FINAL JEE–MAIN EXAMINATION – APRIL, 2024

(Held On Thursday 04 April, 2024) TIME : 3 : 00 PM to 6 : 00 PM

MATHEMATICS

TEST PAPER WITH SOLUTION

SECTION-A

1. If the function $f(x) = \begin{cases} \dfrac{72^x - 9^x - 8^x + 1}{\sqrt{2} - \sqrt{1+\cos x}}, & x \neq 0 \\ a\log_e 2 \log_e 3, & x = 0 \end{cases}$ is continuous at $x = 0$, then the value of a^2 is equal to

(1) 968 (2) 1152

(3) 746 (4) 1250

Ans. (2)

Sol. $\lim\limits_{x\to 0} f(x) = a \ln 2 \ln 3$

$\lim\limits_{x\to 0} \dfrac{72^x - 9^x - 8^x + 1}{\sqrt{2} - \sqrt{1+\cos x}} = \lim\limits_{x\to 0} \dfrac{(8^x - 1)(9^x - 1)}{\sqrt{2} - \sqrt{1+\cos x}}$

$\lim\limits_{x\to 0} \left(\dfrac{8^x - 1}{x}\right)\left(\dfrac{9^x - 1}{x}\right)\left(\dfrac{x^2}{1-\cos x}\right)\left(\sqrt{2} + \sqrt{1+\cos x}\right)$

$= \ln 8 \cdot \ln 9 \cdot 2 \cdot 2\sqrt{2} = 24\sqrt{2} \ln 2 \ln 3$

$\Rightarrow a = 24\sqrt{2}$, $a^2 = 576 \times 2 = 1152$

2. If $\alpha > 0$, let θ be the angle between the vectors $\vec a = \hat i + \alpha\hat j + 3\hat k$ and $\vec b = 3\hat i - \hat j - 2\hat k$. If the vectors $\vec a + \vec b$ and $\vec a - \vec b$ are mutually perpendicular, then the value of $(14\cos\theta)^2$ is equal to

(1) 25 (2) 20

(3) 50 (4) 40

Ans. (1)

Sol. $(\vec a + \vec b)\cdot(\vec a - \vec b) = 0, \Rightarrow |\vec a|^2 - |\vec b|^2 = 0$

$(1 + \alpha^2 + 9) = 9 + 1 + 4$

$\Rightarrow \alpha = 2$, $\cos\theta = \dfrac{\vec a \cdot \vec b}{|\vec a||\vec b|} = \dfrac{3 - \alpha - 6}{\sqrt{14}\sqrt{14}}$

$14\cos\theta = 3 - 8 = -5$

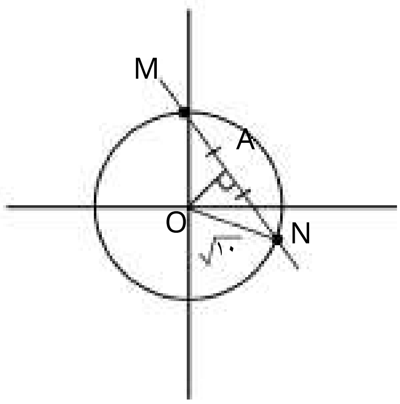
$\Rightarrow (14\cos\theta)^2 = 25$

3. Let C be a circle with radius $\sqrt{10}$ units and centre at the origin. Let the line $x + y = 2$ intersects the circle C at the points P and Q. Let MN be a chord of C of length 2 unit and slope -1. Then, a distance (in units) between the chord PQ and the chord MN is

(1) $2 - \sqrt{3}$ (2) $3 - \sqrt{2}$

(3) $\sqrt{2} - 1$ (4) $\sqrt{2} + 1$

Ans. (2)



$C : x^2 + y^2 = 10$

$AN = \dfrac{MN}{2} = 1$

$\Rightarrow$ In $\triangle OAN$, $(ON)^2 = (OA)^2 + (AN)^2$

$10 = (OA)^2 + 1 \Rightarrow OA = 3$

Perpendicular distance of center from PQ $= \dfrac{|0 + 0 - 2|}{\sqrt{2}} = \sqrt{2}$

Perpendicular distance between MN and PQ $= OA - \sqrt{2}$ or $OA + \sqrt{2}$

$\Rightarrow 3 - \sqrt{2}$ or $3 + \sqrt{2}$

٤. Let a relation R on $N \times N$ be defined as :

$(x_١, y_١)\, R\, (x_٢, y_٢)$ if and only if $x_١ \le x_٢$ or $y_١ \le y_٢$

Consider the two statements :

(I) R is reflexive but not symmetric.

