M ATHEM ATICS

TEST PAPER WITH SOLUTION

SECTION–A

١. Let f(x) = |٢x+٥|x|–٣| ، x∈R. If m and n denote the number of points where f is not continuous and not differentiable respectively، then m + n is equal to :

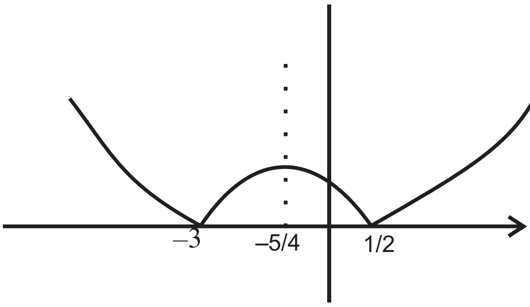
(١) ٥ (٢) ٢

(٣) ٠ (٤) ٣

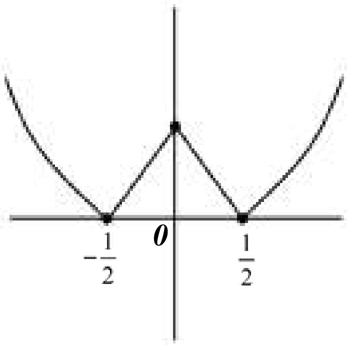
Ans. (٤)

Sol. f(x) = |٢x²+٥|x|–٣|

Graph of y = |٢x²+ ٥x – ٣|



Graph of f(x)



Number of points of discontinuity = ٠ = m

Number of points of non–differentiability = ٣ = n

٢. Let α and β be the roots of the equation px² + qx – r = ٠، where p ≠ ٠. If p، q and r be the consecutive terms of a non–constant G.P and $\frac{1}{\alpha}+\frac{1}{\beta}=\frac{3}{4}$ ، then the value of (α – β)² is :

(١) $\frac{80}{9}$ (٢) ٩

(٣) $\frac{20}{3}$ (٤) ٨

Ans. (١)

Sol. px² + qx – r = ٠

p = A، q = AR، r = AR²

Ax² + ARx – AR² = 0

x² + Rx – R = ٠

$\because \frac{1}{\alpha}+\frac{1}{\beta}=\frac{3}{4}$

$\Rightarrow \frac{\alpha+\beta}{\alpha\beta}=\frac{3}{4} \Rightarrow \frac{-R}{-R^2}=\frac{3}{4} \Rightarrow R=\frac{4}{3}$

$(\alpha-\beta)^2=(\alpha+\beta)^2-4\alpha\beta = R^2-4(-R) = \frac{16}{9}+\frac{16}{3}$

= ٨٠ /٩

٣. The number of solutions of the equation ٤ sin²x – ٤ cos³x + ٩ – ٤ cosx = ٠، x ∈ [–٢π، ٢π] is :

(١) ١

(٢) ٣

(٣) ٢

(٤) ٠

Ans. (٤)

Sol. ٤sin²x – ٤cos³x + ٩ – ٤ cosx = ٠ ; x ∈ [–٢π، ٢π]

٤ – ٤cos²x – ٤cos³x + ٩ – ٤ cosx = ٠

٤cos³x + ٤cos²x + ٤ cosx – ١٣ = ٠

٤cos³x + ٤cos²x + ٤cosx = ١٣

L.H.S. ≤ ١٢ can't be equal to ١٣.

٤. The value of $\int_0^1 (2x^3-3x^2-x+1)^{\frac{1}{3}}dx$ is equal to :

(١) ٠

(٢) ١

(٣) ٢

(٤) –١

Ans. (١)

Sol. $I=\int_0^1 (2x^3-3x^2-x+1)^{\frac{1}{3}}dx$

Using $\int_0^{2a} f(x)dx$ where f(٢a–x) = –f(x)

Here f(١–x) = f(x)

⇒ I = ٠

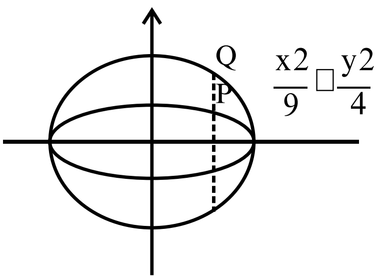
٥. Let P be a point on the ellipse $\frac{x^{٢}}{٩}+\frac{y^{٢}}{٤}=١$. Let the line passing through P and parallel to y-axis meet the circle $x^{٢}+y^{٢}=٩$ at point Q such that P and Q are on the same side of the x-axis. Then، the eccentricity of the locus of the point R on PQ such that PR : RQ = ٤ : ٣ as P moves on the ellipse، is :

(١) $\frac{١١}{١٩}$ (٢) $\frac{١٣}{٢١}$

(٣) $\frac{\sqrt{١٣٩}}{٢٣}$ (٤) $\frac{\sqrt{١٣}}{٧}$

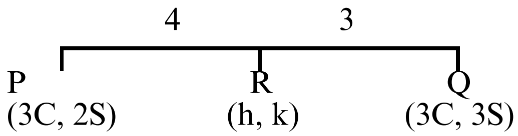
Ans. (٤)

Sol.



$\frac{x2}{9}+\frac{y2}{4}=1$

P(3cosθ, 2sinθ)
Q(3cosθ, 3sinθ)



$h=٣\cos\theta$;

$k=\frac{18}{7}\sin\theta$

$\therefore$ locus $=\frac{x2}{9}+\frac{49y2}{324}=1$

$e=\sqrt{1-\frac{324}{49\times 9}}=\frac{\sqrt{117}}{21}=\frac{\sqrt{13}}{7}$

٦. Let m and n be the coefficients of seventh and thirteenth terms respectively in the expansion of

$\left(\frac{١}{٣}x^{\frac{١}{٣}}+\frac{١}{٢x^{\frac{٢}{٣}}}\right)^{١٨}$. Then $\left(\frac{n}{m}\right)^{\frac{١}{٣}}$ is :

(١) $\frac{٤}{٩}$ (٢) $\frac{١}{٩}$

(٣) $\frac{١}{٤}$ (٤) $\frac{٩}{٤}$

Ans. (٤)

Sol. $\left(\frac{x^{\frac{١}{٣}}}{٣}+\frac{x^{\frac{-2}{٣}}}{٢}\right)^{١٨}$

$t_{٧}={}^{١٨}C_{٦}\left(\frac{x^{\frac{١}{٣}}}{3}\right)^{١٢}\left(\frac{x^{\frac{-2}{٣}}}{٢}\right)^{٦}={}^{١٨}C_{٦}\frac{١}{3^{١٢}}\cdot\frac{١}{٢^{٦}}$

$t_{١٣}={}^{١٨}C_{١٢}\left(\frac{x^{\frac{١}{٣}}}{٣}\right)^{٦}\left(\frac{x^{\frac{-2}{٣}}}{٢}\right)^{١٢}={}^{١٨}C_{١٢}\frac{١}{3^{٦}}\cdot\frac{١}{٢^{١٢}}\cdot x^{-٦}$

$m={}^{١٨}C_{٦}\cdot ٣^{-١٢}\cdot ٢^{-٦}$; $n={}^{١٨}C_{١٢}\cdot ٢^{-١٢}\cdot ٣^{-٦}$

$\left(\frac{n}{m}\right)^{\frac{١}{3}}=\left(\frac{٢^{-١٢}\cdot ٣^{-٦}}{٣^{-١٢}\cdot ٢^{-٦}}\right)^{\frac{١}{٣}}=\left(\frac{٣}{٢}\right)^{٢}=\frac{٩}{٤}$

٧. Let λ be a non-zero real number. Suppose $f: R\rightarrow R$ is a differentiable function such that $f(٠)=٢$ and $\lim_{x\to-\infty}f(x)=١$. If $f'(x)=\lambda f(x)+٣$، for all $x\in R$، then $f(-\log_e ٢)$ is equal to______.

(١) ٣ (٢) ٥

(٣) ٩ (٤) ٧

Ans. (٣ OR BONUS)

Sol. $f(٠)=٢$، $\lim_{x\to-\infty}f(x)=١$

$f'(x)-\lambda.f(x)=٣$

$I.F=e^{-\lambda x}$

$y(e^{-\lambda x})=\int ٣.e^{-\lambda x}dx$

$f(x).(e^{-\lambda x})=\frac{٣e^{-\lambda x}}{-\lambda}+c$

$x=٠\Rightarrow ٢=\frac{-3}{\lambda}+c\Rightarrow\frac{٣}{\lambda}=c-٢$ (١)

$f(x)=\frac{-3}{\lambda}+c.e^{\lambda x}$

$x\to-\infty\Rightarrow ١=\frac{-3}{\lambda}+c(٠)$

$\lambda=-٣\Rightarrow c=١$

$f(-\ln ٢)=\frac{-3}{\lambda}+c.e^{٣\ln ٢}$

$=١+e^{٣\ln٢}=٩$

(But λ should be greater than ٠ for finite value of c)

٨. Let P and Q be the points on the line $\frac{x-3}{8} = \frac{y-4}{2} = \frac{z-1}{2}$ which are at a distance of ٦ units from the point R(١،٢،٣). If the centroid of the triangle PQR is (α, β, γ), then $\alpha^2+\beta^2+\gamma^2$ is:

(١) ٢٦

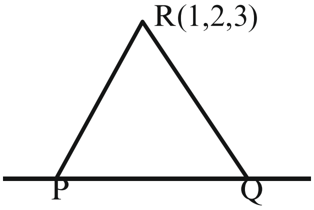
(٢) ٣٦

(٣) ١٨

(٤) ٢٤

Ans. (٣)

Sol.



$P(8\lambda - 3, 2\lambda + 4, 2\lambda - 1)$

PR = ٦

$(8\lambda - 4)^2 + (2\lambda + 2)^2 + (2\lambda - 4)^2 = 36$

$\lambda = 0, 1$

Hence P(−٣، ٤، −١) & Q(٥، ٦، ١)

Centroid of ΔPQR = (١، ٤، ١) ≡ (α, β, γ)

$\alpha^2 + \beta^2 + \gamma^2 = 18$

٩. Consider a ΔABC where A(١،٢،٣)، B(−٢،٨،٠) and C(٣،٦،٧). If the angle bisector of ∠BAC meets the line BC at D, then the length of the projection of the vector $\vec{AD}$ on the vector $\vec{AC}$ is:

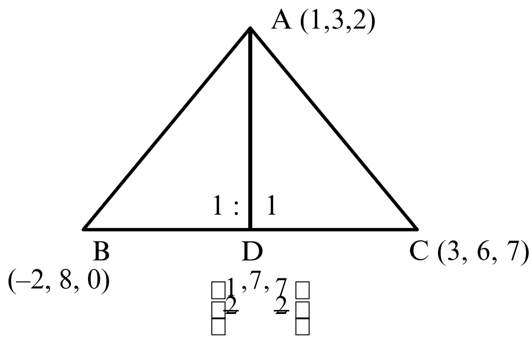
(١) $\frac{37}{2\sqrt{38}}$

(٢) $\frac{\sqrt{38}}{2}$

(٣) $\frac{39}{2\sqrt{38}}$

(٤) $\sqrt{19}$

Ans. (١)



Sol.

