FINAL JEE–MAIN EXAMINATION – JANUARY, 2024

(Held On Wednesday 31 January, 2024) TIME : 3 : 00 PM to 6 : 00 PM

M ATHEM ATICS

TEST PAPER WITH SOLUTION

SECTION-A

१. The number of ways in which २१ identical apples can be distributed among three children such that each child gets at least २ apples, is

(१) ४०६

(२) १३०

(३) १४२

(४) १३६

Ans. (४)

Sol. After giving २ apples to each child १५ apples left now १५ apples can be distributed in $^{15+3-1}C_{3-1}$ ways

$\frac{17 \times 16}{2} = 136$

२. Let A (a, b), B(३, ४) and (−६, −८) respectively denote the centroid, circumcentre and orthocentre of a triangle. Then, the distance of the point P(२a + ३, ७b + ५) from the line २x + ३y − ४ = ० measured parallel to the line x − २y − १ = ० is

(१) $\frac{15\sqrt{5}}{7}$

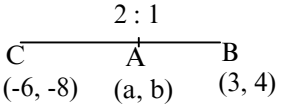
(२) $\frac{17\sqrt{5}}{6}$

(३) $\frac{17\sqrt{5}}{7}$

(४) $\frac{\sqrt{5}}{17}$

Ans. (३)

Sol. A(a, b), B(३, ४), C(−६, −८)



$a = 0, b = 0 \Rightarrow P(3,5)$

Distance from P measured along x − २y − १ = ०

$x = 3 + r\cos\theta, y = 5 + r\sin\theta$

Where $\tan\theta = \frac{1}{2}$

$r = 2\cos\theta + 3\sin\theta \Rightarrow r = \sqrt{17}\cos(\theta - \alpha)$

$\Rightarrow r = \pm\sqrt{17}\frac{5}{7} \Rightarrow r = \pm\frac{17}{7}\sqrt{5}$

३. Let z_1 and z_2 be two complex number such that $z_1 + z_2 = 5$ and $z_1^3 z_2^3 + z_1^2 z_2^2 = 20 + 15i$. Then $z_1^4 + z_2^4$ equals-

(१) ३० 3

(२) ७५

(३) १५ 15

(४) २५ 3

Ans. (२)

Sol. $z_1 + z_2 = 5$

$z_1^3 z_2^3 + z_1^2 z_2^2 = 20 + 15i$

$z_1^3 z_2^3 = (z_1 z_2)^2 + (z_1 z_2)^3 - 3z_1 z_2(z_1 + z_2)$

$z_1^3 z_2^3 = 125 - 3z_1 z_2 \cdot 5$

$20 + 15i = 125 - 15z_1 z_2$

$3z_1 z_2 = 105 - 20 - 15i$

$z_1 z_2 = 21 - 3i$

$z_1 + z_2 = 7 + i$

$z_1^2 + z_2^2 = 25$

$(z_1 + z_2)^2 = 25 + 2(7 + i)$

$z_1^2 + z_2^2 = 39 + 14i$

$z_1^4 + z_2^4 = (39 + 14i)^2 - 2(21 - 3i)^2$

$z_1^4 + z_2^4 = 1521 + 392i - 882 + 126i$

$z_1^4 + z_2^4 = 639 + 518i$

٤. Let a variable line passing through the centre of the circle $x^2 + y^2 - 16x - 4y = 0$, meet the positive co-ordinate axes at the point A and B. Then the minimum value of OA + OB, where O is the origin, is equal to

(١) ١٢

(٢) ١٨

(٣) ٢٠

(٤) ٢٤

Ans. (٢)

Sol.- $y = mx + \frac{2}{m} - 8$

x-intercept

$= \frac{2}{m} - 8$

y-intercept

$= 2 - 8m$

$OA + OB = \frac{2}{m} - 8m + 2$

$f'(m) = \frac{2}{m^2} - 8 = 0$

$m^2 = \frac{1}{4}$

$m = \frac{1}{2}$

$f\left(\frac{1}{2}\right) = 18$

Minimum = ١٨

٥. Let $f, g : (0, \infty) \to R$ be two functions defined by

$f(x) = \int_{\frac{1}{x}}^{x} \frac{e^{-t}}{t^2} dt$ and $g(x) = \int_{0}^{x^2} \frac{e^{-t}}{t^{1/2}} dt$.

Then the value of $f(\log_e 9) + g(\log_e 9)$ is

equal to

(١) ٦

(٢) ٩

(٣) ٨

(٤) ١٠

Ans. (٣)

Sol.-

$$f(x) = \int_{\frac{1}{x}}^{x} \frac{e^{-t}}{t^2} dt$$

$$f'(x) = 2x \cdot \frac{e^{-x^2}}{x^2} - \frac{e^{-x}}{\frac{1}{x^2}} \cdot \left(-\frac{1}{x^2}\right)$$

$$f'(x) = 2e^{-x^2} - e^{-x} \quad \ldots\ldots\ldots (1)$$

$$g(x) = \int_{0}^{x^2} \frac{e^{-t}}{t^{1/2}} dt$$

$$g'(x) = \frac{e^{-x^2}}{x} \cdot 2x = 2e^{-x^2}$$

$$f'(x) + g'(x) = 2xe^{-x^2} + 2x2e^{-x^2} + 2x2e^{-x^2}$$

Integrating both sides w.r.t.x

$$f(x) + g(x) = \int 2xe^{-x^2} dx$$

$x^2 = t$

$$= \int e^{-t} dt = -e^{-t} + 0$$

$$= -e^{-\log_e 9} = -\frac{1}{9}$$

$$f(x) + g(x) = -1 + \frac{8}{9}$$

٦. Let (α, β, γ) be mirror image of the point $(1, 2, 3)$ in the line $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$.

Then $\alpha + \beta + \gamma$ is equal to

(١) ٣٢

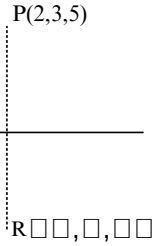
(٢) ٣٣

(٣) ٣١

(٤) ٣٤

Ans. (٢)

Sol.



$\because \vec{PR} \perp (2, 3, 4)$

$\vec{PR} \cdot (2, 3, 4) = 0$

$2(\alpha - 2) + 3(\beta - 3) + 4(\gamma - 5) = 0$

$2\alpha + 3\beta + 4\gamma = 49 - 20 = 33$

٧. Let P be a parabola with vertex (٢، ٣) and directrix $2x + y = 6$. Let an ellipse $E: \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1, a > b$ of eccentricity $\frac{1}{\sqrt{2}}$ pass through the focus of the parabola P. Then the square of the length of the latus rectum of E، is

(١) $\frac{385}{8}$

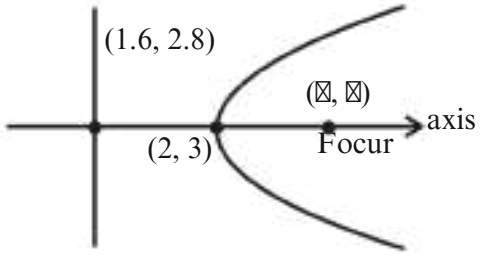
(٢) $\frac{347}{8}$

(٣) $\frac{512}{25}$

(٤) Ans. $\frac{656}{25}$

(٤)

Sol.-



Slope of axis $= \frac{1}{2}$

$y - 3 = \frac{1}{2}(x - 2)$

$2y - 6 = x - 2$

$2y - x = 4$

$2x - y + 6 = 0$

$4x - 2y + 12 = 0$

$d = 1.6 + 4 = 2.4$

$y = 2.8 + 6 = 3.2$

Ellipse passes through (٢.٤، ٣.٢)

$\frac{(2.4)^2}{a^2} + \frac{(3.2)^2}{b^2} = 1$ ... ...(١)

Also $e = \sqrt{1 - \frac{b^2}{a^2}} = \frac{1}{2} = \frac{b}{a} = \frac{1}{2}$

$a^2 = 2b^2$

Put in (١) $b^2 = \frac{328}{25}$

$\left(\frac{2b}{a}\right)^2 = \frac{4b^2}{a^2} = b^2 = 4 \times \frac{1}{2} \times \frac{328}{25} = \frac{656}{25}$

٨. The temperature T(t) of a body at time t = ٠ is ٩٦٠ F and it decreases continuously as per the differential equation $\frac{dT}{dt} = -K(T - 80)$، where K is positive constant. If T(١٥) = ١٢٠°F، then T(٤٥) is equal to

(١) ٨٥F°

(٢) ٩٥F°

(٣) ٩٠F°

(٤) ٨٠F°

Ans. (٣)

Sol.-

$\frac{dT}{dt} = -k(T - 80)$

$\int_{160}^{T} \frac{dT}{T - 80} = \int_0^t -k\,dt$

$\ln|T - 80| - \ln|160 - 80| = -kt$

$\ln|T - 80| - \ln 80 = -kt$

$\ln\left|\frac{T - 80}{80}\right| = -kt$

$T - 80 = 80e^{-kt}$

$120 - 80 = 80e^{-k.15}$

$\frac{40}{80} = e^{-k15} = \frac{1}{2}$

$T - 45 = 80 - 80e^{-k.45}$

$= 80 - 80e^{-k.15} \times e^{-k.30}$

$= 80 - 80 \times \frac{1}{8}$

$= 90$

٩. Let 2^{nd}, 8^{th} and 44^{th} terms of a non-constant A.P. be respectively the 1^{st}, 2^{nd} and 3^{rd} terms of G.P. If the first term of A.P. is 1 then the sum of first 20 terms is equal to-

(١) ٩٨٠ (٢) ٩٦٠

(٣) ٩٩٠ (٤) ٩٧٠

Ans. (٤)

Sol. - $1+d, 1+7d, 1+43d$ are in GP

$(1+7d)^2 = (1+d)(1+43d)$

$1+49d^2+14d = 1+44d+43d^2$

$6d^2-30d = 0$

$d = 5$

$$S_{20} = \frac{20}{2}[2(1)+(20-1)5]$$

$$= 10[2+95]$$

$$= 970$$

١٠. Let $f: R \to (0,\infty)$ be strictly increasing function such that $\lim_{x\to\infty}\frac{f(7x)}{f(x)} = 1$. Then, the value of $\lim_{x\to\infty}\left(\frac{f(5x)}{f(x)}\right)$ is equal to

(١) ٤

(٢) ٠

(٣) ٧/٥

(٤) ١

Ans. (٢)

Sol.- $f: R \to (0,\infty)$

$$\lim_{x\to\infty}\frac{f(7x)}{f(x)} = 1$$

$\because$ f is increasing

$$f(x) < f(5x) < f(7x)$$

$$\because \frac{f(x)}{f(x)} < \frac{f(5x)}{f(x)} < \frac{f(7x)}{f(x)}$$

$$1 < \lim\frac{f(5x)}{f(x)} < 1$$

$$\lim\frac{f(5x)}{f(x)} = 1$$

$$1 \times 1 = 0$$

١١. The area of the region enclosed by the parabola $y = 4x - x^2$ and $2y = (x-4)^2$ is equal to

(١) $\frac{32}{9}$

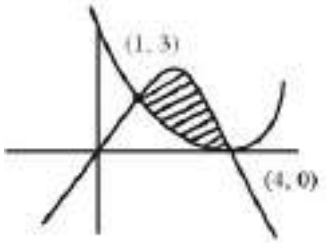
(٢) ٤

(٣) ٦

(٤) $\frac{14}{3}$

Ans. (٣)

