FINAL JEE–MAIN EXAMINATION – APRIL, 2023

(Held On Thursday 13 April, 2023) TIME : 9 : 00 AM to 12 : 00 NOON

MATHEMATICS

SECTION-A

১. $\int_0^{\infty} \frac{6}{e^{3x}+6e^{2x}+11e^{x}+6}dx$

(১) $\log_e\left(\frac{512}{81}\right)$

(২) $\log_e\left(\frac{32}{27}\right)$

(৩) $\log_e\left(\frac{256}{81}\right)$

(৪) $\log_e\left(\frac{64}{27}\right)$

Official Ans. by NTA (২)

Sol.

$$I=\int_0^{\infty}\frac{6}{(e^x+1)(e^x+2)(e^x+3)}dx$$

$$=6\int_0^{\infty}\left(\frac{\frac{1}{2}}{e^x+1}+\frac{-1}{e^x+2}+\frac{\frac{1}{2}}{e^x+3}\right)dx$$

$$=3\int_0^{\infty}\frac{e^{-x}}{1+e^{-x}}dx-6\int_0^{\infty}\frac{e^{-x}dx}{1+2e^{-x}}+3\int_0^{\infty}\frac{e^{-x}}{1+3e^{-x}}dx$$

$$=3\left[-\ln\left(1+e^{-x}\right)\right]_0^{\infty}+6\left[\frac{1}{2}\ln\left(1+2e^{-x}\right)\right]_0^{\infty}$$

$$-\frac{3}{3}\left[\ln\left(1+3e^{-x}\right)\right]_0^{\infty}$$

$$=3\ln 2-3\ln 3+\ln 4$$

$$=\frac{3\ln 2}{3}+\ln 4$$

$$=\frac{\ln 32}{27}$$

২. $\max_{0\le x\le p}\left\{x-2\sin x\cos x+\frac{1}{3}\sin 3x\right\}=$

(১) $\frac{5p+2+3\sqrt{3}}{6}$

(২) $\frac{p+2-3\sqrt{3}}{6}$

(৩) p

(৪) ০

TEST PAPER WITH SOLUTION

Official Ans. by NTA (১)

Sol.

$$f(x)=x-\sin 2x+\frac{1}{3}\sin 3x$$

$$f'(x)=1-2\cos 2x+\cos 3x=0$$

$$x=\frac{p}{6},\frac{5p}{6}$$

$$f''(x)=4\sin 2x-3\sin 3x$$

$$f''\left(\frac{5p}{6}\right)<0$$

$\frac{5p}{6}$ is point of maxima

$$f\left(\frac{5p}{6}\right)=\frac{5p}{6}+\frac{3}{2}+\frac{1}{3}$$

৩. The set of all $a\in R$ for which the equation $x|x-1|+|x+2|+a=0$, has exactly one real root is :

(১) $(-6,-3)$

(২) $(-\infty,\infty)$

(৩) $(-6,\infty)$

(৪) $(-\infty,-3)$

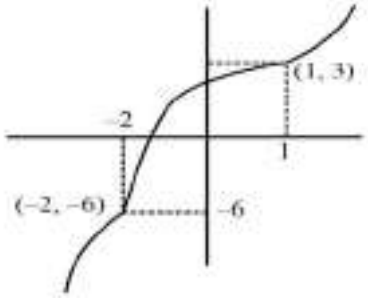
Official Ans. by NTA (২)

Sol.

$$f(x)=x|x-1|+|x+2|$$

$$x|x-1|+|x+2|+a=0$$

$$x|x-1|+|x+2|=-a$$



All values are increasing.

٤. The negation of the statement

((A Ù (B Ú C)) Þ (A Ú B)) is

(١) equivalent to ~ A

(٢) equivalent to ~ C

(٣) equivalent to B Ú ~ C

(٤) a fallacy

Official Ans. by NTA (١)

Sol.

p : ((A Ù (B Ú C)) Þ (A Ú B))

~p : (A Ù (B Ú C)) Ù ~ (A Ú B)

[(A Ù (B Ú C)) Ù ~ (A Ú B)]

(f Ú A) = A

~ p º ~ A

٥. The distance of the point (-١، ٢، ٣) from the plane $\vec{r}.(\hat{i}-2\hat{j}+3\hat{k})=10$ parallel to the line of the shortest distance between the lines $\vec{r}=(\hat{i}-\hat{j})+\lambda(2\hat{i}+\hat{k})$ and $\vec{r}=(2\hat{i}-\hat{j})+m(\hat{i}-\hat{j}+\hat{k})$ is :

(١) $3\sqrt{2}$

(٢) $6\sqrt{2}$

(٣) $3\sqrt{6}$

(٤) $5\sqrt{2}$

Official Ans. by NTA (٣)

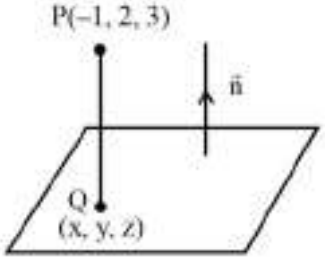
Sol.

Let $L_1: \vec{r}=(\hat{i}-\hat{j})+\lambda(2\hat{i}+\hat{k})$

$L_2: \vec{r}=(2\hat{i}-\hat{j})+m(\hat{i}-\hat{j}+\hat{k})$

$$\vec{n}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 2 & 0 & 1\\ 1 & -1 & 1\end{vmatrix}$$

$\vec{n}=\hat{i}-\hat{j}-2\hat{k}$



Equation of line along shortest distance of L_1 and L_2

$$\frac{x-1}{1}=\frac{y-2}{-1}=\frac{z-3}{-2}=r$$

Þ (x,y,z)º(r-1,2-r,3-2r)

Þ (r-1)-2(2-r)+3(3-2r)=10

Þ r=-2

Þ Q(x,y,z)º(-3,4,7)

Þ PQ= $\sqrt{4+4+16}=2\sqrt{6}$

٦. A coin is biased so that the head is 3 times as likely to occur as tail. This coin is tossed until a head or three tails occur. If X denotes the number of tosses of the coin, then the mean of X is

(١) $\frac{21}{16}$

(٢) $\frac{81}{64}$

(٣) $\frac{15}{16}$

(٤) $\frac{37}{16}$

Official Ans. by NTA (١)

Sol.

$P(H)=\frac{3}{4}$

$P(T)=\frac{1}{4}$

X	1	2	3
P(X)	$\frac{3}{4}$	$\frac{1}{4}\times\frac{3}{4}$	$\left(\frac{1}{4}\right)^3+\left(\frac{1}{4}\right)^2\times\frac{3}{4}$

Mean $\bar{X}=\frac{3}{4}+\frac{3}{8}+3\left(\frac{1}{64}+\frac{3}{64}\right)$

$=\frac{3}{4}+\frac{3}{8}+\frac{3}{16}$

$=3\left(\frac{7}{16}\right)$

$=\frac{21}{16}$

৭. For the system of linear equations

$2x+4y+2az=b$

$x+2y+3z=4$

$2x-5y+2z=8$

which of the following is NOT correct ?

(১) It has infinitely many solutions if a = ৩, b = ৬

(২) It has unique solution if a = b = ৬

(৩) It has unique solution if a = b = ৮

(৪) It has infinitely many solution if a = ৩, b = ৮

Official Ans. by NTA (১)

Sol.

$$D=\begin{vmatrix}2 & 4 & 2a\\ 1 & 2 & 3\\ 2 & -5 & 2\end{vmatrix}=18(3-a)$$

$$D_x=\begin{vmatrix}b & 4 & 2a\\ 4 & 2 & 3\\ 8 & -5 & 2\end{vmatrix}=(64+19b-72a)$$

For unique solution D=০

Þa¹৩ and bÎR

For Infinitely many solution :

D = Dx=Dy=Dz=0

Þ a = 3 Q D = 0

and b = ৮ QD x = 0

৮. For the differentiable function

$f:\mathbb{R}-\{0\}\to\mathbb{R}$, let $3f(x)+2f\left(\frac{1}{x}\right)=\frac{1}{x}-10$, then

$\left|f(3)+f'\left(\frac{1}{4}\right)\right|$ is equal to

(১) ৭

(২) $\frac{33}{5}$

(৩) $\frac{29}{5}$

(৪) ১৩

Official Ans. by NTA (৪)

Sol.

$$3f(x)+2f\left(\frac{1}{x}\right)=\frac{1}{x}-10 \quad \times 3$$

$$2f(x)+3f\left(\frac{1}{x}\right)=x-10 \quad \times 2$$

$$5f(x)=\frac{3}{x}-2x-10$$

$$f(x)=\frac{1}{5}\left(\frac{3}{x}-2x-10\right)$$

$$f'(x)=\frac{1}{5}\left(-\frac{3}{x^2}-2\right)$$

$$\left|f(3)+f'\left(\frac{1}{4}\right)\right|=\left|\frac{1}{5}(1-6-10)+\frac{1}{5}(-48-2)\right|$$

$$=|-3-10|=13$$

৯. Let the tangent and normal at the point $(3\sqrt{3},1)$ on the ellipse $\frac{x^2}{36}+\frac{y^2}{4}=1$ meet the y-axis at the points A and B respectively. Let the circle C be drawn taking AB as a diameter and the line $x=2\sqrt{5}$ intersect C at the points P and Q. If the tangents at the points P and Q on the circle intersect at the point (a,b) , then a^2-b^2 is equal to

(১) $\frac{314}{5}$

(২) $\frac{304}{5}$

(৩) ৬০

(৪) ৬১

Official Ans. by NTA (২)

Sol.

Given ellipse $\frac{x^2}{36}+\frac{y^2}{4}=1$

$$\frac{x}{4\sqrt{3}}+\frac{y}{4}=1$$

$$y=4$$

$$\frac{x}{4}-\frac{4}{4\sqrt{3}}=\frac{2}{\sqrt{3}}$$

$$y=-8$$

$x^2 + y^2 + 4y - 32 = 0$

$hx + ky + 2(y + k) - 32 = 0$

$k = -2$

$hx + 2k - 32 = 0$

$hx = 36$

$a = h = \frac{36}{2\sqrt{5}}$

$b = k = -2$

$a^2 - b^2 = \frac{304}{5}$

١٠. The area of the region enclosed by the curve $f(x) = \max\{\sin x, \cos x\}$, $x \in [0, \pi]$ and the x–axis is

(١) $2(\sqrt{2}+1)$

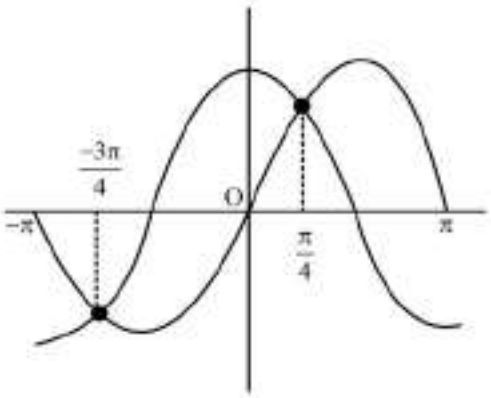
(٢) $2\sqrt{2}(\sqrt{2}+1)$

(٣) $4(\sqrt{2})$

(٤) ٤

Official Ans. by NTA (٤)

Sol.