A(١، ٣، ٢)؛ B(−٢، ٨، ٠)؛ C(٣، ٦، ٧)؛

$\vec{AC} = 2\hat{i} + 3\hat{j} + 5\hat{k}$

AB = $\sqrt{9+25+4} = \sqrt{38}$

AC = $\sqrt{4+9+25} = \sqrt{38}$

$\vec{AD} = \frac{1}{2}\hat{i} + 4\hat{j} + \frac{3}{2}\hat{k} = \frac{1}{2}(\hat{i} + 8\hat{j} + 3\hat{k})$

Length of projection of $\vec{AD}$ on $\vec{AC}$

$= \left|\frac{\vec{AD}\cdot\vec{AC}}{|\vec{AC}|}\right| = \frac{37}{2\sqrt{38}}$

١٠. Let Sn denote the sum of the first n terms of an arithmetic progression. If $S_{10} = 390$ and the ratio of the tenth and the fifth terms is ١٥ : ٧, then $S_{15} - S_5$ is equal to:

(١) ٨٠٠

(٢) ٨٩٠

(٣) ٧٩٠

(٤) ٦٩٠

Ans. (٣)

Sol. $S_{10} = 390$

$\frac{10}{2}\left[2a + (10-1)d\right] = 390$

$\Rightarrow 2a + 9d = 78$ (١)

$\frac{t_{10}}{t_5} = \frac{15}{7} \Rightarrow \frac{a+9d}{a+4d} = \frac{15}{7} \Rightarrow 8a = 3d$ (٢)

From (١) & (٢) $a = 3$ & $d = 8$

$S_{15} - S_5 = \frac{15}{2}\left[6 + 14 \times 8\right] - \frac{5}{2}\left[6 + 4 \times 8\right]$

$= \frac{15 \times 118 - 5 \times 38}{2} = 790$

١١. If $\int_0^{\pi/3} \cos^4 x\,dx = a\pi + b\sqrt{3}$, where a and b are rational numbers, then ٩a + ٨b is equal to :

(١) ٢ (٢) ١
(٣) ٣ (٤) $\frac{٣}{٢}$

Ans. (١)

Sol. $\int_0^{\pi/3} \cos^4 x\,dx$

$= \int_0^{\pi/3} \left(\frac{1+\cos 2x}{2}\right)^2 dx$

$= \frac{1}{4}\int_0^{\pi/3} (1+2\cos 2x+\cos^2 2x)\,dx$

$= \frac{1}{4}\left[\int_0^{\pi/3} dx + 2\int_0^{\pi/3} \cos 2x\,dx + \int_0^{\pi/3} \frac{1+\cos 4x}{2}\,dx\right]$

$= \frac{1}{4}\left[\frac{\pi}{3} + (\sin 2x)_0^{\pi/3} + \frac{1}{2}\left(\frac{\pi}{3}\right) + \frac{1}{8}(\sin 4x)_0^{\pi/3}\right]$

$= \frac{1}{4}\left[\frac{\pi}{3} + (\sin 2x)_0^{\pi/3} + \frac{1}{2}\cdot\frac{\pi}{3} + \frac{1}{8}(\sin 4x)_0^{\pi/3}\right]$

$= \frac{1}{4}\left[\frac{\pi}{2} + \frac{\sqrt{3}}{2} + \frac{1}{8}\left(-\frac{\sqrt{3}}{2}\right)\right]$

$= \frac{\pi}{8} + \frac{7\sqrt{3}}{64}$

$\therefore a = \frac{1}{8};\ b = \frac{7}{64}$

$\therefore ٩a + ٨b = \frac{9}{8} + \frac{7}{8} = 2$

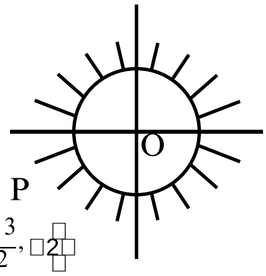
١٢. If z is a complex number such that $|z| = ١$, then the minimum value of $\left|z - \frac{١}{٤}(٣ + ٤i)\right|$ is :

(١) $\frac{٥}{٤}$ (٢) ٢

(٣) ٣ (٤) $\frac{٣}{٢}$

Ans. (Bonus)

Sol. $|z| = ١$



$\left(\frac{3}{2}, 2\right)$

Min. value of $\left|z - \frac{3}{2} - 2i\right|$ is actually zero.

١٣. If the domain of the function $f(x) = \frac{\sqrt{x^٢ - ٢٥}}{(٤ - x^٢)} + \log_{١٠}(x^٢ + ٢x - ١٥)$ is $(-\infty, \alpha) \cup [\beta, \infty)$, then $\alpha^٢ + \beta^٣$ is equal to :

(١) ١٤٠ (٢) ١٧٥
(٣) ١٥٠ (٤) ١٢٥

Ans. (٣)

Sol. $f(x) = \frac{\sqrt{x^2 - 25}}{4 - x^2} + \log_{١٠}(x^٢ + ٢x - ١٥)$

Domain : $x^٢ - ٢٥ \ge ٠ \Rightarrow x \in (-\infty, -٥] \cup [٥, \infty)$

$٤ - x^٢ \ne ٠ \Rightarrow x \ne \{-٢, ٢\}$

$x^٢ + ٢x - ١٥ > ٠ \Rightarrow (x + ٥)(x - ٣) > ٠$

$\Rightarrow x \in (-\infty, -٥) \cup (٣, \infty)$

$\therefore x \in (-\infty, -٥) \cup [٥, \infty)$

$\therefore \alpha = -٥;\ \beta = ٥$

$\therefore \alpha^2 + \beta^3 = 150$

١٤. Consider the relations $R_١$ and $R_٢$ defined as $aR_١b \Leftrightarrow a^٢ + b^٢ = ١$ for all a, b, $\in$ R and $(a, b) R_٢ (c, d) \Leftrightarrow a + d = b + c$ for all $(a, b), (c, d) \in N \times N$. Then

(١) Only $R_١$ is an equivalence relation
(٢) Only $R_٢$ is an equivalence relation
(٣) $R_١$ and $R_٢$ both are equivalence relations
(٤) Neither $R_١$ nor $R_٢$ is an equivalence relation

Ans. (٢)

Sol. $aR_١ b \Leftrightarrow a^٢ + b^٢ = ١;\ a, b \in R$

$(a, b) R_٢ (c, d) \Leftrightarrow a + d = b + c;\ (a, b), (c, d) \in N$

for $R_١$: Not reflexive symmetric not transitive

for $R_٢$: $R_٢$ is reflexive, symmetric and transitive

Hence only $R_٢$ is equivalence relation.

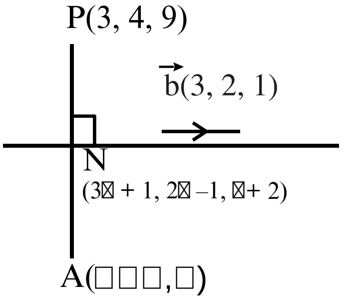
١٥. If the mirror image of the point P(٣، ٤، ٩) in the line $\frac{x-1}{3}=\frac{y+1}{2}=\frac{z-2}{1}$ is (α, β, γ), then ١٤(α + β + γ) is :

(١) ١٠٢ (٢) ١٣٨

(٣) ١٠٨ (٤) ١٣٢

Ans. (٣)

Sol.



$\overrightarrow{PN}\cdot\vec{b}=0$

$3(3\lambda-2)+2(2\lambda-5)+(\lambda-7)=0$

$14\lambda=23 \Rightarrow \lambda=\frac{23}{14}$

$\Rightarrow N\left(\frac{83}{14},\frac{32}{14},\frac{51}{14}\right)$

$\Rightarrow \frac{\alpha+3}{2}=\frac{83}{14} \Rightarrow \alpha=\frac{62}{7}$

$\Rightarrow \frac{\beta+4}{2}=\frac{32}{14} \Rightarrow \beta=\frac{4}{7}$

$\Rightarrow \frac{\gamma+9}{2}=\frac{51}{14} \Rightarrow \gamma=-\frac{12}{7}$

Ans. ١٤(α + β + γ) = ١٠٨

١٦. Let $f(x)=\begin{cases} x-1, & x \text{ is even},\\ 2x, & x \text{ is odd},\end{cases}$ $x \in N$. If for some $a \in N$, $f(f(f(a)))=21$, then $\lim_{x\to a^-}\left\{\frac{|x|^3}{a}-\left[\frac{x}{a}\right]\right\}$, where [t] denotes the greatest integer less than or equal to t, is equal to :

(١) ١٢١

(٢) ١٤٤

(٣) ١٦٩

(٤) ٢٢٥

Ans. (٢)

Sol. $f(x)=\begin{cases} x-1; & x \in \text{even}\\ 2x; & x \in \text{odd}\end{cases}$

$f(f(f(a)))=21$

C–1: If a = even

$f(a)=a-1=\text{odd}$

$f(f(a))=2(a-1)=\text{even}$

$f(f(f(a)))=2a-3=21 \Rightarrow a=12$

C–2: If a = odd

$f(a)=2a=\text{even}$

$f(f(a))=2a-1=\text{odd}$

$f(f(f(a)))=4a-2=21$ (Not possible)

Hence a = 12

Now

$\lim_{x\to 12^-}\left(\frac{|x|^3}{12}-\left[\frac{x}{12}\right]\right)$

$=\lim_{x\to 12^-}\frac{|x|^3}{12}-\lim_{x\to 12^-}\left[\frac{x}{12}\right]$

= 144 – 0 = 144.