(II) R is transitive

Then which one of the following is true ؟

(١) Only (II) is correct.

(٢) Only (I) is correct.

(٣) Both (I) and (II) are correct.

(٤) Neither (I) nor (II) is correct.

Ans. (٢)

Sol. All $((x_١, y_١), (x_١, y_١))$ are in R where

$x_١, y_١ \in N \Rightarrow$ R is reflexive

$((١,١),(٢,٣)) \in R$ but $((٢,٣),(١,١)) \notin R$

$\Rightarrow$ R is not symmetric

$((٢,٤),(٣,٣)) \in R$ and $((٣,٣),(١,٣)) \in R$ but $((٢,٤),(١,٣)) \notin R$

$\Rightarrow$ R is not transitive

٥. Let three real numbers a, b, c be in arithmetic progression and a + ١, b, c + ٣ be in geometric progression. If a > ١٠ and the arithmetic mean of a, b and c is ٨, then the cube of the geometric mean of a, b and c is

(١) ١٢٠ (٢) ٣١٢

(٣) ٣١٦ (٤) ١٢٨

Ans. (١)

Sol. $٢b = a + c$, $b^٢ = (a+١)(c+٣)$,

$\frac{a+b+c}{٣} = ٨ \Rightarrow b = ٨, a + c = ١٦$

$٦٤ = (a+١)(١٩-a) = ١٩ + ١٨a - a^٢$

$a^٢ - ١٨a - ٤٥ = ٠ \Rightarrow (a-١٥)(a+٣) = ٠, (a > ١٠)$

$a = ١٥, c = ١, b = ٨$

$((abc)^{١/٣})^٣ = abc = ١٢٠$

٦. Let A = [illegible matrix] and $B = I + adj(A) + (adj A)^٢ + \dots + (adj A)^{١٠}$. Then, the sum of all the elements of the matrix B is :

(١) −١١٠ (٢) ٢٢

(٣) −٨٨ (٤) −١٢٤

Ans. (٣)

Sol. Adj(A) = $\begin{bmatrix} ١ & -2 \\ ٠ & 1 \end{bmatrix}$

$(AdjA)^٢ = \begin{bmatrix} ١ & -4 \\ ٠ & 1 \end{bmatrix}$

$(AdjA)^{١٠} = \begin{bmatrix} ١ & -20 \\ ٠ & ١ \end{bmatrix}$

$B = \begin{bmatrix} 1 & ٠ \\ 0 & ١ \end{bmatrix} + \begin{bmatrix} ١ & -2 \\ ٠ & 1 \end{bmatrix} + \begin{bmatrix} ١ & -4 \\ ٠ & 1 \end{bmatrix} + \dots + \begin{bmatrix} ١ & -٢٠ \\ ٠ & ١ \end{bmatrix}$

$B = \begin{bmatrix} ١١ & -١١٠ \\ ٠ & ١١ \end{bmatrix} \Rightarrow$ sum of elements of B $= -٨٨$

٧. The value of $\frac{١ \times ٢^٢ + ٢ \times 3^٢ + \dots + ١٠٠ \times (١٠١)^٢}{١^٢ \times ٢ + ٢^٢ \times ٣ + \dots + ١٠٠^٢ \times ١٠١}$ is

(١) $\frac{٣٠٦}{٣٠٥}$ (٢) $\frac{٣٠٥}{٣٠١}$

(٣) $\frac{٣٢}{٣١}$ (٤) $\frac{٣١}{٣٠}$

Ans. (٢)

Sol. $\frac{١ \times ٢^٢ + ٢ \times 3^٢ + \dots + ١٠٠ \times (١٠١)^٢}{١^٢ \times ٢ + ٢^٢ \times ٣ + \dots + ١٠٠^٢ \times ١٠١} = \frac{\sum_{r=1}^{١٠٠} r(r+١)^٢}{\sum_{r=١}^{١٠٠} r^٢(r+١)}$

$= \frac{\sum_{r=1}^{١٠٠} (r^3 + ٢r^٢ + r)}{\sum_{r=1}^{١٠٠} (r3 + r2)} = \frac{\left(\frac{n(n+1)}{٢}\right)^٢ + \frac{٢ \cdot n(n+١)(٢n+١)}{٦} + \frac{n(n+١)}{٢}}{\left(\frac{n(n+١)}{٢}\right)^٢ + \frac{n(n+١)(٢n+1)}{٦}}$

