Choose the correct answer from the options given below: (١) (A) only (٢) (C) only (٣) (B) only (٤) (D) and (E) only

Ans. (٣)

Sol. Heterolytic cleavage of Bond lead to formation of ions.

٧٤. Arrange the bonds in order of increasing ionic character in the molecules. LiF، K٢O، N٢، SO٢ and ClF٣.

(١) ClF٣ > N٢ > SO٢ > K٢O > LiF

(٢) LiF > K٢O > ClF٣ > SO٢ > N٢

(٣) N٢ > SO٢ > ClF٣ > K٢O > LiF

(٤) N٢ > ClF٣ > SO٢ > K٢O > LiF

Ans. (٣)

Sol. Increasing order of ionic character

N٢ > SO٢ > ClF٣ > K٢O > LiF

Ionic character depends upon difference of electronegativity (bond polarity).

٧٥. We have three aqueous solutions of NaCl labelled as 'A'، 'B' and 'C' with concentration ٠.١ M، ٠.٠١M & ٠.٠٠١ M، respectively. The value of van t' Haft factor (i) for these solutions will be in the order.

(١) iA > iB > iC

(٢) iA > iC > iB

(٣) iA = iB = iC

(٤) iA < iB < iC

Ans. (١)

Sol.

Salt	Values of i (for different conc. of a Salt)		
	٠.١ M	٠.٠١ M	٠.٠٠١ M
NaCl	١.٨٧	١.٩٤	١.٩٤

i approach ٢ as the solution become very dilute.

٧٦. In Kjeldahl's method for estimation of nitrogen, $CuSO_4$ acts as :

(١) Reducing agent (٢) Catalytic agent
(٣) Hydrolysis agent (٤) Oxidising agent

Ans. (٢)

Sol. Kjeldahl's method is used for estimation of Nitrogen where $CuSO_4$ acts as a catalyst.

٧٧. Given below are two statements :

Statement (I) : Potassium hydrogen phthalate is a primary standard for standardisation of sodium hydroxide solution.

Statement (II) : In this titration phenolphthalein can be used as indicator.

In the light of the above statements, choose the most appropriate answer from the options given below:

(١) Both Statement I and Statement II are correct
(٢) Statement I is correct but Statement II is incorrect
(٣) Statement I is incorrect but Statement II is correct
(٤) Both Statement I and Statement II are incorrect

Ans. (١)

Sol. Statement (I) : Potassium hydrogen phthalate is a primary standard for standardisation of sodium hydroxide solution as it is economical and its concentration does not changes with time.
Phenophthalin can acts as indicator in acid base titration as it shows colour in pH range ٨.٢ to ١٠.

٧٨. Match List – I with List – II.

	List – I (Reactions)		List – II (Reagents)
(A)	$CH_3(CH_2)_5-\overset{O}{\overset{\|}{C}}-OC_2H_5 \rightarrow CH_3(CH_2)_5CHO$	(I)	CH_3MgBr, H_2O
(B)	$C_6H_5COC_6H_5 \rightarrow C_6H_5CH_2C_6H_5$	(II)	Zn(Hg) and conc. HCl
(C)	$C_6H_5CHO \rightarrow C_6H_5CH(OH)CH_3$	(III)	$NaBH_4$, H^+
(D)	$CH_3COCH_2COOC_2H_5 \rightarrow CH_3\underset{H}{\underset{\|}{C}}(OH)CH_2COOC_2H_5$	(IV)	DIBAL–H, H_2O

Choose the correct answer from options given below:

(١) A-(III), (B)-(IV), (C)-(I), (D)-(II)
(٢) A-(IV), (B)-(II), (C)-(I), (D)-(III)
(٣) A-(IV), (B)-(II), (C)-(III), (D)-(I)
(٤) A-(III), (B)-(IV), (C)-(II), (D)-(I)

Ans. (٢)