١٧. Let the system of equations x + ٢y + ٣z = ٥، ٢x + ٣y + z = ٩، ٤x + ٣y + λz = μ have infinite number of solutions. Then λ + ٢μ is equal to :

(١) ٢٨ (٢) ١٧

(٣) ٢٢ (٤) ١٥

Ans. (٢)

Sol. x + ٢y + ٣z = ٥ ٢x + ٣y + z = ٩

٤x + ٣y + λz = μ for infinite following Δ = Δ١ = Δ٢ = Δ٣ = ٠

$\Delta=\begin{vmatrix}1&2&3\\2&3&1\\4&3&\lambda\end{vmatrix}=0 \Rightarrow \lambda=13$

$\Delta_1=\begin{vmatrix}5&2&3\\9&3&1\\\mu&3&13\end{vmatrix}=0 \Rightarrow \mu=15$

$\Delta_2=\begin{vmatrix}1&5&3\\2&9&1\\4&15&13\end{vmatrix}=0$

$$\begin{vmatrix} 1 & 2 & 5 \\ 2 & 3 & 9 \\ 4 & 3 & 15 \end{vmatrix} = ٠$$

for □ = −١٣، μ = ١٥ system of equation has infinite solution hence □ + ٢μ = ١٧

١٨. Consider ١٠ observation x١، x٢، ... ، x١٠. such that $\sum_{i=1}^{١٠}$ (xi − □) = ٢ and $\sum_{i=1}^{١٠}$ (xi − □) = ٤٠، where □، □ are positive integers. Let the mean and the variance of the observations be $\frac{٦}{٥}$ and $\frac{٨٤}{٢٥}$ respectively. The $\frac{□}{□}$ is equal to :

(١) ٢ (٢) $\frac{٣}{٢}$

(٣) $\frac{٥}{٢}$ (٤) ١

Ans. (١)

Sol. x١، x٢ ... ... .x١٠

$$\sum_{i=1}^{10} (xi - □) = 2 \quad □ \sum_{i=1}^{10} xi - 10□ = 2$$

Mean □ μ □ $\frac{6}{5}$ □ $\frac{\sum x_i}{10}$ □

□□□ □xi□□□□□□

□ □□□□□□□□□□□□□□□□□□□□□□□□□□□

Now $\sum_{i=1}^{10} (x_i - □)^2 = 40$ Let yi = xi − □

$$□^2 = \frac{1}{10} \sum y_i^2 - (\bar{y})^2$$

$$□^2 = \frac{1}{10} \sum (x_i - □)^2 - \left(\frac{\sum_{i=1}^{10} (x_i - □)}{10} \right)^2$$

$$\frac{84}{25} = 4 - \left(\frac{12 - 10□}{10} \right)^2$$

$$\left(\frac{6 - 5□}{5} \right)^2 = 4 - \frac{84}{25} = \frac{16}{25}$$

٦ − ٥□ = ±٤ □ □ = $\frac{2}{5}$ (not possible) or □ = ٢

Hence $\frac{□}{□} = 2$

١٩. Let Ajay will not appear in JEE exam with probability p = $\frac{٢}{٧}$، while both Ajay and Vijay will appear in the exam with probability q = $\frac{١}{٥}$. Then the probability، that Ajay will appear in the exam and Vijay will not appear is :

(١) $\frac{٩}{٣٥}$

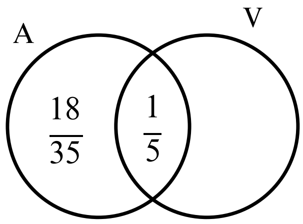
(٢) $\frac{١٨}{٣٥}$

(٣) $\frac{٢٤}{٣٥}$

(٤) $\frac{٣}{٣٥}$

Ans. (٢)

Sol.



$P(\bar{A}) = \frac{2}{7} = p$

$P(A \cap V) = \frac{1}{5} = q$

$P(A) = \frac{5}{7}$

Ans. $P(A \cap \bar{V}) = \frac{18}{35}$

٢٠. Let the locus of the mid points of the chords of circle x٢+(y−١)٢=١ drawn from the origin intersect the line x+y = ١ at P and Q. Then، the length of PQ is :

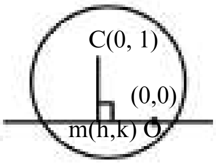
(١) $\frac{١}{\sqrt{٢}}$

(٢) $\sqrt{٢}$

(٣) $\frac{١}{٢}$

(٤) ١

Ans. (١)

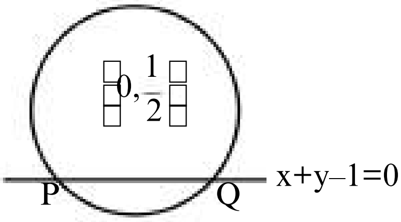
Sol. 

mOM . mCM = −١

$\frac{k}{h}\cdot\frac{k-1}{h} = -1$

∴ locus is x² + y(y − ١) = ٠

x² + y² − y = ٠



$p = \left|\frac{1/2}{\sqrt{2}}\right|$ $p = \frac{1}{2\sqrt{2}}$

PQ = $2\sqrt{r^2-p^2}$

= $2\sqrt{\frac{1}{4}-\frac{1}{8}}$ = $\frac{1}{\sqrt{2}}$

SECTION–B

٢١. If three successive terms of a G.P. with common ratio r(r > ١) are the lengths of the sides of a triangle and [r] denotes the greatest integer less than or equal to r, then ٣[r] + [−r] is equal to : Ans. (١)

Sol. a, ar, ar² G.P.

Sum of any two sides > third side

a + ar > ar², a + ar² > ar, ar + ar² > a

r² − r − ١ < ٠

$r \in \left(\frac{1-\sqrt{5}}{2}, \frac{1+\sqrt{5}}{2}\right)$ (١)

r² − r + ١ > ٠

always true

r² + r − ١ > ٠

$r \in \left(-\infty, \frac{-1-\sqrt{5}}{2}\right) \cup \left(\frac{-1+\sqrt{5}}{2}, \infty\right)$ (٢)

Taking intersection of (١), (٢)

$r \in \left(\frac{\sqrt{5}-1}{2}, \frac{1+\sqrt{5}}{2}\right)$

As r > ١

$r \in \left(1, \frac{1+\sqrt{5}}{2}\right)$

[r] = ١, [−r] = −٢

٣[r] + [−r] = ١

٢٢. Let A = I₂ − MMᵀ, where M is real matrix of order ٢ × ١ such that the relation MᵀM = I₁ holds. If λ is a real number such that the relation AX = λX holds for some non-zero real matrix X of order ٢ × ١, then the sum of squares of all possible values of λ is equal to :

Ans. (٢)

Sol. A = I₂ − ٢MMᵀ

A² = (I₂ − ٢MMᵀ)(I₂ − ٢MMᵀ)

= I₂ − ٢MMᵀ − ٢MMᵀ + ٤MMᵀMMᵀ

= I₂ − ٤MMᵀ + ٤MMᵀ

= I₂

AX = λX

AX = λAX

X = λ(λX)

X = λ²X

X(λ² − ١) = ٠

λ = ±١

Sum of square of all possible values = ٢

23. Let $f:(0,\infty)\to R$ and $F(x)=\int_0^x t f(t)\,dt$. If $F(x^2)=x^4+x^5$, then $\sum_{r=1}^{12} f(r^2)$ is equal to :

Ans. (219)

Sol. $F(x)=\int_0^x t f(t)\,dt$

$F'(x)=xf(x)$

Given $F(x^2)=x^4+x^5$ let $x^2=t$

$F(t)=t^2+t^{5/2}$

$F'(t)=2t+5/2\,t^{3/2}$

$t\cdot f(t)=2t+5/2\,t^{3/2}$

$f(t)=2+5/2\,r^{1/2}$

$\sum_{r=1}^{12} f(r^2)=\sum_{r=1}^{12}\left(2+\frac{5}{2}r\right)$

$=24+5/2\left(\frac{12(13)}{2}\right)$

$=219$

24. If $y=\frac{(\sqrt{x}+1)(x^2-\sqrt{x})}{x\sqrt{x}+x+\sqrt{x}}+\frac{1}{15}(3\cos^2x-5)\cos^3x$, then $96\,y'\left(\frac{\pi}{6}\right)$ is equal to :

Ans. (105)

Sol. $y=\frac{(\sqrt{x}+1)(x^2-\sqrt{x})}{x\sqrt{x}+x+\sqrt{x}}+\frac{1}{15}(3\cos^5x-5\cos^3x)$

$y=\frac{(\sqrt{x}+1)\sqrt{x}\left((\sqrt{x})^3-1\right)}{\sqrt{x}\left(x+\sqrt{x}+1\right)}+\frac{1}{5}\cos^5x-\frac{1}{3}\cos^3x$

$y=(\sqrt{x}-1)(\sqrt{x}+1)+\frac{1}{5}\cos^5x-\frac{1}{3}\cos^3x$

$y'=1-\cos^4x\cdot(\sin x)+\cos^2x(\sin x)$

$y'\left(\frac{\pi}{6}\right)=1-\frac{9}{16}\cdot\frac{1}{2}+\frac{3}{4}\cdot\frac{1}{2}$

$=\frac{32-9+12}{32}=\frac{35}{32}$

$=96\,y'\left(\frac{\pi}{6}\right)=105$

25. Let $\vec{a}=\hat{i}+\hat{j}+\hat{k}$, $\vec{b}=-\hat{i}-8\hat{j}+2\hat{k}$ and $\vec{c}=4\hat{i}+c_2\hat{j}+c_3\hat{k}$ be three vectors such that $\vec{b}\times\vec{a}=\vec{c}\times\vec{a}$. If the angle between the vector $\vec{c}$ and the vector $3\hat{i}+4\hat{j}+\hat{k}$ is θ, then the greatest integer less than or equal to $\tan^2\theta$ is :

Ans. (38)

Sol. $\vec{a}=\hat{i}+\hat{j}+\hat{k}$

$\vec{b}=-\hat{i}-8\hat{j}+2\hat{k}$

$\vec{c}=4\hat{i}+c_2\hat{j}+c_3\hat{k}$

$\vec{b}\times\vec{a}=\vec{c}\times\vec{a}$

$(\vec{b}-\vec{c})\times\vec{a}=0$

$\vec{b}-\vec{c}=\lambda\vec{a}$

$\vec{b}=\vec{c}+\lambda\vec{a}$

$-\hat{i}-8\hat{j}+2\hat{k}=(4\hat{i}+c_2\hat{j}+c_3\hat{k})+\lambda(\hat{i}+\hat{j}+\hat{k})$

$\lambda+4=-1 \Rightarrow \lambda=-5$

$\lambda+c_2=-8 \Rightarrow c_2=-3$

$\lambda+c_3=2 \Rightarrow c_3=7$

$\vec{c}=4\hat{i}-3\hat{j}+7\hat{k}$

$\cos\theta=\frac{12-12+7}{\sqrt{26}\sqrt{74}}=\frac{7}{\sqrt{26}\sqrt{74}}=\frac{7}{2\sqrt{481}}$

$\tan^2\theta=\frac{625\times3}{49}$

$[\tan^2\theta]=38$

٢٦. The lines $L_{١}, L_{٢}, \ldots, L_{٢٠}$ are distinct. For n = ١, ٢, ٣, …, ١٠ all the lines $L_{٢n-١}$ are parallel to each other and all the lines $L_{٢n}$ pass through a given point P. The maximum number of points of intersection of pairs of lines from the set $\{L_{١}, L_{٢}, \ldots, L_{٢٠}\}$ is equal to :

Ans. (١٠١)

Sol. $L_{١}, L_{٣}, L_{٥}, --L_{١٩}$ are Parallel

$L_{٢}, L_{٤}, L_{٦}, --L_{٢٠}$ are Concurrent

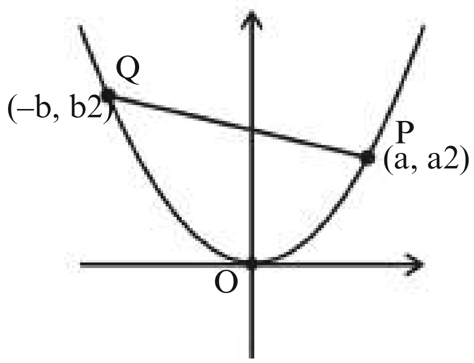
Total points of intersection $= {}^{٢٠}C_{٢} - {}^{١٠}C_{٢} - {}^{١٠}C_{٢} + ١$