$= \frac{\frac{n(n+١)}{٢} + \frac{٢}{٣}(٢n+١) + 1}{\frac{n(n+1)}{٢} + \frac{(٢n+1)}{٣}}$, Put n = ١٠٠

$= \frac{\frac{١٠٠ \times ١٠١}{٢} + \frac{٢}{٣}(201) + 1}{\frac{١٠٠ \times ١٠١}{٢} + \frac{٢٠١}{٣}} = \frac{٥١٨٥}{٥١١٧} = \frac{٣٠٥}{٣٠١}$

٨. Let $f(x)=\int_{0}^{x}\left(t+\sin\left(1-e^{t}\right)\right)dt,\ x\in\mathbb{R}$.

Then $\lim\limits_{x\to 0}\dfrac{f(x)}{x^{3}}$ is equal to

(١) $\frac{1}{6}$ (٢) $-\frac{1}{6}$

(٣) $-\frac{2}{3}$ (٤) $\frac{2}{3}$

Ans. (٢)

Sol. $\lim\limits_{x\to 0}\dfrac{f(x)}{x^{3}}$

Using L Hopital Rule.

$\lim\limits_{x\to 0}\dfrac{f'(x)}{3x^{2}}=\lim\limits_{x\to 0}\dfrac{x+\sin\left(1-e^{x}\right)}{3x^{2}}$ (Again L Hopital)

Using L.H. Rule

$=\lim\limits_{x\to 0}\dfrac{\left(-\sin\left(1-e^{x}\right)\right)\left(e^{x}\right)\cdot e^{x}-\cos\left(1-e^{x}\right)\cdot e^{x}}{6}$

$=-\frac{1}{6}$

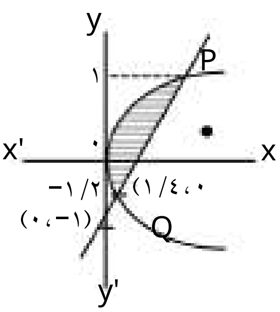
٩. The area (in sq. units) of the region described by $\{(x,y): y^{2}\le 2x,\ \text{and}\ y\ge 4x-1\}$ is

(١) $\frac{11}{32}$ (٢) $\frac{8}{9}$

(٣) $\frac{11}{12}$ (٤) $\frac{9}{32}$

Ans. (٤)

Sol.



Shaded area $=\int_{-1/2}^{1}\left(x_{Right}-x_{Left}\right)dy$

$y^{2}=2x$
$y=4x-1$ Solve
$y=1,\ y=-\frac{1}{2}$

Shaded area $=\int_{-1/2}^{1}\left(\left(\frac{y+1}{4}\right)-\frac{y^{2}}{2}\right)dy$

$=\left(\frac{1}{4}\left(\frac{y^{2}}{2}+y\right)-\frac{y^{3}}{6}\right)_{-1/2}^{1}=\frac{9}{32}$

١٠. The area (in sq. units) of the region $S=\{z\in\mathbb{C}: |z-1|\le 2;\ (z+\bar{z})+i(z-\bar{z})\le 2,\ \operatorname{Im}(z)\ge 0\}$ is

(١) $\frac{7\pi}{3}$ (٢) $\frac{3\pi}{2}$

(٣) $\frac{17\pi}{8}$ (٤) $\frac{7\pi}{4}$

Ans. (٢)

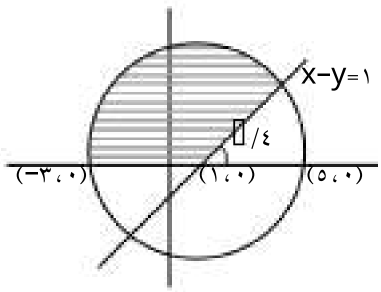
Sol. Put $z=x+iy$

$|z-1|\le 2 \Rightarrow (x-1)^{2}+y^{2}\le 4$ ...(١)

$(z+\bar{z})+i(z-\bar{z})\le 2 \Rightarrow 2x+i(2iy)\le 2$

$\Rightarrow x-y\le 1$ ...(٢)

$\operatorname{Im}(z)\ge 0 \Rightarrow y\ge 0$ ...(٣)



