

FINAL JEE–MAIN EXAMINATION – APRIL, 2024

(Held On Thursday 04 April, 2024)

TIME : 9 : 00 AM to 12 : 00 NOON

MATHEMATICS

SECTION-A

1. Let $f : \mathbb{R} \rightarrow \mathbb{R}$ be a function given by

$$f(x) = \begin{cases} \frac{1 - \cos x}{x^2}, & x \neq 0 \\ \sqrt{\cos x}, & x = 0 \end{cases}$$

f is continuous at $x = 0$, then $\lim_{x \rightarrow 0} f(x)$ is equal to :

- (1) $\frac{1}{2}$ (2) $\frac{1}{\sqrt{2}}$
(3) $\frac{1}{2}$ (4) $\frac{1}{\sqrt{2}}$

Ans. (2)

Sol. $f(0) = \lim_{x \rightarrow 0} \frac{1 - \cos x}{x^2} = \frac{1}{2}$

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2. Three urns A, B and C contain 4 red, 5 black, 6 red, 4 black and 7 red, 3 black balls, respectively. One of the urn is selected at random and a ball is drawn from it. If the ball drawn is black, then the probability that it is drawn from urn A is :

- (1) $\frac{5}{14}$ (2) $\frac{5}{14}$
(3) $\frac{5}{14}$ (4) $\frac{5}{14}$

Ans. (2)

Sol. A B C
4R, 5B 6R, 4B 7R, 3B

$$P(B) = \frac{1}{3} \cdot \frac{5}{9} + \frac{1}{3} \cdot \frac{4}{10} + \frac{1}{3} \cdot \frac{3}{10}$$

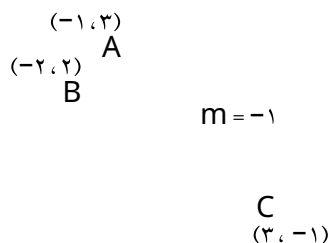
$$\text{required probability} = \frac{\frac{1}{3} \cdot \frac{5}{9}}{\frac{1}{3} \cdot \frac{5}{9} + \frac{1}{3} \cdot \frac{4}{10} + \frac{1}{3} \cdot \frac{3}{10}} = \frac{5}{14}$$

TEST PAPER WITH SOLUTION

3. The vertices of a triangle are $A(-1, 2)$, $B(-2, 2)$ and $C(2, -1)$. A new triangle is formed by shifting the sides of the triangle by one unit inwards. Then the equation of the side of the new triangle nearest to origin is :

- (1) $x - y - 1 = 0$
(2) $-x + y - 1 = 0$
(3) $x + y - 1 = 0$
(4) $x + y + 1 = 0$

Ans. (3)



Sol.

equation of AC $x + y = 1$
equation of line parallel to AC $x + y = d$
 $d = 2$
eqn of new required line
 $x + y = -1$

4. If the solution $y = y(x)$ of the differential equation $(x^2 + 2x + 2y + 2)dy - (2x + 2y + 2)dx = 0$ satisfies $y(-1) = \frac{1}{2}$, then $y(0)$ is equal to :

- (1) $\frac{1}{2}$ (2) $\frac{1}{2}$
(3) $\frac{1}{2}$ (4) $\frac{1}{2}$

Ans. (3)

Sol. $\frac{dy}{dx} = \frac{2xy - 3x^2}{x^2 - 2x + 2}$

$$y = \frac{2x^2 - 3x}{x^2 - 2x + 2}$$

$$y = \frac{dx}{x^2 - 2x + 2} = \frac{dx}{x^2 - 2x + 1 + 1}$$

$$y = \tan(x+1) + \tan x + C$$

$$y(-1) = \frac{1}{e}$$

$$\frac{1}{e} = 0 + \frac{1}{e} + C \Rightarrow C = 0$$

5. $y = \tan(x+1) + \tan x$

$$y(0) = \tan^{-1} 1 = \frac{\pi}{4}$$

Let the sum of the maximum and the minimum

values of the function $f(x) = \frac{mx^2 - nx}{x^2 - 1}$ be $\frac{m}{n}$,

where $\gcd(m, n) = 1$. Then $m+n$ is equal to :

- (1) 182 (2) 217
(3) 190 (4) 201

Ans. (4)

Sol. $y = \frac{mx^2 - nx}{x^2 - 1}$

$$x(2y - 2) + x(3y + 3) + 4y - 4 = 0$$

use D

$$(3y + 3) - 2(2y - 2) + 4y - 4 = 0$$

$$(11y - 6)(5y - 11) = 0$$

$$y = \frac{11}{5} \text{ or } \frac{6}{11}$$

6. $y = 1$ is also included

One of the points of intersection of the curves

$$y = 1 + 2x - x^2 \text{ and } y = \frac{1}{x} \text{ is } \left(\frac{1}{2}, 2\right)$$

of the region enclosed by these curves be

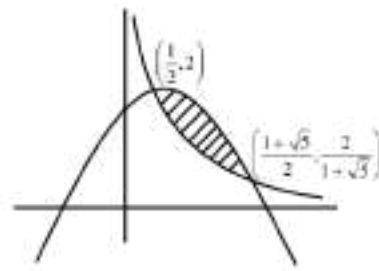
$$\frac{1}{2} \ln \frac{m}{n} \text{ where } m, n \in \mathbb{N}$$

N. Then $m+n$ is equal to

- (1) 32 (2) 30
(3) 29 (4) 31

Ans. (3)

Sol.



$$A = \int_{1/2}^{1+\sqrt{5}/2} \left(1 + 2x - x^2 - \frac{1}{x}\right) dx$$

$$A = \left[x + x^2 - \frac{x^3}{3} - \ln x\right]_{1/2}^{1+\sqrt{5}/2}$$

$$A = \left[1 + \frac{1}{2} + \frac{1}{8} - \frac{1}{24} - \ln \frac{1}{2}\right] - \left[\frac{1}{8} + \frac{1}{4} - \frac{1}{24} - \ln \frac{1}{2}\right]$$

$$A = \frac{1}{2} + \frac{1}{8} - \frac{1}{24} = \frac{11}{24}$$

$$A = \frac{1}{2} + \frac{1}{8} - \frac{1}{24} = \frac{11}{24}$$

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$$A = \frac{1}{2} + \frac{1}{8} - \frac{1}{24} = \frac{11}{24}$$

7.

If the system of equations

$$x + \cos \alpha y + \sin \alpha z = 0$$

$$x + (\cos \alpha)y + (\sin \alpha)z = 0$$

$$x + (\sin \alpha)y - (\cos \alpha)z = 0$$

has a non-trivial solution, then α is equal to :

- (1) $\frac{\pi}{4}$ (2) $\frac{\pi}{2}$
(3) $\frac{3\pi}{4}$ (4) $\frac{5\pi}{4}$

Ans. (3)

Sol.
$$\begin{vmatrix} \sqrt{y} \sin x & \sqrt{y} \cos x \\ \sin x & \cos x \\ \cos x & \sin x \end{vmatrix} = 0$$

$$1 \cdot \sqrt{y} \sin x \cdot \cos x - \sqrt{y} \cos x \cdot \sin x = 0$$

$$1 \cdot \sqrt{y} \cos x \cdot \sin x - \sqrt{y} \sin x \cdot \cos x = 0$$

$$\cos x \cdot \sin x - \sin x \cdot \cos x = 0$$

$$\cos x \cdot \sin x = \sin x \cdot \cos x$$

$$2 \sin x \cos x = 2 \sin x \cos x$$

$$\sin 2x = \sin 2x$$

$$n = 0$$

$$x = \frac{\pi}{2} - \frac{\pi}{2} = 0$$

8. There are n points $P_1, P_2, P_3, P_4, \dots, P_n$ on the side AB , excluding A and B , of a triangle ABC . Similarly there are r points P_1, P_2, \dots, P_r on the side BC and s points P_1, P_2, \dots, P_s on the side CA of the triangle. The number of triangles that can be formed using the points $P_1, P_2, \dots, P_{n+r+s}$ as vertices is :

- (1) 776 (2) 751
(3) 796 (4) 771

Ans. (2)

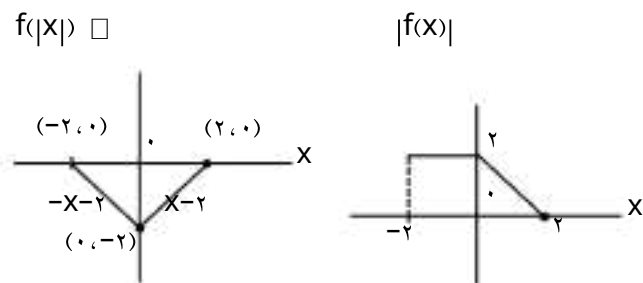
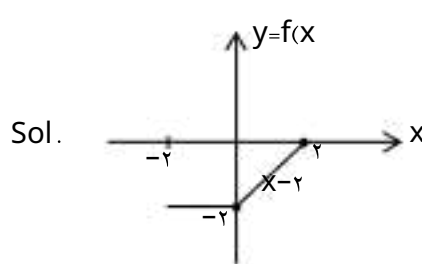
Sol. ${}^{18}C_r - {}^nC_r - {}^nC_r - {}^nC_r$
 $= 751$

9. Let $f(x) = \begin{cases} x^2, & x \geq 0 \\ x^2, & x < 0 \end{cases}$ and $h(x) = f(|x|) + |f(x)|$.

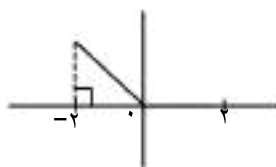
Then $\int_0^1 x^2 dx$ is equal to :

- (1) 2 (2) 4
(3) 1 (4) 6

Ans. (1)



$h(x) = \begin{cases} x^2 + x^2, & x \geq 0 \\ x^2 + x^2, & x < 0 \end{cases}$
 $h(x) = 2x^2$



$\int_0^1 h(x) dx$ and $\int_0^1 h(x) dx$

10. The sum of all rational terms in the expansion of $(\sqrt{5} + \sqrt{5})^{10}$ is equal to :

- (1) 3133 (2) 633
(3) 931 (4) 6131

Ans. (1)

$= C_{10}^0 + C_{10}^2 + C_{10}^4 + C_{10}^6 + C_{10}^8 + C_{10}^{10}$

$= {}^{10}C_0 + {}^{10}C_2 + {}^{10}C_4 + {}^{10}C_6 + {}^{10}C_8 + {}^{10}C_{10}$

$R = \sqrt{5} + \sqrt{5}$

$r = 0, 10$

7 rational terms

${}^{10}C_0 + {}^{10}C_2 + {}^{10}C_4 + {}^{10}C_6 + {}^{10}C_8 + {}^{10}C_{10}$

$= 1 + 45 + 105 + 105 + 45 + 1 = 3133$

11. Let a unit vector which makes an angle of 60° with $\hat{i} + \hat{j} + \hat{k}$ and an angle of 45° with $\hat{i} + \hat{k}$ be \vec{C} .