Sol. -



$$\text{Area} = \left|\int_1^4 \left(4x - x^2 - \frac{(x-4)^2}{3}\right)dx\right|$$

$$\text{Area} = \left|\frac{4x^2}{2} - \frac{x^3}{3} - \frac{(x-4)^3}{9}\right|_1^4$$

$$\left|\left(\frac{64}{2} - \frac{64}{3} - \frac{4}{2} - \frac{1}{3} - \frac{27}{9}\right)\right|$$

$$= 27 - 21 = 6$$

١٢. Let the mean and the variance of 6 observation a, b, 68, 44, 48, 60 be 55 and 194, respectively if $a < b$, then $a + 3b$ is

(١) ٢٠٠

(٢) ١٩٠

(٣) ١٨٠

(٤) ٢١٠

Ans. (٣)

Sol. - a, b, 68, 44, 48, 60

Mean = 55 $a < b$

Variance = 194 $a + 3b$

$$\frac{a+b+68+44+48+60}{6} = 55$$

$$220 + a + b = 330$$

$$a + b = 110 \ldots\ldots(1)$$

Also,

$$\sum \frac{(x_i - \bar{x})^2}{n} = 194$$

$$\sum a_i - 55 = 0 \quad \sum b_i - 55 = 0 \quad \sum (a_i - 68 - 55)^2 + \sum (b_i - 44 - 55)^2$$

$$= 48 \times 55 + 60 \times 55 = 194 \times 6$$

$$\sum a_i - 55 = 0 \quad \sum b_i - 55 = 0 \quad \sum (a_i - 55)^2 + \sum (b_i - 55)^2 = 169 + 121 + 49 + 25 = 1164$$

$$\sum a_i - 55 = 0 \quad \sum b_i - 55 = 0 \quad \sum (a_i - 55)^2 + \sum (b_i - 55)^2 = 1164 + 364 + 800$$

$a^2 + 3025 - 110a + b2 + 3025 - 110b = 800$

$a2 + b2 + 6050 + 12100 = 800$

$a2 + b2 = 6850$.....(2)

Solve (1) & (2);

$a=75, b=35$

$a + 3b = 75 + 3 \times 35 = 75 + 105 = 180$

१३. If the function $f: (-\infty, -1] \to (a, b)$ defined by $f(x) = e^{x^3 - 3x + 1}$ is one-one and onto, then the distance of the point $P(2b + 4, a + 2)$ from the line $x + e^{-3}y = 4$ is :

(१) $2\sqrt{1 + e^6}$ (2) $4\sqrt{1 + e^6}$

(३) $3\sqrt{1 + e^6}$ (४) $\sqrt{1 + e^6}$

Ans. (१)

Sol.- $f(x) = e^{x^3 - 3x + 1}$

$f'(x) = e^{x^3 - 3x + 1} \cdot (3x^2 - 3)$

$= e^{x^3 - 3x + 1} \cdot 3(x - 1)(x + 1)$

For $f'(x) < 0$

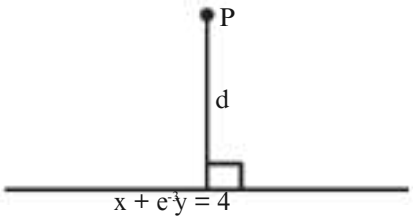
$f(x)$ is increasing function

$a < e < -1 \Rightarrow 0 < f(e) < f(-1)$

$b = e^{-1 - 3 + 1} < e^{3 - f} < e^{1 + 3 + 1}$

$P(2b + 4, a + 2)$

$P(2e^3 + 4, 2)$



$$d = \frac{|2e^3 + 4 - 2e^3 - 4|}{\sqrt{1 + e^{-6}}} = 2\sqrt{1 + e^6}$$

१४. Consider the function $f: (0, \infty) \to R$ defined by $f(x) = e^{-|\log_e x|}$. If m and n be respectively the number of points at which f is not continuous and f is not differentiable, then $m + n$ is

(१) ०

(२) ३

(३) १

(४) २

Ans. (३)

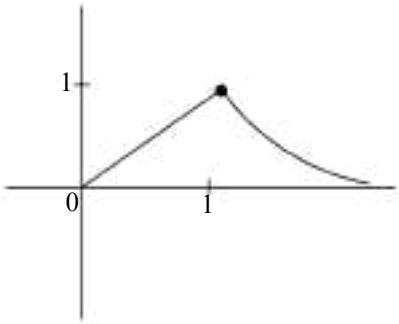
Sol. -

$f: (0, \infty) \to R$

$f(x) = e^{-|\log_e x|}$

$$f'(x) = \frac{1}{e^{|\ln x|}} \begin{cases} \frac{1}{e^{-\ln x}}; & 0 < x < 1 \\ \frac{1}{e^{\ln x}}; & x > 1 \end{cases}$$

$$\begin{cases} 1, & x; 0 < x < 1 \\ x, & \\ \frac{1}{x}, & x > 1 \end{cases}$$



$m = ०$ (No point at which function is not continuous)

$n = १$ (Not differentiable)

$m + n = १$

१५. The number of solutions, of the equation $e^{\sin x} - 2e^{-\sin x} = 2$ is

(१) २ (२) more than २ (३) १ (४) ०

Ans. (४)

Sol.- Take $e^{\sin x} = t$, $t > 0$

$$t = \frac{2}{t}$$

$$\frac{t^2 - 2}{t} = 2$$

$$t^2 - 2t - 2 = 0$$

$$t^2 - 2t + 1 = 3$$

$$(t-1)^2 = 3$$

$$t - 1 = \sqrt{3}$$

$$t = 1 + 1.73$$

$t = ٢.٧٣$ or -٠.٧٣ (rejected as $t > ٠$)

$$e^{\sin x} = 2.73$$

$$\log_e e^{\sin x} = \log_e 2.73$$

$$\sin x = \log_e 2.73 < 1$$

So no solution.

١٦. If $a = \sin^{-1}(\sin ٥)$ and $b = \cos^{-1}(\cos ٥)$ then $a^2 + b^2$ is equal to

(١) ٤٠٢٥

(2) $8 + 2 \times 40 = 50$

(3) $4 + 2 \times 20 = 50$

(٤) ٢٥

Ans. (٢)

Sol. $a = \sin^{-1}(\sin 5) = 5 - 2\pi$

and $b = \cos^{-1}(\cos 5) = 2\pi - 5$

$a^2 + b^2 = (5 - 2\pi)^2 + (2\pi - 5)^2$

$= 8 + 2 \times 40 = 50$

١٧. If for some m, n, $^{6}C_{m} + 2 \cdot {}^{6}C_{m-1} + {}^{6}C_{m-2} = {}^{8}C_{3}$ and $^{n+1}P_3 : {}^{n}P_4 = 1:8$, then ${}^{n}P_{m+1} \cdot {}^{n+1}C_{m}$ is equal to

(١) ٣٨٠

(٢) ٣٧٦

(٣) ٣٨٤

(٤) ٣٧٢

Ans. (٤)

Sol.- ${}^{6}C_{m} + 2 \cdot {}^{6}C_{m-1} + {}^{6}C_{m-2} = {}^{8}C_{3}$

${}^{7}C_{m} + {}^{7}C_{m-1} = {}^{8}C_{3}$

${}^{8}C_{m} = {}^{8}C_{3}$

$m = 2$

And $^{n+1}P_3 : {}^{n}P_4 = 1:8$

$$\frac{(n+1)(n)(n-1)}{n(n-1)(n-2)(n-3)} = \frac{1}{8}$$

$n = 8$

$${}^{n}P_{m+1} \cdot {}^{n+1}C_{m} = {}^{8}P_{3} \cdot {}^{9}C_{2}$$

$$= 8 \times 7 \times 6 \times \frac{9 \times 8}{2}$$

$= ٣٧٢$

١٨. A coin is based so that a head is twice as likely to occur as a tail. If the coin is tossed ٣ times, then the probability of getting two tails and one head is-

(١) $\frac{2}{9}$

(٢) $\frac{1}{9}$

(٣) $\frac{2}{27}$

(٤) $\frac{1}{27}$

Ans. (١)

Sol. Let probability of tail is $\frac{1}{3}$

$\therefore$ Probability of getting head $= \frac{2}{3}$

$\therefore$ Probability of getting ٢ tails and ١ head

$$= \left(\frac{1}{3}\right)^2 \left(\frac{2}{3}\right)^1 \left(\frac{1}{3}\right)^0 \times 3$$

$$= \frac{2}{27} \times 3$$

$$= \frac{2}{9}$$

١٩. Let A be a 3×3 real matrix such that

$$A\begin{bmatrix}1\\0\\1\end{bmatrix}=\begin{bmatrix}1\\2\\1\end{bmatrix}, A\begin{bmatrix}1\\1\\0\end{bmatrix}=\begin{bmatrix}0\\4\\0\end{bmatrix}, A\begin{bmatrix}1\\1\\1\end{bmatrix}=\begin{bmatrix}0\\2\\1\end{bmatrix}.$$

Then, the system $A\begin{bmatrix}x\\y\\z\end{bmatrix}=\begin{bmatrix}1\\2\\3\end{bmatrix}$ has

(١) unique solution

(٢) exactly two solutions

(٣) no solution

(٤) infinitely many solutions

Ans. (١)

Sol. – Let $A=\begin{bmatrix}x_1 & y & z_1\\ x_2 & 1 & z_2\\ x_3 & y & z_3\end{bmatrix}$

Given $A\begin{bmatrix}1\\0\\1\end{bmatrix}=\begin{bmatrix}1\\2\\3\end{bmatrix}$ ... (١)

$$\begin{bmatrix}x_1+z_1\\ x_2+z_2\\ x_3+z_3\end{bmatrix}=\begin{bmatrix}2\\0\\2\end{bmatrix}$$

$x_1+z_1=2$ ... (٢)

$x_2+z_2=0$ ... (٣)

$x_3+z_3=0$ ... (٤)

Given $A\begin{bmatrix}1\\1\\0\end{bmatrix}=\begin{bmatrix}0\\4\\0\end{bmatrix}$

$$\begin{bmatrix}x_1+z_1\\ x_2+z_2\\ x_3+z_3\end{bmatrix}=\begin{bmatrix}4\\0\\4\end{bmatrix}$$

$x_1+z_1=4$ ... (٥)

$x_2+z_2=0$ ... (٦)

$x_3+z_3=4$

Given $A\begin{bmatrix}1\\1\\1\end{bmatrix}=\begin{bmatrix}0\\2\\1\end{bmatrix}$

$$\begin{bmatrix}y_1\\ y_2\\ y_3\end{bmatrix}=\begin{bmatrix}0\\2\\0\end{bmatrix}$$

$y_1=0, y_2=2, y_3=0$

from (٢), (٣), (٤), (٥), (٦) and (٧)

$x_1=3x, x_2=0, x_3=1$

$y=0, y_2=2, y_3=0$

$1=1, z_2=0, z_3=3$

$$z=\begin{bmatrix}3 & 0 & 1\\ 0 & 2 & 0\\ 1 & 0 & 3\end{bmatrix}$$

Now $A\begin{bmatrix}x\\y\\z\end{bmatrix}=\begin{bmatrix}1\\2\\3\end{bmatrix}$

$$\begin{bmatrix}3 & 0 & 1\\ 0 & 1 & 0\\ 1 & 0 & 3\end{bmatrix}\begin{bmatrix}x\\y\\z\end{bmatrix}=\begin{bmatrix}1\\2\\3\end{bmatrix}$$