Required area

= Area of semi-circle − area of sector A

$=\frac{1}{2}\pi(2)^{2}-\frac{\pi}{2}$

$=\frac{3\pi}{2}$

١١. If the value of the integral $\int_{-1}^{1} \frac{\cos \alpha x}{1+3^{x}} dx$ is $\frac{٢}{\pi}$. Then، a value of α is

(١) $\frac{\pi}{٦}$ (٢) $\frac{\pi}{٢}$

(٣) $\frac{\pi}{٣}$ (٤) $\frac{\pi}{٤}$

Ans. (٢)

Sol. Let $I = \int_{-1}^{1} \frac{\cos \alpha x}{١+٣^{x}} dx$ ...(I)

$I = \int_{-1}^{1} \frac{\cos \alpha x}{١+٣^{-x}} dx$

$\left(\text{using} \int_a^b f(x)dx = \int_a^b f(a+b-x)dx \right)$ ...(II)

Add (١) and (II)

$٢I = \int_{-1}^{1} \cos(\alpha x)dx = ٢\int_{0}^{1} \cos(\alpha x)dx$

$I = \frac{\sin \alpha}{\alpha} = \frac{٢}{\pi}$ (given)

$\Rightarrow \alpha = \frac{\pi}{٢}$

١٢. Let $f(x) = ٣\sqrt{x-2} + \sqrt{٤-x}$ be a real valued function. If α and β are respectively the minimum and the maximum values of f، then $\alpha^{٢} + ٢\beta^{٢}$ is equal to

(١) ٤٤ (٢) ٤٢

(٣) ٢٤ (٤) ٣٨

Ans. (٢)

Sol. $f(x) = ٣\sqrt{x-2} + \sqrt{٤-x}$

$x-٢ \ge ٠$ & $٤-x \ge ٠$

$\Rightarrow x \in [٢، ٤]$

Let $x = ٢\sin^{٢}\theta + ٤\cos^{٢}\theta$

$\Rightarrow f(x) = ٣\sqrt{٢}|\cos\theta| + \sqrt{٢}|\sin\theta|$

$= \sqrt{٢}\left(٣|\cos\theta| + |\sin\theta|\right) \le \sqrt{٢}\sqrt{٩+١}$

$\sqrt{٢}\cdot ... \le \sqrt{٢}\cos\theta + \sqrt{٢}\sin\theta \le \sqrt{٢٠}$

$\alpha = \sqrt{٢}, \beta = \sqrt{٢٠}$

$\alpha^{٢} + ٢\beta^{٢} = ٢ + ٤٠ = ٤٢$

١٣. If the coefficients of $x^{٤}$، $x^{٥}$ and $x^{٦}$ in the expansion of $(١+x)^{n}$ are in the arithmetic progression، then the maximum value of n is :

(١) ١٤ (٢) ٢١

(٣) ٢٨ (٤) ٧

Ans. (١)

Sol. Coeff. of $x^{٤} = {}^{n}C_{٤}$

Coeff. of $x^{٥} = {}^{n}C_{٥}$

Coeff. of $x^{٦} = {}^{n}C_{٦}$

${}^{n}C_{٤}، {}^{n}C_{٥}، {}^{n}C_{٦}$ .... AP

$٢.{}^{n}C_{٥} = {}^{n}C_{٤} + {}^{n}C_{٦}$

$2 = \frac{{}^{n}C_{٤}}{{}^{n}C_{٥}} + \frac{{}^{n}C_{٦}}{{}^{n}C_{٥}} \quad \left(\frac{{}^{n}C_{r}}{{}^{n}C_{r-1}} = \frac{n-r+1}{r} \right)$

$2 = \frac{٥}{n-٤} + \frac{n-٥}{٦}$

$١٢(n-٤) = ٣٠ + n^{٢} - ٩n + ٢٠$

$n^{٢} - ٢١n + ٩٨ = ٠$

$(n-١٤)(n-٧) = ٠$

nmax = ١٤ nmin = ٧

١٤. Consider a hyperbola H having centre at the origin and foci and the x-axis. Let $C_{١}$ be the circle touching the hyperbola H and having the centre at the origin. Let $C_{٢}$ be the circle touching the hyperbola H at its vertex and having the centre at one of its foci. If areas (in sq. units) of $C_{١}$ and $C_{٢}$ are ٣٦π and ٤π، respectively، then the length (in units) of latus rectum of H is