Then \vec{C} is $\frac{1}{\sqrt{3}}\hat{i} + \frac{1}{\sqrt{3}}\hat{j} + \frac{\sqrt{2}}{\sqrt{3}}\hat{k}$

$$(1) \frac{\sqrt{2}}{\sqrt{3}}\hat{i} + \frac{\sqrt{2}}{\sqrt{3}}\hat{j} + \frac{1}{\sqrt{3}}\hat{k} \quad (2) \frac{1}{\sqrt{3}}\hat{i} + \frac{1}{\sqrt{3}}\hat{j} + \frac{\sqrt{2}}{\sqrt{3}}\hat{k}$$

$$(3) \frac{1}{\sqrt{3}}\hat{i} + \frac{1}{\sqrt{3}}\hat{j} + \frac{1}{\sqrt{3}}\hat{k} \quad (4) \frac{\sqrt{2}}{\sqrt{3}}\hat{i} + \frac{1}{\sqrt{3}}\hat{j} + \frac{1}{\sqrt{3}}\hat{k}$$

$$(5) \frac{1}{\sqrt{3}}\hat{i} + \frac{1}{\sqrt{3}}\hat{j} + \frac{1}{\sqrt{3}}\hat{k} \quad (6) \frac{1}{\sqrt{3}}\hat{i} + \frac{1}{\sqrt{3}}\hat{j} + \frac{1}{\sqrt{3}}\hat{k}$$

$$(7) \frac{\sqrt{2}}{\sqrt{3}}\hat{i} + \frac{1}{\sqrt{3}}\hat{j} + \frac{1}{\sqrt{3}}\hat{k}$$

Ans. (5)

Sol. $\vec{C} = C_1\hat{i} + C_2\hat{j} + C_3\hat{k}$

$$C_1^2 + C_2^2 + C_3^2 = 1$$

$$C_1 + C_2 + C_3 = \frac{1}{\sqrt{3}}$$

$$C_1 - C_3 = 1$$

$$C_1 + C_2 = \frac{1}{\sqrt{3}}$$

$$C_1 = \frac{\sqrt{2}}{\sqrt{3}} \quad C_2 = \frac{1}{\sqrt{3}}$$

$$C_3 = \frac{1}{\sqrt{3}}$$

$$C_3 = \frac{\sqrt{2}}{\sqrt{3}} \quad C_4 = \frac{1}{\sqrt{3}}$$

12. Let the first three terms a , p and q , with $q < a$, of a G.P. be respectively the m^{th} , n^{th} and 1^{th} terms of an A.P. If the m^{th} term of the G.P. is the n^{th} term of the A.P., then n is equal to

- (1) 101 (2) 169
(3) 177 (4) 163

Ans. (4)

Sol. $p^2 = aq$

$$a = a + rd \quad \dots (i)$$

$$= a + rd \quad \dots (ii)$$

$$q = a + rd \quad \dots (iii)$$

$$p - a = d \quad \dots (ii) - (i)$$

$$q - p = rd \quad \dots (iii) - (ii)$$

$$q - p = r(p - a)$$

$$q = rp - ra$$

$$p^2 = r(rp - ra)$$

$$p^2 - rp + ra = 0$$

$$p = 10, r = 2$$

$$p = 10, q = 50$$

$$d = 8$$

$$a = -10$$

$$10, 10, 50, 20, 10, 50, 10, 50$$

$$a_n = a + (n - 1)d$$

$$10 = -10 + (n - 1)8$$

$$n = 163$$

13. Let $a, b \in \mathbb{R}$. Let the mean and the variance of n observations $-3, -1, 1, 3, \dots, a, b$ be μ and σ^2 respectively. The mean deviation about the mean of these n observations is :

$$(1) \frac{13}{n}$$

$$(2) \frac{16}{n}$$

$$(3) \frac{11}{n}$$

$$(4) \frac{14}{n}$$

Ans. (1)

Sol. $\frac{\sum x_i}{n} = \mu$ and $\frac{\sum x_i^2}{n} = \sigma^2 + \mu^2$

$$\mu = 10$$

$$\sigma^2 = 8$$

solving we get $\mu = 10, \sigma^2 = 8$

$$\frac{\sum |x_i - \mu|}{n} = \frac{0 + 0 + 0 + 0 + 2 + 4 + \dots}{n}$$

14. If α and β are the roots of the equation $ax^2 + bx + c = 0$, then the quadratic equation whose roots are $\frac{1}{\alpha} + \frac{1}{\beta}$ and $\frac{1}{\alpha}\frac{1}{\beta}$ is :

- (1) $2X^2 + 11X + 12 = 0$ (2) $2X^2 + 14X + 12 = 0$
 (3) $X^2 + 10X + 16 = 0$ (4) $X^2 + 18X + 12 = 0$

Ans. (4)

Sol. Sum = $-\frac{b}{a}$

Product = $\frac{c}{a} = \frac{1}{a}$ $a = \frac{1}{12}$

$b = \frac{1}{2}$

$\frac{1}{\alpha} + \frac{1}{\beta} = \frac{2}{12} = \frac{1}{6}$

$\frac{1}{\alpha}\frac{1}{\beta} = \frac{1}{12} = \frac{1}{12}$

sum = $-\frac{1}{6}$

P = $\frac{1}{12}$

$X^2 + 18X + 12 = 0$

15. Let S and P be the sum and the product of all the non-zero solutions of the equation $z^n + z^n + \dots + z^n = 0$, $z \neq 0$.

Then $\frac{S}{P}$ is equal to :

- (1) 6 (2) 4
(3) 8 (4) 2

Ans. (2)

Sol. $z = x + iy$
 $z^n + z^n + \dots + z^n = 0$

$z^n + z^n + \dots + z^n = 0$

$x^n + y^n + \dots + x^n + y^n = 0$

$x^n + y^n = 0$ or $y^n = -x^n$
 $-y^n + |y|^n = 0$ $|x|^n = |y|^n$

$|y| = |x|$
 $y = \pm x$

$i, -i$ $i = i - i = 0$
 are roots $i = i(-i) = 1$
 $i(1 + i) = i + i^2 = -1$

Let the point P on the line passing through the points $P(1, -2, 3)$ and $Q(0, -1, 4)$, farther from the origin and at a distance of 9 units from the point P . Then $\frac{OP}{PQ}$ is equal to :

- (1) 100 (2) 100
(3) 160 (4) 160

Ans. (1)

Sol. PQ line

$\frac{x-1}{1-0} = \frac{y+2}{-1-3} = \frac{z-3}{4-3}$

pt $(1+t, -2-t, 3+t)$

distance = $\sqrt{1+t^2 + (-2-t)^2 + (3+t)^2} = 9$

$t = \frac{1}{2}$

pt $(\frac{3}{2}, -\frac{5}{2}, \frac{7}{2})$
 $\frac{OP}{PQ} = 100$

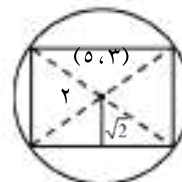
option (1)

17. A square is inscribed in the circle $x^2 + y^2 - 10x + 3 = 0$. One side of this square is parallel to $y = x + 3$. If (x_i, y_i) are the vertices of the square, then $\sum x_i^2 + \sum y_i^2$ is equal to :

- (1) 148 (2) 106
(3) 160 (4) 102

Ans. (4)

Sol.



$y = x + c$ &

$x + y + d = 0$

$\frac{|0 - 3 + c|}{\sqrt{2}} = \sqrt{2}$

$\frac{|5 + d|}{\sqrt{2}} = \sqrt{2}$

$|c + 3| = 2$

$|5 + d| = 2$

$c = 0, -5$

$d = -1, -7$

pts $(0, 0), (3, 3), (7, 3), (0, 7)$

$\sum x_i^2 + \sum y_i^2 = 0^2 + 3^2 + 7^2 + 0^2 + 0^2 + 3^2 + 7^2 + 7^2 = 102$

Option (4)

18. If the domain of the function $\sin^{-1} \frac{x^2 - 2}{x^2 + 1}$ is $(-\infty, \infty)$, then $r + 1 \cdot 1$ is equal to :

- (1) 97 (2) 100
(3) 90 (4) 98

Ans. (1)

Sol. $\sin^{-1} \frac{x^2 - 2}{x^2 + 1} \in [-\frac{\pi}{2}, \frac{\pi}{2}]$

$x^2 - 2 \in [-1, 1]$
 $x^2 \in [1, 3]$
 $x \in [1, \sqrt{3}] \cup [-\sqrt{3}, -1]$
 $r + 1 \cdot 1 = 97$

Option (1)

19. Let $f(x) = x + e^{x/\xi}$ for all $x \in \mathbb{R}$. Consider a function $g(x)$ such that $(g \circ f)(x) = x$ for all $x \in \mathbb{R}$. Then the value of $g(2)$ is :

- (1) 16 (2) 8
(3) 4 (4) 2

Ans. (1)

Sol. $f(x) = x + e^{x/\xi}$
when $x = 0$,
 $g(f(x)) = f(x)$

$g(2) = 16$

$f(x) = x + e^{x/\xi}$

$g(2) = 16$

Ans = 16

Option (1)

20. Let $A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$ and $B = \begin{pmatrix} 4 & 3 \\ 2 & 1 \end{pmatrix}$.

If $\det(\text{adj}(A - B) \cdot \text{adj}(A + B)) = 2$, then $(\det(A))^2$ is equal to :

- (1) 1 (2) 49
(3) 16 (4) 36

Ans. (3)

Sol. $|\text{adj}(A - B) \cdot \text{adj}(A + B)| = 2$

$|(A - B)^T (A + B)| = 2$

$|A - B| |A + B| = 2$

$(A - B)^T = A^T - B^T$

$|A - B| = |A - B^T|$

$|A - B| = 2$

$|A - B| = 2$

$\begin{vmatrix} 1 & 2 \\ 3 & 4 \end{vmatrix} = 2$

$\begin{vmatrix} 1 & 2 \\ 3 & 4 \end{vmatrix} = 2$

$1 + 2 = 3$

$3 = 3$

$3 = 1$

$|A| = \begin{vmatrix} 1 & 2 \\ 3 & 4 \end{vmatrix} = 1 \cdot 4 - 2 \cdot 3 = -2$

$|A|^2 = 4$

SECTION-B

21. If $\lim_{x \rightarrow 1} \frac{x^2 - 1}{x - 1} = 2$, where m, n are coprime, then $am + bn$ is equal to _____

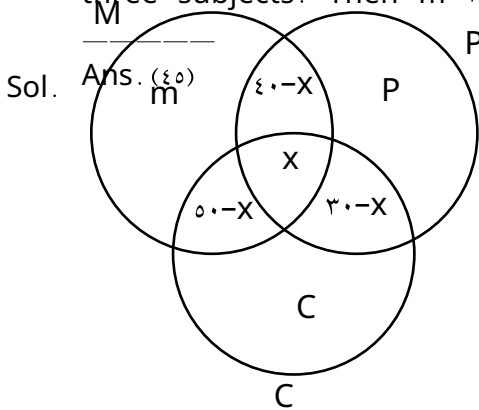
Ans. (100)