$= ١٠١$

٢٧. Three points O(٠, ٠), P($a, a^{٢}$), Q($-b, b^{٢}$), a > ٠, b > ٠, are on the parabola $y = x^{٢}$. Let $S_{١}$ be the area of the region bounded by the line PQ and the parabola, and $S_{٢}$ be the area of the triangle OPQ. If the minimum value of $\frac{S_{١}}{S_{٢}}$ is $\frac{m}{n}$, gcd(m, n) = ١, then m + n is equal to :

Ans. (٧)

Sol.



$$S_{٢} = ١/٢ \left| \begin{matrix} ٠ & ٠ & ١ \\ a & a^{٢} & ١ \\ -b & b^{٢} & ١ \end{matrix} \right| = ١/٢\,(ab^{٢} + a^{٢}b)$$

$$PQ : y - a^{٢} = \frac{a^{٢} - b^{٢}}{a + b}(x - a)$$

$y - a^{٢} = (a - b)x - (a - b)a$

$y = (a - b)x + ab$

$$S_{١} = \int_{-b}^{a} \left((a - b)x + ab - x^{٢}\right)dx$$

$$= \left[(a - b)\frac{x^{٢}}{٢} + abx - \frac{x^{٣}}{٣}\right]_{-b}^{a}$$

$$= \frac{(a - b)^{٢}(a + b)}{٢} + ab(a + b) - \frac{(a^{٣} + b^{٣})}{٣}$$

$$\frac{S_{١}}{S_{٢}} = \frac{\frac{(a - b)^{٢}}{٢} + ab - \frac{(a^{٢} + b^{٢} - ab)}{٣}}{\frac{ab}{٢}}$$

$$= \frac{٣(a - b)^{٢} + ٦ab - ٢(a^{٢} + b^{٢} - ab)}{٣ab}$$

$$= \frac{١}{٣}\left(\underbrace{\frac{a}{b} + \frac{b}{a}}_{\min = ٢} + 2\right)$$

$$= \frac{٤}{٣} = \frac{m}{n} \qquad m + n = ٧$$

٢٨. The sum of squares of all possible values of k, for which area of the region bounded by the parabolas $ky^{٢} = x$ and $ky = ٢(y - x)$ is maximum, is equal to :

Ans. (٨)

Sol. $ky^2 = 2(y - x)$ $\qquad$ $2y^2 = kx$

Point of intersection

$$ky^2 = 2\left(y - \frac{2y^2}{k}\right)$$

$y = 0$ $\qquad$ $ky = 2\left(1 - \frac{2y}{k}\right)$

$$ky + \frac{4y}{k} = 2$$

$$y = \frac{2}{k + \frac{4}{k}} = \frac{2k}{k^2+4}$$

$$A = \int_0^{\frac{2k}{k^2+4}} \left(y - \frac{ky^2}{2} - \frac{2y^2}{k}\right) dy$$

$$= \left[\frac{y^2}{2} - \left(\frac{k}{2} + \frac{2}{k}\right)\frac{y^3}{3}\right]_0^{\frac{2k}{k^2+4}}$$

$$= \left(\frac{2k}{k^2+4}\right)^2\left(\frac{1}{2} - \frac{k^2+4}{2k}\cdot\frac{1}{3}\cdot\frac{2k}{k^2+4}\right)$$

$$= \frac{1}{6}\cdot 4\left(\frac{1}{k + \frac{4}{k}}\right)^2$$

$$A.M. \ge G.M. \quad \frac{\left(k + \frac{4}{k}\right)}{2} \ge 2$$

$$k + \frac{4}{k} \ge 4$$

Area is maximum when $k = \frac{4}{k}$

$k = 2, -2$

29. If $\frac{dx}{dy} = \frac{1 + x - y^2}{y}$, $x(1) = 1$, then $5x(2)$ is equal to :

Ans. (5)

Sol. $\frac{dx}{dy} - \frac{x}{y} = \frac{1 - y^2}{y}$

Integrating factor $= e^{-\int \frac{1}{y} dy} = \frac{1}{y}$

$$x \cdot \frac{1}{y} = \int \frac{1 - y^2}{y^2} dy$$

$$\frac{x}{y} = -\frac{1}{y} - y + c$$

$x = -1 - y^2 + cy$

$x(1) = 1$

$1 = -1 - 1 + c \Rightarrow c = 3$

$x = -1 - y^2 + 3y$

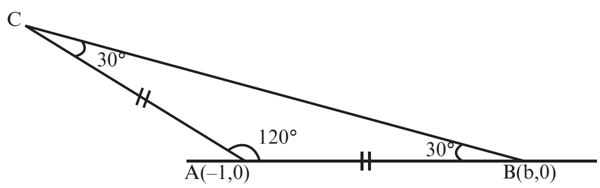
$5x(2) = 5(-1 - 4 + 6)$

$= 5$

30. Let ABC be an isosceles triangle in which A is at $(-1, 0)$, $\angle A = \frac{2\pi}{3}$, AB = AC and B is on the positive x-axis. If $BC = 4\sqrt{3}$ and the line BC intersects the line $y = x + 3$ at (α, β), then $\frac{\beta^4}{\alpha^2}$ is :

Ans. (36)

Sol.



$$\frac{c}{\sin 30^\circ} = \frac{4\sqrt{3}}{\sin 120^\circ} \quad \text{(By sine rule)}$$

$2c = 8 \Rightarrow c = 4$

AB□ □b□1٤□□

b = ٣ ، mAB = ٠

$mBC □ \frac{□١}{\sqrt{٣}}$

$BC: - y □ \frac{□١}{\sqrt{٣}} (x □ ٣)$

$\sqrt{٣} y □ x □ ٣$

Point of intersection : $y = \sqrt{٣} x$ ، $\sqrt{٣} y + x □ ٣$

$□ \sqrt{٣} □ ١ □ y □ ٦$

y

$□ \frac{٦}{\sqrt{٣} □ ١}$

$x □ \frac{٦}{\sqrt{٣} □ ١} □ 3$

$□ \frac{٦ □ \sqrt{٣} □ ٣}{\sqrt{٣} □ ١}$

$□ ٣ \frac{□ ١ □ \sqrt{٣} □}{□ ١ □ \sqrt{٣} □} □ \frac{□ ٦}{□ 1 □ \sqrt{3} □^{٢}}$

$\frac{□ 4}{□ 2} □ ٣٦$

PHYSICS   TEST PAPER WITH SOLUTION

SECTION-A

٣١. In an ammeter، ٥٪ of the main current passes through the galvanometer. If resistance of the galvanometer is G، the resistance of ammeter will be :

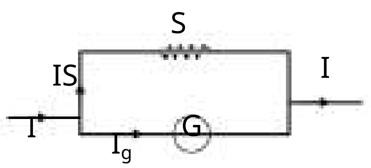
(١) $\frac{G}{200}$

(٢) $\frac{G}{199}$

(٣) ١٩٩ G

(٤) ٢٠٠ G

Ans. (Bonus)

Sol. 

$I_S S = I_g G$

$\frac{٩٥}{١٠٠} I_S = \frac{٥I}{١٠٠} G$

$S = \frac{G}{١٩}$

$R_A = \frac{SG}{S+G} = \frac{\frac{G^٢}{١٩}}{\frac{٢٠G}{١٩}}$

$R_A = \frac{G}{20}$

٣٢. To measure the temperature coefficient of resistivity α of a semiconductor، an electrical arrangement shown in the figure is prepared. The arm BC is made up of the semiconductor. The experiment is being conducted at ٢٥°C and resistance of the semiconductor arm is ٣ mΩ. Arm BC is cooled at a constant rate of ٢°C/s. If the galvanometer G shows no deflection after ١٠s، then α is :

B
0.8 mΩ
A
G
C
1 mΩ
3 mΩ
D

V = 5mV

(١) −٢ × ١٠$^{-٢}$ °C$^{-١}$

(٢) −١.٥ × ١٠$^{-٢}$ °C$^{-١}$

(٣) −١ × ١٠$^{-٢}$ °C$^{-١}$

(٤) −٢.٥ × ١٠$^{-٢}$ °C$^{-١}$

Ans. (٣)

Sol. For no deflection $\frac{0.8}{1} = \frac{R}{3}$

$\Rightarrow R = ٢.٤ m\Omega$

Temperature fall in ١٠s = ٢٠°C

$\Delta R = R\alpha\Delta t$

$\Rightarrow \alpha = \frac{\Delta R}{R\Delta t} = \frac{-0.6}{3 \times 20}$

= −١٠$^{-٢}$ C$^{-١}$

٣٣. From the statements given below :

(A) The angular momentum of an electron in n^{th} orbit is an integral multiple of h.

(B) Nuclear forces do not obey inverse square law.

(C) Nuclear forces are spin dependent.

(D) Nuclear forces are central and charge independent.

(E) Stability of nucleus is inversely proportional to the value of packing fraction.

Choose the correct answer from the options given below :

(١) (A), (B), (C), (D) only

(٢) (A), (C), (D), (E) only

(٣) (A), (B), (C), (E) only

(٤) (B), (C), (D), (E) only

Ans. (٣)

Sol. Part of theory

٣٤. A diatomic gas (γ = ١.٤) does ٢٠٠ J of work when it is expanded isobarically. The heat given to the gas in the process is :

(١) ٨٥٠ J (٢) ٨٠٠ J

(٣) ٦٠٠ J (٤) ٧٠٠ J

Ans. (٤)

Sol. γ = ١ + $\frac{2}{f}$ = ١.٤ $\Rightarrow \frac{2}{f} = 0.4$

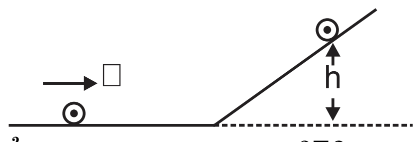
$\Rightarrow$ f = ٥

W = n R ΔT = ٢٠٠ J

$$Q = \left(\frac{f+2}{2}\right) nR\Delta T$$

$= \frac{7}{2} \times 200$ = ٧٠٠ J

٣٥. A disc of radius R and mass M is rolling horizontally without slipping with speed v. It then moves up an inclined smooth surface as shown in figure. The maximum height that the disc can go up the incline is :



(١) $\frac{v^2}{g}$ (٢) $\frac{3v^2}{4g}$

(٣) $\frac{1}{2}\frac{v^2}{g}$ (٤) $\frac{2}{3}\frac{v^2}{g}$

Ans. (٣)

Sol. Only the translational kinetic energy of disc changes into gravitational potential energy. And rotational KE remains unchanged as there is no friction.