Area =

$$\left|\int_{-\pi}^{\frac{-3\pi}{4}} \sin x\,dx\right| + \left|\int_{\frac{-3\pi}{4}}^{\frac{-\pi}{2}} \cos x\,dx\right| + \int_{\frac{-\pi}{2}}^{\frac{\pi}{4}} \cos x\,dx + \int_{\frac{\pi}{4}}^{\pi} \sin x\,dx$$

= ٤

١١. The number of symmetric matrices of order ٣, with all the entries from the set $\{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$, is :

(١) ٦١٠

(٢) ٩١٠

(٣) ١٠٩

(٤) ١٠٦

Official Ans. by NTA (٤)

Sol.

$$A = \begin{bmatrix} a & b & c \\ b & d & e \\ c & e & f \end{bmatrix}, a,b,c,d,e,f \in \{0,1,2,....9\}$$

Number of matrices = 106

١٢. Among :

$$(S1) : \lim_{n\to\infty} \frac{1}{n^2}(2+4+6+\ldots\ldots+2n) = 1$$

$$(S2) : \lim_{n\to\infty} \frac{1}{n^{16}}(1^{15}+2^{15}+3^{15}+\ldots\ldots+n^{15}) = \frac{1}{16}$$

(١) Both (S١) and (S٢) are true

(٢) Both (S١) and (S٢) are false

(٣) Only (S٢) is true

(٤) Only (S١) is true

Official Ans. by NTA (١)

Sol.

$$S_1 : \lim_{n\to\infty} \frac{n(n+1)}{n^2} = 1 \Rightarrow \text{True}$$

$$S_2 : \lim_{n\to\infty} \frac{1}{n^{16}} \sum_{r=1}^{n} r^{15} = \int_0^1 x^{15}\,dx = \frac{1}{16} \Rightarrow \text{True}$$

١٣. Let PQ be a focal chord of the parabola $y^2 = ٣٦x$ of length ١٠٠, making an acute angle with the positive x–axis. Let the ordinate of P be positive and M be the point on the line segment PQ such that PM : MQ = ٣ : ١. Then which of the following points does NOT lie on the line passing through M and perpendicular to the line PQ?

(1) (-3,43)

(2) (-6,45)

(3) (3,33)

(٤) (٦, ٢٩)

Official Ans. by NTA (١)

Sol.

$$9\left(t + \frac{1}{t}\right)^2 = 100$$

$t = ٣$

$\Rightarrow P(81,54) \& Q(1,-6)$

M (٢١, ٩)

PL is $(y - 9) = \frac{-4}{3}(x - 21)$

$3y - 27 = -4x + 84$

$4x + 3y = 111$

For x ¡, two real valued functions f(x) and g(x) are such that, g(x)=$\sqrt{x}$+1 and fog(x)=x+3-$\sqrt{x}$. Then f(०) is equal to

(१) १

(२) -३

(३) ५

(४) ०

Official Ans. by NTA (३)

Sol.

$g(x) = \sqrt{x} + 1$

$fog(x) = x + 3 - \sqrt{x}$

$= (\sqrt{x})^2 - 3\sqrt{x} + 5$

$= g(x)^2 - 3g(x) + 5$

$\therefore f(x) = x^2 - 3x + 5$

$\therefore f(0) = 5$

But, if we consider the domain of the composite function fog(x) then in that case f(०) will be not defined as g(x) cannot be equal to zero.

१५. Fractional part of the number $\frac{4^{2022}}{15}$ is equal to

(१) $\frac{4}{15}$

(२) $\frac{1}{15}$

(३) $\frac{14}{15}$

(४) $\frac{8}{15}$

Official Ans. by NTA (२)

Sol.

$$\left\{\frac{4^{2022}}{15}\right\} = \left\{\frac{2^{4044}}{15}\right\} = \left\{\frac{(1+15)^{1011}}{15}\right\}$$

$$= \frac{1}{15}$$

१६. Let $\vec{a} = \hat{i} + 4\hat{j} + 2\hat{k}, \vec{b} = 3\hat{i} - 2\hat{j} + 7\hat{k}$ and $\vec{c} = 2\hat{i} - \hat{j} + 4\hat{k}$. If a vector $\vec{d}$ satisfies $\vec{d} \times \vec{b} = \vec{c} \times \vec{b}$ and $\vec{d} \cdot \vec{a} = 24$, then $|\vec{d}|^2$ is equal to

(१) ४१३

(२) ४२३

(३) ३२३

(४) ३१३

Official Ans. by NTA (१)

Sol.:

$\vec{d} \times \vec{b} = \vec{c} \times \vec{b}$

$\Rightarrow (\vec{d} - \vec{c}) \times \vec{b} = \vec{0}$

$\Rightarrow \vec{d} = \vec{c} + l\vec{b}$

Also $\vec{d} \cdot \vec{a} = 24$

$\Rightarrow (\vec{c} + l\vec{b}) \cdot \vec{a} = 24$

$l = \frac{24 - \vec{c} \cdot \vec{a}}{\vec{b} \cdot \vec{a}} = \frac{24 - 6}{9} = 2$

$\Rightarrow \vec{d} = \vec{c} + 2(\vec{b})$

$= 8\hat{i} - 5\hat{j} + 18\hat{k}$

$\Rightarrow |\vec{d}|^2 = 64 + 25 + 324 = 413$

१७. Let $B = \begin{bmatrix} 1 & 3 & a \\ 1 & 2 & 3 \\ a & a & 4 \end{bmatrix}, a > 2$ be the adjoint of a matrix A and $|A| = 2$, then $[a \ -2a \ a]B\begin{bmatrix} a \\ -2a \\ a \end{bmatrix}$ is equal to

(१) १६ (२) ३२

(३) -१६

(४) ०

Official Ans. by NTA (३)

Sol.

Given, $B = \begin{bmatrix} 1 & 3 & a \\ 1 & 2 & 3 \\ a & a & 4 \end{bmatrix}$

$|B| = 4$

$1(8 - 3a) - 3(4 - 3a) + a(a - 2a) = 4$

$-a^2 + 6a - 8 = 0$

$a = 2, 4$

Given, $a > 2$

So, $a = 2$ is rejected

$$[4 \ -8 \ 4]\begin{bmatrix} 1 & 3 & 4 \\ 1 & 2 & 3 \\ 4 & 4 & 4 \end{bmatrix}\begin{bmatrix} 4 \\ -8 \\ 4 \end{bmatrix}^{-1}$$

١٨. Let $S_1, S_2, S_3, \ldots, S_{10}$ respectively be the sum to ١٢ terms of ١٠ A.P.s whose first terms are ١، ٢، ٣، ... ، ١٠
and the common differences are ١، ٣، ٥، ... ، ١٩. Then $\sum_{i=1}^{10} S_i$ is equal to

(١) ٧٣٨٠

(٢) ٧٢٢٠

(٣) ٧٣٦٠

(٤) ٧٢٦٠

Official Ans. by NTA (٤)

Sol.

$$S_k = 6(2k + (11)(2k-1))$$

$$S_k = 6(2k + 22k-11)$$

$$S_k = 144k - 66$$

$$\sum_{1}^{10} S_k = 144\sum_{k=1}^{10} k - 66'10$$

$$= 144' \frac{10'11}{2} - 660$$

$$= 7920 - 660$$

$$= 7260$$

١٩. Let $y = y_1(x)$ and $y = y_2(x)$ be the solution curves of the differential equation $\frac{dy}{dx} = y + 7$ with initial conditions $y_1(0) = 0, y_2(0) = 1$ respectively. Then the curves $y = y_1(x)$ and $y = y_2(x)$ intersect at

(١) Two points

(٢) no point

(٣) infinite number of points

(٤) one point

Official Ans. by NTA (٢)

Sol.

$$\frac{dy}{dx} = y + 7 \Rightarrow \frac{d}{y} - y = 7$$

$$I.F. = e^{-x}$$

$$ye^{-x} = \int 7e^{-x} dx$$

$$\Rightarrow ye^{-x} = -7e^{-x} + c$$

$$\Rightarrow y = -7 + cex$$

$$-7 + 7ex = -7 + 8ex$$

$$\Rightarrow e^{x} = 0$$

No solution

٢٠. Let the equation of plane passing through the line of intersection of the planes $x+2y+az=2$ and $x-y+z=3$ be $5x-11y+bz=6a-1$. For $c \in \mathbb{R}$, if the distance of this plane from the point (a, c, c) is $\frac{2}{\sqrt{a}}$ then $\frac{a+b}{c}$ is equal to

(١) -2

(٢) ٢

(٣) -4

(٤) ٤

Official Ans. by NTA (٣)

Sol.

$$(x + 2y + az - 2) + \lambda (x - y + z - 3) = 0$$

$$\frac{1+\lambda}{5} = \frac{2-\lambda}{-11} = \frac{a+\lambda}{b} = \frac{2+3\lambda}{6a-1}$$

$$\lambda = -\frac{7}{2}, a = 3, b = 1$$

$$\frac{2}{\sqrt{a}} = \left|\frac{5a + 11c + bc - 6a + 1}{\sqrt{25 + 121 + 1}}\right|$$

$c = -1$

$$\therefore \frac{a+b}{c} = \frac{3+1}{-1} = -4$$

SECTION-B

21. Let a be the constant term in the binomial expansion of $\left(\sqrt{x} - \frac{6}{x^{\frac{1}{3}}}\right)^n$, $n \in 15$. If the sum of the coefficients of the remaining terms in the expansion is ٦٤٩ and the coefficient of x^{-n} is λa, then λ is equal to _____.

Official Ans. by NTA (٣٦)

Sol.

$$T_{k+1} = {}^{n}C_k (x)^{\frac{n-k}{2}} (-6)^k (x)^{-\frac{3}{2}k}$$

$$\frac{n-k}{2} - \frac{3}{2}k = 0$$

$$n - 4k = 0$$

$$(-5)^n \left({}^{n}C_{\frac{n}{4}}\right)(-6)^{\frac{n}{4}} = 649$$

By observation $(625 + 24 = 649)$, we get $n = 4$

Q $n = 4$ & $k = 1$

Required is coefficient of x^{-4} is $\left(\sqrt{4} - \frac{6}{x^{\frac{3}{2}}}\right)^4$

$${}^{4}C_1 (-6)^3$$

By calculating we will get $\lambda = 36$

২২. If

$$S = \left\{x \in \mathbb{R} : \sin^{-1}\left(\frac{x+1}{\sqrt{2x^2+2x+2}}\right) - \sin^{-1}\left(\frac{x}{\sqrt{x^2+1}}\right) = \frac{\pi}{4}\right\},$$

then

$$\sum_{x \in R} \left(\sin\left(\sqrt{x^2+x+5} - \frac{\pi}{2}\right) - \cos\left(\sqrt{x^2+x+5}\right)\pi\right)^2$$

is equal to _____.