Sol. $\lim_{x \rightarrow 1} \frac{x^2 - 1}{x - 1} = 2$

$\frac{m}{n} = 2$

$am + bn = 100$

22. In a survey of 220 students of a higher secondary school, it was found that at least 120 and at most 130 students studied Mathematics; at least 80 and at most 90 studied Physics; at least 70 and at most 90 studied Chemistry; 30 studied both Physics and Chemistry; 50 studied both Chemistry and Mathematics; 40 studied both Mathematics and Physics and 10 studied none of these subjects. Let m and n respectively be the least and the most number of students who studied all the three subjects. Then m + n is equal to



$$120 \leq m + 90 - x \leq 130$$

$$80 \leq P + 70 - x \leq 90$$

$$70 \leq C + 80 - x \leq 90$$

$$m + P + C + 120 - 2x = 220$$

$$120 - x \leq 40 + 30 - x \leq 130$$

$$120 - x \leq 40 + 30 - x \leq 130$$

$$30 + 10 = 40$$

23. Let the solution $y = y(x)$ of the differential

equation $\frac{dy}{dx} - y = 1 + \sin x$ satisfy $y(0) = 1$. Then

$y\left(\frac{\pi}{2}\right) + 1$ is equal to _____

Ans. (7)

Sol. $y e^{-x} = \int e^{-x} (1 + \sin x) dx$

$$y e^{-x} = -e^{-x} (\sin x + \cos x) + C$$

$$y = -1 - (\sin x + \cos x) + C e^x$$

$$y(0) = 1 \implies C = 2$$

$$y\left(\frac{\pi}{2}\right) = -1 - 2 = -3 \implies \text{Ans} = 10 - 3 = 7$$

- If the shortest distance between the lines $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$ and $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 2$ is

$\frac{1}{\sqrt{a^2 + b^2 + c^2}}$ and $\frac{1}{\sqrt{a^2 + b^2 + c^2}}$ where $\lfloor x \rfloor$ denotes the greatest integer function, then $\lfloor x \rfloor$ is equal to _____

Ans. (4)

Sol. $\frac{1}{\sqrt{a^2 + b^2 + c^2}}$

$$\frac{1}{\sqrt{a^2 + b^2 + c^2}}$$

$$k = \frac{1}{\sqrt{a^2 + b^2 + c^2}}$$

$$k = \frac{1}{\sqrt{a^2 + b^2 + c^2}}$$

$$\frac{1}{\sqrt{a^2 + b^2 + c^2}}$$

$$\frac{1}{\sqrt{a^2 + b^2 + c^2}}$$

$$\frac{1}{\sqrt{a^2 + b^2 + c^2}}$$

$$\frac{1}{\sqrt{a^2 + b^2 + c^2}}$$

$$\frac{1}{\sqrt{a^2 + b^2 + c^2}}$$

24. Let A be a square matrix of order 2 such that $|A| = 2$ and the sum of its diagonal elements is -2 . If the points (x, y) satisfying $A + xA + yI = 0$ lie on a hyperbola, whose transverse axis is parallel to the x-axis, eccentricity is e and the length of the latus rectum is l, then $e + \frac{1}{e}$ is equal to _____

Ans. (Bouns)

NTA Ans. (20)

Sol. Given $|A| = 2$

$$\text{trace } A = -2$$

$$\text{and } A + xA + yI = 0$$

$$x = 2, y = 2$$

so, information is incomplete to determine eccentricity of hyperbola (e) and length of latus rectum of hyperbola (l)

$$26. \text{ Let } a = 1 - \frac{{}^r C_2}{2} + \frac{{}^r C_4}{2^2} - \frac{{}^r C_6}{2^3} + \dots$$

$$b = 1 - \frac{{}^r C_1}{1} + \frac{{}^r C_2}{2} - \frac{{}^r C_3}{2^2} + \frac{{}^r C_4}{2^3} - \frac{{}^r C_5}{2^4} + \dots$$

Then $\frac{b}{a}$ is equal to _____

Sol. Ans. (A)

$$f(x) = 1 + \frac{(1-x)}{1!} + \frac{(1-x)^2}{2!} + \frac{(1-x)^3}{3!} + \dots$$

$$\frac{e^{(1-x)}}{1-x} = 1 + \frac{(1-x)}{1!} + \frac{(1-x)^2}{2!} + \frac{(1-x)^3}{3!} + \dots$$

coef x in RHS : $1 + \frac{{}^r C_2}{2} - \frac{{}^r C_4}{2^2} + \dots = a$

coeff. x in L.H.S.

$$e^{(1-x)} = 1 - x + \frac{x^2}{2!} - \frac{x^3}{3!} + \dots$$

is $e^{(1-x)} = e^{(1-x)}$

$$b = 1 - \frac{{}^r C_1}{1} + \frac{{}^r C_2}{2} - \frac{{}^r C_3}{2^2} + \dots = e^{-x}$$

$$\frac{b}{a} = 1$$

27. Let A be a 3×3 matrix of non-negative real

elements such that $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$. Then the

maximum value of $\det(A)$ is _____

Ans. (27)

Sol. Let $A = \begin{bmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{bmatrix}$

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

$$a_1 + a_2 + a_3 = 3 \quad \dots (1)$$

$$b_1 + b_2 + b_3 = 3 \quad \dots (2)$$

$$c_1 + c_2 + c_3 = 3 \quad \dots (3)$$

Now,

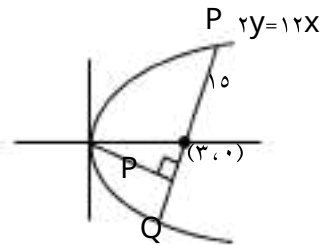
$$|A| = (a_1 b_2 c_3 + a_2 b_3 c_1 + a_3 b_1 c_2) - (a_3 b_2 c_1 + a_2 b_3 c_2 + a_1 b_3 c_3)$$

From above information, clearly $|A|_{\max} = 27$, when $a_1 = 3, b_2 = 3, c_3 = 3$

28. Let the length of the focal chord PQ of the parabola $y = 4x$ be 10 units. If the distance of PQ from the origin is p, then \sqrt{p} is equal to _____

Ans. (28)

Sol.



length of focal chord = $4a \csc \theta = 10$

$$4 \csc \theta = 10$$

$$\sin \theta = \frac{4}{5}$$

$$\tan \theta = \frac{4}{3}$$

$$\tan \theta = \frac{4}{3}$$

equation $y = 4x - 4$

$$y = 4x - 4$$

$$4x - y - 4 = 0$$

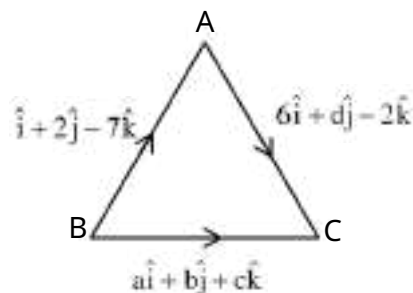
$$P = \frac{4}{\sqrt{20}}$$

$$\sqrt{p} = \frac{4}{\sqrt{20}} = \frac{2}{\sqrt{5}}$$

29. Let ABC be a triangle of area $10\sqrt{2}$ and the vectors $\vec{AB} = \hat{i} + \hat{j} + \hat{k}$, $\vec{BC} = \hat{a} + \hat{b} + \hat{c}$ and $\vec{AC} = \hat{i} + \hat{d} + \hat{k}$, $d < 0$. Then the square of the length of the largest side of the triangle ABC is

Ans. (29)

Sol.



$$\text{Area} = \frac{1}{2} \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 3 \\ d & 1 & 2 \end{vmatrix} = \frac{1}{2} \sqrt{2}$$

$$(-\hat{x} + \sqrt{2}\hat{y})\hat{i} - \hat{j}(-\sqrt{2} + \hat{x}) + \hat{k}(d - \sqrt{2})$$

$$(\sqrt{2}d - \hat{x})\hat{i} + (\hat{x} - \sqrt{2})\hat{j} + (d - \sqrt{2})\hat{k} = \sqrt{2}\hat{k}$$

$$\Rightarrow d\sqrt{2} - \sqrt{2} = 0 \Rightarrow d = 1$$

$$\Rightarrow d\sqrt{2} - \sqrt{2} = 0 \Rightarrow d = 1$$

$$\Rightarrow d\sqrt{2} - \sqrt{2} = 0 \Rightarrow d = 1$$

$$\Rightarrow d(d - \sqrt{2}) + \sqrt{2}(d - \sqrt{2}) = 0$$

$$d = \sqrt{2} \text{ or } d = 1$$

$$\Rightarrow d < 1, d = 1$$

$$(a + 1)\hat{i} + (b + 2)\hat{j} + (c - 3)\hat{k} = 3\hat{i} + 2\hat{j} - 3\hat{k}$$

$$a + 1 = 3 \Rightarrow a = 2, b + 2 = 2 \Rightarrow b = 0, c - 3 = -3 \Rightarrow c = 0$$

$$a = 2, b = 0, c = 0$$

$$|AB| = \sqrt{1^2 + 2^2 + 3^2} = \sqrt{14}$$

$$|BC| = \sqrt{2^2 + 0^2 + 0^2} = \sqrt{4} = 2$$

$$|AC| = \sqrt{1^2 + 2^2 + 3^2} = \sqrt{14}$$

$$\text{Ans. } \sqrt{14}$$

30. If $\int_0^{\pi/2} \frac{\sin^2 x}{1 + \sin x \cos x} dx = a \log_e \frac{a}{b}$, where a

$$b \in \mathbb{N}, \text{ then } a + b \text{ is equal to } \underline{\hspace{2cm}}$$

$$\text{Ans. } (1)$$

Sol. $\int_0^{\pi/2} \frac{\sin^2 x}{1 + \sin x \cos x} dx = \int_0^{\pi/2} \frac{\cos^2 x}{\sin x} dx$

$$(I_1) = \int_0^{\pi/2} \frac{\cos^2 x}{\sin x} dx$$

$$(I_1) = \int_0^{\pi/2} \frac{dx}{\tan x}$$

$$(I_1) = \int_0^{\pi/2} \frac{dx}{\tan x}$$

$$\int_0^{\pi/2} \frac{\sec^2 x}{\tan x} dx = \int_0^{\pi/2} \frac{dx}{\tan x}$$

$$\tan x = t$$

$$2 \int_0^1 \frac{dt}{t \sqrt{1-t^2}} = \frac{1}{\sqrt{1-t^2}}$$

$$I_2 = \int_0^{\pi/2} \frac{\cos^2 x}{\sin x} dx = \int_0^{\pi/2} \frac{1}{\tan x} dx$$

$$I_1 = \int_0^{\pi/2} \frac{1}{\tan x} dx = \int_0^{\pi/2} \frac{1}{\tan x} dx$$

$$a = 2, b = 1$$

$$\text{Ans. } 1$$

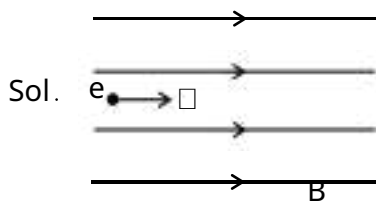
PHYSICS

SECTION-A

31. An electron is projected with uniform velocity along the axis inside a current carrying long solenoid. Then :

- the electron will be accelerated along the axis.
- the electron will continue to move with uniform velocity along the axis of the solenoid.
- the electron path will be circular about the axis.
- the electron will experience a force at 90° to the axis and execute a helical path.

Ans. (2)



Sol.

Since $\vec{v} \parallel \vec{B}$ so force on electron due to magnetic field is zero. So it will move along axis with uniform velocity.