$\frac{1}{2}mv^2 = mgh$

$h = \frac{v^2}{2g}$

٣٦. Conductivity of a photodiode starts changing only if the wavelength of incident light is less than ٦٦٠ nm. The band gap of photodiode is found to be $\left(\frac{X}{8}\right)$ eV. The value of X is :

(Given, h = ٦.٦ × ١٠ $^{-34}$ Js, e = 1.6 × 10^{-19} C)

(١) ١٥ (٢) ١١

(٣) ١٣ (٤) ٢١

Ans. (١)

Sol. $E_g = \frac{hc}{\lambda} = \frac{6.6 \times 10^{-34} \times 3 \times 10^{8}}{660 \times 10^{-9}}$ J

$= \frac{6.6 \times 10^{-34} \times 3 \times 10^{8}}{660 \times 10^{-9} \times 1.6 \times 10^{-19}} eV$

$= \frac{15}{8} eV$

So x = ١٥

٣٧. A big drop is formed by coalescing ١٠٠٠ small droplets of water. The surface energy will become :

(١) ١٠٠ times (٢) ١٠ times

(٣) $\frac{1}{100}th$ (٤) $\frac{1}{10}th$

Ans. (٤)

Sol. Lets say radius of small droplets is r and that of big drop is R

$\frac{4}{3}\pi R^3 = 1000 \times \frac{4}{3}\pi r^3$

R = ١٠r

Ui = ١٠٠٠ (٤πr²S)

Uf = ٤πR²S

= ١٠٠ (٤πr²S)

Uf = $\frac{1}{10} U_i$

٣٨. If frequency of electromagnetic wave is ٦٠ MHz and it travels in air along z direction then the corresponding electric and magnetic field vectors will be mutually perpendicular to each other and the wavelength of the wave (in m) is

(١) ٢.٥ (٢) ١٠

(٣) ٥ (٤) ٢

Ans. (٣)

Sol. $\lambda = \frac{c}{f} = \frac{3\times10^8}{60\times10^6} = 5m$

٣٩. A cricket player catches a ball of mass ١٢٠ g moving with ٢٥ m/s speed. If the catching process is completed in ٠.١ s then the magnitude of force exerted by the ball on the hand of player will be (in SI unit):

(١) ٢٤ (٢) ١٢

(٣) ٢٥ (٤) ٣٠

Ans. (٤)

Sol. $F_{av} = \frac{\Delta p}{\Delta t}$

$= \frac{0.12 \times 25}{0.1} = 30N$

٤٠. Monochromatic light of frequency ٦ × ١٠ Hz is produced by a laser. The power emitted is ٢ × ١٠ W. How many photons per second on an average, are emitted by the source?
(Given h = ٦.٦٣ × ١٠ ^-٣٤ Js)

(١) ٩ × ١٠^١٨ (٢) ٦ × ١٠^١٥

(٣) ٥ × ١٠^١٥ (٤) ٧ × ١٠^١٦

Ans. (٣)

Sol. P = nhν

$n = \frac{P}{h\nu} = \frac{2\times10^{-3}}{6.63\times10^{-34}\times6\times10^{14}}$

= ٥ × ١٠^١٥

٤١. A microwave of wavelength ٢.٠ cm falls normally on a slit of width ٤.٠ cm. The angular spread of the central maxima of the diffraction pattern obtained on a screen ١.٥ m away from the slit, will be:

(١) ٣٠° (٢) ١٥°

(٣) ٦٠° (٤) ٤٥°

Ans. (٣)

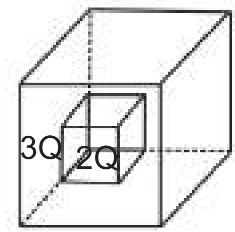
Sol. For first minima a sinθ = λ

$\sin\theta = \frac{\lambda}{a} = \frac{1}{2}$

θ = ٣٠°

Angular spread = ٦٠°

٤٢. C١ and C٢ are two hollow concentric cubes enclosing charges ٢Q and ٣Q respectively as shown in figure. The ratio of electric flux passing through C١ and C٢ is:



(١) ٢ : ٥ (٢) ٥ : ٢

(٣) ٢ : ٣ (٤) ٣ : ٢

Ans. (١)

Sol. $\phi_{smaller\ cube} = \frac{2Q}{\varepsilon_0}$

$\phi_{bigger\ cube} = \frac{5Q}{\varepsilon_0}$

$\frac{\phi_{smaller\ cube}}{\phi_{bigger\ cube}} = \frac{2}{5}$

٤٣. If the root mean square velocity of hydrogen molecule at a given temperature and pressure is ٢ km/s, the root mean square velocity of oxygen at the same condition in km/s is:

(١) ٢.٠ (٢) ٠.٥

(٣) ١.٥ (٤) ١.٠

Ans. (٢)

Sol. $V_{rms} = \sqrt{\frac{3RT}{M}}$

$\frac{V_1}{V_2} = \sqrt{\frac{M_2}{M_1}} \Rightarrow \frac{2}{V_2} = \sqrt{\frac{32}{2}}$

V٢ = ٠.٥ km/s

٤٤. Train A is moving along two parallel rail tracks towards north with speed ٧٢ km/h and train B is moving towards south with speed ١٠٨ km/h. Velocity of train B with respect to A and velocity of ground with respect to B are (in ms⁻¹):

(١) –٣٠ and ٥٠

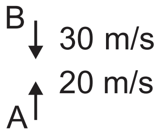
(٢) –٥٠ and –٣٠

(٣) –٥٠ and ٣٠

(٤) ٥٠ and –٣٠

Ans. (٣)

Sol.



VA = ٢٠ m/s VB = –٣٠ m/s

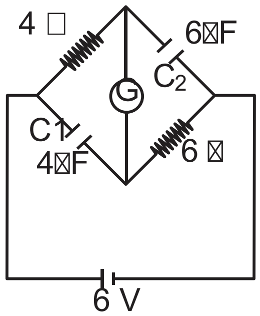
Velocity of B w.r.t. A

VB/A = – ٥٠ m/s

Velocity of ground w.r.t. B

VG/B = ٣٠ m/s

٤٥. A galvanometer (G) of ٢Ω resistance is connected in the given circuit. The ratio of charge stored in C١ and C٢ is:



(١) $\frac{2}{3}$

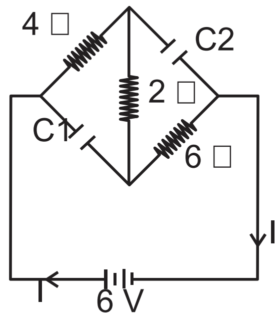
(٢) $\frac{3}{2}$

(٣) ١

(٤) $\frac{1}{2}$

Ans. (٤)

Sol.



In steady state

Req = ١٢Ω

$$I = \frac{6}{12} = 0.5A$$

P.D across C١ = ٣V

P.D acoross C٢ = ٤V

q١ = C١V١ = ١٢ μC

q٢ = C٢V٢ = ٢٤ μC

$$\frac{q_1}{q_2} = \frac{1}{2}$$

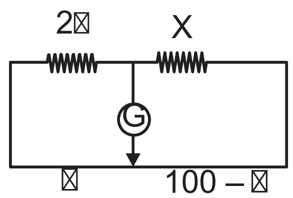
٤٦. In a metre-bridge when a resistance in the left gap is ٢Ω and unknown resistance in the right gap, the balance length is found to be ٤٠ cm. On shunting the unknown resistance with ٢Ω, the balance length changes by:

(١) ٢٢.٥ cm (٢) ٢٠ cm

(٣) ٦٢.٥ cm (٤) ٦٥ cm

Ans. (١)

Sol.



First case $\frac{2}{40} = \frac{X}{60} \Rightarrow X = 3\ \Omega$

In second case $X' = \frac{2 \times 3}{2 + 3} = 1.2\ \Omega$

$$\frac{2}{\ell} = \frac{1.2}{100 - \ell}$$

٢٠٠ – ٢ℓ = ١.٢ ℓ

$$\therefore \quad \ell = \frac{200}{3.2} = 62.5cm$$

Balance length changes by ٢٢.٥ cm

٤٧. Match List – I with List – II.

List – I (Number)	List – II (Significant figure)
(A) ١٠٠١	(I) ٣
(B) ٠١٠.١	(II) ٤
(C) ١٠٠.١٠٠	(III) ٥
(D) ٠.٠٠١٠٠١٠	(IV) ٦

Choose the correct answer from the options given below : (١) (A)-(III), (B)-(IV), (C)-(II), (D)-(I) (٢) (A)-(IV), (B)-(III), (C)-(I), (D)-(II) (٣) (A)-(II), (B)-(I), (C)-(IV), (D)-(III) (٤) (A)-(I), (B)-(II), (C)-(III), (D)-(IV)

Ans. (٣)

Sol. Theoretical A transformer has an efficiency of

٤٨. ٨٠٪ and works at ١٠ V and ٤ kW. If the secondary voltage is ٢٤٠ V, then the current in the secondary coil is :

(١) ١.٥٩ A (٢) ١٣.٣٣ A

(٣) ١.٣٣ A (٤) ١٥.١ A

Ans. (٢)

Sol. Efficiency = $\frac{E_S I_S}{E_P I_P}$

$0.8 = \frac{240 I_S}{4000}$

$I_S = \frac{3200}{240}$ = ١٣.٣٣A

٤٩. A light planet is revolving around a massive star in a circular orbit of radius R with a period of revolution T. If the force of attraction between planet and star is proportional to $R^{-3/2}$ then choose the correct option :

(١) $T^2 \propto R^{5/2}$ (٢) $T^2 \propto R^{7/2}$

(٣) $T^2 \propto R^{3/2}$ (٤) $T^2 \propto R^{3}$

Ans. (١)

Sol. $F = \frac{GMm}{R^{3/2}} = m\omega^2 R$

$\omega^2 \propto \frac{1}{R^{5/2}} \quad \because T = \frac{2\pi}{\omega}$ so

$T^2 \propto R^{5/2}$

٥٠. A body of mass ٤ kg experiences two forces $\vec{F}_1 = 5\hat{i} + 8\hat{j} + 7\hat{k}$ and $\vec{F}_2 = 3\hat{i} - 4\hat{j} - 3\hat{k}$. The acceleration acting on the body is :

(1) $-2\hat{i} - \hat{j} - \hat{k}$

(2) $4\hat{i} + 2\hat{j} + 2\hat{k}$

(3) $2\hat{i} + \hat{j} + \hat{k}$

(4) $2\hat{i} + 3\hat{j} + 3\hat{k}$

Ans. (٣)

Sol. Net force = $8\hat{i} + 4\hat{j} + 4\hat{k}$

$\vec{a} = \frac{\vec{F}}{m} = 2\hat{i} + \hat{j} + \hat{k}$

SECTION-B

٥١. A mass m is suspended from a spring of negligible mass and the system oscillates with a frequency f١. The frequency of oscillations if a mass ٩ m is suspended from the same spring is f. The value of $\frac{f_1}{f_2}$ is ____.