Official Ans. by NTA (৫)

Sol.

$$\sin^{-1}\left(\frac{(x+1)}{\sqrt{(x+1)^2+1}}\right) - \sin^{-1}\left(\frac{x}{\sqrt{x^2+1}}\right) = \frac{\pi}{4}$$

$$\alpha \quad \frac{t}{\sqrt{t^2+1}} \in (-1,1)$$

$$\sin^{-1}\left(\frac{(x+1)}{\sqrt{(x+1)^2+1}}\right) = \sin^{-1}\left(\frac{x}{\sqrt{x^2+1}}\right) + \frac{\pi}{4}$$

$$\frac{(x+1)}{\sqrt{(x+1)^2+1}} = \frac{1}{\sqrt{2}}\cos\sec\sin^{-1}\left(\frac{x}{\sqrt{x^2+1}}\right) + \frac{1}{\sqrt{2}}\left(\frac{x}{\sqrt{x^2+1}}\right)$$

$$= \frac{1}{\sqrt{2}}\left(\sqrt{1 - \frac{x^2}{x^2+1}} + \frac{x}{\sqrt{x^2+1}}\right)$$

$$\frac{(x+1)}{\sqrt{(x+1)^2+1}} = \frac{1}{\sqrt{2}}\left(\frac{1+x}{\sqrt{x^2+1}}\right)$$

After solving this equation, we get

$$x = -1 \text{ or } x = 0$$

$$S = \{-1, 0\}$$

$$\sum_{x \in R} \left(\sin\left(\sqrt{x^2+x+5} - \frac{\pi}{2}\right) - \cos\left(\sqrt{x^2+x+5}\right)\pi\right)^2$$

$$= \left[\sin\left(\frac{\pi}{2}\right) - \cos(\pi)\right]^2 + \left[\sin\left(\frac{\pi}{2}\right) - \cos(\pi)\right]^2$$

$$= 4$$

২৩. Let $w = zz + k_1 z + k_2 iz + l(1+i), k_1, k_2 \in \mathbb{I}$. Let $\mathrm{Re}(w) = 0$ be the circle C of radius 1 in the first quadrant touching the line $y = 1$ and the y-axis. If the curve $\mathrm{Im}(w) = 0$ intersects C at A and B, then $30(AB)^2$ is equal to _____.

Official Ans. by NTA (২৪)

Sol.

$$w = zz + k_1 z + k_2 iz + l(1+i)$$

$$\mathrm{Re}(w) = x^2 + y^2 + k_1 x - k_2 y + l = 0$$

Centre $\equiv \left(-\frac{k_1}{2}, \frac{k_2}{2}\right) \equiv (1, 2)$

$k_1 = -2, \; k_2 = 4$

radius $= \sqrt{l} = 4$

$\mathrm{Im} = k_1 y + k_2 x + l = 0$

$\therefore 2x - y + 2 = 0$

$d = \frac{2}{\sqrt{5}}$

$\frac{l}{4} = 1 - \frac{4}{5} = \frac{1}{5}$

$\therefore 30l^2 = 24$

٢٤. Let for $x \in [0,1]$, $S_0(x) = x$,

$S_k(x) = C_k x + k\int_0^x S_{k-1}(t)\,dt$, where

$C_0 = 1, C_k = 1 - k\int_0^1 S_{k-1}(x)\,dx$, $k = ١, ٢, ٣, \ldots$. Then

$S_٢(٣) + ٦C_٣$ is equal to _____.

Official Ans. by NTA (١٨)

Sol.

Given,

$S_k(x) = C_k x + k\int_0^x S_{k-1}(t)\,dt$,

Put $k = ٢$ and $x = ٣$

$S_2(3) = C_2(3) + 2\int_0^3 S_1(t)\,dt \quad \ldots(١)$

Also,

$S_1(x) = C_1(x) + \int_0^x S_0(t)\,dt$

$= C_1 x + \frac{x^2}{2}$

$S_2(3) = 3C_2 + 2\int_0^3 \left(C_1 t + \frac{t^2}{2}\right) dt$

$= ٣C_٢ + ٩C_١ + ٩$

Also,

$C_1 = 1 - \int_0^1 S_0(x)\,dx = \frac{1}{2}$

$C_2 = 1 - \int_0^1 S_1(x)\,dx = 0$

$C_3 = 1 - \int_0^1 S_2(x)\,dx$

$= 1 - \int_0^1 \left(C_2 x + C_1 \frac{x^2}{2} + \frac{x^3}{3}\right) dx$

$= \frac{3}{4}$

$S_2(x) = C_2 x + 2\int_0^x S_1(t)\,dt$

$= C_2 x + C_1 x^2 + \frac{x^3}{3}$

$\therefore S_2(3) + 6C_3 = 3C_2 + 9C_1 + 9$

$= 18$

٢٥. The sum to ٢٠ terms of the series

$٢.٢٢ - ٣٢ + ٢.٤٢ - ٥٢ + ٢.٦٢ - \ldots\ldots$ is equal to _____.

Official Ans. by NTA (١٣١٠)

Sol.

$(22 - 3^2 + 4^2 - 52 + 20\text{terms}) +$

$(22 + 42 + \ldots + 10\text{terms})$

$-(2 + 3 + 4 + 5 + \ldots\ldots + 11) + 4[1 + 22 + \ldots\ldots 102]$

$= \frac{21 \cdot 22}{2}(40 + 1 - 23) + 4 \cdot \frac{10 \cdot 11 \cdot 21}{6}$

$= 1310 - 231 + 14 \cdot 11 \cdot 10$

$=$

$=$

٢٦. The number of seven digit positive integers formed using the digits ١, ٢, ٣ and ٤ only and sum of the digits equal to ١٢ is _____.

Official Ans. by NTA (٤١٣)

Sol.

$x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 = 12, x_i \in \{1,2,3,4\}$

No. of solutions $= {}^{7}C_{7-1} - \frac{7!}{6!} - \frac{7!}{5!} = 413$

٢٧. Let $m_١$ and $m_٢$ be the slopes of the tangents drawn from the point $P(٤, ١)$ to the hyperbola $H: \frac{y^2}{25} - \frac{x^2}{16} = ١$. If Q is the point from which the tangents drawn to H have slopes $|m_١|$ and $|m_٢|$ and they make positive intercepts a and b on the x-axis, then $\frac{(PQ)^2}{ab}$ is equal to _____.

Official Ans. by NTA (٨)

Sol.

Equation of tangent to the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$

$y=mx\pm\sqrt{a^2m^2-b^2}$

passing through $(4,1)$

$1=4m\pm\sqrt{25-16m^2}\Rightarrow 4m^2-m-3=0$

$\Rightarrow m=1,\ \frac{-3}{4}$

Equation of tangent with positive slopes $\frac{3}{4}$ &

$$\left.\begin{aligned}4y&=3x-16\\ y&=x-3\end{aligned}\right\}\Rightarrow \text{with positive intercept on x-axis.}$$

$a=\frac{16}{3}, b=3$

Intersection points :

$Q:(-4,-7)$

$P:(4,1)$

$PQ^2=128$

$\frac{PQ^2}{ab}=\frac{128}{16}=8$

२८. Let the image of the point $\left(5,\frac{5}{3},\frac{8}{3}\right)$ in the plane $x-2y+z-2=0$ be P. If the distance of the point $Q(6,-2,a), a<0$, from P is 13, then a is equal to _____.

Official Ans. by NTA (15)

Sol.

Image of point $\left(5,\frac{5}{3},\frac{8}{3}\right)$

$$\frac{x-5}{1}=\frac{y-\frac{5}{3}}{-2}=\frac{z-\frac{8}{3}}{1}=\frac{-2\left(5-\frac{5}{3}+(-2)\frac{8}{3}+1-\frac{8}{3}\right)}{1^2+2^2+1^2}$$

$$=\frac{2}{3}$$

$\therefore x=2, y=1, z=3$

$132=(6-2)^2+(-2-1)^2+(a-3)^2$

$\Rightarrow (a-3)^2=144 \Rightarrow a=15\ (\because a>0)$

२९. Let $\vec{a}=3\hat{i}+\hat{j}-\hat{k}$ and $\vec{c}=2\hat{i}-3\hat{j}+3\hat{k}$. If $\vec{b}$ is a vector such that $\vec{a}=\vec{b}\times\vec{c}$ and $|\vec{b}|^2=50$, then $|72-|\vec{b}+\vec{c}|^2|$ is equal to _____.

Official Ans. by NTA (66)

Sol.

$|\vec{a}|=\sqrt{11}, |\vec{c}|=\sqrt{22}$

$|\vec{b}\times\vec{c}|=|\vec{b}||\vec{c}|\sin\theta$

$\sqrt{11}=\sqrt{50}\sqrt{22}\sin\theta$

$\Rightarrow \sin\theta=\frac{1}{10}$

$|\vec{b}+\vec{c}|^2=|\vec{b}|^2+|\vec{c}|^2+2\vec{b}\cdot\vec{c}$

$=|\vec{b}|^2+|\vec{c}|^2+2|\vec{b}||\vec{c}|\cos\theta$

$=50+22+2\sqrt{50}\sqrt{22}\frac{\sqrt{99}}{10}$

$=72+66$

$|72-|\vec{b}+\vec{c}|^2|=66$

३०. Let the mean of the data

x	१	३	५	७	९
Frequency(f)	४	२४	२८	a	८

be ५. If m and s^2 are respectively the mean deviation about the mean and the variance of the data, then $\frac{3a}{m+s}$ is equal to _____.

Official Ans. by NTA (8)

Sol.

$5=\bar{x}=\frac{\sum x_i f_i}{\sum f_i}=\frac{4+72+140+7a+72}{64+a}$

$\Rightarrow 320+5a=288+7a\Rightarrow 2a=32\Rightarrow a=16$

$M.D.(\bar{x})=\frac{\sum f_i|x_i-\bar{x}|}{\sum f_i}$ where $\sum f_i=64+16=80$

$M.D.(\bar{x})=\frac{4+24\times 2+28\times 0+16\times 2+8\times 4}{80}$

$=\frac{8}{5}$

$\text{Variance}=\frac{\sum f_i(x_i-\bar{x})^2}{\sum f_i}$

$=\frac{4\times 16+24\times 4+0+16\times 4+8\times 16}{80}=\frac{352}{80}$

$\therefore \frac{3a}{m+s^2}=\frac{3\times 16}{\frac{128}{80}+\frac{352}{80}}=8$

SECTION-A

३१. Which of the following Maxwell's equations is valid for time varying conditions but not valid for static conditions :

(1) $\oint B.dl = \mu_0 I$ (2) $\oint E.dl = 0$

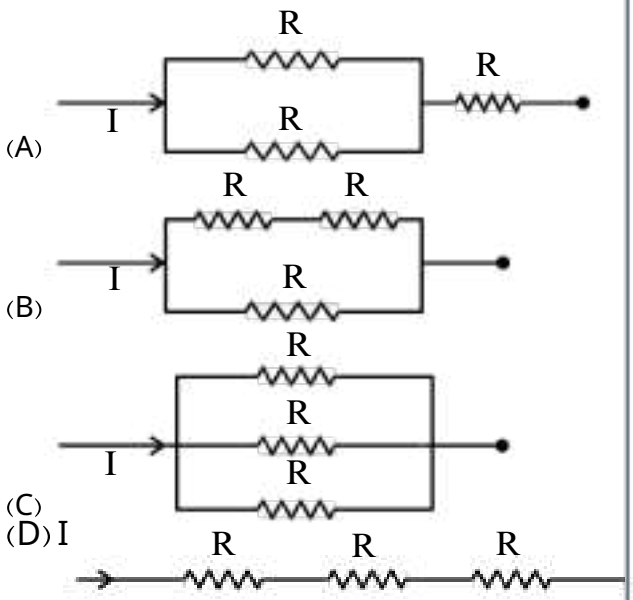
(3) $\oint E.dl = -\frac{dB}{dt}$ (4) $\oint D.dA = Q$

Official Ans. by NTA (३)

Sol. Based on equations of Maxwell

३२. Different combination of ३ resistors of equal resistance R are shown in the figures.

The increasing order for power dissipation is :



(१) $P_A > P_B > P_C > P_D$

(२) $P_C > P_D > P_A > P_B$

(३) $P_B > P_C > P_D > P_A$

(४) $P_C > P_B > P_A > P_D$

Official Ans. by NTA (४)

Sol. $P = I^2R$

$R_1 = \frac{3R}{2}, R_2 = \frac{2R}{3}, R_3 = \frac{R}{3}, R_4 = 3R$

Since i is same, hence $P \propto R$ so options (४) is correct

३३. A vessel of depth 'd' is half filled with oil of refractive index n_1 and the other half is filled with water of refractive index n_2. The apparent depth of this vessel when viewed from above will be-

(१) $\frac{d n_1 n_2}{n_1 + n_2}$

(२) $\frac{d(n_1 + n_2)}{2 n_1 n_2}$

(३) $\frac{d n_1 n_2}{2(n_1 + n_2)}$

(४) $\frac{2d(n_1 + n_2)}{n_1 n_2}$

Official Ans. by NTA (२)

Sol. Formula used $d_{app} = \frac{d_1}{n_1} + \frac{d_2}{n_2}$

$d_{app} = \frac{d}{2}\left(\frac{n_1 + n_2}{n_1 n_2}\right)$

३४. The source of time varying magnetic field may be

(A) a permanent magnet

(B) an electric field changing linearly with time

(C) direct current

(D) a decelerating charge particle

(E) an antenna fed with a digital signal

Choose the correct answer from the options given below :

(१) (D) only

(२) (C) and (E) only

(३) (A) only

(४) (B) and (D) only

Official Ans. by NTA (१)

Sol. Source of time varying magnetic field may be

• accelerated or retarded charge which produces varying electric and magnetic fields.