$z=1, y=2, x=3$

٢٠. The shortest distance between lines L_1 and L_2, where $L_1: \frac{x-1}{2}=\frac{y-1}{-3}=\frac{z-4}{-2}$ and L_2 is the line passing through the points $A(٤, ٤, ٣)$ and B and perpendicular to the line $\frac{x-3}{-2}=\frac{y}{3}=\frac{z-1}{1}$, is

(١) $\frac{121}{\sqrt{221}}$ (٢) $\frac{24}{\sqrt{117}}$

(٣) $\frac{141}{\sqrt{221}}$ (٤) $\frac{42}{\sqrt{117}}$

Ans. (٣)

Sol.-

$$L_2 \equiv \frac{x-4}{3} = \frac{y-4}{2} = \frac{z-3}{0}$$

$$\text{S.D} = \frac{\begin{vmatrix} x_2-x_1 & y_2-y_1 & z_2-z_1 \\ 2 & -3 & 2 \\ 3 & 2 & 0 \end{vmatrix}}{|\vec{n}_1 \times \vec{n}_2|}$$

$$= \frac{\begin{vmatrix} 5 & -5 & 7 \\ 2 & -3 & 2 \\ 3 & 2 & 0 \end{vmatrix}}{|\vec{n}_1 \times \vec{n}_2|}$$

$$= \frac{141}{|-4\hat{i}-6\hat{j}+13\hat{k}|}$$

$$= \frac{141}{\sqrt{16+36+169}}$$

$$= \frac{141}{\sqrt{221}}$$

SECTION-B

21. $\left| \frac{120}{3} \int_0^{\pi} \frac{x^2 \sin x \cos x}{\sin^4 x + \cos^4 x} dx \right|$ is equal to _____.

Ans. (15)

Sol.- $\int_0^{\pi} \frac{x^2 \sin x \cdot \cos x}{\sin^4 x + \cos^4 x} dx$

$$= \int_0^{\frac{\pi}{2}} \frac{\sin x \cdot \cos x}{\sin^4 x + \cos^4 x} \{x^2 + (\pi - x)^2\} dx$$

$$= \int_0^{\frac{\pi}{2}} \frac{\sin x \cdot \cos x \cdot \pi^2}{\sin^4 x + \cos^4 x} dx$$

$$= 2\int_0^{\frac{\pi}{2}} \frac{x \sin x \cos x}{\sin^4 x + \cos^4 x} dx + \pi^2 \int_0^{\frac{\pi}{2}} \frac{\sin x \cos x}{\sin^4 x + \cos^4 x} dx$$

$$= 2 \cdot \frac{\pi}{4} \int_0^{\frac{\pi}{2}} \frac{\sin x \cos x}{\sin^4 x + \cos^4 x} dx + \pi^2 \int_0^{\frac{\pi}{2}} \frac{\sin x \cos x}{\sin^4 x + \cos^4 x} dx$$

$$= \frac{\pi}{2} \int_0^{\frac{\pi}{2}} \frac{\sin x \cos x}{\sin^4 x + \cos^4 x} dx + \pi^2 \int_0^{\frac{\pi}{2}} \frac{\sin x \cos x}{\sin^4 x + \cos^4 x} dx$$

$$= \frac{2\pi}{2} \int_0^{\frac{\pi}{2}} \frac{\sin 2x}{1 - \frac{1}{2}\sin^2 2x} dx$$

$$= \pi \int_0^{\frac{\pi}{2}} \frac{\sin 2x}{1 - \frac{1}{2}(1 - \cos^2 2x)} dx$$

$$= \pi \int_0^{\frac{\pi}{2}} \frac{\sin 2x}{\frac{1}{2} + \frac{1}{2}\cos^2 2x} dx$$

Let $\cos 2x = t$

22. Let a, b, c be the length of three sides of a triangle satisfying the condition $(a+b)x^2 - 2b(a+c)x + b(b+c) = 0$. If the set of all possible values of x is the interval (α, β), then $12\alpha\beta$ is equal to _____.

Ans. (36)

Sol.- $(a+b)^2 x^2 - 2b(a+c)x + b(b+c) = 0$

$$a^2x^2 + 2abx + b^2 + b^2x^2 + 2bcx + bc = 0$$

$$(ax+b)^2 + (bx+c)(x) = 0$$

$$ax + b = 0,\ bx + c = 0$$

a, b, c $\quad b + c > a \quad c + a > b$

$a + ax > bx$	$ax + bx > a$	$ax^2 + a > ax$
$a + ax > ax^2$	$ax + ax^2 > a$	$x^2 - x + 1 > 0$
$x^2 - x - 1 < 0$	$x^2 + x - 1 > 0$	always true

$$\frac{1-\sqrt{5}}{2} < x < \frac{1+\sqrt{5}}{2}$$

$$x \in \left(\frac{-1-\sqrt{5}}{2}, \text{ or } x \in \frac{-1+\sqrt{5}}{2}\right)$$

$\left(\frac{\sqrt{5}+1}{2}\right)x\left(\frac{\sqrt{5}+1}{2}\right)$

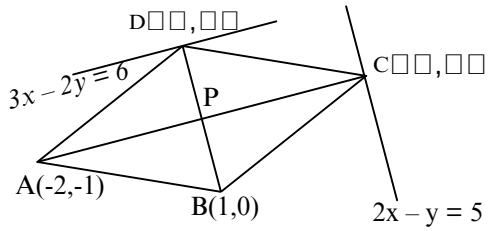
$\left(\frac{\sqrt{5}+1}{2}\right),\left(\frac{\sqrt{5}+1}{2}\right)$

$12\left(\frac{\left(\sqrt{5}+1\right)^2}{4}\right)=36$

२३. Let A(−२, −१), B(१, ०), C(□, □) and D(□, □) be the vertices of a parallelogram ABCD. If the point C lies on $2x-y=5$ and the point D lies on $3x-2y=6$, then the value of |□□□□□□□| is equal to _____.

Ans. (३२)

Sol.–



$P\left(\frac{\square+2}{2},\frac{\square+1}{2}\right)$ $\left(\frac{\square+1}{2},\frac{\square-2}{2}\right)$

$\frac{\square+2}{2}-\frac{\square+1}{2}$ and $\frac{\square+1}{2}-\frac{\square-2}{2}$

□ − □ = 3.....(1), □ − □ = 1.....(2)

Also, (□, □) lies on $3x-2y=6$

$3\square-2\square=6$.....(3)

and (□, □) lies on $2x-y=5$

$2\square-\square=5$.....(4)

Solving (१), (२), (३), (४)

□ = 3, □ = 1, □ = 6, □ = 2

|□□□□□□| = 32

२४. Let the coefficient of x in the expansion of

$(x+3)^{n+1}+(x+3)^{n+2}+(x+3)^{n+3}+\ldots+(x+3)^{2n+1}$

be $\square_r$. If $\sum_{r=0}^{n}\square_r=\square(n^3-n),\square,\square\in\mathbb{N}$, then the value of $\square^2+\square^{2n+1}$ equals _____.

Ans. (४२)

Sol.–

$(x+3)^{n+1}+(x+3)^{n+2}+(x+3)^{n+3}+\ldots+(x+3)^{2n+1}$

$=(x+2)^{n+1}+\ldots+(x+2)^{2n+1}$

$=\square^{n+1}+\square^{n+2}+\square^{n+3}+\ldots+\square^{2n+1}$

$=\square^{n+1}\left(\frac{\square^{n}-1}{\square-1}\right)$

$=(x+2)^{n+1}\left(\frac{(x+2)^{n}-1}{x+1}\right)$

$=\frac{(x+2)^{2n+1}-(x+2)^{n+1}}{x+1}$

$\square_r=\square(n^3-n)$

$\square=4,\ \square=3$

$\square^2+\square^2=16+9=25$

२५. Let A be a 3×3 matrix and det(A) = २. If

$n=\det(\underbrace{\text{adj}(\text{adj}(\ldots(\text{adj}A))\ldots)}_{2024\text{-times}})$

Then the remainder when n is divided by ९ is equal to _____.

Ans. (७)

Sol.– $|A|=2$

$\det(\underbrace{\text{adj}(\text{adj}(\text{adj}\ldots)\ldots)}_{2024\text{ times}}A)=|A|^{(3-1)^{2024}}$

$=|A|^{2^{2024}}$

$=2^{2^{2024}}$

$2^{2024} = 2^2 \cdot 2^{2022} = 4 \cdot 8^{674} = 4 \cdot 9 - 1^{674}$

$\equiv 2^{2024} \equiv 4 \mod 9$

$\equiv 9m + 4, m$

2^{2024} even

$2^{9m+4} = 16 \cdot 2^{3m} \equiv 16 \mod 9$

$\equiv 7$

२६. Let $\vec{a} = 3\hat{i} + 2\hat{j} + \hat{k}, \vec{b} = 2\hat{i} + \hat{j} + 3\hat{k}$ and $\vec{c}$ be a vector such that $\vec{a} \times \vec{c} = \vec{b} \times \vec{c}, \vec{c} \cdot (\vec{a} + \vec{b}) = 24$ and $|\vec{a} \times \vec{b}|^2 = 13$. Then $|\vec{c}|^2$ is equal to _____.

Ans. (३८)

Sol.- $\vec{a} \times \vec{b} = \vec{b} \times \vec{c} = 24\hat{j} + 6\hat{k}$

$\vec{a} + \vec{b} + \vec{c} = 5\hat{i} + \hat{j} + 4\hat{k}$

$\vec{c} \cdot (\vec{a} + \vec{b} + \vec{c}) = 24 \Rightarrow 7\hat{j} + 7\hat{k}$

$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 5 & 1 & 4 \\ x & y & z \end{vmatrix} = 14\hat{i} + 10\hat{j} + 20\hat{k}$

$\Rightarrow \hat{i}(z - 4y) + \hat{j}(5z - 4x) + \hat{k}(5y - x) = 14\hat{i} + 10\hat{j} + 20\hat{k}$

$z - 4y = 14, 4x - 5z = 10, 5y - x = 20$

$\vec{a} \cdot \vec{b} = \vec{b} \cdot \vec{c} = \vec{c} \cdot \vec{a} = 3$

$\vec{c} = -2\hat{i} - 3\hat{j} - 2\hat{k} + \lambda(\vec{a} \times \vec{b})$

$2x + 3y + 2z = 3$

$\Rightarrow x = 5, y = -3, z = 0$

$|\vec{c}|^2 = 25 + 9 + 4 = 38$

२७. If $\lim_{x \to 0} \frac{ax^2 e^{-x} - b\log_e(1+x) + cxe^{x^3}}{x^2 \sin x} = 1$,

then $11(a + b + c)$ is equal to _____.