Sol. $CH_3(CH_2)_5COOC_2H_5 \xrightarrow{DIBAL-H,\ H_2O} CH_3(CH_2)_5CHO$

$C_6H_5COC_6H_5 \xrightarrow{Zn(Hg)\ \&\ conc.\ HCl} C_6H_5CH_2C_6H_5$

$C_6H_5CHO \xrightarrow[H_2O]{CH_3MgBr} C_6H_5CHOHCH_3$

$CH_3COCH_2COOC_2H_5 \xrightarrow{NaBH_4,\ H^+} CH_3CH(OH)CH_2COOC_2H_5$

٧٩. Choose the correct option for free expansion of an ideal gas under adiabatic condition from the following :

(١) $q = 0$, $\Delta T \neq 0$, $w = 0$
(٢) $q = 0$, $\Delta T < 0$, $w \neq 0$
(٣) $q \neq 0$, $\Delta T = 0$, $w = 0$
(٤) $q = 0$, $\Delta T = 0$, $w = 0$

Ans. (٤)

Sol. During free expansion of an ideal gas under adiabatic condition $q = 0$, $\Delta T = 0$, $w = 0$.

٨٠. Given below are two statements:

Statement (I) : The NH_2 group in Aniline is ortho and para directing and a powerful activating group.

Statement (II) : Aniline does not undergo Friedel-Craft's reaction (alkylation and acylation).

In the light of the above statements, choose the most appropriate answer from the options given below :

(١) Both Statement I and Statement II are correct
(٢) Both Statement I and Statement II are incorrect
(٣) Statement I is incorrect but Statement II is correct
(٤) Statement I is correct but Statement II is incorrect

Ans. (١)

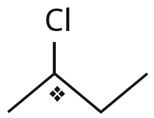
Sol. The NH_2 group in Aniline is ortho and para directing and a powerful activating group as NH_2 has strong +M effect.
Aniline does not undergo Friedel-Craft's reaction (alkylation and acylation) as Aniline will form complex with $AlCl_3$ which will deactivate the benzene ring.

SECTION-B

٨١. Number of optical isomers possible for

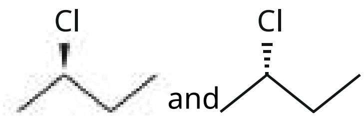
٢ – chlorobutane … … …

Ans. (٢)

Sol. 

There is one chiral centre present in given compound.

So, Total optical isomers = ٢



٨٢. The potential for the given half cell at ٢٩٨K is

(–) … … … … × ١٠ V$^{-٢}$

٢H^{+}(aq) + ٢e^{-} → $H_٢$ (g)

[H^{+}] = ١M, $P_{H_٢}$ = ٢ atm

(Given : ٢.٣٠٣ RT/F = ٠.٠٦V, log٢ = ٠.٣)

Ans. (١)

Sol. $E = E^{o}_{H^{+}/H_٢} - \frac{٠.٠٦}{٢} \log \frac{P_{H_٢}}{[H^{+}]^٢}$

$E = ٠.٠٠ - \frac{٠.٠٦}{٢} \log \frac{٢}{[١]^٢}$

E = –٠.٠٣ × ٠.٣ = –٠.٠٩ × ١٠$^{-٢}$V

٨٣. The number of white coloured salts among the following is … … … … … .

(A) $SrSO_٤$ (B) $Mg(NH_٤)PO_٤$ (c) $BaCrO_٤$

(D) $Mn(OH)_٢$ (E) $PbSO_٤$ (F) $PbCrO_٤$

(G) AgBr (H) $PbI_٢$ (I) $CaC_٢O_٤$

(J) $[Fe(OH)_٢(CH_٣COO)]$

Ans. (٥)

Sol. $SrSO_٤$ – white

$Mg(NH_٤)PO_٤$ – white

$BaCrO_٤$ – yellow

$Mn(OH)_٢$ – white

$PbSO_٤$ – white

$PbCrO_٤$ – yellow

AgBr – pale yellow

$PbI_٢$ – yellow

$CaC_٢O_٤$ – white

$[Fe(OH)_٢(CH_٣COO)]$ – Brown Red

٨٤. The ratio of $\frac{^{١٤}C}{^{١٢}C}$ in a piece of wood is $\frac{١}{٨}$ part that of atmosphere. If half life of C is ٥٧٣٠ years, the age of wood sample is … . . years.