• An electric field varying linearly with time will not produce variable magnetic field as current will be constant

٣٥. Two trains 'A' and 'B' of length 'l' and '٤l' are travelling into a tunnel of length 'L' in parallel tracks from opposite directions with velocities ١٠٨ km /h and ٧٢ km /h، respectively. If train 'A' takes ٣٥s less time than train 'B' to cross the tunnel then، length 'L' of tunnel is : (Given L = ٦٠ l)

(١) ١٢٠٠ m

(٢) ٢٧٠٠ m

(٣) ١٨٠٠ m

(٤) ٩٠٠ m

Official Ans. by NTA (٣)

Sol. $\frac{10\ell + 4\ell}{20} - \frac{6\ell}{30} = 35$

$\ell = \frac{1050}{35}$

$L = 60\ell = \frac{1050}{35} \times 60 = 1800\text{m}$

٣٦. The ratio of powers of two motors is $\frac{3\sqrt{x}}{\sqrt{x+1}}$، that are capable of raising ٣٠٠ kg water in ٥ minutes and ٥٠ kg water in ٢ minutes respectively from a well of ١٠٠ m deep. The value of x will be

(١) ٢

(٢) ٤

(٣) ٢.٤

(٤) ١٦

Official Ans. by NTA (٤)

Sol. Average Power $= \frac{\text{total work done}}{\text{total time}}$

$$\text{So } P = \frac{mgh}{t}$$

$$\frac{P_1}{P_2} = \frac{\frac{m_1 gh}{t_1}}{\frac{m_2 gh}{t_2}} = \frac{m_1}{t_1}\frac{t_2}{m_2}$$

$$\frac{P_1}{P_2} = \frac{300}{5} \times \frac{2}{50} = \frac{12}{5} = \frac{3\sqrt{x}}{\sqrt{x+1}}$$

$$12\sqrt{x} + 12 = 15\sqrt{x}$$

$$3\sqrt{x} = 12$$

$$x = 16$$

٣٧. A planet having mass ٩ Me and radius ٤Re، where Me and Re are mass and radius of earth respectively، has escape velocity in km/s given by :

(Given escape velocity on earth $v_e = 11.2 \times 10^3 \text{m/s}$)

(١) ٦٧.٢

(٢) ١٦.٨

(٣) ٣٣.٦

(٤) ١١.٢

Official Ans. by NTA (٢)

Sol. $V_p = \sqrt{\frac{2GM_p}{R_p}}$ $V_E = \sqrt{\frac{2GM_E}{R_E}}$

$$\frac{V_P}{V_E} = \frac{\sqrt{\frac{2GM_P}{R_P}}}{\sqrt{\frac{2GM_E}{R_E}}} = \sqrt{\frac{R_E}{R_P}\frac{M_P}{M_E}}$$

$$V_P = \sqrt{\frac{1}{4}} \times 9 \times V_E = \frac{3}{2} V_E$$

$$V_P = \frac{3}{2} \times 11.2 \text{ km/sec}$$

$$= 16.8 \text{km/sec}$$

٣٨. The difference between threshold wavelengths for two metal surfaces A and B having work function in nm is :

$\phi_A = 9\text{eV}$ $\phi_B = 4.5\text{eV}$

(Given، hc = ١٢٤٢ eV nm)

(١) ٢٦٤

(٢) ١٣٨

(٣) ٢٧٦

(٤) ٥٤٠

Official Ans. by NTA (٢)

Sol. $\lambda_A = \frac{1242}{9} = 138 \text{ nm}$

$\lambda_B = \frac{1242}{4.5} = 276 \text{ nm}$

$\lambda_B - \lambda_A = 138 \text{ nm}$

٣٩. A bullet ١٠ g leaves the barrel of gun with a velocity of ٦٠٠ m/s. If the barrel of gun is ٥٠ cm long and mass of gun is ٣ kg, then value of impulse supplied to the gun will be :

(١) ١٢ Ns (٢) ٦ Ns

(٣) ٣٦ Ns (٤) ٣ Ns

Official Ans. by NTA (٢)

Sol. By momentum conservation

$0 = 3v + 0.01 \times 600 \times v$

$v \sim -2 m/s$

Impulse on gun $= 3 \times 2 = 6$ Ns

٤٠. Two charges each of magnitude ٠.٠١ C and separated by a distance of ٠.٤ mm constitute an electric dipole. If the dipole is placed in an uniform electric field 'E' of ١٠ dyne/C making ٣٠ angle with E, the magnitude of torque acting on dipole is :

(1) 4.0×10^{-10} Nm (2) 2.0×10^{-10} Nm

(3) 1.0×10^{-8} Nm (4) 1.5×10^{-9} Nm

Official Ans. by NTA (٢)

Sol. $|\vec{P}| = qd$

$= 0.01 \times 0.4 \times 10^{-3}$

$= 4 \times 10^{-6}$

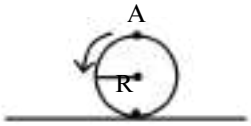
$|\vec{\tau}| = PE\sin\theta$

$= 4 \times 10^{-6} \times 10^{-10} \times 10^{5} \times \sin 30$

$= 4 \times 10^{-6} \times 5 \times 1 \times \frac{1}{2}$

$= 2 \times 10^{-10}$

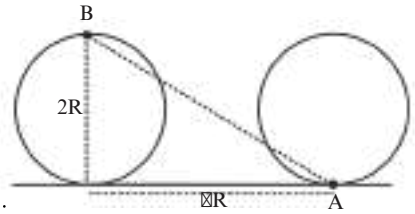
٤١. A disc is rolling without slipping on a surface. The radius of the disc is R. At t = ٠, the top most point on the disc is A as shown in figure. When the disc completes half of its rotation, the displacement of point A from its initial position is



(١) $R\sqrt{٢+٤}$ (٢) $R\sqrt{2+1}$

(٣) ٢R (٤) $2R\sqrt{1+4}$

Official Ans. by NTA (١)



Sol.

٤٢. Displacement $\sqrt{(2R)^2 + (\pi R)^2}$

Match List – I with List – II

List – I (Layer of atmosphere)	List – II (Approximate height over earth's surface)
(A) F١ – Layer	(I) ١٠ km
(B) D – Layer	(II) ١٧٠ – ١٩٠ km
(C) Troposphere	(III) ١٠٠ km
(D) E–layer	(IV) ٦٥ – ٧٥ km

Choose the correct answer from the options given below :

(١) A – III, B – IV, C – I, D – II

(٢) A – II, B – IV, C – III, D – I

(٣) A – II, B – I, C – IV, D – III

(٤) A – II, B – IV, C – I, D – III

Official Ans. by NTA (٤)

Sol. Based on Theory

٤٣. The rms speed of oxygen molecule in a vessel at particular temperature is $\frac{n}{x}\left(\frac{3RT}{M}\right)^{\frac{1}{2}}$, where n is the average speed of the molecule. The value of x will be : (Take $\pi = \frac{22}{7}$)

(١) ٢٨

(٢) ٢٧

(٣) ٨

(٤) ٤

Official Ans. by NTA (١)

Sol. $\sqrt{\frac{3RT}{M}} = \frac{n}{x}\left(\frac{5}{2}\right)\sqrt{\frac{8RT}{\pi M}}$

$= \frac{3\pi}{7\pi 8} = 1 - \frac{5}{x}$

$x = 28$

٤٤. A body of mass (٥٠٠٠٥) kg is moving with a velocity of (٢٠٠٠٠٤) m/s. Its kinetic energy will be

(1) 1000140J (2) 10000.14J (3) 5000.14J (4) 500140J

Official Ans. by NTA (١)

Sol. $k = \frac{1}{2}mv^2$

$k = \frac{1}{2} \times 5 \times 400 \times 5 \times 200 \times 1000J$

$\frac{k}{2k} = \frac{m}{m} = \frac{2v}{v} = \frac{0.5}{5} = \frac{2 \times 0.4}{20}$

$k = 1000 \times \frac{1}{10} \times \frac{4}{100} = 1000 \times \frac{10 \times 4}{100} = 140J$

٤٥. Two bodies are having kinetic energies in the ratio ١٦ : ٩. If they have same linear momentum, the ratio of their masses respectively is :

(١) ٤ : ٣ (٢) ٣ : ٤

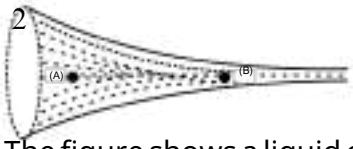
(٣) ١٦ : ٩ (٤) ٩ : ١٦

Official Ans. by NTA (٤)

Sol. $\frac{K}{1} = \frac{p^2}{2m} \times \frac{2m}{p^2} \times \frac{m}{2} \times \frac{16}{9}$

$\frac{K}{1} = \frac{m}{2} \times \frac{9}{16}$ $\frac{m}{1}$

٤٦. $\frac{m}{2}$



The figure shows a liquid of given density flowing steadily in horizontal tube of varying cross-section. Cross sectional areas at A is ١.٥ cm^2 and B is ٢٥ mm^2, if the speed of liquid at B is ٦٠ cm/s then (PA – PB) is :

(Given PA and PB are liquid pressures at A and B points.