Ans. (८१)

Sol.- $ax^2\left(1 - x + \frac{x^2}{2!} - \frac{x^3}{3!} + \dots\right) - b\left(x - \frac{x^2}{2} + \frac{x^3}{3} - \dots\right)$

$+ cx\left(1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots\right)$

$\lim_{x \to 0} \frac{}{x^3 \cdot \frac{\sin x}{x}}$

$= \lim_{x \to \infty} \frac{(c - b)x + \left(\frac{b}{2} + c + a\right)x^2 + \left(a - \frac{b}{3} + \frac{c}{2}\right)x^3 + \dots}{x^3} = 1$

$c - b = 0, \quad \frac{b}{2} + c + a = 0$

$a - \frac{b}{3} - \frac{c}{2} = 1 \quad a = \frac{3}{4} \quad b = c = -\frac{3}{2}$

$a^2 + b^2 + c^2 = \frac{9}{16} + \frac{9}{4} + \frac{9}{4}$

$16(a^2 + b^2 + c^2) = 81$

२८. A line passes through A(४, −६, −२) and B(१६, −२, ४). The point P(a, b, c) where a, b, c are non-negative integers, on the line AB lies at a distance of २१ units, from the point A. The distance between the points P(a, b, c) and Q(४, −१२, ३) is equal to _____.

Ans. (२२)

Sol.-

$\frac{x - 4}{12} = \frac{x - 6}{4} = \frac{z - 2}{6}$

$\frac{x - 4}{6} = \frac{y - 6}{2} = \frac{z - 2}{3} = 21$

$\left(21 \cdot \frac{6}{7} + 4, 21 \cdot \frac{2}{7} + 6, 21 \cdot \frac{3}{7} + 2\right)$

$(22, 0, 7) \equiv (a, b, c)$

$\sqrt{324 + 144 + 16} = 22$

२९. Let $y = y(x)$ be the solution of the differential equation

$\sec^2 x\,dx + e^{2y}\tan^2 x\,dy = 0, x \tan x \neq 0$

$0 < x < \frac{\pi}{2}, y\left(\frac{\pi}{4}\right) = 0$. If $y\left(\frac{\pi}{6}\right) = \alpha$,

Then $e^{8\alpha}$ is equal to _____.

Ans. (९)

Sol.-

$$\sec^2 x\frac{dx}{dy} + e^{2y}\tan^2 x + \tan x = 0$$

Put $\tan x = t \Rightarrow \sec^2 x \frac{dx}{dy} = \frac{dt}{dy}$

$$\frac{dt}{dy} + e^{2y}t^2 + t = 0$$

$$\frac{dt}{dy} + t = -t^2.e^{2y}$$

$$\frac{1}{t^2}\frac{dt}{dy} + \frac{1}{t} = -e^{2y}$$

Put $\frac{1}{t} = u \Rightarrow -\frac{1}{t^2}\frac{dt}{dy} = \frac{du}{dy}$

$$\frac{du}{dy} - u = e^{2y}$$

$$\frac{du}{dy} - u = e^{2y}$$

I.F. $= e^{\int -dy} = e^{-y}$

$$ue^{-y} = \int e^{y}e^{2y}\,dy$$

$$\frac{1}{\tan x}e^{-y} = e^{y} + c$$

$x = \frac{\pi}{4}, y = 0, c = 0$

$x = \frac{\pi}{6}, y = ?$

$$\sqrt{3}e^{?} = e^{?}$$

$$e^{2} = \sqrt{3}$$

$$e^{8} = 9$$

३०. Let $A = \{१, २, ३, \ldots\ldots\ldots १००\}$. Let R be a relation on A defined by $(x, y) \in R$ if and only if $२x = ३y$

Let $R_१$ be a symmetric relation on A such that $R \subset R_१$ and the number of elements in $R_१$ is n.

Then, the minimum value of n is _____

Ans. (६६)

Sol.-

$$R = \{(3,2), (6,4), (9,6), (12,8), \ldots\ldots\}$$

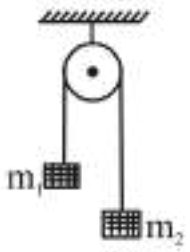
$n(R) = 33$

$= 66$

PHYSICS

SECTION-A

٣١. A light string passing over a smooth light fixed pulley connects two blocks of masses m_1 and m_2. If the acceleration of the system is g/٨, then the ratio of masses is



(١) $\frac{9}{7}$ (٢) $\frac{8}{1}$

(٣) $\frac{4}{3}$ (٤) $\frac{5}{3}$

Ans. (١)

Sol. $a = \frac{(m1-m2)g}{(m1+m2)} \cdot \frac{g}{8}$

$8m1+8m2=m1+m2$

$7m1=9m2$

$\frac{m1}{m2} = \frac{9}{7}$

٣٢. A uniform magnetic field of 2×10^{-3} T acts along positive Y-direction. A rectangular loop of sides ٢٠ cm and ١٠ cm with current of ٥ A is Y-Z plane. The current is in anticlockwise sense with reference to negative X axis. Magnitude and direction of the torque is :

(1) 2×10^{-4} N-m along positive Z-direction

(2) 2×10^{-4} N-m along negative Z-direction

(٣) 2×10^{-4} N-m along positive X-direction

(4) 2×10^{-4} N-m along positive Y-direction

Ans. (٢)

TEST PAPER WITH SOLUTION

Sol.

$\vec{M} = i\vec{A}$

$= 5(0.2)(0.1)\hat{i}$

$= 0.1\hat{i}$

$\vec{\tau} = \vec{M}\times\vec{B} = 0.1\hat{i}\times2\times10^{-3}\hat{k}$

$= 2\times10^{-4}\hat{k}\ N-m$

٣٣. The measured value of the length of a simple pendulum is ٢٠ cm with ٢ mm accuracy. The time for ٥٠ oscillations was measured to be ٤٠ seconds with ١ second resolution. From these measurements, the accuracy in the measurement of acceleration due to gravity is N%. The value of N is :

(١) ٤ (٢) ٨

(٣) ٦ (٤) ٥

Ans. (٣)

Sol. $T = 2\pi\sqrt{\frac{\ell}{g}}$

$g = \frac{4\pi^2\ell}{T^2}$

$\frac{\Delta g}{g} = \frac{\Delta\ell}{\ell} + \frac{2\Delta T}{T}$

$= \frac{0.2}{20} + 2\left(\frac{1}{40}\right)$

$= \frac{0.3}{20}$

Percentage change $= \frac{0.3}{20}\times100 = 6\%$

٣٤. Force between two point charges q_1 and q_2 placed in vacuum at 'r' cm apart is F. Force between them when placed in a medium having dielectric K = ٥ at 'r/٥' cm apart will be:

(١) F/٢٥ (٢) ٥F

(٣) F/٥ (٤) ٢٥F

Ans. (٢)

Sol. In air $F = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^2}$

In medium

$$F' = \frac{1}{4\pi K\varepsilon_0 r'^2} = \frac{25}{4\pi\varepsilon_0 r^2} = 25F$$

٣٥. An AC voltage $V = ٢٠\sin ٢٠٠t$ is applied to a series LCR circuit which drives a current $I = 10\sin\left(200t + \frac{\pi}{3}\right)$. The average power dissipated is:

(١) ٢١.٦ W (٢) ٢٠٠ W

(٣) ١٧٣.٢ W (٤) ٥٠ W

Ans. (٤)

Sol. $P = IV\cos\phi$

$$= \frac{20}{\sqrt{2}} \cdot \frac{10}{\sqrt{2}}\cos 60^o$$

$= 50$ W

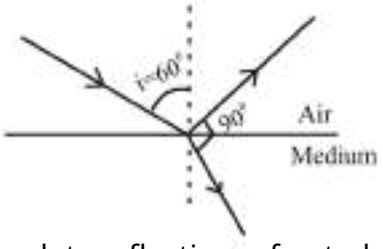
٣٦. When unpolarized light is incident at an angle of ٦٠° on a transparent medium from air. The reflected ray is completely polarized. The angle of refraction in the medium is

(١) ٣٠° (٢) ٦٠°

(٣) ٩٠° (٤) ٤٥°

Ans. (١)

Sol. By Brewster's law



At complete reflection refracted ray and reflected ray are perpendicular.

٣٧. The speed of sound in oxygen at S.T.P. will be approximately:

(Given, $R = 8.3\ JK^{-1}$, $\gamma = 1.4$)

(1) $310\ m/s$

(٢) ٣٣٣ m/s

(٣) ٣٤١ m/s

(٤) ٣٢٥ m/s

Ans. (١)

Sol. $v = \sqrt{\frac{\gamma RT}{M}} = \sqrt{\frac{1.4 \times 8.3 \times 273}{32 \times 10^{-3}}}$

$= 314.8541 \simeq 315 m/s$

٣٨. A gas mixture consists of ٨ moles of argon and ٦ moles of oxygen at temperature T. Neglecting all vibrational modes, the total internal energy of the system is

(١) ٢٩ RT

(٢) ٢٠ RT

(٣) ٢٧ RT

(٤) ٢١ RT

Ans. (٣)

Sol. $U = nCVT$

$U = n_1 CVT + n_2 CVT$

$= 8 \cdot \frac{3R}{2} \cdot T + 6 \cdot \frac{5R}{2} \cdot T$

$= ٢٧RT$

٣٩. The resistance per centimeter of a meter bridge wire is r, with X = resistance in left gap. Balancing length from left end is at ٤٠ cm with ٢٥ Ω resistance in right gap. Now the wire is replaced by another wire of ٢r resistance per centimeter. The new balancing length for same settings will be at

(١) ٢٠ cm

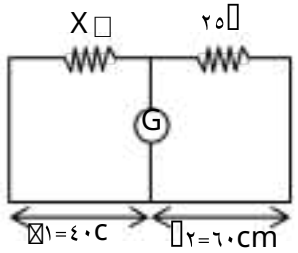
(٢) ١٠ cm

(٣) ٨٠ cm

(٤) ٤٠ cm

Ans. (٤)

Sol.



$$\frac{25}{r\ell_1} = \frac{X}{r\ell_2 X} \quad \ldots\ldots(i)$$

$$\frac{25}{2r'\ell_1} = \frac{}{2r'\ell_2} \quad \ldots\ldots(ii)$$

From (i) and (ii)

$$\ell'_2 = \ell_2 = 40cm$$

٤٠. Given below are two statements :
Statement I : Electromagnetic waves carry energy as they travel through space and this energy is equally shared by the electric and magnetic fields.
Statement II : When electromagnetic waves strike a surface, a pressure is exerted on the surface.
In the light of the above statements, choose the most appropriate answer from the options given below : (١) Statement I is incorrect but Statement II is correct (٢) Both Statement I and Statement II are correct. (٣) Both Statement I and Statement II are incorrect.
(٤) Statement I is correct but Statement II is incorrect

Ans. (٣)

Sol. $\frac{1}{2}\varepsilon E^2 = \frac{B^2}{2\mu_0}$

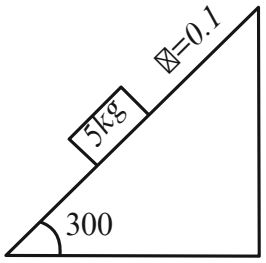
$\therefore E = CB \text{ and } C = \frac{1}{\sqrt{\mu_0\varepsilon_0}}$

٤١. In a photoelectric effect experiment a light of frequency ١.٥ times the threshold frequency is made to fall on the surface of photosensitive material. Now if the frequency is halved and intensity is doubled, the number of photo electrons emitted will be :
(١) Doubled (٢) Quadrupled
(٣) Zero (٤) Halved

Ans. (٣)