(١) $\frac{٢٨}{٣}$ (٢) $\frac{١٤}{٣}$

(٣) $\frac{١٠}{٣}$ (٤) $\frac{١١}{٣}$

Ans. (١)

Sol. Let H : $\frac{x^٢}{a^٢} - \frac{y^٢}{b^٢} = ١$ $\quad (b^٢ = a^٢(e^٢ - ١))$

$\therefore$ eq of $C_١ = x^٢ + y^٢ = a^٢$

Ar. $= ٣٦\pi$

$\therefore \pi a^٢ = ٣٦\pi$

$a = ٦$

Now radius of $C_٢$ can be $a(e - ١)$ or $a(e + ١)$

for $r = a(e - ١)$	for $r = a(e + ١)$
Ar. $= ٤\pi$	$\pi r^٢ = ٤\pi$
$\pi a^٢(e - ١)^٢ = ٤\pi$	$a^٢(e + ١)^٢ = ٤$
$٣٦\pi(e - ١)^٢ = ٤\pi$	$٣٦(e + ١)^٢ = ٤$
$e - ١ = \frac{١}{٣}$	$e + ١ = \frac{١}{٣}$
$e = \frac{٤}{٣}$	$\Rightarrow -\frac{٢}{٣}$ Not possible

$\therefore b^٢ = ٣٦\left(\frac{١٦}{٩} - 1\right) = ٢٨$

$\therefore LR = \frac{٢b^٢}{a} = \frac{٢ \times ٢٨}{٦} = \frac{٢٨}{٣}$

١٥. If the mean of the following probability distribution of a random variable X:

X	٠	٢	٤	٦	٨
P(X)	a	٢a	a + b	٢b	٣b

is $\frac{٤٦}{٩}$, then the variance of the distribution is

(١) $\frac{٥٨١}{٨١}$ (٢) $\frac{٥٦٦}{٨١}$

(٣) $\frac{١٧٣}{٢٧}$ (٤) $\frac{١٥١}{٢٧}$

Ans. (٢)

Sol. $\Sigma P_i = ١$

$a + ٢a + a + b + ٢b + ٣b = ١$

$٤a + ٦b = ١$ .... (I)

$E(x) = \text{mean} = \frac{٤٦}{٩}$

$\Sigma P_i X_i = \frac{٤٦}{٩} \Rightarrow ٤a + ٤a + ٤b + ١٢b + ٢٤b = \frac{٤٦}{٩}$

$٨a + ٤٠b = \frac{٤٦}{٩}$

$٤a + ٢٠b = \frac{٢٣}{٩}$ .... (II)

Subtract (I) from (II) we get

$b = \frac{١}{٩}$ & $a = \frac{١}{١٢}$

Variance $= E(x_i^٢) - E(x_i)^٢$

$E(x_i^٢) = ٠ \times a + ٢^٢ \times ٢a + ٤^٢(a + b) + ٦^٢(٢b) + ٨^٢(٣b)$

$= ٢٤a + ٢٨٠b$

Put $a = \frac{١}{١٢}$, $b = \frac{١}{٩}$

$E(x_i^٢) = ٢ + \frac{٢٨٠}{٩} = \frac{٢٩٨}{٩}$

$\therefore \sigma^٢ = E(x_i^٢) - E(x_i)^٢$

$= \frac{٢٩٨}{٩} - \left(\frac{٤٦}{9}\right)^2$

$= \frac{٢٩٨}{٩} - \frac{٢١١٦}{٨١}$

$= \frac{٥٦٦}{٨١}$

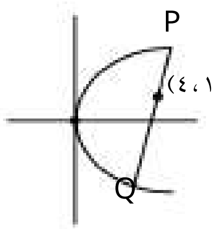
١٦. Let PQ be a chord of the parabola $y^٢ = ١٢x$ and the midpoint of PQ be at (٤, ١). Then, which of the following point lies on the line passing through the points P and Q?

(١) (٣, −٣) (٢) $\left(\frac{3}{2}, -١٦\right)$

(٣) (٢, −٩) (٤) $\left(\frac{1}{2}, -٢٠\right)$

Ans. (٤)

Sol.



$T = S_١$

$y - ٦(x + ٤) = ١ - ٤٨$

$٦x - y = ٢٣$

Option ٤ $\left(\frac{١}{٢}, -٢٠\right)$ will satisfy

١٧. Given the inverse trigonometric function assumes principal values only. Let x، y be any two real numbers in ﴾−١، ١﴿ such that

$\cos^{-١}x - \sin^{-١}y = □$، $\frac{□□}{٢}$□□□□.