Density = 1000 kg m^{-3}

A and B are on the axis of tube

(١) ١٧٥ Pa

(٢) ٢٧ Pa

(٣) ١٣٥ Pa

(٤) ٣٦ Pa

Official Ans. by NTA (١)

Sol. From continuity theorem $A_1V_1 = A_2V_2$

$1.5 \times V_1 = 25 \times 10^{-2} \times 60$

$V_1 = \frac{25 \times 60 \times 10^{-2} \times 10}{1.5}$

$V_1 = 10 cm/s$

By Bernoulli's theorem

$P_1 + \frac{1}{2} \times 1000 \times 0.1^2 = P_2 + \frac{1}{2} \times 1000 \times 6^2$

$P_1 + 5 = P_2 + \frac{1}{2} \times 1000 \times 36 \times 10^{-2}$

$P_1 + 5 = P_2 + 180$

$P_1 - P_2 = 175 Pa$

٤٧. Under isothermal condition, the pressure of a gas is given by $P = aV^{-3}$, where a is a constant and V is the volume of the gas. The bulk modulus at constant temperature is equal to

(١) $\frac{P}{2}$ (٢) ٣P

(٣) ٢P (٤) P

Official Ans. by NTA (٢)

Sol. $B = -\frac{dP}{dv/v}$

$Pv^{-3} = a$

Differentiating w.r.t to pressure

$v^{-3} + P(-3)v^{-2}\frac{dv}{dP} = 0$

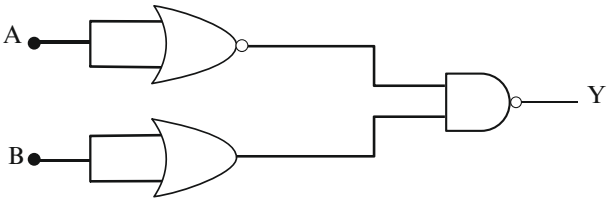
$v - \frac{3Pdv}{dP} = 0$

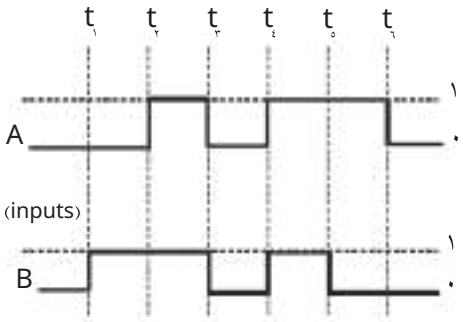
$v = -\frac{3Pdv}{dP}$

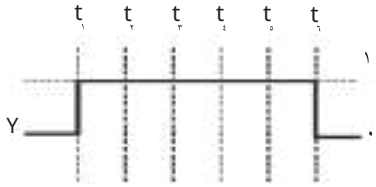
$\frac{dP.v}{dv} = -3P$

$B = -\frac{dPv}{dv} = -(-3P) = 3P$

٤٨. For the following circuit and given inputs A and B choose the correct option for output 'Y'





(١)


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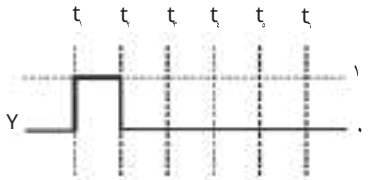
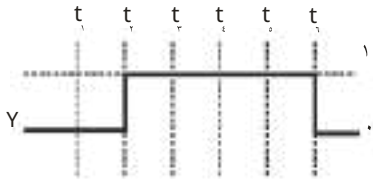
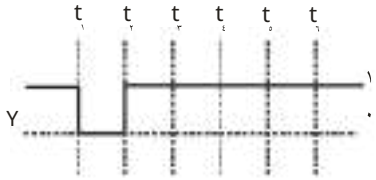


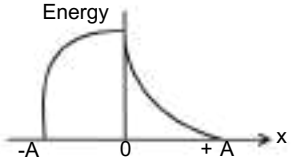
(٣)


(٤)


Official Ans. by NTA (٤)

Sol. Y = $\overline{A+B} = \overline{A}\,\overline{B}$

٤٩. Which graph represents the difference between total energy and potential energy of a particle executing SHM Vs it's distance from mean position?

(١)


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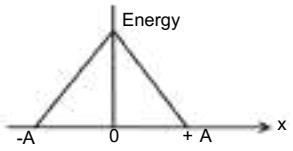
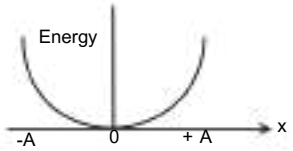
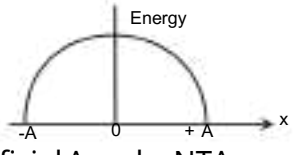


(٣)


(٤)


Official Ans. by NTA (٤)

Sol. T.E. = P.E. + K.E.

$K.E. = \frac{1}{2}m\omega^2(A^2 - x^2)$

Which is the equation of downward parabola.

٥٠. $^{238}_{92}A \rightarrow {}^{234}_{90}B + {}^{4}_{2}Q$

In the given nuclear reaction, the approximate amount of energy released will be :

[Given, mass of $^{238}_{92}A = 238.05079$ u, 1 u = 931.5 MeV/c^2,

mass of $^{234}_{90}B = 234.04363$ u,

mass of $^{4}_{2}D = 4.00260$ u, 1 u = 931.5 MeV/c^2]

(١) ٣.٨٢ MeV (٢) ٥.٩ MeV

(٣) ٢.١٢ MeV (٤) ٤.٢٥ MeV

Official Ans. by NTA (٤)

Sol. $Q = (m_A - m_B - m_D)c^2$ MeV

$= (238.05079 - 234.04363 - 4.00260)$ u

$= 4.25$ MeV

Section - B

٥١. The elastic potential energy stored in a steel wire of length ٢٠ m stretched through ٢ cm is ٨٠ J. The cross sectional area of the wire is_____ mm^2.

(Given, y = 2.0 × 10^{11} Nm^{-2})

Official Ans. by NTA (٤٠)

Sol. Energy per unit volume = $\frac{1}{2}$ stress × strain

$$\text{Energy} = \frac{1}{2} \text{stress} \times \text{strain} \times \text{volume}$$

$$80 = \frac{1}{2} \times Y \times \text{strain}^2 \times A \times \ell$$

$$80 = \frac{1}{2} \times 2 \times 10^{11} \times \left(\frac{2 \times 10^{-2}}{400}\right)^2 \times A \times 20$$

$$20 = \frac{20}{10^{-7}} \times A$$

$$40 \times 10^{-6}\ m^2 = A$$

$$A = 40\ mm^2$$

٥٢. A potential V٠ is applied across a uniform wire of resistance R. The power dissipation is P١. The wire is then cut into two equal halves and a potential of V٠ is applied across the length of each half. The total power dissipation across two wires is P٢. The ratio P٢ : P١ is x : ١. The value of x is _____.

Official Ans. by NTA (١٦)

Sol. $P = VI = I^2R = \frac{V^2}{R}$

Now $R = \frac{\rho l}{A}$

If wire is cut in two equal half

$$R' = \frac{R}{2}$$

Initial $P_1 = \frac{V_0^2}{R}$

After $P_2 = \frac{V_0^2}{R'} = 2 \times \frac{V_0^2}{R} = 4$

$$\frac{P_2}{P_1} = 4 = \frac{\sqrt{x}}{1}$$

$$x = 16$$

٥٣. At a given point of time the value of displacement of a simple harmonic oscillator is given as $y = A \cos(\omega t)$. If amplitude is ٤٠ cm and kinetic energy at that time is ٢٠٠ J, the value of force constant is ١.٠ × ١٠^x Nm^{-1}. The value of x is _____.

Official Ans. by NTA (٤)

Sol. General equation for displacement is given by

$$x = A \sin(\omega t + \phi)$$

at given time

$$\omega t + \phi = 300$$

$$x = 40 \times \frac{\sqrt{3}}{2} = 20\sqrt{3}\ cm$$

$$A = 40cm$$

$$K.E = \frac{1}{2} k \times A^2 - x^2 = 200$$

$$200 = \frac{1}{2} k \times \frac{1600 \times 1200}{100 \times 100}$$

$$400 \times 100 \times 100 = k \times 400$$

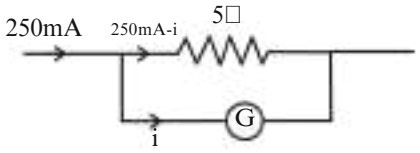
$$k = 10^4$$

$$x = 4$$

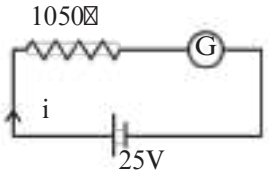
٥٤. When a resistance of ٥ Ω is shunted with a moving coil galvanometer, it shows a full scale deflection for a current of ٢٥٠ mA, however when ١٠٥٠ Ω resistance is connected with it in series, it gives full scale deflection for ٢٥ volt. The resistance of galvanometer is _____ Ω.

Official Ans. by NTA (٥٠)

Sol.



$$\frac{250mA \times 5}{5 + R_G} = i \qquad \ldots\ldots\ldots (i)$$



$$i = \frac{25}{1050 + R_G} \qquad \ldots\ldots\ldots (ii)$$

From (i) and (ii)

$$\frac{25}{1050 + R_G} = \frac{5}{45 + R_G}$$

$$100 + 5R_G = 1050 + 5R_G$$

$$95R_G = 4750$$

$$R_G = 50\Omega$$

٥٥. The radius of 9^{th} orbit of He^+ of Bohr's model is r_1 and that of fourth orbit of Be^{3+} is represented as r_2.

Now the ratio $\frac{r_2}{r_1}$ is x : ١. The value of x is

_____

Official Ans. by NTA (٢)

Sol. $r = \frac{n^2}{Z}$

$$\frac{r_{He^+}}{r_{Be^{3+}}} = \frac{2^2}{2 \times 4} = \frac{1}{2}$$

٥٦. A thin infinite sheet charge and an infinite line charge of respective charge densities σ and λ are placed parallel at ٥ m distance from each other.

Points 'P' and 'Q' are at $\frac{3}{\sqrt{2}}$ m and $\frac{4}{\sqrt{2}}$ m perpendicular distance from line charge towards sheet charge, respectively. 'EP' and 'EQ' are the magnitudes of resultant electric field intensities at point 'P' and 'Q', respectively. If $\frac{E_P}{E_Q} = \frac{a}{6}$ for $\frac{\lambda}{\sigma} = 2\pi\varepsilon_0$. Then the value of a is _____.

Official Ans. by NTA (٦)

Sol. $E_A = \frac{\sigma}{2\varepsilon_0} + \frac{\lambda}{2\pi\varepsilon_0 r_A} = \frac{3}{2\varepsilon_0}$

$$= \frac{1}{2\varepsilon_0}\left(\frac{\lambda}{\pi} + \sigma\right)$$

$$E_B = \frac{\sigma}{2\varepsilon_0} + \frac{\lambda}{2\pi\varepsilon_0 r_B} = \frac{4}{2\varepsilon_0}$$

$$= \frac{1}{2\varepsilon_0}\left(\frac{\lambda}{\pi} + \sigma\right)$$

$$\frac{E_A}{E_B} = \frac{3}{4} = \frac{\frac{\lambda}{\pi} + \sigma}{\frac{\lambda}{\pi} + \sigma}$$

$$= \frac{2\frac{\lambda}{\pi} + \sigma}{2\frac{\lambda}{\pi} + \sigma}$$

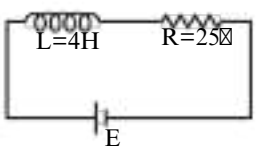
$$= \frac{4}{3}$$

$$= \frac{4}{6}$$

٥٧. In the given figure, an inductor and a resistor are connected in series with a battery of emf E volt.

$\frac{E^2}{2b}$ J/s represents the maximum rate at which the energy is stored in the magnetic field (inductor).