Sol. Since $\frac{f}{2} < f_0$
i.e. the incident frequency is less than threshold frequency. Hence there will be no emission of photoelectrons.
$\therefore$ current = 0

٤٢. A block of mass ٥ kg is placed on a rough inclined surface as shown in the figure.



If F_1 is the force required to just move the block up the inclined plane and F_2 is the force required to just prevent the block from sliding down, then the value of $|F_1 - F_2|$ is : (Use $g = 10m/s^2$)

(١) ٢٥٣N (٢) ٥٠٣N

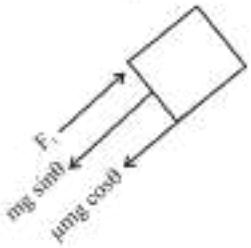
(٣) $\frac{5\sqrt{3}}{2}N$ (٤) ١٠ N

Ans. (—) BONUS

Sol. $f_K = \mu mg\cos\theta$

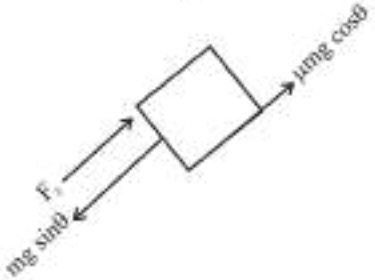
$= 0.1 \times \frac{50\sqrt{3}}{2}$

$= 2.5\sqrt{3}N$



$F_1 = mg\sin\theta - f_K$

$= 25 - 2.5\sqrt{3}$



$F_2 = mg\sin\theta + f_k$

$= 25 + 2.5\sqrt{3}$

$\therefore |F_1 - F_2| = 5\sqrt{3}N$

٤٣. By what percentage will the illumination of the lamp decrease if the current drops by ٢٠٪?

(١) ٤٦٪ (٢) ٢٦٪

(٣) ٣٦٪ (٤) ٥٦٪

Ans. (٣)

Sol. P∝i2R

$P_{int} = I^2_{int} R$

Pfinal = 0.8Pint × R

% change in power =

$\frac{P_{final} - P_{int}}{P_{int}} \times 100 = (0.64 - 1) \times 100 = -36\%$

٤٤. If two vectors $\vec{A}$ and $\vec{B}$ having equal magnitude R are inclined at an angle θ, then

(١) $|\vec{A} + \vec{B}| = \sqrt{2}\, R \sin\left(\frac{\theta}{2}\right)$

(٢) $|\vec{A} - \vec{B}| = 2R \sin\left(\frac{\theta}{2}\right)$

(٣) $|\vec{A} + \vec{B}| = 2R \cos\left(\frac{\theta}{2}\right)$

(٤) $|\vec{A} - \vec{B}| = 2R \cos\left(\frac{\theta}{2}\right)$

Ans. (٣)

Sol. The magnitude of resultant vector

$R' = \sqrt{a^2 + b^2 + 2ab\cos\theta}$

Here a = b = R

Then $R' = \sqrt{R^2 + R^2 + 2R^2\cos\theta}$

$= R\sqrt{2}\sqrt{1 + \cos\theta}$

$= \sqrt{2}R\sqrt{2\cos^2\frac{\theta}{2}}$

$= 2R\cos\frac{\theta}{2}$

٤٥. The mass number of nucleus having radius equal to half of the radius of nucleus with mass number ١٩٢ is:

(١) ٢٤ (٢) ٣٢

(٣) ٤٠ (٤) ٢٠

Ans. (١)

Sol. $R_1 = \frac{R_2}{2}$

$R_0 A_1^{1/3} = \frac{R_0}{2}(A_2)^{1/3}$

$A_1 = \frac{1}{8} A_2$

$A_1 = \frac{192}{8} = 24$

٤٦. The mass of the moon is ١/١٤٤ times the mass of a planet and its diameter ١/١٦ times the diameter of a planet. If the escape velocity on the planet is v, the escape velocity on the moon will be:

(١) $\frac{v}{3}$ (٢) $\frac{v}{4}$

(٣) $\frac{v}{12}$ (٤) $\frac{v}{6}$

Ans. (١)

Sol. $V_{escape} = \sqrt{\frac{2GM}{R}}$

$V_{planet} = \sqrt{\frac{2GM}{R}} = V$

$V_{Moon} = \sqrt{\frac{2GM \times 16}{144R}} = \frac{1}{3}\sqrt{\frac{2GM}{R}}$

$V_{Moon} = \frac{V_{Planet}}{3} = \frac{V}{3}$

٤٧. A small spherical ball of radius r, falling through a viscous medium of negligible density has terminal velocity 'v'. Another ball of the same mass but of radius 2r, falling through the same viscous medium will have terminal velocity:

(١) $\frac{v}{2}$ (٢) $\frac{v}{4}$

(٣) ٤v (٤) ٢v

Ans. (١)

Sol. Since density is negligible hence Buoyancy force will be negligible

At terminal velocity.

$Mg = 6\pi\eta rv$

$V = \frac{1}{r}$ (as mass is constant)

Now, $\frac{v}{v'} = \frac{r'}{r}$

$r' = 2r$

So, $v' = \frac{v}{2}$

٤٨. A body of mass ٢ kg begins to move under the action of a time dependent force given by $\vec{F} = (٦t\hat{i} + ٦t^{٢}\hat{j})$ N. The power developed by the force at the time t is given by :

(1) $(6t^{4} + 9t^{5})$ W

(2) $(3t^{3} + 6t^{5})$ W

(3) $(9t^{5} + 6t^{3})$ W

(4) $(9t^{3} + 6t^{5})$ W

Ans. (٤)

Sol. $\vec{F} = (6t\hat{i} + 6t^{2}\hat{j})$ N

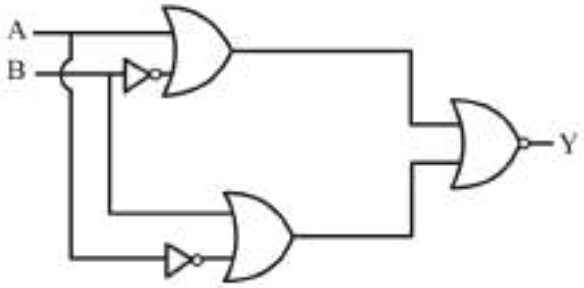
$\vec{F} = m\vec{a} = (6\hat{i} + 6t^{2}\hat{j})$

$\vec{a} = \frac{\vec{F}}{m} = (3t\hat{i} + 3t^{2}\hat{j})$

$\vec{v} = \int_{0}^{t} \vec{a}\,dt = \frac{3t^{2}}{2}\hat{i} + t^{3}\hat{j}$

$P = \vec{F}.\vec{v} = (9t^{5} + 6t^{5})$ W

٤٩.



The output of the given circuit diagram is

(١)

A	B	Y
0	0	0
1	0	0
0	1	0
1	1	1

(٢)

A	B	Y
0	0	0
1	0	1
0	1	1
1	1	0

(٣)

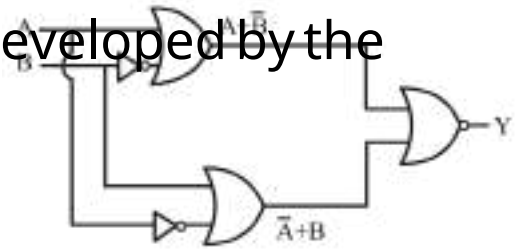
A	B	Y
0	0	0
1	0	0
0	1	0
1	1	0

(٤)

A	B	Y
0	0	0
1	0	0
0	1	1
1	1	0

Ans. (٣)

Sol.



If A = ٠ ; $A = ١$

A = ١ ; $A = ٠$

B = ٠ ; $B = ١$

B = ١ ; $B = ٠$

$Y = \overline{A\bar{B} + \bar{A}B} = \overline{١ + ١} = ٠$

٥٠. Consider two physical quantities A and B related to each other as $E = \frac{B - x^{2}}{At}$ where E, x and t have dimensions of energy, length and time respectively. The dimension of AB is

(1) $L^{2}M^{1}T^{0}$

(2) $L^{2}M^{-1}T^{1}$

(3) $L^{-2}M^{-1}T^{1}$

(4) $L^{0}M^{-1}T^{1}$

Ans. (٢)

Sol. $[B] = [L^{2}]$

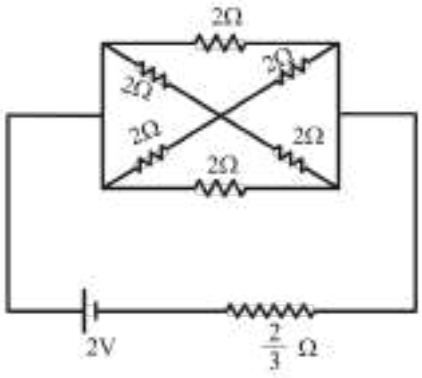
$A = \frac{x^{2}}{tE} = \frac{L^{2}}{TML^{2}T^{-2}} = \frac{1}{MT^{-1}}$

$[A] = [M^{-1}T]$

$[AB] = [L^{2}M^{-1}T^{1}]$

SECTION-B

٥١. In the following circuit, the battery has an emf of ٢ V and an internal resistance of $\frac{2}{3}\Omega$. The power consumption in the entire circuit is _____ W



Ans. (٣)

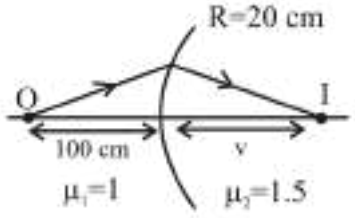
Sol. $R_{eq} = \frac{4}{3}\Omega$

$P = \frac{V^2}{R_{eq}} = \frac{4}{4/3} = 3\,W$

٥٢. Light from a point source in air falls on a convex curved surface of radius ٢٠ cm and refractive index ١.٥. If the source is located at ١٠٠ cm from the convex surface, the image will be formed at _____ cm from the object.

Ans. (٢٠٠)

Sol.



$\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$

$\frac{1.5}{v} - \frac{1}{-100} = \frac{1.5-1}{20}$

$v = 100cm$

Distance from object

= ١٠٠+١٠٠

= ٢٠٠ cm

٥٣. The magnetic flux ϕ (in weber) linked with a closed circuit of resistance ٨ Ω varies with time (in seconds) as $\phi = 5t^2 - 36t + 1$. The induced current in the circuit at t = ٢s is _____ A.

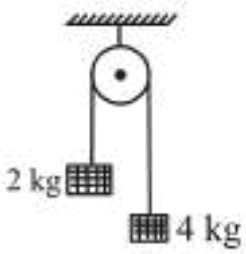
Ans. (٤)

Sol. $\left|\frac{d\phi}{dt}\right| = |10t - 36|$

at $t = 2, = 16V$

٥٤. $i = \frac{}{R} = \frac{16}{8} = 2\,A$

Two blocks of mass ٢ kg and ٤ kg are connected by a metal wire going over a smooth pulley as shown in figure. The radius of wire is ٤.٠×١٠^{-٥} m and Young's modulus of the metal is $2.0 \times 1011N/m2.$ The longitudinal strain developed in the wire is $\frac{1}{}$. The value of is _____. (Use $g = ١٠\,m/s^2$)



Ans. (١٢)

Sol. $T = \frac{2mm_1m_2}{m_1+m}g = \frac{80}{3}N$

$A = \pi r^2 = 16 \times 10^{-10} m^2$

Strain = $\frac{\Delta\ell}{\ell} = \frac{F}{AY} = \frac{T}{AY}$

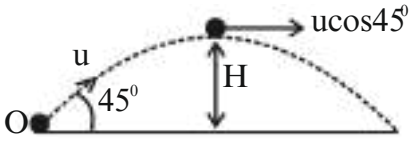
$= \frac{80/3}{16 \times 10^{-10} \times 2 \times 1011} = \frac{1}{12}$

٥٥. 12

A body of mass 'm' is projected with a speed 'u' making an angle of ٤٥° with the ground. The angular momentum of the body about the point of projection, at the highest point is expressed as $\frac{\sqrt{2}mu^3}{Xg}$. The value of 'X' is _____.