32. The electric field in an electromagnetic wave is

given by $\vec{E} = \hat{i} \cdot \cos \left(\frac{z}{c} - \omega t \right)$. The

magnetic field induction of this wave is (in SI unit):

- $\vec{B} = \hat{i} \cdot \cos \left(\frac{z}{c} - \omega t \right) \frac{Z}{c}$
- $\vec{B} = \hat{j} \cdot \cos \left(\frac{z}{c} - \omega t \right) \frac{Z}{c}$
- $\vec{B} = \frac{k \cdot \hat{i}}{c} \cos \left(\frac{z}{c} - \omega t \right) \frac{Z}{c}$
- $\vec{B} = \hat{j} \cdot \frac{\hat{i}}{c} \cos \left(\frac{z}{c} - \omega t \right) \frac{Z}{c}$

Ans. (2)

TEST PAPER WITH SOLUTION

Sol. $\vec{E} = \hat{i} \cdot \cos \left(\frac{z}{c} - \omega t \right)$

\vec{E} is along +x direction

\vec{v} is along +z direction

So direction of \vec{B} will be along +y and magnitude

of \vec{B} will be $\frac{E}{c}$

So answer is $\hat{j} \cdot \cos \left(\frac{z}{c} - \omega t \right) \frac{Z}{c}$

33. Which of the following nuclear fragments corresponding to nuclear fission between neutron and uranium isotope is correct:

- ${}^{141}_{54}\text{Ba} + {}^{92}_{36}\text{Kr} + 3\text{n}$
- ${}^{141}_{54}\text{Xe} + {}^{92}_{38}\text{Sr} + 3\text{n}$

Ans. (2) ${}^{141}_{54}\text{Sb} + {}^{92}_{41}\text{Nb} + 3\text{n}$ (2) ${}^{141}_{56}\text{Ba} + {}^{92}_{36}\text{Kr} + 3\text{n}$
Sol. Balancing mass number and atomic number

${}^{235}_{92}\text{U} + {}^1_0\text{n} \rightarrow {}^{141}_{54}\text{Ba} + {}^{92}_{36}\text{Kr} + 3\text{n}$

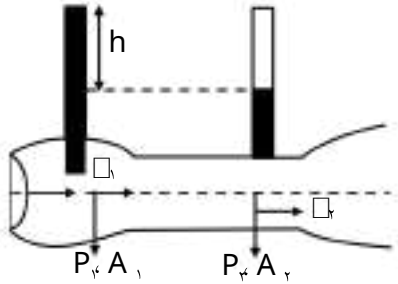
34. In an experiment to measure focal length (f) of convex lens, the least counts of the measuring scales for the position of object (u) and for the position of image (v) are Δu and Δv , respectively. The error in the measurement of the focal length of the convex lens will be :

- $\frac{\Delta u}{u} + \frac{\Delta v}{v}$
- $f \left(\frac{\Delta u}{u} + \frac{\Delta v}{v} \right)$
- $\frac{f}{2} \left(\frac{\Delta u}{u} + \frac{\Delta v}{v} \right)$
- $f \left(\frac{\Delta u}{u} + \frac{\Delta v}{v} \right)^2$

Ans. (2)

Sol. $\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$
 $d\left(\frac{1}{f}\right) = d\left(\frac{1}{v}\right) - d\left(\frac{1}{u}\right)$
 $\frac{1}{f^2} df = -\frac{1}{v^2} dv - \frac{1}{u^2} du$
 $df = f^2 \left(\frac{dv}{v^2} + \frac{du}{u^2} \right)$
 $df = f \left(\frac{dv}{v} + \frac{du}{u} \right)$

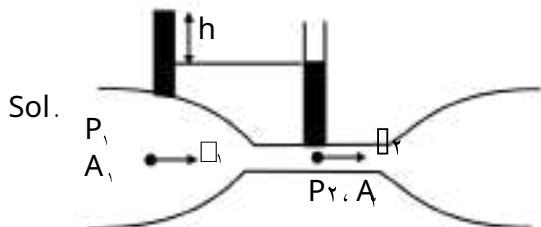
35. Given below are two statements :
- Statement I : When speed of liquid is zero everywhere, pressure difference at any two points depends on equation $P_1 - P_2 = \rho g (h_2 - h_1)$
- Statement II : In venturi shown
- $$2gh = \frac{\rho}{2} (v_1^2 - v_2^2)$$



In the light of the above statements, choose the most appropriate answer from the options given below.

- (1) Both Statement I and Statement II are correct.
- (2) Statement I is incorrect but Statement II is correct.
- (3) Both Statement I and Statement II are incorrect.
- (4) Statement I is correct but Statement II is incorrect.

Ans. (4)



Applying Bernoulli's equation

$$P_1 + \rho gh_1 + \frac{1}{2} \rho v_1^2 = P_2 + \rho gh_2 + \frac{1}{2} \rho v_2^2$$

h_1 & h_2 are height of point from any reference level.

Given $v_1 = v_2 = 0$ (for statement-1)

$$P_1 - P_2 = \rho g (h_2 - h_1)$$

For statement-2

$$P_1 + \frac{1}{2} \rho v_1^2 = P_2 + \frac{1}{2} \rho v_2^2$$

$$P_1 - P_2 = \rho gh$$

$$P_1 + \frac{1}{2} \rho v_1^2 = P_2 + \frac{1}{2} \rho v_2^2$$

$$\rho gh = \frac{1}{2} \rho (v_2^2 - v_1^2)$$

$$2gh = v_2^2 - v_1^2$$

Hence answer (4)

36. The resistances of the platinum wire of a platinum resistance thermometer at the ice point and steam point are $\lambda \Omega$ and $\gamma \Omega$ respectively. After inserting in a hot bath of temperature $\xi \text{ } ^\circ\text{C}$, the resistance of platinum wire is :

- (1) $\gamma \Omega$ (2) $\gamma \lambda \Omega$ (3) $\lambda \Omega$ (4) $\gamma \Omega$

Ans. (2)

Sol. Given $R_0 = \lambda \Omega$, $R_{100} = \gamma \Omega$

$$\lambda R_{100} = R_0 (1 + \alpha T)$$

$$\text{Also, } R_{\xi} = R_0 (1 + \alpha T)$$

$$\lambda \gamma = \lambda (1 + \alpha \times 100) \quad \lambda \gamma \Omega = \frac{\lambda}{\xi}$$

$$R_{\xi} = \lambda (1 + \alpha \xi) = \lambda (1 + \gamma) = \gamma \lambda \Omega$$

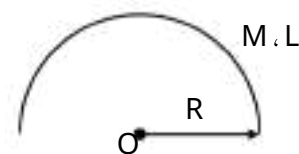
Hence option (2)

37. A metal wire of uniform mass density having length L and mass M is bent to form a semicircular arc and a particle of mass m is placed at the centre of the arc. The gravitational force on the particle by the wire is:

- (1) $\frac{GMm}{\sqrt{L^2}}$ (2) $\frac{GMm}{L^2}$
- (3) $\frac{GmM}{L^2}$ (4) $\frac{\sqrt{2}GmM}{L^2}$

Ans. (4)

Sol.



We have $R = \frac{L}{\dots}$

$$g = \frac{\frac{2GM}{L}}{R} = \frac{2GM}{L^2}$$

$$F_m = mg = \frac{2GMm}{L^2}$$

Hence option (ε)

38. On Celsius scale the temperature of body increases by 5°C . The increase in temperature on Fahrenheit scale is:

- (1) 9°F
- (2) 18°F
- (3) 27°F
- (4) 50°F

Ans. (3)

Sol. We know that per $^\circ\text{C}$ change is equivalent to 1.8° change in $^\circ\text{F}$.

$\therefore 5^\circ$ change on Celsius scale will correspond to 9° change on Fahrenheit scale.

Hence option (3)

39. An effective power of a combination of 5 identical convex lenses which are kept in contact along the principal axis is 20 D . Focal length of each of the convex lens is:

- (1) 20 cm
- (2) 50 cm
- (3) 500 cm
- (4) 20 cm

Ans. (1)

Sol. We know that $P_{eq} = \sum P_i$

\therefore given all lenses are identical

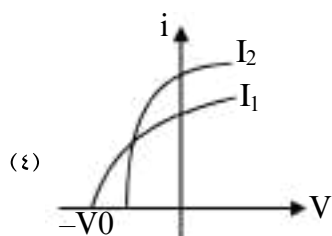
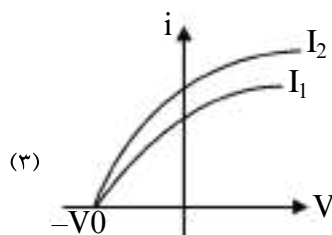
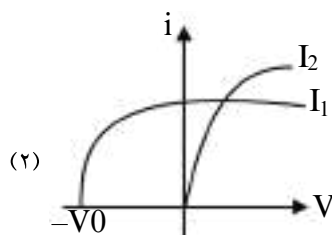
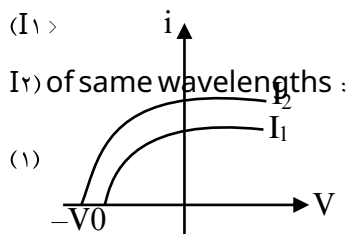
$$20\text{ D} = 5D$$

$$P = 4\text{ D}$$

$$\frac{1}{f} = 5 \times \frac{1}{f} \Rightarrow f = 20\text{ cm}$$

Hence option (1)

40. Which figure shows the correct variation of applied potential difference (V) with photoelectric current (I) at two different intensities of light



Ans. (3)

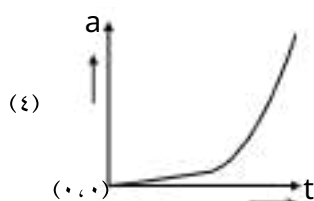
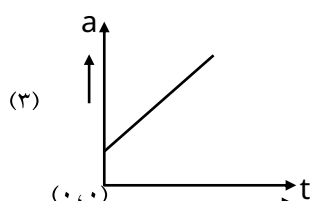
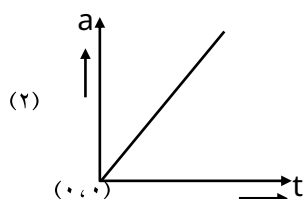
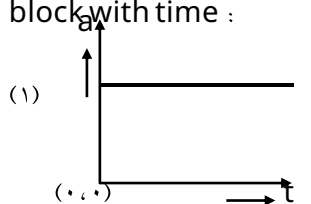
Sol. Given lights are of same wavelength.

Hence stopping potential will remain same.

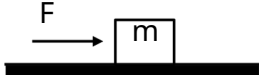
Since $I_2 < I_1$, hence saturation current corresponding to I_2 will be greater than that corresponding to I_1 .

Hence option (3)

ε 1. A wooden block, initially at rest on the ground, is pushed by a force which increases linearly with time t . Which of the following curve best describes acceleration of the block with time :



Ans. (2)

Sol. 

$$F = ma \Rightarrow a = \frac{F}{m} = \frac{k}{t}$$

a vs t will be a straight line passing through origin. Since option (2).