Then، the minimum value of $x^٢ + y^٢ + ٢xy \sin□$ is

(١) −١ (٢) ٠

(٣) $\frac{□1}{}$ (٤) $\frac{١}{٢}$

Ans. (٢)

Sol. $\cos^{-١}x - \frac{□}{٢} □ \cos^{□١}y □ = □$

$\cos^{-١}x + \cos^{-١}y = \frac{□}{٢}$ □ □

□ □□ $\frac{□}{2}$ □ □ $\frac{□}{2}$ □ □ , □ $\frac{٣□□}{٢}$ □ □

$\cos□_١□xy□\sqrt{١□x^٢}\sqrt{١□y^٢}□ = \frac{□}{٢}$□□

$xy□\sqrt{١□x^٢}\sqrt{١□y^٢}□□\sin□$

$(xy + \sin□)^٢ = (١ - x^٢)(١ - y^٢)$

$x^٢y^٢ + ٢xy\sin a + \sin^٢ a = ١ - x^٢ - y^٢ + x^٢y^٢$

$x^٢ + y^٢ + ٢xy \sin□ = ١ - \sin^٢□$

$x^٢ + y^٢ + ٢xy\sin□ = \cos^٢□$

Min. value of $\cos^٢□ = ٠$

At $□□ \frac{□}{٢}$

Option (٢) is correct

١٨. Let $y = y(x)$ be the solution of the differential equation

$(x^٢ + ٤)dy + (٢x^٣y + ٨xy - ٢)dx = ٠$. If $y(٠) = ٠$، then $y(٢)$ is equal to

(١) $\frac{□}{٨}$ (٢) $\frac{□}{١٦}$

(٣) ٢□ (٤) $\frac{□}{٣٢}$

Ans. (٤)

Sol. $\frac{dy}{dx} □ y □\frac{٢x^٣ □ ٨x}{□x^٢□4□} □ \frac{٢}{□x^٢ □ ٤□}$

$\frac{dy}{dx} □ y □ \frac{٢x}{x^٢ □ 4} □ \frac{٢}{□x^٢ □ ٤□}$

$IF = e^{□\frac{٢x}{x^٢ □ ٤}dx}$

$IF = x^٢ + ٤$

$y \times (x^٢ + ٤) = □\frac{٢}{□x^٢ □ 4□} □ □x^٢ □ ٤□ dx$

$y(x^٢ + ٤) = ٢ □ \frac{dx}{x^٢ □ ٢^٢}$

$y□x^٢□٤□ = \frac{٢}{٢}\tan^{□١}□\frac{x}{٢}□ □ c$

٠ = ٠ + c = c = ٠

$y(x^٢ + ٤) = \tan^{-١}□\frac{x}{٢}□$

y at x = ٢

$y(٤ + ٤) = \tan^{-١}(١)$

$y □ 2 □ \frac{□}{٣٢}$

Option (٤) is correct

١٩. Let $\vec{a} □ \hat{i} □ \hat{j} □ \hat{k}$، $\vec{b} □ ٢\hat{i} □ ٤\hat{j} □ ٥\hat{k}$ and $\vec{c} □ x\hat{i} □ ٢\hat{j} □ ٣\hat{k}$، $x □ \mathbb{R}$. If $\vec{d}$ is the unit vector in the direction of $\vec{b} □ \vec{c}$ such that $\vec{a}.\vec{d} □ ١$، then $(\vec{a} □ \vec{b}) \cdot \vec{b}$ is equal to

(١) ٩ (٢) ٦

(٣) ٣ (٤) ١١

Ans. (٤)

Sol. $\vec{d} = \lambda(\vec{b} \times \vec{c})$

$\vec{a}\cdot\vec{d} = \lambda[\vec{b}\ \vec{c}\ \vec{a}]$

$١ = \lambda(١ + x + ٥)$

$١ = \lambda(x + ٦)$ ....(١)