Ans. (٨)

Sol.



$$L = mu\cos\theta \frac{u^2 \sin^2\theta}{2g}$$

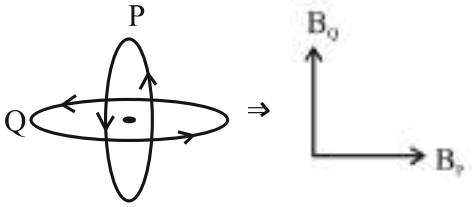
$$= mu^3 \frac{1}{4\sqrt{2}\, g} \times 8$$

٥٦. Two circular coils P and Q of ١٠٠ turns each have same radius of cm. The currents in P and R are ١ A and ٢ A respectively. P and Q are placed with their planes mutually perpendicular with their centers coincide. The resultant magnetic field induction at the center of the coils is $x\sqrt{mT}$, where x = _____.

[Use $\frac{\mu_0}{4\pi} = 10^{-7} TmA^{-1}$]

Ans. (٢٠)

Sol.



$$B_P = \frac{\mu_0 N i_1}{2r} = \frac{\mu_0}{2} \times \frac{1 \times 100}{\pi} = 2 \times 10^{-3} T$$

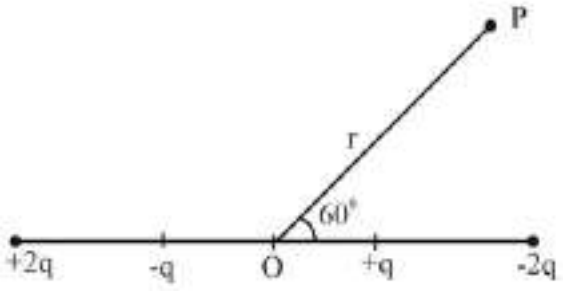
$$B_Q = \frac{\mu_0 N i_2}{2r} = \frac{\mu_0}{2} \times \frac{2 \times 100}{\pi} = 4 \times 10^{-3} T$$

$$B_{net} = \sqrt{B_P^2 + B_Q^2}$$

$$= \sqrt{20}\, mT$$

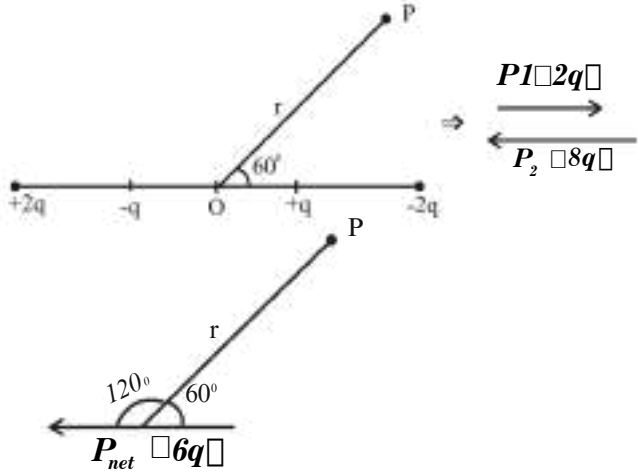
x = ٢٠

٥٧. The distance between charges +q and −q is $2l$ and between +٢q and −٢q is $4l$. The electrostatic potential at point P at a distance r from centre O is $\frac{ql}{4\pi\varepsilon_0 r^2} \times 10^9 V$, where the value of is _____. (Use $\frac{1}{4\pi\varepsilon_0} = 9 \times 10^9 Nm^2C^{-2}$)



Ans. (٢٧)

Sol.



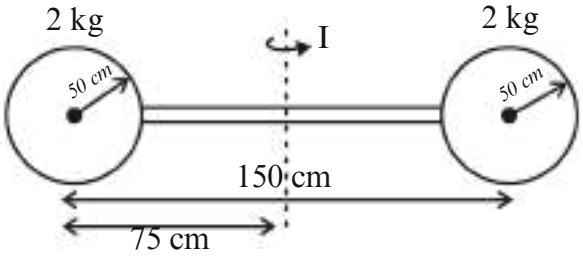
$$V = \frac{Kp.r}{r^3} = \frac{9 \times 10^9 \times 6q \ell \cos 120^0}{r^2}$$

$$= \frac{27 \times 10^9 q\ell}{r^2}$$

٥٨. Two identical spheres each of mass ٢ kg and radius ٥٠ cm are fixed at the ends of a light rod so that the separation between the centers is ١٥٠ cm. Then moment of inertia of the system about an axis perpendicular to the rod and passing through its middle point is $\frac{x}{20}$ kgm٢, where the value of x is _____.

Ans. (٥٣)

Sol.

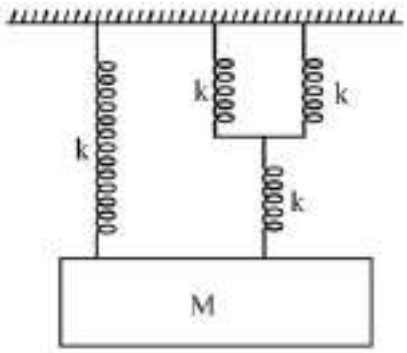


$$I = \frac{2}{5}mR^2 + md^2 + 2$$

$$I = 2 \times \frac{2}{5} \times 1 \times \left(\frac{1}{2}\right)^2 + 2 \times \frac{3}{4} \times \left(\frac{3}{2}\right)^2 = \frac{53}{20}\ kg\ m^2$$

X = ٥٣

٥٩. The time period of simple harmonic motion of mass M in the given figure is $\pi\sqrt{\frac{M}{5K}}$ s, where the value of π is _____.



Ans. (١٢)

Sol. $k_{eq} = \frac{2k \cdot k}{3k} + k = \frac{5k}{3}$

Angular frequency of oscillation $= \sqrt{\frac{k_{eq}}{m}}$

$= \sqrt{\frac{5k}{3m}}$

Period of oscillation $= \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{3m}{5k}}$

$= \sqrt{\frac{12m}{5k}}$

٦٠. A nucleus has mass number A_1 and volume V_1. Another nucleus has mass number A_2 and volume V_2. If relation between mass number is $A_2 = 4A_1$, then $\frac{V_2}{V_1}$ = _____.

Ans. (٤)

Sol. For a nucleus

Volume : $V = \frac{4}{3}\pi R^3$

$R = R_0 A^{1/3}$

$V = \frac{4}{3}\pi R_0^3 A$

$\frac{V_2}{V_1} = \frac{A_2}{A_1} = 4$

CHEMISTRY

SECTION–A

६१. Match List I with List II

LIST – I (Complex ion)		LIST – II (Electronic Configuration
A.	$[Cr(H_2O)_6]^{3+}$	I. $t_{2g}^2 e_g^0$
B.	$[Fe(H_2O)_6]^{3+}$	II. $t_{2g}^3 e_g^0$
C.	$[Ni(H_2O)_6]^{2+}$	III. $t_{2g}^3 e_g^2$
D.	$[V(H_2O)_6]^{3+}$	IV. $t_{2g}^6 e_g^2$

Choose the correct answer from the options given below :

(१) A–III, B–II, C–IV, D–I

(२) A–IV, B–I, C–II, D–III

(३) A–IV, B–III, C–I, D–II

(४) A–II, B–III, C–IV, D–I

Ans. (४)

Sol :- $[Cr(H_2O)_6]^{3+}$ Contains $Cr^{3+}: Ar\ 3d^3 : t_{2g}^3 e_g^0$

$[Fe(H_2O)_6]^{3+}$ Contains $Fe^{3+}: Ar\ 3d^5 : t_{2g}^5 e_g^0$

$[Ni(H_2O)_6]^{2+}$ Contains $Ni^{2+}: Ar\ 3d^8 : t_{2g}^6 e_g^2$

$[V(H_2O)_6]^{3+}$ Contains $V^{3+}: Ar\ 3d^2 : t_{2g}^2 e_g^0$

TEST PAPER WITH SOLUTION

६२. A sample of $CaCO_3$ and $MgCO_3$ weighed २.२१ g is ignited to constant weight of १.१५२ g. The composition of mixture is :

(Given molar mass in g mol^{-1} $CaCO_3$:100, $MgCO_3$:84)

(१) 1.187g $CaCO_3$ & 1.023g $MgCO_3$

(२) 1.023g $CaCO_3$ & 1.023g $MgCO_3$

(३) 1.187g $CaCO_3$ & 1.187g $MgCO_3$

(४) 1.023g $CaCO_3$ & 1.187g $MgCO_3$

Ans. (१)

Sol :- $CaCO_3(s) \rightarrow CaO(s) + CO_2(g)$

$MgCO_3(s) \rightarrow MgO(s) + CO_2(g)$

Let the weight of $CaCO_3$ be x gm

$\therefore$ weight of $MgCO_3 = 2.21 - x$ gm

Moles of $CaCO_3$ decomposed = moles of CaO formed

$\frac{x}{100}$ = moles of CaO formed

$\therefore$ weight of CaO formed $= \frac{x}{100} \times 56$

Moles of $MgCO_3$ decomposed = moles of MgO formed

$\frac{2.21 - x}{84}$ = moles of MgO formed

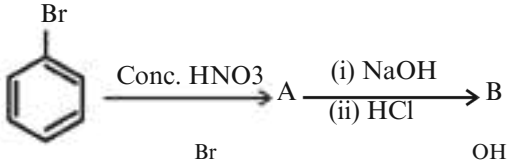
$\therefore$ weight of MgO formed $= \frac{2.21 - x}{84} \times 40$

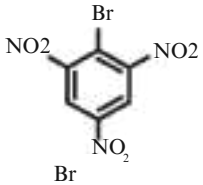
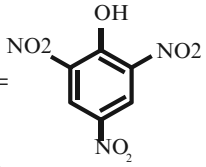
$\therefore \frac{2.21 - x}{84} \times 40 + \frac{x}{100} \times 56 = 1.152$

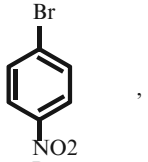
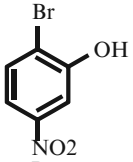
$\therefore x = 1.1886$ g = weight of $CaCO_3$

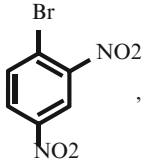
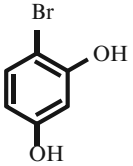
& weight of $MgCO_3 = 1.0214$g

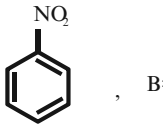
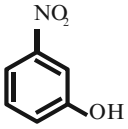
٦٣. Identify A and B in the following reaction sequence.