ε 2. If a rubber ball falls from a height h and rebounds upto the height of $h/2$. The percentage loss of total energy of the initial system as well as velocity ball before it strikes the ground, respectively, are :

- (1) 50%, $\sqrt{\frac{gh}{2}}$ (2) 50%, \sqrt{gh}
 (3) 40%, \sqrt{gh} (4) 50%, $\sqrt{\frac{gh}{2}}$

Ans. (4)

Sol. Velocity just before collision = $\sqrt{2gh}$

Velocity just after collision = $\sqrt{2g \cdot \frac{h}{2}}$

$$\Delta KE = \frac{1}{2} m \sqrt{2gh} - \frac{1}{2} m \sqrt{gh}$$

$$= \frac{1}{2} mgh$$

\therefore loss in energy

$$= \frac{\Delta KE}{KE_i} = \frac{\frac{1}{2} mgh}{\frac{1}{2} m \sqrt{2gh}} = 50\%$$

Hence option (4)

ε 3. The equation of stationary wave is :

$$y = a \sin \frac{2\pi nt}{\lambda} \cos \frac{2\pi x}{\lambda}$$

Which of the following is NOT correct

- (1) The dimensions of nt is L
 (2) The dimensions of n is LT
 (3) The dimensions of n/λ is T
 (4) The dimensions of x is L

Ans. (3)

Sol. Comparing the given equation with standard

equation of standing $\frac{2\pi nt}{\lambda} = k$ & $\frac{2\pi x}{\lambda} = k$

$$\frac{n}{\lambda} = \frac{T}{\lambda} = \frac{1}{T}$$

$$nt = \frac{\lambda}{T} = L$$

$$n = \frac{\lambda}{T} = LT$$

$$x = \frac{\lambda}{2} = L$$

Hence option (3)

44. A body travels 102.0 m in n^{th} second and 110.0 m in $(n+2)^{\text{th}}$ second. The acceleration is :

- (1) 9 m/s^2 (2) 6.20 m/s^2
(3) 12.0 m/s^2 (4) 0 m/s^2

Ans. (2)

Sol. Given: $102.0 = u + \frac{a}{2} n^2$ &

$$110 = u + \frac{a}{2} (n+2)^2$$

$$102.0 = u + \frac{a}{2} n^2$$

$$110 = u + \frac{a}{2} n^2 + 2an + 2a$$

$$12.0 = 2an + 2a \Rightarrow a = 6.20 \text{ m/s}^2$$

Hence option (2)

45. To measure the internal resistance of a battery,

potentiometer is used. For $R = 10 \Omega$, the balance

point is observed at $l = 50.0 \text{ cm}$ and for $R = 1 \Omega$ the balance point is observed at $l = 40.0 \text{ cm}$. The

internal resistance of the battery is approximately :

Ans. (4)

Sol. Let potential gradient be x .

$$i \times 10 = x \times 50.0 = E - ir_s$$

$$500x = E - 50x r_s$$

Also,

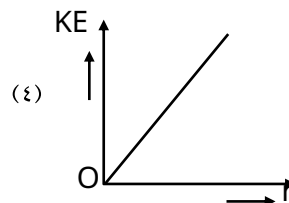
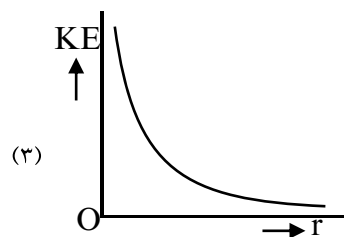
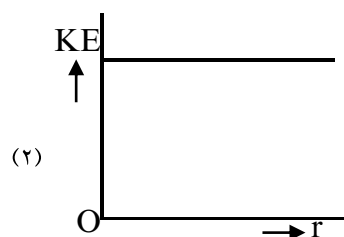
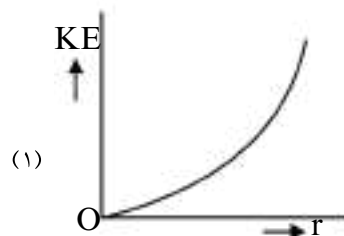
$$i' \times 1 = x \times 40.0 = E - i' r_s$$

$$400x = E - 400x r_s$$

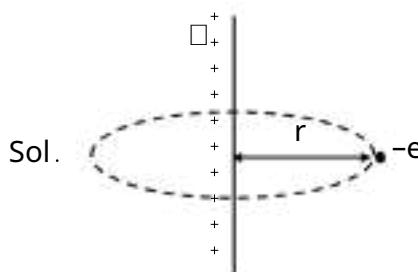
$$100x = 350x r_s \Rightarrow r_s = \frac{10}{350} = 0.0286 \approx 0.03 \Omega$$

Hence option (4)

46. An infinitely long positively charged straight thread has a linear charge density $\lambda \text{ Cm}^{-1}$. An electron revolves along a circular path having axis along the length of the wire. The graph that correctly represents the variation of the kinetic energy of electron as a function of radius of circular path from the wire is :



Ans. (2)



Electric field E at a distance r due to infinite long wire is $E = \frac{\lambda k}{r}$

Force of electron $F = eE$

$$F = \frac{1}{4\pi\epsilon_0} \frac{e^2}{r^2}$$

$$F = \frac{1}{4\pi\epsilon_0} \frac{e^2}{r^2}$$

This force will provide required centripetal force

$$F = \frac{mv^2}{r} = \frac{1}{4\pi\epsilon_0} \frac{e^2}{r^2}$$

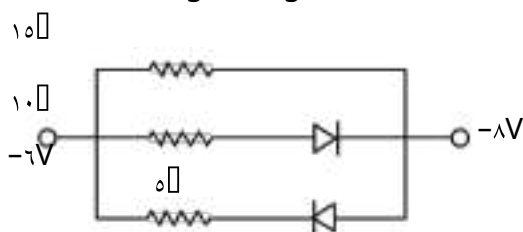
$$v = \sqrt{\frac{1}{4\pi\epsilon_0} \frac{e^2}{mr}}$$

$$KE = \frac{1}{2} mv^2 = \frac{1}{2} m \left(\sqrt{\frac{1}{4\pi\epsilon_0} \frac{e^2}{mr}} \right)^2$$

$$= \frac{1}{4\pi\epsilon_0} \frac{e^2}{2r}$$

This is constant so option (r) is correct.

Q. The value of net resistance of the network as shown in the given figure is :



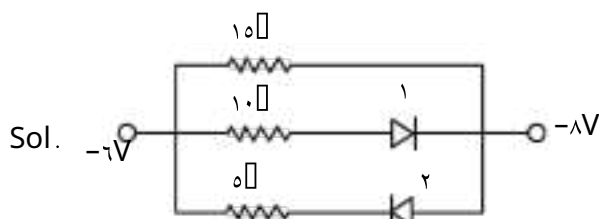
(1) $\frac{10}{3}$ ohm

(2) $\frac{10}{2}$ ohm

(3) 6 ohm

(4) $\frac{10}{4}$ ohm

Ans. (3)

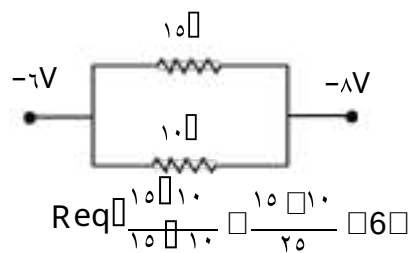


Diode 1 is in reverse bias

So current will not flow in branch of 1 diode. So we can assume it to be broken wire.

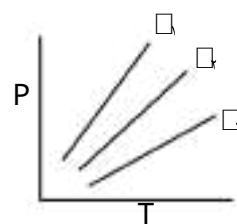
Diode 2 is in forward bias

So it will behave like conducting wire. So new circuit will be



Correct answer (3)

Q. P-T diagram of an ideal gas having three different densities ρ_1, ρ_2, ρ_3 (in three different cases) is shown in the figure. Which of the following is correct :



(1) $\rho_1 > \rho_2$

(2) $\rho_1 < \rho_2$

(3) $\rho_1 > \rho_3$

(4) $\rho_1 = \rho_2 = \rho_3$

Ans. (2)

Sol. For ideal gas

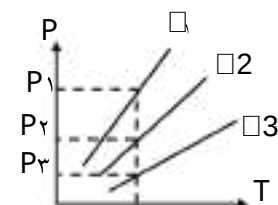
$$PV = nRT$$

$$PV = \frac{M}{M_0} RT$$

$$P = \frac{M}{M_0} \frac{RT}{V}$$

$$P = \frac{RT}{M}$$

(Where m is mass of gas and M is molecular mass of gas)



for same temperature $P_1 < P_2 < P_3$

$$\text{So } \rho_1 < \rho_2 < \rho_3$$

So correct answer is (2)

49. The co-ordinates of a particle moving in x-y plane are given by :

$$x = r + \epsilon t, y = r t + \lambda t^2$$

The motion of the particle is :

- (1) non-uniformly accelerated.
- (2) uniformly accelerated having motion along a straight line.
- (3) uniform motion along a straight line.
- (4) uniformly accelerated having motion along a parabolic path.

Ans. (4)

Sol. $x = r + \epsilon t$

$$\frac{dx}{dt} = v_x = \epsilon$$

$$\frac{dv_x}{dt} = a_x = 0$$

$$y = r t + \lambda t^2$$

$$\frac{dy}{dt} = v_y = r + 2\lambda t$$

$$\frac{dv_y}{dt} = a_y = 2\lambda$$

the motion will be uniformly accelerated motion.

For path

$$x = r + \epsilon t$$

$$t = \frac{x - r}{\epsilon}$$

Put this value of t in equation of y

$$y = r \left(\frac{x - r}{\epsilon} \right) + \lambda \left(\frac{x - r}{\epsilon} \right)^2$$

this is a quadratic equation so path will be parabola.

Correct answer (4)

In an ac circuit, the instantaneous current is zero, when the instantaneous voltage is maximum. In this case, the source may be connected to : A. pure inductor. B. pure capacitor. C. pure resistor. D. combination of an inductor and capacitor. Choose the correct answer from the options given below : (1) A, B and C only (2) A and B only

(3) B, C and D only

(4) A, B and D only

Ans. (4)

Sol. This is possible when phase difference is $\frac{\pi}{2}$

between current and voltage so correct answer will be (4)

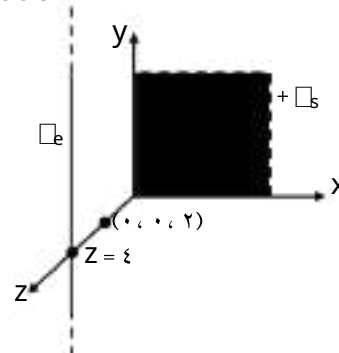
SECTION-B

51. An infinite plane sheet of charge having uniform surface charge density $+\sigma$ C/m² is placed on x-y plane. Another infinitely long line charge having

uniform linear charge density $+\lambda$ C/m is placed at $z = \epsilon$ m plane and parallel to y-axis. If the magnitude of electric field at point (1, 1, 1) is n times that of line charge is

The value of n is _____.

Ans. (4) the ratio of magnitudes of electric field values due



Sol.

$$E_s = \frac{\sigma}{2\epsilon_0}$$

$$E_l = \frac{\lambda}{2\pi\epsilon_0 r}$$

$$\frac{E_s}{E_l} = \frac{\sigma}{\lambda} \cdot \frac{2\pi r}{2}$$

$$n = 16$$

Q2. A hydrogen atom changes its state from $n = 3$ to $n = 2$. Due to recoil, the percentage change in the wave length of emitted light is approximately _____.