$|\vec{d}| = ١$ $\quad \boxed{\lambda = \frac{١}{x + ٦}}$

$|\lambda\, \vec{b} \times \vec{c}| = 1$

$|\lambda((x + ٢)\hat{i} - ٦\hat{j} + ٢\hat{k})| = ١$

$\lambda^٢((x + ٢)^٢ + ٦ + ٢)^٢ = ١$

$x^٢ + ٤x + ٤ + ٣٦ + ٤ = (x + ٦)^٢$

$x^٢ + ٤x + ٤٤ = x^٢ + ١٢x + ٣٦$

$٨x = ٨،\ x = ١$

$$\begin{vmatrix} ١ & ١ & ١ \\ ٢ & ٤ & -٥ \\ x & ٢ & ٣ \end{vmatrix} = [\vec{a}\ \vec{b}\ \vec{c}]$$

$$\begin{vmatrix} ٠ & ٠ & ١ \\ -2 & ٩ & -4 \\ x-٢ & -1 & ٣ \end{vmatrix} = ٢ - ٩(x - ٢)$$

$= ٢٠ - ٩x$

at $x = ١$

$٢٠ - ٩ = ١١$

Option ٤ is correct

٢٠. Let P the point of intersection of the lines

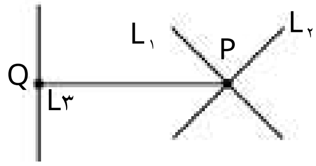
$$\frac{x-٢}{١} = \frac{y-٤}{٥} = \frac{z-٢}{١} \quad \text{and} \quad \frac{x-٣}{٢} = \frac{y-٢}{٣} = \frac{z-٣}{٢}.$$

Then، the shortest distance of P from the line $٤x = ٢y = z$ is

(١) $\frac{٥\sqrt{١٤}}{٧}$ (٢) $\frac{\sqrt{١٤}}{٧}$

(٣) $\frac{٣\sqrt{١٤}}{٧}$ (٤) $\frac{٦\sqrt{١٤}}{٧}$

Ans. (٣)



$L_١ : \frac{x-٢}{١} = \frac{y-٤}{٥} = \frac{z-٢}{١} = \lambda$

$P(\lambda + ٢، ٥\lambda + ٤، \lambda + 2)$

$L_2 : \frac{x-٣}{٢} = \frac{y-٢}{٣} = \frac{z-٣}{٢} = \mu$

$P(٢\mu + ٣، ٣\mu + ٢، ٢\mu + ٣)$

$\lambda + ٢ = ٢\mu + ٣ \qquad ٣\mu + ٢ = ٥\lambda + ٤$

$\lambda = ٢\mu + ١ \qquad ٣\mu = ٥\lambda + ٢$

$٣\mu = ٥(٢\mu + ١) + ٢$

$٣\mu = ١٠\mu + ٧$

$\mu = -١ \quad \lambda = -١$

Both satisfies (P)

P(١، −١، ١)

$L_3 : \frac{x}{١/٤} = \frac{y}{١/٢} = \frac{z}{١} = k$

$L_3 : \frac{x}{١} = \frac{y}{٢} = \frac{z}{٤}$

Coordinates of Q(k، ٢k، ٤k)

DR's of PQ = <k−١، ٢k + ١، ٤k − ١>

PQ ⊥ to L٣

$(k - ١) + ٢(٢k + ١) + ٤(٤k - ١) = ٠$

$k - ١ + ٤k + ٢ + ١٦k - ٤ = ٠$

$k = \frac{١}{٧}$

$Q\left(\frac{1}{7}، \frac{٢}{٧}، \frac{٤}{٧}\right)$

$PQ = \sqrt{\left(1 - \frac{١}{٧}\right)^٢ + \left(-1 - \frac{٢}{٧}\right)^٢ + \left(1 - \frac{٤}{٧}\right)^٢}$

$= \sqrt{\frac{٣٦}{٤٩} + \frac{٨١}{٤٩} + \frac{٩}{٤٩}} = \frac{\sqrt{١٢٦}}{٧}$

$PQ = \frac{٣\sqrt{١٤}}{٧}$

Option-٣ will satisfy

SECTION–B

٢١. Let S = {sin ٢θ : (sin⁴θ + cos⁴θ)x + (sin ٢θ)x + (sin⁶θ + cos⁶θ) = ٠ has real roots}. If α and β be the smallest and largest elements of the set S, respectively, then ٣((α – ٢)² + (β – ١)²) equals ... . .