The numerical value of $\frac{b}{a}$ will be _____



Official Ans. by NTA (٢٥)

Sol. $E = \frac{1}{2}LI^2$

Rate of energy storing $= \frac{dE}{dt} = LI\frac{dI}{dt}$

Now we Know for $R-L$ circuit

$$I = \frac{E}{R}\left(1-e^{-\frac{Rt}{L}}\right)$$

$$\frac{dI}{dt} = \frac{E}{L}e^{-\frac{tR}{L}}$$

$$\frac{dE}{dt} = \frac{E^2}{R}\left(1-e^{-\frac{tR}{L}}\right)e^{-\frac{tR}{L}}$$

Time at which rate of power storing will be max,

$$t = \frac{L}{R\ln 2}$$

$$\frac{dE}{dt} = \frac{E^2}{R}\left(1-\frac{1}{2}\right)\frac{1}{2} = \frac{E^2}{4R} = \frac{E^2}{100} = \frac{E^2}{2\times 50}$$

$a = 2, b = 50$

$$\frac{b}{a} = 25$$

५८. A fish rising vertically upward with a uniform velocity of $८\ ms^{-१}$ observes that a bird is diving vertically downward towards the fish with the velocity of $१२\ ms^{-१}$. If the refractive index of water is $\frac{४}{३}$, then the actual velocity of the diving bird to pick the fish, will be _____ $ms^{-१}$.

Official Ans. by NTA (३)

Sol. $\frac{V_{b/f}}{\frac{4}{3}} = \frac{8}{\frac{4}{3}} = \frac{v_b - v_f}{1}$

$$\frac{12}{\frac{4}{3}} = \frac{8}{\frac{4}{3}} = \frac{v_b}{1}$$

$v = 3\ m/s$

५९. A solid sphere is rolling on a horizontal plane without slipping. If the ratio of angular momentum about axis of rotation of the sphere to the total energy of moving sphere is ७ : २२ then, the value of its angular speed will be _____ rad/s.

Official Ans. by NTA (४)

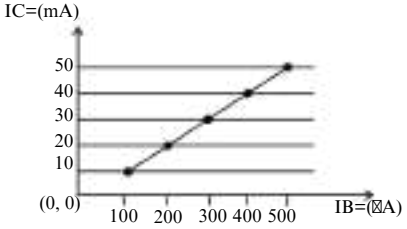
Sol. $L = I_{com}\omega$ and $K = \frac{1}{2}I_{com}\omega^2$

$$L = \frac{2}{5}MR^2\frac{V_{com}}{R} \qquad K = \frac{1}{2}\cdot\frac{2}{5}MR^2\cdot\frac{V_{com}^2}{R^2} = \frac{1}{2}MV_{com}^2$$

$$L = \frac{2MRV_{com}}{5} \qquad K = \frac{7}{10}MV_{com}^2$$

$$\text{Ratio } \frac{L}{K} = \frac{4}{7}\frac{R}{V_{com}} = \frac{4}{7}\cdot\frac{22}{22} = 4$$

६०. From the given transfer characteristic of a transistor in CE configuration, the value of power gain of this configuration is $x\times १०$, for $R_B = १०\ k\Omega$, and $R_C = १\ k\Omega$. The value of x is _____.



Official Ans. by NTA (३)

Sol. Power gain

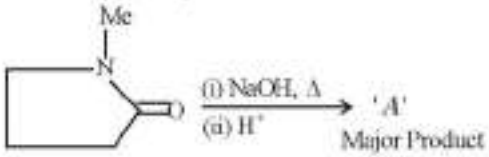
$$= A_V \cdot A_I = \frac{R_C}{R_B}\cdot\beta\cdot\beta = \frac{R_C}{R_B}\beta^2$$

$$= \frac{20\times 10^3}{10\times 3\times 10^3}\times 100\times 100 = \frac{200\times 100\times 10^6}{6\times 10^3}$$

Hence $x = 3$

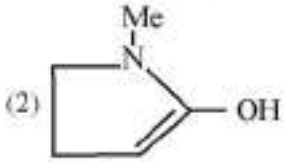
SECTION-A

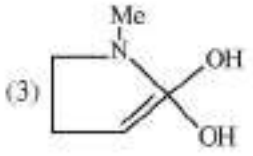
61. In the reaction given below



'A' is

(1) 

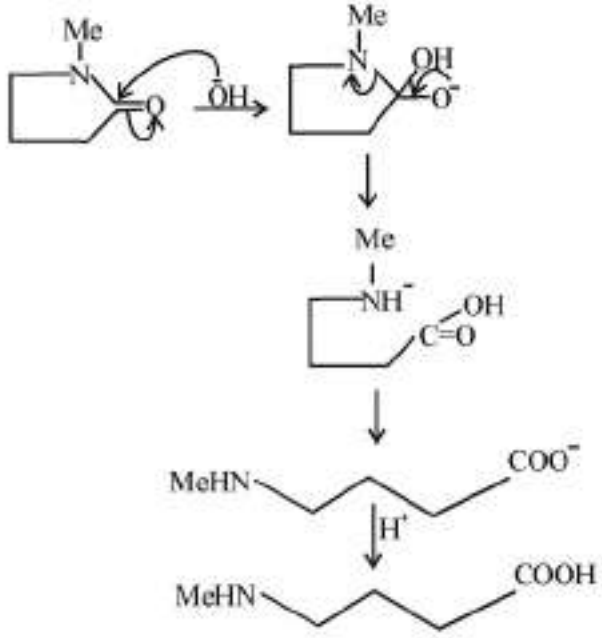
(2) 

(3) 

(4) 

Official Ans. by NTA (1)

Sol.



62. Given below are two statements:

Statement-I Permutit process is more efficient compared to the synthetic resin method for the softening of water.

Statement-II: Synthetic resin method results in the formation of soluble sodium salts.

In the light of the above statements, choose the most appropriate answer from the options given below:

(1) Both the Statements I and II are correct

(2) Statement I is correct but Statement II is incorrect

(3) Statement I is incorrect but Statement II is correct

(4) Both the Statements I and II are incorrect

Official Ans. by NTA (4)

Sol. Nowadays hard water is softened by using synthetic ion exchangers. This method is more efficient than zeolite process/Permutit process

63. The mismatched combinations are

A. Chlorophyll - Co

B. Water hardness - EDTA

C. Photography - $\left[Ag(CN)_2\right]^-$

D. Wilkinson catalyst - $\left[(Ph_3P)_3\,RhCl\right]$

E. Chelating ligand – D - Penicillamine

Choose the correct answer from the options given below:

(1) A and C Only

(2) A and E Only

(3) D and E Only

(4) A, C and E Only

Official Ans. by NTA (1)

Sol. Mg is present in chlorophyll and in black and white photography the developed film is fixed by washing with hypo solution which dissolves the undecomposed AgBr to form a complex ion $[Ag(S_2O_3)_2]^{3-}$

64. In which of the following processes, the bond order increases and paramagnetic character changes to diamagnetic one?

(1) $O_2 \rightarrow O_2^{2-}$

(2) $NO \rightarrow NO^+$

(3) $N_2 \rightarrow N_2^+$

(4) $O_2 \rightarrow O_2^+$

Official Ans. by NTA (2)

Sol. NO is paramagnetic with BO = 2.5, NO^+ is diamagnetic with BO = 3

65. The incorrect statement from the following for borazine is:

(1) It has electronic delocalization

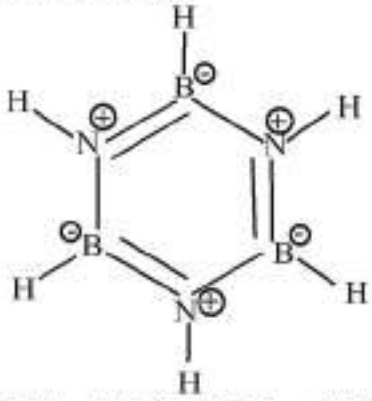
(2) It contains banana bonds.

(3) It can react with water.

(4) It is a cyclic compound.

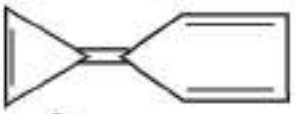
Official Ans. by NTA (3)

Sol. Borazine is $B_3N_3H_6$

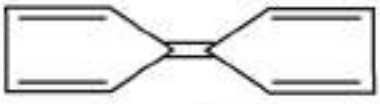


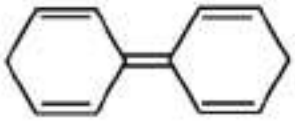
$$B_3N_3H_6 + 9H_2O \rightarrow 3NH_3 + 3H_3BO_3 + 3H_2$$

66. Among the following compounds, the one which shows highest dipole moment is

(1) 

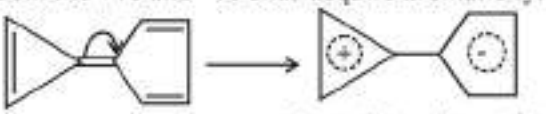
(2) 

(3) 

(4) 

Official Ans. by NTA (1)

Sol. Among the given compounds, the following compound has the highest dipole moment because both the +ve and –ve ends acquire aromaticity.



(Aromatic) (Aromatic)

67. Match the following

Column –A		Column-B	
a	Nylon 6	I	Natural Rubber
b	Vulcanized Rubber	II	Cross Linked
c	cis-1,4-polyisoprene	III	Caprolactam
d	Polychloroprene	IV	Neoprene

Choose the correct answer from option given below:

(1) a → IV, b-III, c → II, d → I

(2) a → III, b → IV, c → I, d → II

(3) a → II, b → III, c → IV, d → I

(4) a → III, b → II, c → I, d → IV

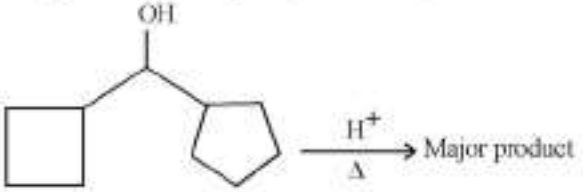
Official Ans. by NTA (4)

Sol. Nylon-6 – Caprolactum (Monomer)

Natural rubber – Isoprene (Monomer)

Vulcanized rubber – Sulphur containing rubber

Neoprene – Chloroprene (Monomer)

68. 

In the above reaction. Left hand side and right hand side rings are named as 'A' and 'B' respectively. They undergo ring expansion. The correct statement for this process is:

(1) Finally both rings will become six membered each.

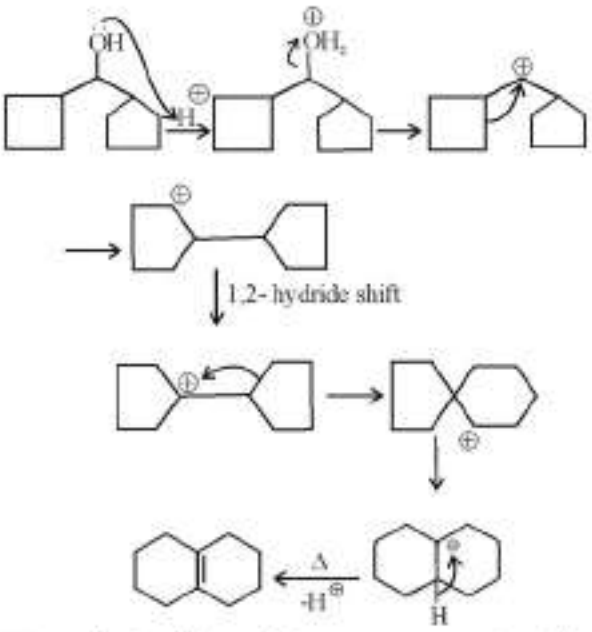
(2) Finally both rings will become five membered each.