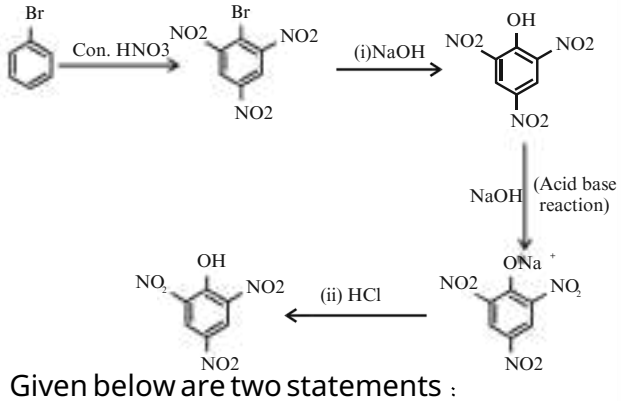
(1) A=  , B= 

(2) A=  , B= 

(3) A=  , B= 

(4) A=  , B= 

Ans. (١)

Sol:- 

٦٤. Given below are two statements :

Statement I : S_8 solid undergoes disproportionation reaction under alkaline conditions to form S^{2-} and $S_2O_3^{2-}$

Statement II : ClO_4^- can undergo disproportionation reaction under acidic condition.

In the light of the above statements, choose the most appropriate answer from the options given below :

(١) Statement I is correct but statement II is incorrect.

(٢) Statement I is incorrect but statement II is correct

(٣) Both statement I and statement II are incorrect

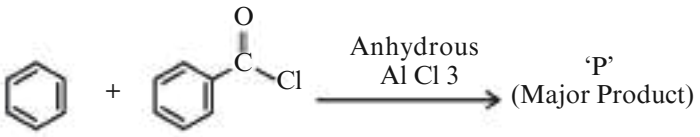
(٤) Both statement I and statement II are correct

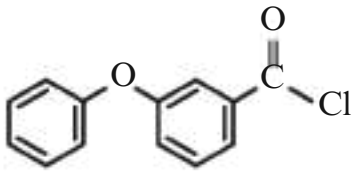
Ans. (١)

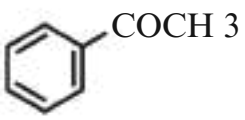
Sol:- S_1: $S_8 + 12\,OH^- \rightarrow 4S^{2-} + 2S_2O_3^{2-} + 6H_2O$

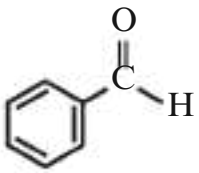
S_2: ClO_4^- cannot undergo disproportionation reaction as chlorine is present in it's highest oxidation state.

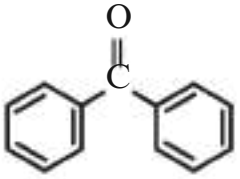
٦٥. Identify major product 'P' formed in the following reaction.



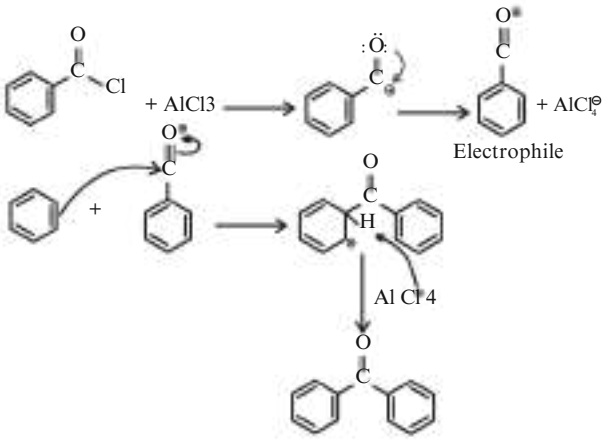
(1) 

(2) 

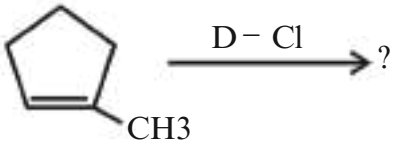
(3) 

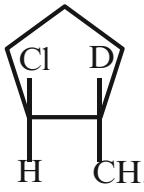
(4) 

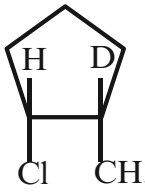
Ans. (٤)

Sol:- 

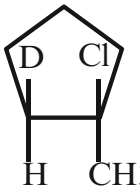
٦٦. Major product of the following reaction is –



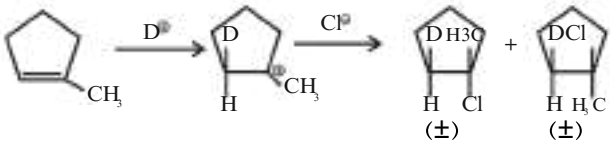
(1) 

(2) 

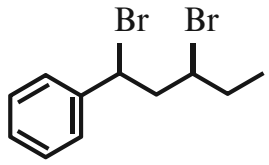
(3) 

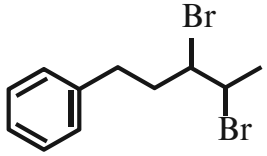
(4) 

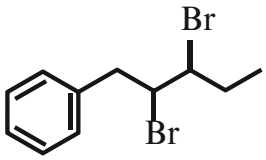
Ans. (३ or ४)

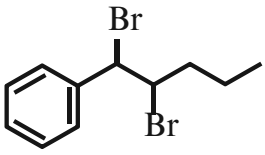
Sol:- 

٦٧. Identify structure of २,३-dibromo-१-phenylpentane.

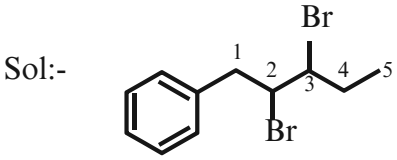
1. 

2. 

3. 

4. 

Ans. (३)

Sol:- 

2, 3-dibromo -1-phenylpentane

٦٨. Select the option with correct property –

(१) $[Ni(CO)_4]$ and $[NiCl_4]^{2-}$ both diamagnetic

(२) $[Ni(CO)_4]$ and $[NiCl_4]^{2-}$ both paramagnetic

(3) $NiCl_4^{2-}$ diamagnetic, $Ni(CO)_4$ paramagnetic

(४) $[Ni(CO)_4]$ diamagnetic, $[NiCl_4]^{2-}$ paramagnetic

Ans. (४)

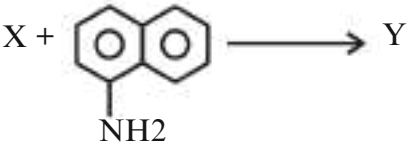
Sol:- $[Ni(CO)_4]$ (४) diamagnetic, sp^3 hybridisati

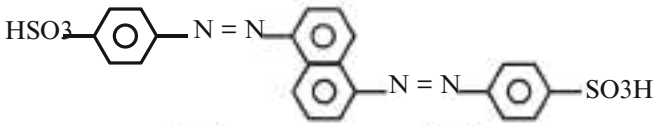
number of unpaired electrons = ०

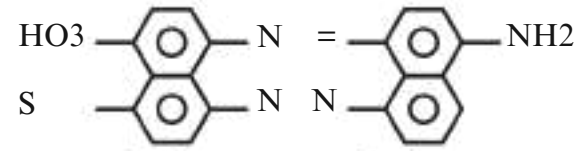
$[NiCl_4]^{2-}$ paramagnetic, sp^3 hybridisat

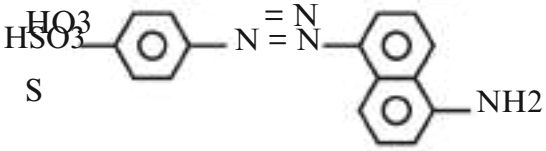
number of unpaired electrons = २

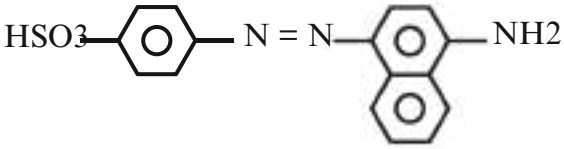
٦٩. The azo-dye (Y) formed in the following reactions is Sulphanilicacid $NaNO_2$ CH_3COOH X



1. 

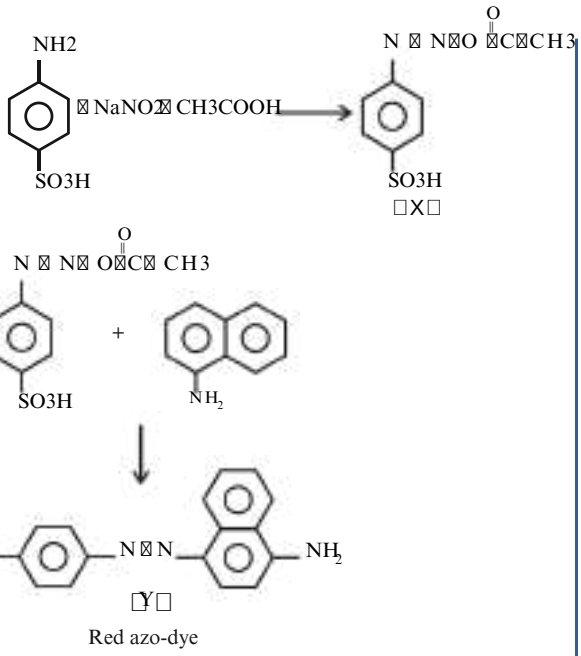
2. 

3. 

4. 

Ans. (४)

Sol:-



This is known as Griess-Ilosvay test.

७०. Given below are two statements :

Statement I : Aniline reacts with con. H_2SO_4 followed by heating at ४५३-४७३ K gives p-aminobenzene sulphonic acid, which gives blood red colour in the 'Lassaigne's test'.

Statement II : In Friedel - Craft's alkylation and acylation reactions, aniline forms salt with the $AlCl_3$ catalyst. Due to this, nitrogen of aniline aquires a positive charge and acts as deactivating group.

In the light of the above statements, choose the correct answer from the options given below :

१. Statement I is false but statement II is true

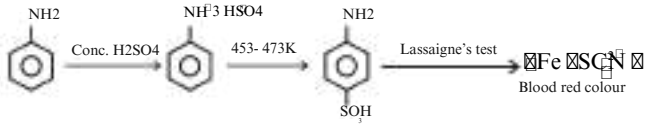
२. Both statement I and statement II are false

३. Statement I is true but statement II is false

४. Both statement I and statement II are true

Ans. (४)

Sol:-



७१. $A_{(g)} \rightleftharpoons B_{(g)} + \frac{C}{2}{}_{(g)}$ The correct relationship between K_P and equilibrium pressure P is

(१) $K_P = \frac{P^{1/2}}{2}$

(२) $K_P = \frac{P^{3/2}}{2}$

(३) $K_P = \frac{P^{3/2}}{2}$

(४) $K_P = \frac{P^{1/2}}{2}$

Ans. (२)

Sol:-

$A_{(g)} \rightleftharpoons B_{(g)} + \frac{C}{2}{}_{(g)}$

$t = t_{eq}$

$P_B = \frac{}{}.P, \quad P_A = \frac{}{}.P, \quad P_C = \frac{}{}.P$

$K_P = \frac{P_B \cdot P_C^{\frac{1}{2}}}{P_A}$

७२. Choose the correct statements from the following

A. All group १६ elements form oxides of general formula EO_2 and EO_3 where E = S, Se, Te and Po. Both the types of oxides are acidic in nature.