Given $Rhc = 13.6 \text{ eV}$, $hc = 1242 \text{ eV nm}$,

$h = 6.6 \times 10^{-34} \text{ Js}$, mass of the hydrogen atom

$= 1.6 \times 10^{-27} \text{ kg}$

Ans. (7)

Sol. $E = 13.6 \left(\frac{1}{2^2} - \frac{1}{3^2} \right) = 1.9 \text{ eV}$

$E = \frac{hc}{\lambda}$

$\lambda = \frac{hc}{E}$

$P_i = P_f$

$\frac{h}{\lambda} = mv$

$v = \frac{h}{m\lambda}$

$E = \frac{1}{2}mv^2 = \frac{hc}{\lambda}$

$\lambda = \frac{hc}{2E} = \frac{hc}{2 \times 1.9}$

Now

$E = \frac{hc}{\lambda}$

$2E = \frac{hc}{\lambda}$

$\lambda = \frac{hc}{2E}$

$\lambda = \frac{hc}{2E}$

$\lambda = \frac{hc}{2E}$

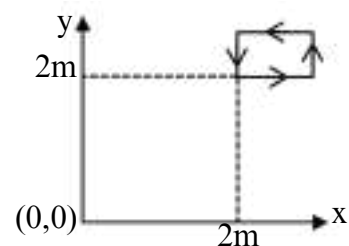
$\lambda = \frac{hc}{2E}$

$\lambda = \frac{1242}{2 \times 1.9} = 322.1 \text{ nm}$

% change = 1%

Correct answer 7

Q3. The magnetic field existing in a region is given by $\vec{B} = 0.2\hat{x} + 0.2\hat{y} \text{ kT}$. A square loop of edge 0.5 cm carrying 1.0 A current is placed in x-y plane with its edges parallel to the x-y axes, as shown in figure. The magnitude of the net magnetic force experienced by the loop is _____ mN.



Ans. (50)

Sol. Force on segment parallel to x-axis will cancel each other. Hence F_{net} will be due to portion parallel to y-axis.

$F = 1.0 \times 0.5 \times 10^{-2} \times 0.2 - 1.0 \times 0.5 \times 10^{-2} \times 0.2$

$= 1.0 \times 0.5 \times 10^{-2}$

$= 0.50 \times 10^{-2}$

$= 0.01 \text{ N}$

$= 50 \text{ mN}$

Q4. An alternating current at any instant is given by

$i = 5 \sin \sqrt{2} \pi t$ A. The rms value of the current is _____ A.

Ans. (A)

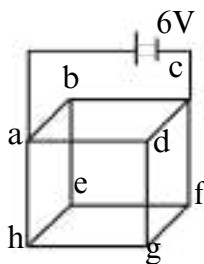
Sol. $I_{rms} = \frac{1}{\sqrt{2}} \sqrt{\int_0^T i^2 dt}$

$$I_{rms} = \frac{1}{\sqrt{2}} \sqrt{\int_0^T 25 \sin^2 \sqrt{2} \pi t dt}$$

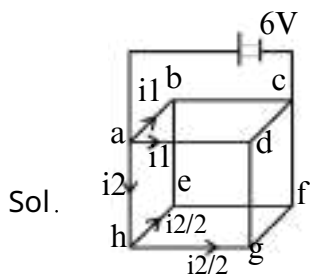
$$= \frac{1}{\sqrt{2}} \sqrt{25 \times \frac{T}{2}}$$

$$= 5 A$$

Q5. Twelve wires each having resistance r are joined to form a cube. A battery of V emf is joined across point a and c. The voltage difference between e and f is _____ V.



Ans. (1)



Sol.

From symmetry, current through e-b & g-d =

$$R_{eq} = \frac{r}{2} \parallel R = \frac{r}{4}$$

Current through battery $= \frac{V}{R_{eq}} = \frac{V}{\frac{r}{4}} = \frac{4V}{r}$

$$i_1 = \frac{4V}{r} \times \frac{1}{4} = \frac{V}{r}$$

V across e-f $= i_2 \times R = \frac{V}{r} \times \frac{r}{2} = \frac{V}{2}$

Q6. A soap bubble is blown to a diameter of y cm. 36960 erg of work is done in blowing it further. If surface tension of soap solution is ϵ dyne/cm then the new radius is _____ cm. Take $\pi = \frac{22}{7}$

Ans. (v)

Sol. $W = \Delta U = S \Delta A$

$$36960 \text{ erg} = \frac{\epsilon \cdot \text{dyne}}{\text{cm}} \times 4\pi r^2 \Delta r$$

$$r = y \text{ cm}$$

Q7. Two wavelengths λ_1 and λ_2 are used in Young's double slit experiment $\lambda_1 = 450 \text{ nm}$ and $\lambda_2 = 600 \text{ nm}$. The minimum order of fringe produced by λ_2 which overlaps with the fringe produced by λ_1 is n . The value of n is _____.

Ans. (9)

Sol. $n\lambda_2 = m\lambda_1$

$$\frac{n}{m} = \frac{\lambda_1}{\lambda_2} = \frac{450}{600} = \frac{3}{4}$$

$$n = 9$$

Q8. An elastic spring under tension of r N has a length a . Its length is b under tension r N. For its length $(ra - rb)$, the value of tension will be _____ N.

Ans. (o)

Sol. $r = K(a - l_0)$

$$r = K(b - l_0) \quad T = K(ra - rb -$$

$$l_0) \quad T = K(r(a - l_0) - r(b - l_0))$$

$$= K \left(\frac{3}{4}r - \frac{2}{4}r \right) = \frac{r}{4}$$

$$= 9 - 9 = 0$$

$$= 0 \text{ N}$$

59. Two forces are acting on a body. One force has magnitude thrice that of the other force and the resultant of the two forces is equal to the force of larger magnitude. The angle between F

and F₂ is $\cos^{-1}\left(\frac{1}{n}\right)$. The value of |n| is _____.

Ans. (6)

Sol. \vec{F}_1, \vec{F}_2

$$|\vec{F}_R| = |\vec{F}_1| = 3F$$

$$F_R^2 = F_1^2 + F_2^2 + 2F_1F_2\cos\theta$$

$$9F^2 = F^2 + F^2 + 2F^2\cos\theta$$

$$\cos\theta = \frac{1}{2}$$

$$\theta = \cos^{-1}\left(\frac{1}{2}\right)$$

$$n = -2$$

$$|n| = 2$$

60. A solid sphere and a hollow cylinder roll up without slipping on same inclined plane with same initial speed v. The sphere and the cylinder reaches

upto maximum heights h_1 and h_2 above the initial level. The ratio $h_1 : h_2$ is. The value of n is _____.

Ans. (4)

Sol Gain in P.E. = Loss in K.E.

$$mgh = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$$

$$h = \frac{v^2}{2g} \left(1 + \frac{I}{mR^2}\right)$$

$$\frac{h_1}{h_2} = \frac{1 + \frac{I_1}{mR^2}}{1 + \frac{I_2}{mR^2}} = \frac{1 + \frac{2}{5}}{1 + \frac{1}{2}} = \frac{7}{5}$$

$$n = 5$$

CHEMISTRY

SECTION-A

61. What pressure (bar) of H_2 would be required to make emf of hydrogen electrode zero in pure water at $25^\circ C$?

(1) 10^{-14} (2) 10^{-7} (3) 1 (4) 0.0

NTA Ans. (3)

Sol. $2e^- + 2H^+(aq) \rightleftharpoons H_2(g)$

$$E = E^\circ - \frac{0.059}{n} \log \frac{P_{H_2}}{[H^+]^2}$$

$$0 = 0 - \frac{0.059}{2} \log \frac{P_{H_2}}{(10^{-7})^2}$$

$$\log \frac{P_{H_2}}{(10^{-7})^2} = 0$$

$$\frac{P_{H_2}}{10^{-14}} = 1$$

$$P_{H_2} = 10^{-14} \text{ bar}$$

62. The correct sequence of ligands in the order of decreasing field strength is :

(1) $CO > H_2O > F^- > S^{2-}$

(2) $OH^- > F^- > NH_3 > CN^-$

(3) $NCS^- < EDTA^{4-} < CN^- < CO$

(4) $S^{2-} < OH^- < EDTA^{4-} < CO^{4-}$

Ans. (1)

Sol. According to spectrochemical series ligand field strength is $CO > H_2O > F^- > S^{2-}$

TEST PAPER WITH SOLUTION

63. Match List -I with List II:

List - I	List - II
Mechanism steps	Effect
$\begin{array}{c} \square \\ \\ NH \end{array}$	$\begin{array}{c} \square \\ \\ NH_2 \end{array}$
	(I) - E effect

(A)

(B) $+H^+$ $\begin{array}{c} H \\ | \\ \square \end{array}$ (II) - R effect

(C) $+CN^-$ $\begin{array}{c} - \\ | \\ CN \end{array}$ (III) + E effect

(D) $O = N - \begin{array}{c} \square \\ | \\ O \end{array} - \begin{array}{c} \square \\ | \\ N \end{array} - O$ (IV) + R effect

Choose the correct answer from the options given below :

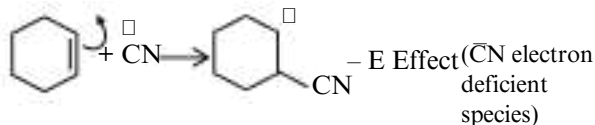
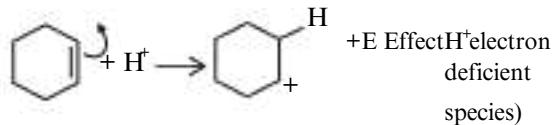
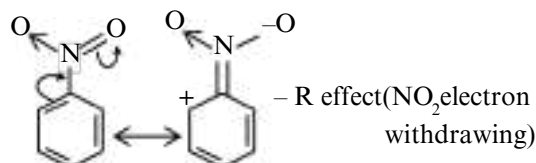
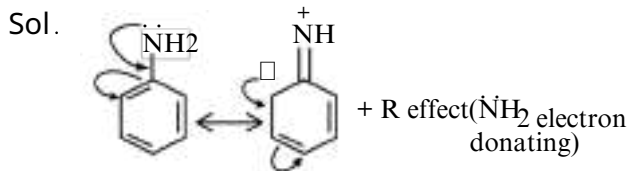
(1) (A) - (IV), (B) - (III), (C) - (I), (D) - (II)

(2) (A) - (III), (B) - (I), (C) - (II), (D) - (IV)

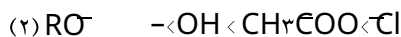
(3) (A) - (II), (B) - (IV), (C) - (III), (D) - (I)

(4) (A) - (I), (B) - (II), (C) - (IV), (D) - (III)

Ans. (1)



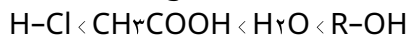
74. What will be the decreasing order of basic strength of the following conjugate bases :



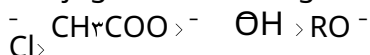
Ans. (2)

Sol. Strong acid have weak conjugate base

Acidic strength :



Conjugate base strength :



75. In the precipitation of the iron group (III) in qualitative analysis, ammonium chloride is added before adding ammonium hydroxide to :