(3) Only 'A' will become 6 membered.

(4) Ring expansion can go upto seven membered rings

Official Ans. by NTA (1)

Sol.



69. The radical which mainly causes ozone depletion in the presence of UV radiations is:

(1) $CH_3^{\bullet}$ (2) $NO^{\bullet}$

(3) $Cl^{\bullet}$ (4) $\dot{O}H$

Official Ans. by NTA (3)

Sol.

$$O_2(g) \xrightarrow{UV} O(g) + O(g)$$

$$O_2(g) + O(g) \longrightarrow O_3(g)$$

$$CF_2Cl_2(g) \xrightarrow{UV} \dot{C}l(g) + \dot{C}F_2Cl(g)$$

$$\dot{C}l(g) + O_3(g) \longrightarrow Cl\dot{O}(g) + O_2(g)$$

$$Cl\dot{O}(g) + O(g) \longrightarrow \dot{C}l(g) + O_2(g)$$

70. In the following reaction 'X' is

$$CH_3(CH_2)_4CH_3 \xrightarrow[HCl,\,\Delta]{Anhy.AlCl_3} \underset{\text{major product}}{X}$$

(1) $CH_3(CH_2)_4CH_2Cl$

(2) $Cl-CH_2-(CH_2)_4-CH_2-Cl$

(3) $CH_3\underset{\displaystyle CH_3}{\underset{|}{C}}H-(CH_2)_2CH_3$

(4) 

Official Ans. by NTA (3)

Sol. n-alkanes on heating in this presence of anhydrous $AlCl_3$ and hydrogen chloride gas isomerise to branched chain alkanes. The major product has one methyl side chain.

$$CH_3-(CH_2)_4-CH_3 \xrightarrow[HCl,\,\Delta]{Anhy.AlCl_3} CH_3-\underset{\displaystyle CH_3}{\underset{|}{C}}H-(CH_2)_2-CH_3$$

2 – methylpentane
(major)

71. 2-Methyl propyl bromide reacts with $C_2H_5O^-$ and gives 'A' whereas on reaction with C_2H_5OH it gives 'B'. The mechanism followed in these reactions and the products 'A' and 'B' respectively are:

(1) S_N2. A = iso-butyl ethyl ether; S_N1, B= tert-butyl ethyl ether

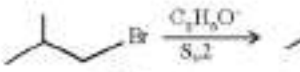
(2) S_N1, A = tert-butyl ethyl ether; S_N1, B = 2-butyl ethyl ether

(3) S_N1, A = tert-butyl ethyl ether; S_N2, B = iso-butyl ethyl ether

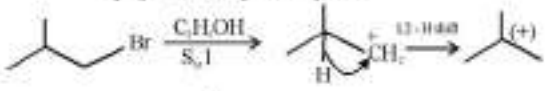
(4) S_N2, A =2-butyl ethyl ether; S_N2, B= iso-butyl ethyl ether

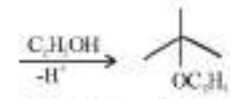
Official Ans. by NTA (1)

Sol.

(i) 

$C_2H_5O^-$ is strong nucleophile.

(ii) 



C_2H_5OH is weak nucleophile.

72. D- (+)- Glyceraldehyde $\xrightarrow[(ii)\ H_2O/H^+]{(i)\ HCN}$ $\xrightarrow{(iii)\ HNO_3}$

The products formed in the above reaction are

(1) Two optically active products

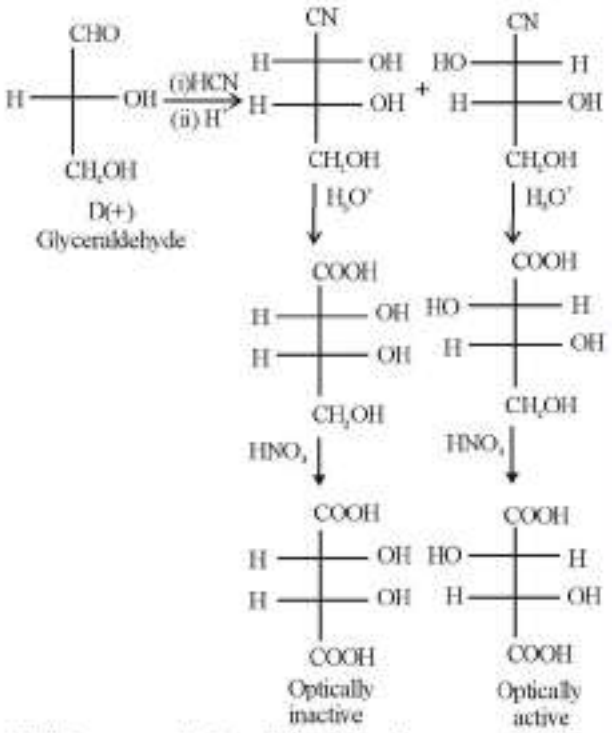
(2) One optically active and one meso product

(3) One optically inactive and one meso product.

(4) Two optically inactive products

Official Ans. by NTA (2)

Sol.



73. Which one of the following is most likely a mismatch?

(1) Zinc- Liquation

(2) Titanium – van Arkel method

(3) Nickel – Mond process

(4) Copper - Electrolysis

Official Ans. by NTA (1)

Sol. Zinc is refined by distillation method, which is used for metals having low boiling point.

74. ClF_5 at room temperature is a:

(1) Colourless gas with trigonal bipyramidal geometry.

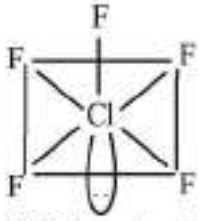
(2) Colourless gas with square pyramidal geometry

(3) Colourless liquid with square pyramidal geometry

(4) Colourless liquid with trigonal bipyramidal geometry.

Official Ans. by NTA (3)

Sol.



ClF_5 is colourless liquid.

75. $Be(OH)_2$ react with $Sr(OH)_2$ to yield an ionic salt. Choose the incorrect option related to this reaction from the following:

(1) Be is tetrahedrally coordinated in the ionic salt.

(2) The reaction is an example of acid - base neutralization reaction.

(3) Both Sr and Be elements are present in the ionic salt.

(4) The elements Be is present in the cationic part of the ionic salt.

Official Ans. by NTA (4)

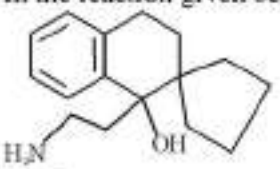
Sol. $Be(OH)_2$ is amphoteric in nature.

$Sr(OH)_2$ is basic in nature.

These two undergo acid – base reaction to form a salt.

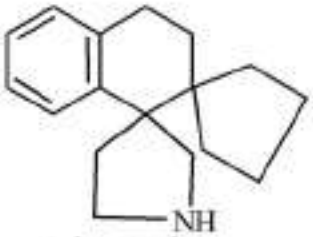
$$Be(OH)_2 + Sr(OH)_2 \rightarrow \underset{\text{(salt)}}{Sr[Be(OH)_4]}$$

76. In the reaction given below

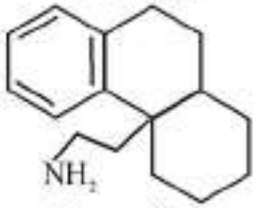


'B' is:

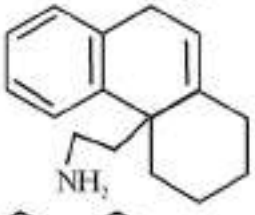
(1)



(2)



(3)



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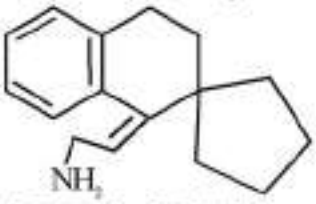
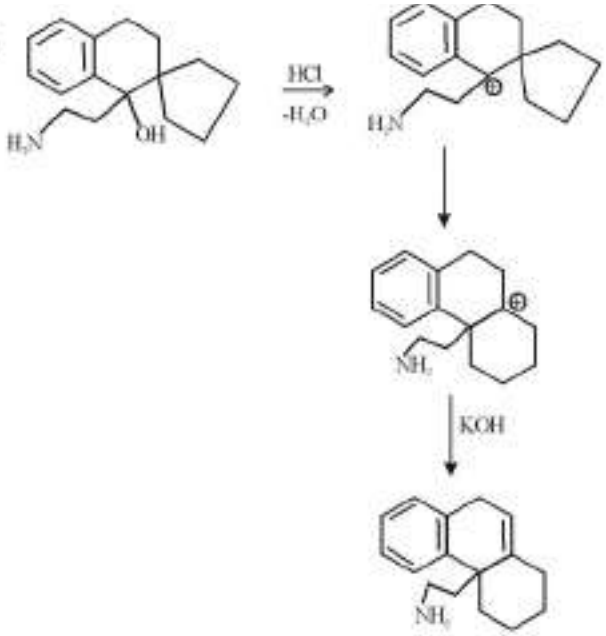


Official Ans. by NTA (3)

Sol.



77. Which of the following statements are **not** correct?

A. The electron gain enthalpy of F is more negative than that of Cl

B. Ionization enthalpy decreases in a group of periodic table

C. The electronegativity of an atom depends upon the atoms bonded to it.

D. Al_2O_3 and NO are examples of amphoteric oxides.

Choose the most appropriate answer from the options given below:

(1) A, B, C, and D

(2) A, C and D Only

(3) B and D Only

(4) A, B and D Only

Official Ans. by NTA (2)

Sol. Electronegativity of an element depends on the atom with which it is attached.

NO = neutral oxide

Al_2O_3 = amphoteric oxide

78. The energy of an electron in the first Bohr orbit of hydrogen atom is $-2.18\times10^{-18}J$. Its energy in the third Bohr orbit is_____.

(1) $\frac{1}{27}$ of this value

(2) One third of this value

(3) Three times of this value

(4) $\frac{1}{9}$ th of this value

Official Ans. by NTA (4)

Sol.

$$E_n = \frac{-2.18\times10^{-18}Z^2}{n^2}$$

$$\text{i.e. } E_n \propto \frac{1}{n^2}$$

79. What happens when a lyophilic sol is added to a lyophobic sol?

(1) Lyophilic sol is dispersed in lyophobic sol.

(2) Film of lyophobic sol is formed over lyophilic sol.

(3) Lyophobic sol is coagulated

(4) Film of lyophilic sol is formed over lyophobic sol.

Official Ans. by NTA (4)

Sol. Lyophilic sol is used as protective action for lyophobic sol. It forms a layer / film around the lyophobic sol.