Ans. (١٧١٩٠)

Sol. $\lambda t = \ln \frac{(^{١٤}C/^{١٢}C)_{atmosphere}}{(^{١٤}C/^{١٢}C)_{wood\ sample}}$

As per the question,

$\frac{(^{١٤}C/^{١٢}C)_{wood}}{(^{١٤}C/^{١٢}C)_{atmosphere}} = \frac{١}{٨}$

So, $\lambda t = \ln ٨$

$\frac{\ln ٢}{t_{١/٢}} t = \ln ٨$

t = ٣ × $t_{١/٢}$ = ١٧١٩٠ years

٨٥. The number of molecules/ion/s having trigonal bipyramidal shape is … … . . .

$PF_٥$, $BrF_٥$, $PCl_٥$, $[PtCl_٤]^{٢-}$, $BF_٣$, $Fe(CO)_٥$

Ans. (٣)

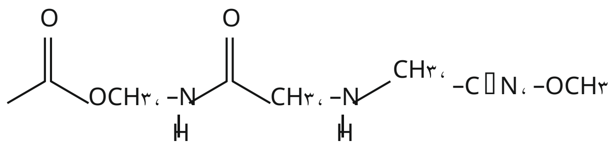
Sol. $PF_٥$, $PCl_٥$, $Fe(CO)_٥$: Trigonal bipyramidal

$BrF_٥$: square pyramidal

$[PtCl_٤]^{٢-}$ square planar

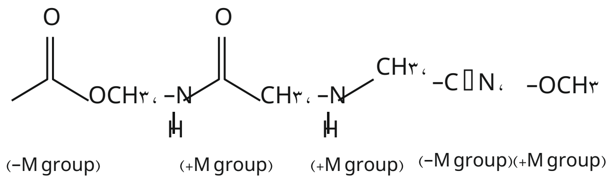
$BF_٣$: Trigonal planar

٨٦. Total number of deactivating groups in aromatic electrophilic substitution reaction among the following is



Ans. (٢)

Sol.



٨٧. Lowest Oxidation number of an atom in a compound A٢B is −٢. The number of an electron in its valence shell is

Ans. (٦)

Sol. A٢B ٢A + B٢− B has complete octet in its di-anionic form، thus in its atomic state it has ٦ electrons in its valence shell. As it has negative charge، it has acquired two electrons to complete its octet.

٨٨. Among the following oxide of p – block elements، number of oxides having amphoteric nature is Cl٢O٧، CO، PbO٢، N٢O، NO، Al٢O٣، SiO٢، N٢O٥، SnO٢

Ans. (٣)

Sol. Acidic oxide: Cl٢O٧، SiO٢، N٢O٥ Neutral oxide: CO، NO، N٢O Amphoteric oxide: Al٢O٣ ، SnO٢ ، PbO٢

٨٩. Consider the following reaction:

٣PbCl٢ + ٢(NH٤)٣PO٤ Pb٣(PO٤)٢ + ٦NH٤Cl

If ٧٢ mmol of PbCl٢ is mixed with ٥٠ mmol of (NH٤)٣PO٤ ، then amount of Pb٣(PO٤)٢ formed is

Ans. (٢٤)...... mmol. (nearest integer)

Sol. Limiting Reagent is PbCl٢

mmol of Pb٣(PO٤)٢ formed

= (mmol of PbCl٢ reacted) / ٣

= ٢٤ mmol

٩٠. Ka for CH٣COOH is ١.٨ × ١٠−٥ and Kb for NH٤OH is ١.٨ × ١٠−٥. The pH of ammonium acetate solution will be

Ans. (٧)

Sol. pH = (pK_w + pK_a − pK_b) / ٢

pKa = pKb

∴ pH = pK_w / ٢ = ٧