Ans. (٣)

Sol. $f_1 = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$

$f_2 = \frac{1}{2\pi}\sqrt{\frac{k}{9m}}$

$\frac{f_1}{f_2} = \sqrt{\frac{9}{1}} = \frac{3}{1}$

٥٢. A particle initially at rest starts moving from reference point. x = ٠ along x-axis, with velocity v that varies as $v = 4\sqrt{x}$ m/s. The acceleration of the particle is ______ ms^{-2}.

Ans. (٨)

Sol. $V = 4\sqrt{x}$

$a = V\frac{dv}{dx}$

$= 4\sqrt{x} \times 4 \times \frac{1}{2}x^{-1/2}$ = ٨ m/s٢

٥٣. A moving coil galvanometer has ١٠٠ turns and each turn has an area of ٢.٠ $cm^{٢}$. The magnetic field produced by the magnet is ٠.٠١ T and the deflection in the coil is ٠.٠٥ radian when a current of ١٠ mA is passed through it. The torsional constant of the suspension wire is $x \times ١٠^{-٥}$ N-m/rad. The value of x is____.

Ans. (٤)

Sol. $\tau = BINA\sin\theta$

$C\theta = BINA\sin ٩٠°$

$$C = \frac{BINA}{\theta} = \frac{٠.٠١ \times ١٠ \times ١٠^{-٣} \times ١٠٠ \times ٢ \times ١٠^{-٤}}{٠.٠٥}$$

$= ٤ \times ١٠^{-٥}$ N-m/rad.

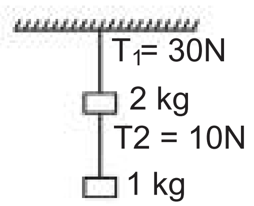
x = ٤

٥٤. One end of a metal wire is fixed to a ceiling and a load of ٢ kg hangs from the other end. A similar wire is attached to the bottom of the load and another load of ١ kg hangs from this lower wire. Then the ratio of longitudinal strain of upper wire to that of the lower wire will be____.

[Area of cross section of wire = ٠.٠٠٠٥ $cm^{٢}$, $Y = ٢ \times ١٠^{١١}$ $Nm^{-٢}$ and $g = ١٠\ ms^{-٢}$]

Ans. (٣)

Sol.



$$\Delta L = \frac{FL}{AY}$$

$$\frac{\Delta L}{L} = \frac{F}{AY}$$

$$\frac{\frac{\Delta L1}{L_1}}{\frac{\Delta L2}{L_2}} = \frac{F_1}{F_2} = \frac{30}{10} = 3$$

٥٥. A particular hydrogen – like ion emits the radiation of frequency ٣×١٠^{١٥} Hz when it makes transition from n = ٢ to n = ١. The frequency of radiation emitted in transition from n = ٣ to n = ١ is

$\frac{x}{9} \times ١٠^{١٥}$ Hz, when x = ________.

Ans. (٣٢)

Sol. $E = ١٣.٦Z^{٢}\left(\frac{١}{n_i^{٢}} - \frac{١}{n_f^{٢}}\right)$

$$E = C\left(\frac{1}{n_f^{٢}} - \frac{١}{n_i^{٢}}\right)$$

$$h\nu = C\left(\frac{1}{n_f^{٢}} - \frac{١}{n_i^{٢}}\right)$$

$$\frac{\nu_٢}{\nu_١} = \frac{\left(\frac{١}{n_f^{٢}} - \frac{١}{n_i^{٢}}\right)_٢}{\left(\frac{١}{n_f^{٢}} - \frac{١}{n_i^{٢}}\right)_١}$$

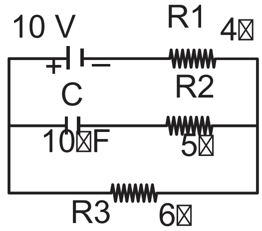
$$= \frac{\left(1 - \frac{١}{٤}\right)}{\left(1 - \frac{١}{٩}\right)} = \frac{٣/٤}{٨/٩}$$

$$= \frac{\frac{1}{1}}{\frac{1}{٣}} = \frac{٩}{٨}$$

$$\frac{\nu_٤}{\nu_٢} = \frac{٢٧}{٣٢}$$

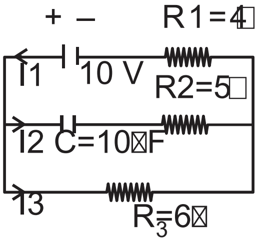
$$\nu_2 = \frac{٣٢}{٢٧}\nu = \frac{٣٢}{٢٧} \times ٣ \times ١٠^{١٥} Hz = \frac{٣٢}{٩} \times ١٠^{١٥} Hz$$

٥٦. In an electrical circuit drawn below the amount of charge stored in the capacitor is ____μC.



Ans. (٦٠)

Sol.



In steady state there will be no current in branch of capacitor، so no voltage drop across R٢ = ٥

I٢ = ٠

I١ I٣ $\frac{١٠}{٤ ٦}$ ١A

VR Vc VR VR ٠

I٣R٣ = Vc

Vc = ١ × ٦ = ٦ volt

qc = CVc = ١٠ × ٦ = ٦٠ C

٥٧. A coil of ٢٠٠ turns and area ٠.٢٠ m is rotated at half a revolution per second and is placed in uniform magnetic field of ٠.٠١ T perpendicular to axis of rotation of the coil. The maximum voltage generated in the coil is $\frac{2}{}$ volt. The value of is___.

Ans. (٥)

Sol. = NAB cos(t)

$= \frac{d}{dt}$ = NAB sin(t)

max = NAB

= ٢٠٠ × ٠.٢ × ٠.٠١ ×

$\frac{4}{١٠}$ $\frac{2}{٥}$ volt

٥٨. In Young's double slit experiment، monochromatic light of wavelength ٥٠٠٠ Å is used. The slits are ١.٠ mm apart and screen is placed at ١.٠ m away from slits. The distance from the centre of the screen where intensity becomes half of the maximum intensity for the first time is___ ×١٠ $^{-٦}$ m

Ans. (١٢٥)

Sol. Let intensity of light on screen due to each slit is I٠

So internity at centre of screen is ٤I٠

Intensity at distance y from centre–

I = I٠ + I٠ + ٢ $\sqrt{I_0 I_0}$ cos

Imax = ٤I٠

$\frac{I_{max}}{2}$ = ٢I٠ = ٢I٠ + ٢I٠ cos

cos = ٠

$\frac{}{2}$

$K x \frac{}{2}$

$\frac{2}{} d sin \frac{}{2}$

$\frac{2}{} d \frac{y}{D} \frac{1}{2}$

$y \frac{D}{4d} \frac{5 10^7 1}{4 10^3}$

= ١٢٥ × ١٠ $^{-٦}$

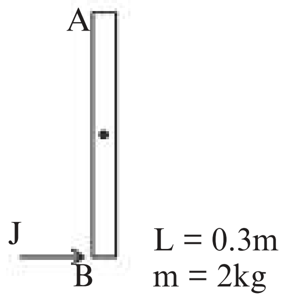
= ١٢٥

٥٩. A uniform rod AB of mass ٢ kg and Length ٣٠ cm at rest on a smooth horizontal surface. An impulse of force ٠.٢ Ns is applied to end B. The time taken by the rod to turn through at right angles will be

$\frac{}{x}$ s، where x = ______.

Ans. (٤)

Sol.



Impulse J = ٠.٢ N-S

$J = \int F\,dt = ٠.٢\ N\text{-}s$

Angular impulse (M)

$M = \int \tau\,dt$

$= \int F \frac{L}{2} dt$

$= \frac{L}{2}\int F\,dt = \frac{L}{2} J$

$= \frac{٠.٣}{٢} \times ٠.٢$

= ٠.٠٣

$I_{cm} = \frac{ML^{٢}}{١٢} = \frac{٢ \times (٠.٣)^{٢}}{١٢} = \frac{٠.٠٩}{٦}$

$M = I_{cm}(\omega_f - \omega_i)$

$٠.٠٣ = \frac{٠.٠٩}{٦}(\omega_f)$

ω_f = ٢ rad/s

$\theta = \omega t$

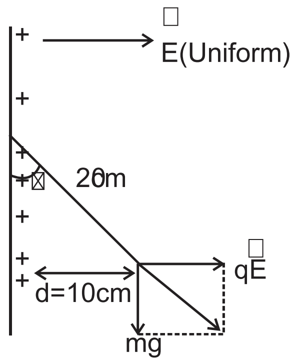
$t = \frac{\theta}{\omega} = \frac{\pi}{٢ \times 2} = \frac{\pi}{٤}$ sec.

X = ٤

٦٠. Suppose a uniformly charged wall provides a uniform electric field of ٢ × ١٠⁴ N/C normally. A charged particle of mass ٢ g being suspended through a silk thread of length ٢٠ cm and remain stayed at a distance of ١٠ cm from the wall. Then the charge on the particle will be $\frac{1}{\sqrt{x}}\ \mu C$ where x = __________. (use g = ١٠ m/s²)

Ans. (٣)

Sol.



$\sin\theta = \frac{10}{20} = \frac{1}{2}$

θ = ٣٠°

$\tan\theta = \frac{qE}{mg}$

$\tan 30° = \frac{q \times 2 \times 10^4}{2 \times 10^{-3} \times 10}$

$\frac{1}{\sqrt{3}} = q \times 10^{6}$

$q = \frac{1}{\sqrt{3}} \times 10^{-6} C$

x = ٣

CHEMISTRY

TEST PAPER WITH SOLUTION

SECTION-A

٦١. The transition metal having highest 3rd ionisation enthalpy is :

(1) Cr (2) Mn

(3) V (4) Fe

Ans. (٢)

Sol. 3rd Ionisation energy : [NCERT Data]

V : 2833 KJ/mol

Cr : 2990 KJ/mol

Mn : 3260 KJ/mol

Fe : 2962 KJ/mol

alternative

Mn : 3d5 4s2

Fe : 3d6 4s2

Cr : 3d5 4s1

V : 3d3 4s2

So Mn has highest 3rd IE among all the given elements due to d5 configuration.

٦٢. Given below are two statements :

Statement (I) : A p bonding MO has lower electron density above and below the inter-nuclear asix.

Statement (II) : The p* antibonding MO has a node between the nuclei.

In the light of the above statements, choose the most appropriate answer from the options given below:

(1) Both Statement I and Statement II are false

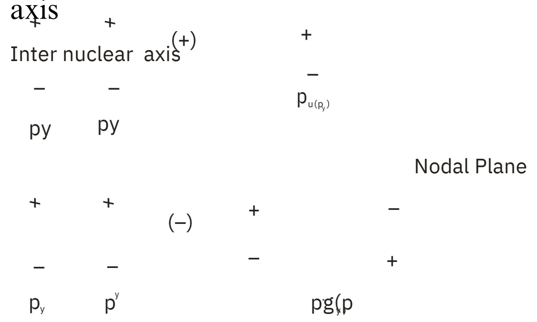
(2) Both Statement I and Statement II are true

(3) Statement I is false but Statement II is true

(4) Statement I is true but Statement II is false

Ans. (٣)

Sol. A p bonding molecular orbital has higher electron density above and below inter nuclear axis



٦٣. Given below are two statements : one is labelled as Assertion (A) and the other is labelled as Reason (R).

Assertion (A) : In aqueous solutions Cr2+ is reducing while Mn3+ is oxidising in nature.