B. TeO_2 is an oxidising agent while SO_2 is reducing in nature.

C. The reducing property decreases from H_2S to H_2Te down the group.

D. The ozone molecule contains five lone pairs of electrons.

Choose the correct answer from the options given below :

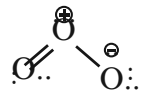
१. A and D only
२. B and C only
३. C and D only
४. A and B only

Ans. (४)

Sol :- (A) All group ١٦ elements form oxides of the EO2andEO3 type where E⊠S,Se,TeorPo.

(B) SO ٢ is reducing while TeO٢ is an oxidising agent.

(C) The reducing property increases from S to H٢Tedown the group.

(D)  have six lone pairs

٧٣. Identify the name reaction.

(١) Stephen reaction

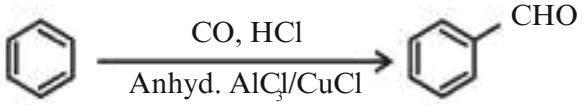
(٢) Etard reaction

(٣) Gatterman-koch reaction

(٤) Rosenmund reduction

Ans. (٣)

Sol :-



Gatterman-Koch reaction

٧٤. Which of the following is least ionic ?

(1) $BaCl_2$ (٢) AgCl

(٣) KCl (4) $CoCl_2$

Ans. (٢)

Sol:- AgCl⊠CoCl ٢⊠BaCl٢⊠KCl (ionic character)

Reason : Ag⊠ has pseudo inert gas configuration.

٧٥. The fragrance of flowers is due to the presence of some steam volatile organic compounds called essential oils. These are generally insoluble in water at room temperature but are miscible with water vapour in vapour phase. A suitable method for the extraction of these oils from the flowers is -

١. crystallisation

٢. distillation under reduced pressure

٣. distillation

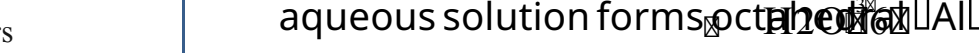
٤. steam distillation

Ans. (٤)

Sol :- Steam distillation technique is applied to separate substances which are steam volatile and are immiscible with water.

Given below are two statements :

Statement I : Group ١٣ trivalent halides get easily hydrolyzed by water due to their covalent nature.

Statement II : AlCl٣ upon hydrolysis in acidified aqueous solution forms octa  ion.

In the light of the above statements, choose the correct answer from the options given below :

١. Statement I is true but statement II is false

٢. Statement I is false but statement II is true

٣. Both statement I and statement II are false

٤. Both statement I and statement II are true

Ans. (٤)

Sol :- In trivalent state most of the compounds being covalent are hydrolysed in water. Trichlorides on hydrolysis in water form tetrahedral species, the hybridisation state of element M is sp^3.

In case of aluminium, acidified aqueous solution forms octahedral ion.

٧٧. The four quantum numbers for the electron in the outer most orbital of potassium (atomic no. ١٩) are

(١) $n = 4,\ l = ٢,\ m = ١,\ s = \frac{1}{2}$

(٢) $n = 4,\ l = ٠,\ m = ٠,\ s = \frac{1}{2}$

(٣) $n = 3,\ l = ٠,\ m = ١,\ s = \frac{1}{2}$

(٤) $n = ٢,\ l = ١,\ m = ٠,\ s = \frac{1}{2}$

Ans. (٢)

Sol :- 19K $1s^2\ 2s^2, 2p^6, 3s^2, 3p^6, 4s^1$.

Outermost orbital of potassium is 4s orbital

$n = 4, l = 0, ml = 0, s = \frac{1}{2}$.

৭৮. Choose the correct statements from the following

A. Mn_2O_7 is an oil at room temperature

B. V_2O_4 reacts with acid to give VO^{2+}

C. CrO is a basic oxide

D. V_2O_5 does not react with acid

Choose the correct answer from the options given below :

১. A, B and D only

২. A and C only

৩. A, B and C only

৪. B and C only

Ans. (২)

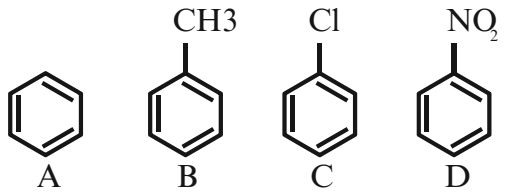
Sol :- (A) Mn_2O_7 is green oil at room temperature.

(B) V_2O_4 dissolve in acids to give V^{4+} salts.

(C) CrO is basic oxide

(D) V_2O_5 is amphoteric it reacts with acid as well as base.

৭৯. The correct order of reactivity in electrophilic substitution reaction of the following compounds is :



১. B > C > A > D

২. D > C > B > A

৩. A > B > C > D

৪. B > A > C > D

Ans. (৪)

Sol :- $-CH_3$ shows $+M$ and $+I$.

$-Cl$ shows $+M$ and $+I$ but inductive effect dominates.

$-NO_2$ shows $-M$ and $-I$.

Electrophilic substitution $\propto \frac{1}{-M \text{ and } -I}$

$\propto +M$ and $+I$

Hence, order is B > A > C > D.

৮০. Consider the following elements.

Group ↓ A'B' → Period
C'D'

Which of the following is/are true about A', B', C' and D' ?

A. Order of atomic radii : B' > A' > D' > C'

B. Order of metallic character : B' > A' > D' > C'

C. Size of the element : D' > C' > B' > A'

D. Order of ionic radii : B' > A' > D' > C'

Choose the correct answer from the options given below :

১. A only

২. A, B and D only

৩. A and B only

৪. B, C and D only

Ans. (২)

Sol :- In general along the period from left to right, size decreases and metallic character decrease.

In general down the group, size increases and metallic character increases.

B' > A' $\propto$ size

D' > C' $\propto$ size

B' > A' $\propto$ metallic character

D' > C' $\propto$ metallic character

B' > A' $\propto$ size

D' > C' $\propto$ size

C statement is incorrect.

SECTION-B

৮১. A diatomic molecule has a dipole moment of ১.২ D. If the bond distance is ১Å, then fractional charge on each atom is _____ ×১০⁻¹esu. (Given ১D = ১০×১০⁻¹⁸ esu cm)

Ans. (০)

Sol:- $\mu = 1.2D = q \times d$

$= 1.2 \times 10^{-10} esu\text{Å} \times q \times 1\text{Å}$

$q = 1.2 \times 10^{-10} esu$

৮২. r = k[A] for a reaction, ৫০% of A is decomposed in ১২০ minutes. The time taken for ৯০% decomposition of A is _____ minutes.

Ans. (৩৯৯)

Sol:- r = k[A]

So, order of reaction = ১

$t_{1/2} = 120 min$

For ৯০% completion of reaction

$k = \frac{2.303}{t} \log \frac{a}{a-x}$

$\frac{0.693}{t_{1/2}} = \frac{2.303}{t} \log \frac{100}{10}$

$t = 399 min.$

৮৩. A compound (X) with molar mass ১০৮ gmol⁻¹ undergoes acetylation to give product with molar mass ১৯২ gmol⁻¹. The number of amino groups in the compound (X) is _____.

Ans. (২)

Sol:- $R-NH_2 + CH_3-\overset{O}{\overset{\|}{C}}-Cl \rightarrow R-NH-\overset{O}{\overset{\|}{C}}-CH_3$

Gain in molecular weight after acylation with one $-NH_2$ group is ৪২.

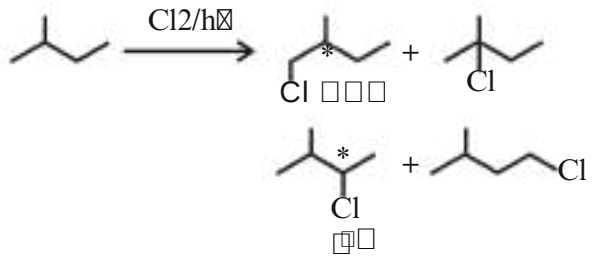
Total increase in molecular weight = ৮৪

$\therefore$ Number of amino group in $X = \frac{84}{42} = 2$

৮৪. Number of isomeric products formed by mono-chlorination of ২-methylbutane in presence of sunlight is _____.

Ans. (৬)

Sol:-



$\therefore$ Number of isomeric products = ৬

৮৫. Number of moles of H^+ ions required by ১ mole of MnO_4^- to oxidise oxalate ion to CO_2 is _____.

Ans. (৮)

Sol:-

$2MnO_4^- + 5C_2O_4^{2-} + 16H^+ \rightarrow 2Mn^{2+} + 10CO_2 + 8H_2O$

$\therefore$ Number of moles of H^+ ions required by ১ mole of MnO_4^- to oxidise oxalate ion to CO_2 = ৮

৮৬. In the reaction of potassium dichromate, potassium chloride and sulfuric acid (conc.), the oxidation state of the chromium in the product is (+)_____.

Ans. (৬)

Sol:- $K_2Cr_2O_7(s) + 4KCl(s) + 6H_2SO_4(conc.) \rightarrow 2CrO_2Cl_2(g) + 6KHSO_4 + 3H_2O$

This reaction is called chromyl chloride test.

Here oxidation state of Cr is +৬.

৮৭. The molarity of ১L orthophosphoric acid having ৭০% purity by weight (specific gravity ১.৫৪ gcm⁻³) is _____M.

(Molar mass of H_3PO_4 = 98 gmol⁻¹)

Ans. (১১)

Sol :- Specific gravity (density) = ١.٥٤ g /cc.

Volume = 1L = 1000ml

Mass of solution = ١.٥٤ × ١٠٠٠

= 1540g

% purity of H_2SO_4 is ٧٠%

So weight of H_3PO_4 = 0.7 × 1540 = 1078g

Mole of H_3PO_4 = $\frac{1078}{98}$ = ١١

Molarity = $\frac{11}{1L}$ = 11

٨٨. The values of conductivity of some materials at ٢٩٨.١٥ K in Sm^{-1} are ٢.١ × ١٠٣, 1.0×10^{-16}, 1.2×10^{10}, 3.91, 1.5×10^{2}, ١ × ١٠ × ٧, ١.٠ × ١٠٣. The number of conductors among the materials is _____.

Ans. (٤)

Sol :-

Conductivity (Sm^{-1})

2.1 × 103

1.2×10

3.91 conductors at 298.15K

1 × 103

1×10^{-16} Insulator at 298.15K

1.5×10^{2}

1×10^{-7} Semiconductor at 298.15K

Therefore number of conductors is ٤.

٨٩. From the vitamins A, D, E and K, the number vitamins that can be stored in our body is _____.

Ans. (٥)

Sol :- Vitamins A, D, E, K and B ١٢ are stored in liver and adipose tissue.

٩٠. If ٥ moles of an ideal gas expands from ١٠ L to a volume of ١٠٠ L at ٣٠٠ K under isothermal and reversible condition then work, w, is x J. The value of x is _____.

(Given R = 8.314 JK^{-1}mol^{-1})

Ans. (٢٨٧٢١)

Sol :- It is isothermal reversible expansion, so work done negative

$$W = -2.303nRT\log\left(\frac{V_2}{V_1}\right)$$

$$= -2.303 \times 5 \times 8.314 \times 300 \log\left(\frac{100}{10}\right)$$

$$= -28720.738$$

$$= -28721J$$