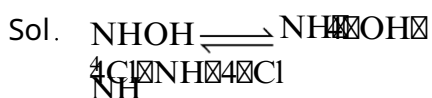
(1) prevent interference by phosphate ions

(2) decrease concentration of OH⁻ ions

(3) increase concentration of Cl⁻ ions

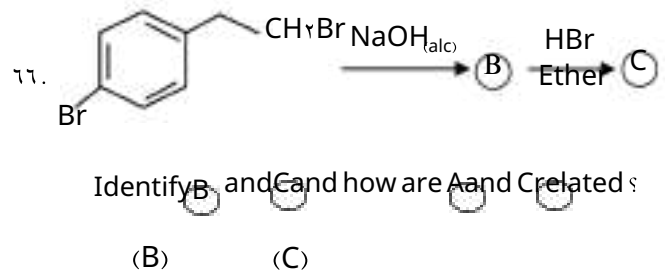
(4) increase concentration of NH₄⁺ ions

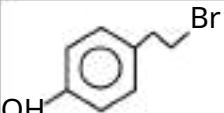
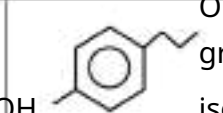
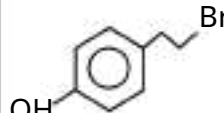
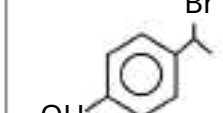
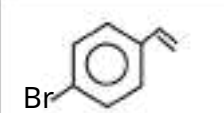
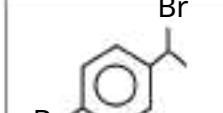
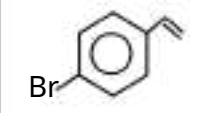
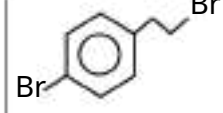
Ans. (2)



Due to common ion effect of NH₄⁺

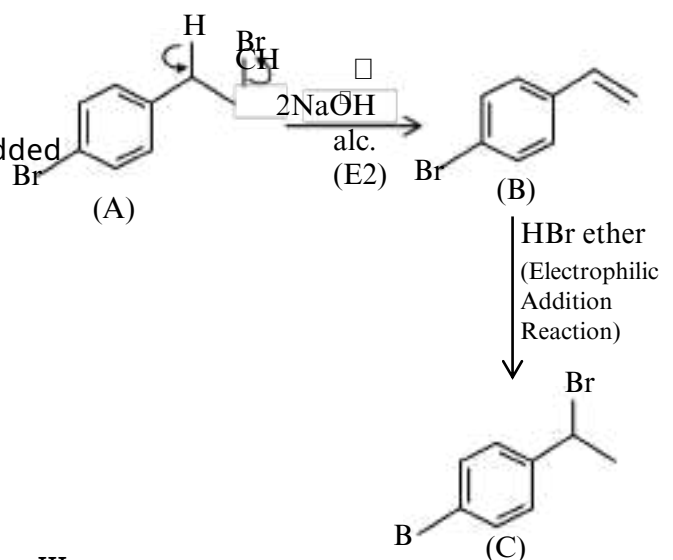
OH⁻ decreases in such extent that only group-III cation can be precipitated, due to their very low K_{sp} in the range of 10⁻³⁰.



(1)			OH functional group isomers
(2)			Derivative
(3)			position isomers
(4)			chain isomers

Ans. (3)

Sol.



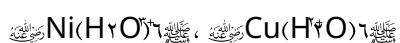
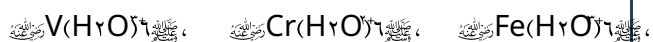
A and C are position isomer.

77. One of the commonly used electrode is calomel electrode. Under which of the following categories calomel electrode comes ?
 (1) Metal – Insoluble Salt – Anion electrodes
 (2) Oxidation – Reduction electrodes
 (3) Gas – Ion electrodes
 (4) Metal ion – Metal electrodes

Ans. (1)

Sol. Theory based

78. Number of complexes from the following with even number of unpaired d electrons is _____.



Given atomic numbers : V = 23, Cr = 24, Fe = 26,

Ni = 28, Cu = 29

(1) 2

(2) 4

(3) 5

(4) 1

Ans. (1)

Sol. $V(H_2O)_6^{3+}$: d^2sp^3

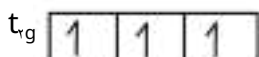
$23V$: $[Ar] 3d^3 4s^2$

V^{3+} : $[Ar] 3d^3$, $n = 3$ (even number of unpaired e)

$Cr(H_2O)_6^{3+}$: d^3sp^3

$24Cr$: $[Ar] 3d^5 4s^1$

Cr^{3+} : $[Ar] 3d^3$, $n = 3$ (even number of unpaired e)



$Fe(H_2O)_6^{3+}$: d^5sp^3

$26Fe$: $[Ar] 3d^6 4s^2$

$n = 6$ (odd number of unpaired e)

$Ni(H_2O)_6^{2+}$: d^8sp^3

$28Ni$: $[Ar] 3d^8 4s^2$

Ni^{2+} : $[Ar] 3d^8$, $n = 3$ (odd number of unpaired e)

$Cu(H_2O)_6^{2+}$: d^9sp^3

$29Cu$: $[Ar] 3d^{10} 4s^1$

$n = 1$ (odd number of unpaired e)

79. Which one of the following molecules has maximum dipole moment ?

(1) NF_3

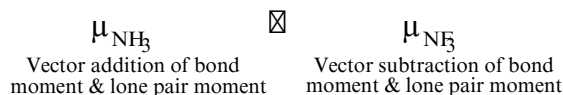
(2) CH_4

(3) NH_3

(4) PF_5

Ans. (3)

Sol. CH_4 & PF_5 , $\mu_{net} = 0$ (non polar)



80. Number of molecules / ions from the following in which the central atom is involved in sp^2 hybridization is _____.

NO_2^+ , BCl_3 , ClO_2^+ , ClO_2^-

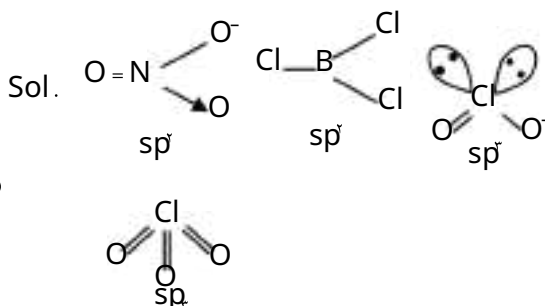
(1) 2

(2) 4

(3) 3

(4) 1

Ans. (1)



81. Which among the following is incorrect statements?

(1) Electromeric effect dominates over inductive effect

(2) The electromeric effect is a temporary effect

(3) The organic compound shows electromeric effect in the presence of the reagent only

(4) Hydrogen ion (H) shows negative electromeric effect

Ans. (4)

Sol. Hydrogen ion (H) shows positive electromeric effect.

v2. Given below are two statements :

Statement I : Acidity of α -hydrogens of aldehydes and ketones is responsible for Aldol reaction.

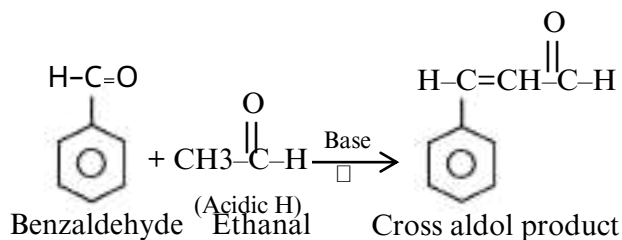
Statement II : Reaction between benzaldehyde and ethanal will NOT give Cross - Aldol product.

In the light of above statements, choose the most appropriate answer from the options given below.

- (1) Both Statement I and Statement II are correct.
 (2) Both Statement I and Statement II are incorrect.
 (3) Statement I is incorrect but Statement II is correct.
 (4) Statement I is correct but Statement II is incorrect.

Ans. (4)

Sol. Aldehyde and ketones having acidic α -hydrogen show aldol reaction



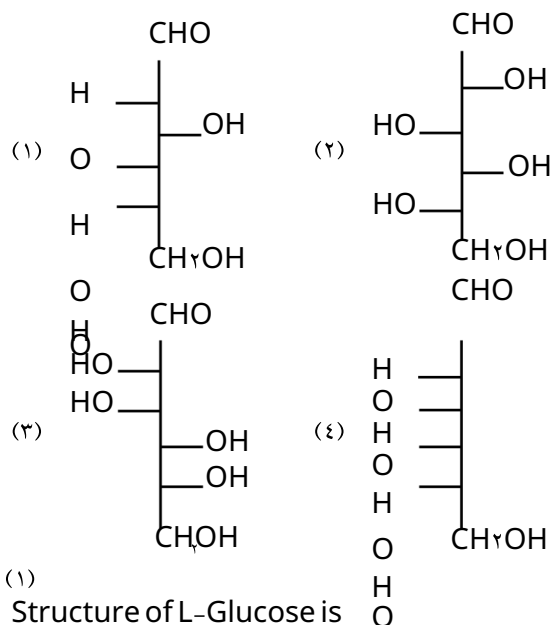
v3. Which of the following nitrogen containing compound does not give Lassaigne's test :

- (1) Phenyl hydrazine (2) Glycine
 (3) Urea (4) Hydrazine

Ans. (4)

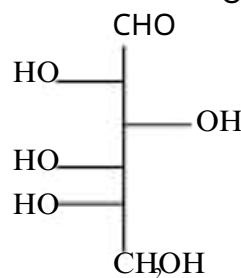
Sol. Hydrazine ($\text{NH}_2\text{--NH}_2$) have no carbon so does not show Lassaigne's test.

v4. Which of the following is the correct structure of L-Glucose :



Ans. (1)

Sol. Structure of L-Glucose is



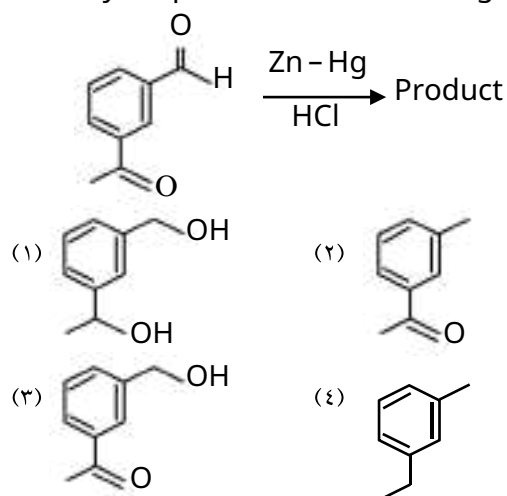
v5. The element which shows only one oxidation state other than its elemental form is :

- (1) Cobalt (2) Scandium
 (3) Titanium (4) Nickel

Ans. (2)

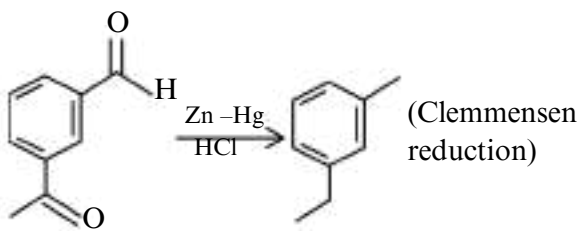
Sol. Co, Ti, Ni can show +2, +3 and +4 oxidation state. But 'Sc' only shows +3 stable oxidation state.

v6. Identify the product in the following reaction :



Ans. (4)

Sol.



vii. Number of elements from the following that CANNOT form compounds with valencies which match with their respective group valencies is _____.