Reason (R) : Extra stability to half filled electronic configuration is observed than incompletely filled electronic configuration.

In the light of the above statement, choose the most appropriate answer from the options given below:

(1) Both (A) and (R) are true and (R) is the correct explanation of (A)

(2) Both (A) and (R) are true but (R) is not the correct explanation of (A)

(3) (A) is false but (R) is true

(4) (A) is true but (R) is false

Ans. (١)

Sol. Cr2+ is reducing as it configuration changes from d4 to d3 due to formation of Cr3+, which has half filled

t on other hand, the change Mn3+ to Mn2+2glevel,

result half filled d5 configuration which has extra

stability.

٦٤. Match List - I with List - II.

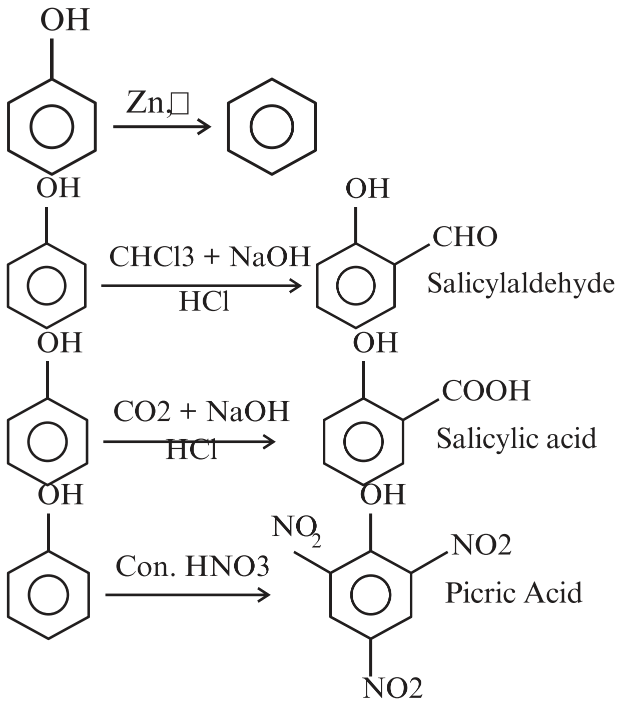
List-I (Reactants)	List-II Products
(A) Phenol، Zn /∆	(I) Salicylaldehyde
(B) Phenol، CHCl٣، NaOH، HCl	(II) Salicylic acid
(C) Phenol، CO٢، NaOH، HCl	(III) Benzene
(D) Phenol، Conc. HNO٣	(IV) Picric acid

Choose the correct answer from the options given below.

(١) (A)-(IV)، (B)، (II)، (C)-(I)، (D)-(III)
(٢) (A)-(IV)، (B)-(I)، (C)-(II)، (D)-(III)
(٣) (A)-(III)، (B)-(I)، (C)-(II)، (D)-(IV)
(٤) (A)-(III)، (B)-(IV)، (C)-(I)، (D)-(II)

Ans. (٣)

Sol.



٦٥. Given below are two statements :

Statement (I) : Both metal and non-metal exist in p and d-block elements.

Statement (II) : Non-metals have higher ionisation enthalpy and higher electronegativity than the metals.

In the light of the above statements، choose the most appropriate answer from the option given below:

(١) Both Statement I and Statement II are false
(٢) Statement I is false but Statement II is true
(٣) Statement I is true but Statement II is false
(٤) Both Statement I and Statement II are true

Ans. (٢)

Sol. I. In p-Block both metals and non metals are present but in d-Block only metals are present.

II. EN and IE of non metals are greater than that of metals

I - False، II-True

٦٦. The strongest reducing agent amont the following is:

(١) NH٣ (٢) SbH٣
(٣) BiH٣ (٤) PH٣

Ans. (٣)

Sol. Strongest reducing agent : BiH٣ explained by its low bond dissociation energy.

٦٧. Which of the following compounds show colour due to d-d transition؟

(١) CuSO٤.٥H٢O (٢) K٢Cr٢O٧
(٣) K٢CrO٤ (٤) KMnO٤

Ans. (١)

Sol. CuSO٤.٥H٢O

Cu²⁺ : ٣d٩٤s٠

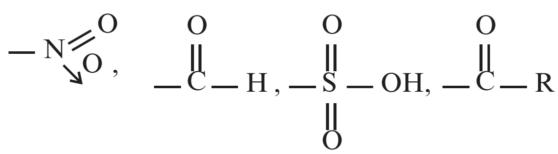
unpaired electron present so it show colour due to d-d transition.

٦٨. The set of meta directing functional groups from the following sets is:

(١) -CN، -NH٢، -NHR، -OCH٣
(٢) -NO٢، -NH٢، -COOH، -COOR
(٣) -NO٢، -CHO، -SO٣H، -COR
(٤) -CN، -CHO، -NHCOCH٣، -COOR

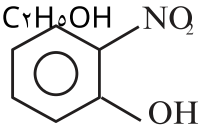
Ans. (٣)

Sol.



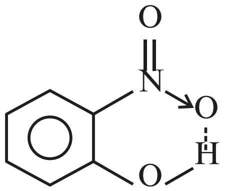
All are -M، Hence meta directing groups.

٦٩. Select the compound from the following that will show intramolecular hydrogen bonding.

(١) H٢O
(٢) NH٣
(٣) C٢H٥OH
(٤) 

Ans. (٤)

Sol. H٢O، NH٣، C٢H٥OH → Intermolecular H-Bonding



→ Intramolecular H-Bonding

٧٠. Lassaigne's test is used for detection of :

(١) Nitrogen and Sulphur only
(٢) Nitrogen، Sulphur and Phosphorous Only
(٣) Phosphorous and halogens only
(٤) Nitrogen، Sulphur، phosphorous and halogens

Ans. (٤)

Sol. Lassaigne's test is used for detection of all element N، S، P، X.

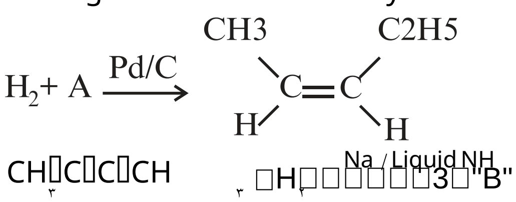
٧١. Which among the following has highest boiling point؟

(١) CH٣CH٢CH٢CH٣
(٢) CH٣CH٢CH٢CH٢-OH
(٣) CH٣CH٢CH٢CHO
(٤) H٥C٢ – O – C٢H٥

Ans. (٢)

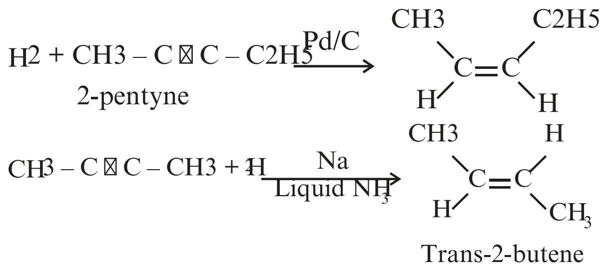
Sol. Due to H-bonding boiling point of alcohol is High.

٧٢. In the given reactions identify A and B.



(١) A : ٢-Pentyne B : trans – ٢ – butene
(٢) A : n – Pentane B : trans – ٢ – butene
(٣) A : ٢ – Pentyne B : Cis – ٢ – butene
(٤) A : n – Pentane B : Cis – ٢ – butene

Ans. (١)

Sol.


٧٣. The number of radial node /s for ٢p orbital is :

(١) ١ (٢) ٤
(٣) ٢ (٤) ٣

Ans. (١)

Sol. For ٢p : n = ٢، ℓ = ١
Number of radial node = n – ℓ – ١
= ٢ – ١ – ١ = ١

٧٤. Match List - I with List - II.

List - I	List - II
Compound	Use
(A) Carbon tetrachloride	(I) Paint remover
(B) Methylene chloride	(II) Refrigerators and air conditioners
(C) DDT	(III) Fire extinguisher
(D) Freons	(IV) Non Biodegradable insecticide

Choose the correct answer from the options given below :

(١) (A)-(I)، (B)، (II)، (C)-(III)، (D)-(IV)
(٢) (A)-(III)، (B)-(I)، (C)-(IV)، (D)-(II)
(٣) (A)-(IV)، (B)-(III)، (C)-(II)، (D)-(I)
(٤) (A)-(II)، (B)-(III)، (C)-(I)، (D)-(IV)

Ans. (٢)

Sol. CCl٤ used in fire extinguisher. CH٢Cl٢ used as paint remover. Freons used in refrigerator and AC. DDT used as non Biodegradable insecticide.

٧٥. The functional group that shows negative resonance effect is :

(١) –NH٢ (٢) –OH
(٣) –COOH (٤) –OR

Ans. (٣)

Sol. $-\overset{\overset{\displaystyle O}{||}}{C}-$ OH shows –R effect، while rest ٣ groups shows +R effect via lone pair.

٧٦. Co(NH٣)٦ and CoF٦ are respectively known as:

(١) Spin free Complex, Spin paired Complex

(٢) Spin paired Complex, Spin free Complex

(٣) Outer orbital Complex, Inner orbital Complex

(٤) Inner orbital Complex, Spin paired Complex

Ans. (٢)

Sol. Co(NH٣)٦ ٣+

Co (strong field ligand) ٣d t٦ 2geg ,

Hybridisation : ٢ dsp

Inner obital complex (spin paired complex)

Pairing will take place.

CoF٦ ٣+

Co (weak field ligand) ٣d t٤g e٢ g

Hybridisation : spd

Outer orbital complex (spin free complex)

no pairing will take place

Given below are two statements :

٧٧. Statement (I) : SiO٢ and GeO٢ are acidic while SnO and PbO are amphoteric in nature.

Statement (II) : Allotropic forms of carbon are due to property of catenation and p-d bond formation.

In the light of the above statements, choose the most appropriate answer from the options given below:

(١) Both Statement I and Statement II are false

(٢) Both Statement I and Statement II are true

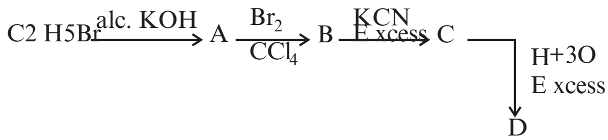
(٣) Statement I is true but Statement II is false

(٤) Statement I is false but Statement II is true

Ans. (٣)

Sol. SiO٢ and GeO٢ are acidic and SnO, PbO are amphoteric.

Carbon does not have d-orbitals so can not form p-d Bond with itself. Due to properties of catenation and p-p bond formation. carbon is able to show allotropic forms.

٧٨. 