80. The pair of lanthanides in which both elements have high third –ionization energy is:

(1) Eu, Gd

(2) Eu, Yb

(3) Lu, Yb

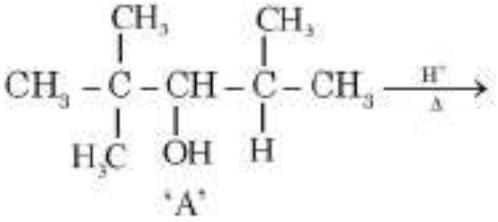
(4) Dy, Gd

Official Ans. by NTA (2)

Sol. $Eu^{+2}:[Xe]4f^7$, $Yb^{+2}:[Xe]4f^{14}$ } High IE due to half filled & fully filled configurations

SECTION-B

81. For the given reaction

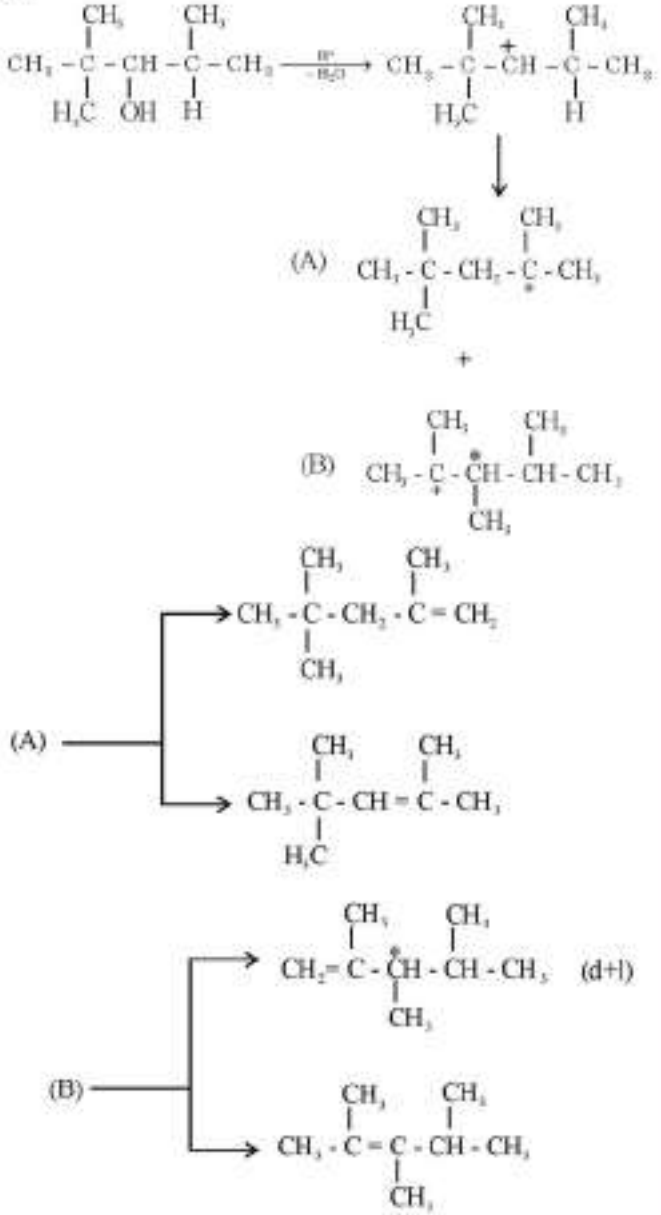


'A'

The total number of possible products formed by tertiary carbocation of A is _____.

Official Ans. by NTA (4)

Sol.



82. Solution of 12 g of non – electrolyte (A) prepared by dissolving it in 1000 mL of water exerts the same osmotic pressure as that of 0.05 M glucose solution at the same temperature. The empirical formula of A is CH_2O. The molecular mass of A is _____ g. (Nearest integer)

Official Ans. by NTA (240)

Sol. For Isotonic solutions

$\pi_1 = \pi_2$

$\Rightarrow C_1 = C_2$

$\frac{12}{x} = 0.05 \left[x \rightarrow \text{Molar Mass of A} \right]$

$X = 240$

83. 25.0 mL of 0.050 M $Ba(NO_3)_2$ is mixed with 25.0 mL of 0.020 M NaF. K_{sp} of BaF_2 is 0.5×10^{-6} at 298 K. The ratio of $\left[Ba^{2+}\right]\left[F^-\right]^2$ and K_{sp} is _____ (Nearest integer)

Official Ans. by NTA (5)

Sol. $\left[Ba^{+2}\right] = \frac{25 \times 0.05}{50} = 0.025M$

$\left[F^-\right] = \frac{25 \times 0.02}{50} = 0.01M$

$\left[Ba^{+2}\right]\left[F^-\right]^2 = 25 \times 10^{-7}$

$K_{sp} = 5 \times 10^{-7}$ (given)

$\text{Ratio} = \frac{\left[Ba^{+2}\right]\left[F^-\right]^2}{K_{sp}} = 5$

84. $A_2 + B_2 \rightarrow 2AB.\ \Delta H_f^0 = -200\ kJmol^{-1}$

AB, A_2 and B_2 are diatomic molecule. If the bond enthalpies of A_2, B_2 and AB are in the ratio 1:0.5:1, then the bond enthalpy of A_2 is _____ $kJmol^{-1}$ (Nearest integer)

Official Ans. by NTA (400)

Sol. $A_2 + B_2 \rightarrow 2AB\ ;\ \Delta H_f^0 = -200\ kJ\ mol^{-1}$

$\Rightarrow \Delta H_f^0(AB) = -200\ kJ\ mol^{-1}$

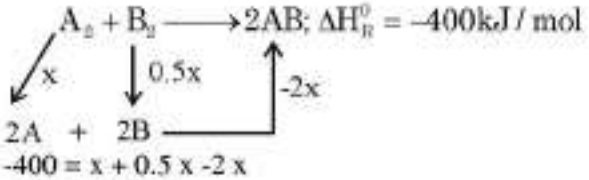
$\therefore \Delta H_R^0$ for reaction $A_2 + B_2 \rightarrow 2AB$ is $-400\ kJ\ mol^{-1}$

Given: Bond Enthalpy of A_2, B_2 and AB is $1:0.5:1$

Assuming bond enthalpy of A_2 be x kJ mol^{-1}

$\therefore$ Bond enthalpy $B_2 = 0.5$ x kJ mol^{-1}

$\therefore$ Bond enthalpy $AB = (x)$ kJ mol^{-1}



$-400 = x + 0.5\ x - 2\ x$

$-400 = -0.5\ x$

$\therefore x = 800$ kJ / mol

85. An organic compound gives 0.220 g of CO_2 and 0.126 g of H_2O on complete combustion. If the % of carbon is 24 then the % hydrogen is _____ $\times 10^{-1}$. (Nearest integer)

Official Ans. by NTA (56)

Sol. Moles of $CO_2 = \frac{0.22}{44} = \frac{1}{200}$

$\therefore$ Moles of carbon

$= (\text{Moles of } CO_2) \times 1$

$= \frac{1}{200}$

$\therefore$ wt. of C $= \frac{1}{200} \times 12 = 0.06$

% of C $= \frac{0.06}{W} \times 100 = 24$

(W = Wt. of Organic Compound)

W = 0.25

Moles of $H_2O = \frac{0.126}{18} = 0.007$

$\therefore$ Moles of H atom $= 2 \times 0.007 = 0.014$

% of Hydrogen $= \frac{0.014 \times 1}{W} \times 100$

$= \frac{0.014 \times 1}{0.25} \times 100$

$= 5.6$

$= 56 \times 10^{-1}$

86. 20 mL of calcium hydroxide was consumed when it was reacted with 10 mL of unknown solution of H_2SO_4. Also 20 mL standard solution of 0.5 M HCl containing 2 drops of phenolphthalein was titrated with calcium hydroxide the mixture showed pink colour when burette displayed the value of 35.5 mL whereas the burette showed 25.5 mL initially. The concentration of H_2SO_4 is _____ M (Nearest integer)

Official Ans. by NTA (1)

Sol. Reaction with HCl

$Ca(OH)_2 + 2HCl \rightarrow CaCl_2 + 2H_2O$

Volume of $Ca(OH)_2 = 10$ ml

Volume of HCl = 20 ml

Concentration of HCl = 0.5 M.

No. of milli moles of HCl = 10

No. of milli moles of $Ca(OH)_2 = 5$.

i.e. $M_{Ca(OH)_2} = \frac{\text{no. of milli moles}}{V(ml)} = \frac{5}{10}$

$= 0.5$ M.

Reaction with H_2SO_4

$Ca(OH)_2 + H_2SO_4 \rightarrow CaSO_4 + 2H_2O$.

No. of milli moles of $Ca(OH)_2 = 20 \times 0.5$

$= 10$

i.e. no. of milli moles of $H_2SO_4 = 10$

$\Rightarrow M_{H_2SO_4} = \frac{\text{no. of mil lim oles}}{V(ml)}$

$= \frac{10}{10}$

$= 1$ M

87. A certain quantity of real gas occupies a volume of 0.15 dm^3 at 100 atm and 500 K when its compressibility factor is 1.07. Its volume at 300 atm and 300K (When its compressibility factor is 1.4) is _____ $\times 10^{-4} dm^3$ (Nearest integer)

Official Ans. by NTA (392)

Sol. $z = \frac{PV}{nRT}$; $n = \frac{PV}{ZRT}$

$Z_1 = 1.07, P_1 = 100\text{atm}, V_1 = 0.15\text{L}, T_1 = 500\text{K}$

$Z_2 = 1.4, P_2 = 300\text{atm}, T_2 = 300\text{K}, V_2 = ?$

$\frac{P_1V_1}{Z_1RT_1} = \frac{P_2V_2}{Z_2RT_2} = n$

$V_2 = \frac{1.4}{1.07} \times .03$

$= 392 \times 10^{-4}\ dm^3$

88. $t_{87.5}$ is the time required for the reaction to undergo 87.5% completion and t_{50} is the time required for the reaction to undergo 50% completion. The relation between $t_{87.5}$ and t_{50} for a first order reaction is $t_{87.5} = x \times t_{50}$

The value of x is_____ (Nearest integer)

Official Ans. by NTA (3)

Sol. $A_t = A_0 \times \frac{12.5}{100} = \frac{A_0}{8}$ [87.5% complete]

$$A_0 \xrightarrow{t_{1/2}} \frac{A_0}{2} \xrightarrow{t_{1/2}} \frac{A_0}{4} \xrightarrow{t_{1/2}} \frac{A_0}{8}$$

$t_{87.5} = 3t_{1/2}$

$\therefore x = 3$

89. $KMnO_4$ is titrated with ferrous ammonium sulphate hexahydrate in presence of dilute H_2SO_4. Number of water molecules produced for 2 molecules of $KMnO_4$ is _____.

Official Ans. by NTA (68)

Sol. By balancing the redox reaction we get

$$10\left[FeSO_4.(NH_4)_2SO_4.6H_2O\right] + 2KMnO_4 + 8H_2SO_4 \downarrow$$

$$5Fe_2(SO_4)_3 + 2MnSO_4 + 10(NH_4)_2SO_4 + K_2SO_4 + 68H_2O$$

90. A metal surface of $100\,cm^2$ area has to be coated with nickel layer of thickness 0.001mm. A current of 2A was passed through a solution of $Ni(NO_3)_2$ for 'x' seconds to coat the desired layer. The value of x is _____ (Nearest integer)

(ρ_{Ni} (density of Nickel) is $10\,gmL^{-1}$, Molar mass of Nickel is $60\,gmol^{-1}$ $F = 96500\,C\,mol^{-1}$)

Official Ans. by NTA (161)

Sol. $W = z \times i \times t.$

$$\text{Density} \times \text{volume} = \frac{E \times i \times t}{96500}$$

$$10 \times 100 \times 0.0001 = \frac{\left(\frac{\text{atomic wt.}}{\text{v.f}}\right) \times 2 \times x}{96500} \quad (\text{v.f} = 2)$$

$\therefore x = 161$ sec.