B, C, N, S, O, F, P, Al, Si

(1) 7 (2) 0 (3) 6 (4) 3

Ans. (4)

Sol. N, O, F can't extend their valencies upto their group number due to the non-availability of vacant $2d$ like orbital.

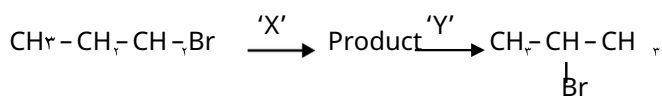
viii. The Molarity (M) of an aqueous solution containing 5.85 g of NaCl in 500 mL water is : (Given : Molar Mass Na : 23 and Cl : 35.5 gmol)

(1) 2.0 (2) 0.2 (3) 2 (4) 2

Ans. (2)

Sol. $M = \frac{n_{NaCl}}{V_{sol}(in L)}$
 5.85 M
 $58.5 \frac{g}{mol} \times 0.01 \text{ L} = 0.2 \text{ M}$

ix. Identify the correct set of reagents or reaction conditions 'X' and 'Y' in the following set of transformation.



(1) X = conc. alc. NaOH, $80^\circ C$, Y = $Br_2/CHCl_3$
 (2) X = dil. aq. NaOH, $25^\circ C$, Y = HBr/acetic acid
 (3) X = conc. alc. NaOH, $80^\circ C$, Y = HBr/acetic acid
 (4) X = dil. aq. NaOH, $25^\circ C$, Y = $Br_2/CHCl_3$

Ans. (3)

Sol. $CH_3-CH_2-CH_2-Br \xrightarrow{KOH, alc., 80^\circ C} CH_3-CH=CH_2$
 $CH_3-CH=CH_2 \xrightarrow{HBr, Acetic acid} CH_3-CH_2-CH_2-Br$

x. The correct order of first ionization enthalpy values of the following elements is : (A) O < C < Be < E < B Choose the correct answer from the options given below : (1) B > D > C > E > A < (2) C > E > A > B > D

(2) E > C > A > B > D (3) A > B > D > C > E

Ans. (2)

Sol. Correct order of I.E

Li > B > Be > C > O > N > F > Ne

$E > C > A > B > D$

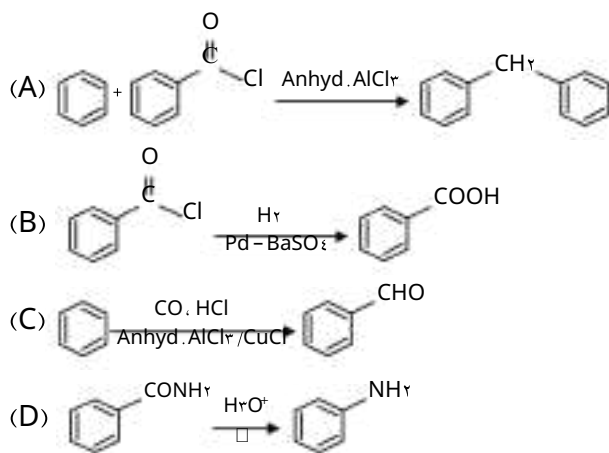
SECTION-B

xi. The enthalpy of formation of ethane (C_2H_6) from ethylene by addition of hydrogen where the bond energies of C-H, C-C, H-H are 414 kJ, 347 kJ, 435 kJ and 430 kJ respectively is - _____

Ans. (120)

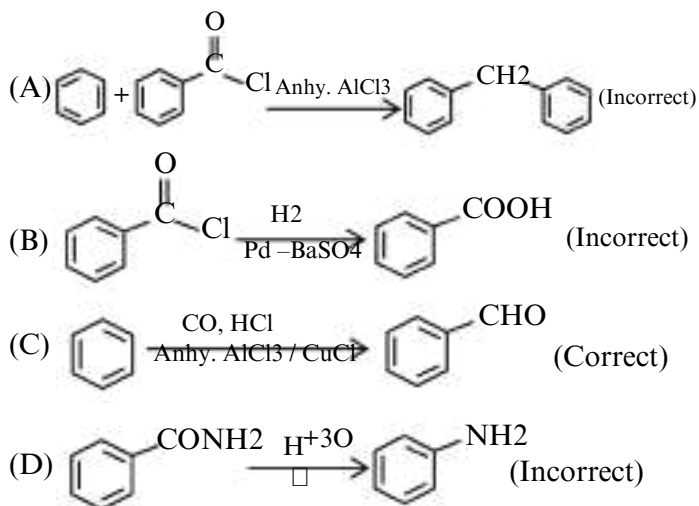
Sol. $C_2H_4(g) + H_2(g) \rightarrow C_2H_6(g)$
 $\Delta H = BE(C=C) + 2BE(C-H) + BE(H-H) - BE(C-C) - 4BE(C-H)$
 $\Delta H = BE(C=C) + BE(H-H) - BE(C-C) - 2BE(C-H)$
 $= 610 + 430 - 347 - 2 \times 414$
 $= -120 \text{ kJ}$

xii. The number of correct reaction(s) among the following is _____.



Ans. (1)

Sol.



Ans. X g of ethylamine is subjected to reaction with NaNO_2/HCl followed by water; evolved dinitrogen gas which occupied 2.24 L volume at STP. X is _____ g.

Ans. (40)

Sol.



given : N_2 evolved is 2.24 L i.e. 0.1 mole.
i.e. $\text{CH}_3\text{CH}_2\text{NH}_2$ (ethyl amine) will be 4.5 g
(= 0.1 mole)

Hence the answer = 40×10^{-3} g

Ans. The de-Broglie's wavelength of an electron in the n^{th} orbit is _____ \AA . (a_0 = Bohr's radius)

Ans. (A)

Sol. $r_n = n^2 a_0$

$$2\pi a_0 \frac{n^2}{Z} = n\lambda$$

$$2\pi a_0 \frac{42}{1} = 4\pi a_0$$

$$\lambda = 8 a_0$$

Ans. Only 2 mL of KMnO_4 solution of unknown molarity is required to reach the end point of a titration of 20 mL of oxalic acid (2 M) in acidic medium. The molarity of KMnO_4 solution should be _____ M.

NTA Ans. (0.0)

Sol. $\text{eq.}(\text{KMnO}_4) = \text{eq.}(\text{H}_2\text{C}_2\text{O}_4)$

$$M \times 2 \times 5 = 2 \times 20 \times 2$$

$$M = 8M$$

Ans. Consider the following reaction $\text{MnO}_2 + \text{KOH} + \text{O}_2 \rightarrow \text{A} + \text{H}_2\text{O}$.

Product 'A' in neutral or acidic medium disproportionate to give products 'B' and 'C' along with water. The sum of spin-only magnetic moment values of B and C is _____ BM.

Ans. (nearest integer)

Sol. (Given atomic number of Mn is 25)



$$n = 4, \quad \ell = 3 \quad \mu = 3.47 \text{ B.M.}$$

Nearest integer is (4)

87. Consider the following transformation involving first order elementary reaction in each step at constant temperature as shown below.

$$A \xrightleftharpoons[\text{Step 2}]{\text{Step 1}} B \xrightarrow{\text{Step 3}} C \xrightarrow{\text{Step 4}} D \xrightarrow{\text{Step 5}} E$$

Some details of the above reaction are listed below.

Step	Rate constant (sec ⁻¹)	Activation energy (kJ mol ⁻¹)
1	k ₁	300
2	k ₂	200
3	k ₃	E _{a3}

If the overall rate constant of the above transformation (k) is given as $k = \frac{k_1 k_3}{k_2}$ and the overall activation energy (E_a) is 100 kJ mol⁻¹, then the value of E_{a3} is _____ kJ mol⁻¹ (nearest integer)

Ans. (100)

Sol. $k = \frac{k_1 k_3}{k_2}$

$$A e^{-\frac{E_a}{RT}} = \frac{A_1 e^{-\frac{E_{a1}}{RT}} A_3 e^{-\frac{E_{a3}}{RT}}}{A_2 e^{-\frac{E_{a2}}{RT}}}$$

$$A e^{-\frac{E_a}{RT}} = \frac{A_1 A_3}{A_2} e^{-\frac{E_{a1} + E_{a3} - E_{a2}}{RT}}$$

$$E_a = E_{a1} + E_{a3} - E_{a2}$$

$$100 = 300 + E_{a3} - 200$$

$$E_{a3} = 100 \text{ kJ/mole}$$

88. 2.0 g of a non-volatile, non-electrolyte is dissolved in 100 g of water at 25°C. The solution showed a boiling point elevation by 1°C. Assuming the solute concentration is negligible with respect to the solvent concentration, the vapour pressure of the resulting aqueous solution is _____ mm of Hg (nearest integer)

Given : Molal boiling point elevation constant of water (K_b) = 0.52 K. kg mol⁻¹, 1 atm pressure = 760 mm of Hg, molar mass of water = 18 g mol⁻¹

Ans. (707)

Sol. $\gamma = 0.02 \times m$

$$m = \frac{2}{0.52}$$

According to question, solution is much diluted

$$SO \quad \frac{P}{P_0} = \frac{n_{\text{solvent}}}{n_{\text{total}}}$$

$$\frac{P}{P_0} = \frac{m}{1000 + m} M_{\text{solvent}}$$

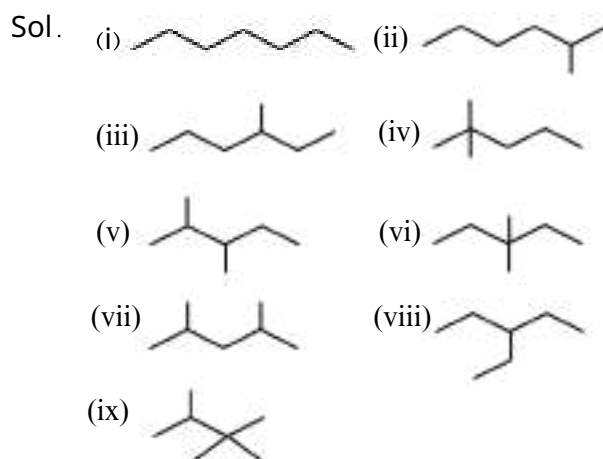
$$\frac{P}{760} = \frac{m}{1000 + m} \times 18$$

$$P = 760 \times \frac{0.52}{1000} \times 18 = 707.115$$

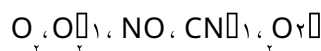
$$P_0 = 760 - 707.115 = 52.885 \text{ mm of Hg}$$

89. The number of different chain isomers for C₇H₁₆ is _____.

Ans. (9)



90. Number of molecules/species from the following having one unpaired electron is _____.



Ans. (2)

Sol. According to M.O.T.

O₂ no. of unpaired electrons = 2

O₂⁺ no. of unpaired electron = 1

NO no. of unpaired electron = 1

CN⁻ no. of unpaired electron = 0

O₂²⁺ no. of unpaired electron = 0