Acid D formed in above reaction is :

(١) Gluconic acid

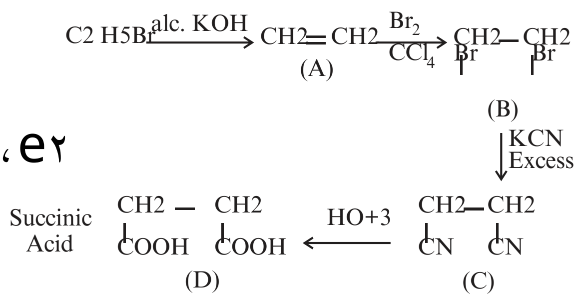
(٢) Succinic acid

(٣) Oxalic acid

(٤) Malonic acid

Ans. (٢)

Sol.



٧٩. Solubility of calcium phosphate (molecular mass, M) in water is Wg per ١٠٠ mL at ٢٥° C. Its solubility product at ٢٥°C will be approximately.

(١) $10^{7}\left(\frac{W}{M}\right)^{3}$ (٢) $10^{7}\left(\frac{W}{M}\right)^{5}$

(٣) $10^{3}\left(\frac{W}{M}\right)^{5}$ (٤) $10^{5}\left(\frac{W}{M}\right)^{5}$

Ans. (٢)

Sol. $S = \frac{W \times 10}{M}$

$Ca_3(PO_4)_2(s) \rightleftharpoons 3Ca^{2+}(aq.) + 2PO_4^{3-}(aq.)$

$\quad\quad\quad\quad\quad\quad\quad\quad 3s \quad\quad\quad\quad 2s$

$S = \frac{W \times 1000}{M \times 100} = \frac{W \times 10}{M}$

$K_{sp} = (3s)^3(2s)^2$

$= 108 s^5$

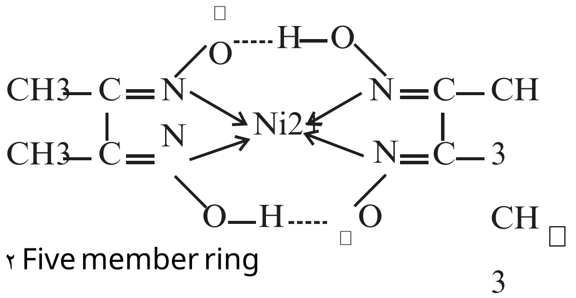
$= 108 \times 10^5 \left(\frac{W}{M}\right)^5$

$= 1.08 \times 10^7 \left(\frac{W}{M}\right)^5$

۸۰. Given below are two statements :
Statement (I) : Dimethyl glyoxime forms a six-membered covalent chelate when treated with $NiCl_2$ solution in presence of NH_4OH.
Statement (II) : Prussian blue precipitate contains iron both in (+۲) and (+۳) oxidation states. In the light of the above statements, choose the most appropriate answer from the options given below:
(۱) Statement I is false but Statement II is true
(۲) Both Statement I and Statement II are true
(۳) Both Statement I and Statement II are false
(٤) Statement I is true but Statement II is false

Ans. (۱)

Sol. Ni^{+2} + NH_4OH + dmg →



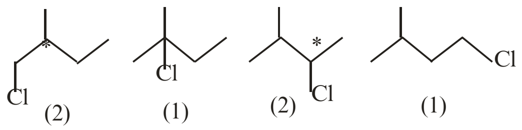
۲ Five member ring

III II
$Fe_4[Fe(CN)_6]_3$
Prussian Blue

SECTION-B

۸۱. Total number of isomeric compounds (including stereoisomers) formed by monochlorination of ۲-methylbutane is____________.

Ans. (٦)

Sol.


۸۲. The following data were obtained during the first order thermal decomposition of a gas A at constant volume:

A(g) → ۲B(g) + C(g)

S.No	Time /s	Total pressure /(atm)
۱.	۰	۰.۱
۲.	۱۱٥	۰.۲۸

The rate constant of the reaction is______ × ۱۰$^{-۲}$ s$^{-۱}$ (nearest integer)

Ans. (۲)

Sol. A(g) → ۲B(g) + C(g)
t = ۰ ۰.۱
t = ۱۱٥ sec. ۰.۱ − x ۲x x

۰.۱ + ۲x = ۰.۲۸
۲x = ۰.۱۸
x = ۰.۰۹

$K = \frac{۱}{۱۱٥} \ell n \frac{۰.۱}{۰.۱ - ۰.۰۹}$

= ۰.۰۲۰۰ sec$^{-۱}$

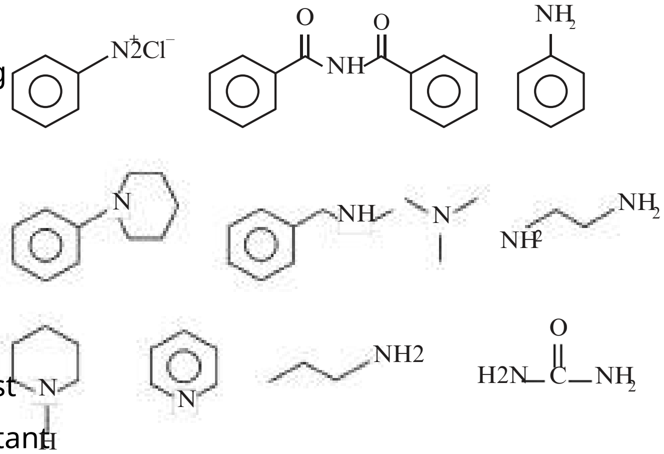
= ۲ × ۱۰$^{-۲}$ sec$^{-۱}$

۸۳. The number of tripeptides formed by three different amino acids using each amino acid once is ________.

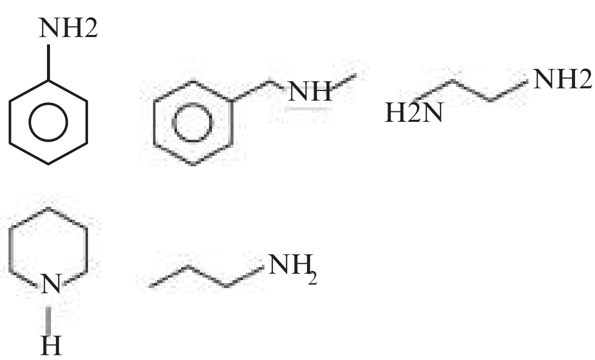
Ans. (٦)

Sol. Let ۳ different amino acid are A, B, C then following combination of tripeptides can be formed-
ABC, ACB, BAC, BCA, CAB, CBA

۸٤. Number of compounds which give reaction with Hinsberg's reagent is__________.



Ans. (٥)

Sol.


٨٥. Mass of ethylene glycol (antifreeze) to be added to ١٨.٦ kg of water to protect the freezing point at $-٢٤°C$ is ____________ kg (Molar mass in g mol$^{-١}$ for ethylene glycol ٦٢, K_f of water = ١.٨٦ K kg mol$^{-١}$)

Ans. (١٥)

Sol. $\Delta T_f = iK_f \times$ molality

$$٢٤ = (١) \times ١.٨٦ \times \frac{W}{٦٢ \times ١٨.٦}$$

W = ١٤٨٨٠ gm

= ١٤.٨٨٠ kg

٨٦. Following Kjeldahl's method, ١g of organic compound released ammonia, that neutralised ١٠ mL of ٢M $H_٢SO_٤$. The percentage of nitrogen in the compound is ____________%.

Ans. (٥٦)

Sol. $H_٢SO_٤ + ٢NH_٣ \rightarrow (NH_٤)_٢SO_٤$

Millimole of $H_٢SO_٤$ = ١٠ × ٢

So Millimole of $NH_٣$ = ٢٠ × ٢ = ٤٠

Organic Compound → $NH_٣$ ٤٠ Millimole

∴ Mole of N = $\frac{٤٠}{١٠٠٠}$

wt. of N = $\frac{٤٠}{١٠٠٠} \times ١٤$

% composition of N in organic compound

$$= \frac{٤٠ \times ١٤}{١٠٠٠ \times ١} \times ١٠٠$$

= ٥٦%

٨٧. The amount of electricity in Coulomb required for the oxidation of ١ mol of $H_٢O$ to $O_٢$ is ________ × ١٠^٥ C.

Ans. (٢)

Sol. $٢H_٢O \rightarrow O_٢ + ٤H^+ + ٤e^-$

$$\frac{W}{E} = \frac{Q}{٩٦٥٠٠}$$

mole × n-factor = $\frac{Q}{٩٦٥٠٠}$

$$١ \times ٢ = \frac{Q}{٩٦٥٠٠}$$

Q = ٢ × ٩٦٥٠٠ C

= ١.٩٣ × ١٠^٥ C

٨٨. For a certain reaction at ٣٠٠K, K = ١٠, then $\Delta G°$ for the same reaction is – ____________ × ١٠$^{-١}$ kJ mol$^{-١}$.

(Given R = ٨.٣١٤ JK$^{-١}$ mol$^{-١}$)

Ans. (٥٧)

Sol. $\Delta G° = -RT \ln K$

= –٨.٣١٤ × ٣٠٠ ln (١٠)

= ٥٧٤٤.١٤ J/mole

= ٥٧.٤٤ × ١٠$^{-١}$ kJ/mole

٨٩. Consider the following redox reaction:

$$MnO_٤^- + H^+ + H_٢C_٢O_٤ \rightleftharpoons Mn^{٢+} + H_٢O + CO_٢$$

The standard reduction potentials are given as below ($E°_{red}$)

$E°_{MnO_٤^-/Mn^{٢+}} = +١.٥١V$

$E°_{CO_٢/H_٢C_٢O_٤} = -٠.٤٩V$

If the equilibrium constant of the above reaction is given as $K_{eq} = ١٠^x$, then the value of x = ____________ (nearest integer)

Ans. (٣٣٨ OR ٣٣٩)

Sol. Cell Rx : $MnO_4^- + H_2C_2O_4 \rightarrow Mn^{2+} + CO_2$

$E_{cell} = E^{\circ}_{op}$ of anode $+ E^{\circ}_{RP}$ of cathode

$= 0.49 + 1.51 = 2.00$ V

At equilibrium

$E_{cell} = 0$,

$E^{\circ}_{cell} = \frac{0.059}{n} \log K$

(As per NCERT $\frac{RT}{F} = 0.059$ But $\frac{RT}{F} = 0.0591$ can also be taken.)

$2 = \frac{0.059}{10} \log K$

$\log K = 338.98$

90. 10 mL of gaseous hydrocarbon on combustion gives 40 mL of $CO_2(g)$ and 50 mL of water vapour. Total number of carbon and hydrogen atoms in the hydrocarbon is ________.

Ans. (14)

Sol. $\underset{10\,ml}{C_xH_y} + O_2 \rightarrow CO_2 + H_2O$

$C_xH_y + \left(x + \frac{y}{4}\right)O_2 \rightarrow \underset{10x}{xCO_2} + \underset{5y}{\frac{y}{2} H_2O}$

$10x = 40$

$x = 4$

$5y = 50$

$y = 10$

C_4H_{10}