Ans. (٤)

Sol. $D \geq 0 \Rightarrow \sin^{2} 2\theta - 4\left(1 - \frac{\sin^2 2\theta}{2}\right)\left(1 - \frac{3}{4}\sin^2 2\theta\right) \geq 0$

$\Rightarrow \sin^2 2\theta - 4\left(1 - \frac{5}{4}\sin^2 2\theta + \frac{3}{8}\sin^4 2\theta\right) \geq 0$

$D = -\frac{3}{2}\sin^4 2\theta + 6\sin^2 2\theta - 4 \geq 0$

$3\sin^4 2\theta - 12\sin^2 2\theta + 8 \leq 0$

$\sin^2 2\theta = \frac{12 \pm \sqrt{144 - 12.8}}{6} = \frac{12 \pm 4\sqrt{3}}{6} = \frac{6 \pm 2\sqrt{3}}{3}$

$\sin^2 2\theta \geq 2 - \frac{2}{\sqrt{3}}$, but $\sin^2 2\theta \in [0, 1]$

$\Rightarrow \alpha = -\sqrt{2 - \frac{2}{\sqrt{3}}}, \beta = 1 \Rightarrow (\alpha - 2)^2 = \frac{4}{\sqrt{3}}, (\beta - 1)^2 = 0$

$\Rightarrow 3\left((\alpha - 2)^2 + (\beta - 1)^2\right) = 4$

٢٢. If $\int \text{cosec}^5 x\,dx = \alpha \cot x \,\text{cosec}\, x\left(\text{cosec}^2 x + \frac{3}{2}\right) + \beta \log_e \left|\tan \frac{x}{2}\right| + C$

where $\alpha, \beta \in \mathbb{R}$ and C is constant of integration, then the value of ٨(α + β) equals ... . .

Ans. (١)

Sol. $\int \text{cosec}^3 x \,\text{cosec}^2 x\,dx = I$

By applying integration by parts

$I = -\cot x\,\text{cosec}^3 x - \int \cot x\left(-3\,\text{cosec}^2 x \cot x\,\text{cosec}\, x\right)dx$

$I = -\cot x\,\text{cosec}^3 x - 3\int \text{cosec}^3 x\left(\text{cosec}^2 x - 1\right)dx$

$I = -\cot x\,\text{cosec}^3 x - 3I + 3\int \text{cosec}^3 x\,dx$

let

$I_1 = \int \text{cosec}^3 x\,dx = -\text{cosec}\, x \cot x - \int \cot^2 x\,\text{cosec}\, x\,dx$

$I_1 = -\text{cosec}\, x \cot x - \int\left(\text{cosec}^2 x - 1\right)\text{cosec}\, x\,dx$

$2I_1 = -\text{cosec}\, x \cot x + \ell n\left|\tan \frac{x}{2}\right|$

$I_1 = -\frac{1}{2}\text{cosec}\, x \cot x + \frac{1}{2}\ell n\left|\tan \frac{x}{2}\right|$

$4I = -\cot x\,\text{cosec}^3 x - \frac{3}{2}\text{cosec}\, x \cot x + \frac{3}{2}\ell n\left|\tan \frac{x}{2}\right| + 4c$

$I = -\frac{1}{4}\text{cosec}\, x \cot x\left(\text{cosec}^2 x + \frac{3}{2}\right) + \frac{3}{8}\ell n\left|\tan \frac{x}{2}\right| + c$

$\Rightarrow \alpha = -\frac{1}{4}, \beta = \frac{3}{8} \Rightarrow 8(\alpha + \beta) = 1$

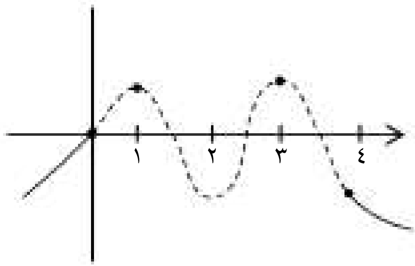
٢٣. Let f : ℝ → ℝ be a thrice differentiable function such that f(٠) = ٠, f(١) = ١, f(٢) = –١, f(٣) = ٢ and f(٤) = –٢. Then, the minimum number of zeros of (٣f' f'' + ff''')(x) is ... . .

Ans. (٥)

Sol. $\Rightarrow 3f'f'' + ff''' = (ff'' + (f')^2)'$

$ff'' + (f')^2 = (ff')'$

$\Rightarrow 3f'f'' + ff''' = (f(x) f'(x))''$



min. roots of f(x) ≥ ٤

⇒ min. roots of f'(x) ≥ ٣

⇒ min. roots of f(x) f'(x) ≥ ٧

⇒ min. roots of (f(x) f'(x))'' ≥ ٥

٢٤. Consider the function f : ℝ → ℝ defined by $f(x) = \frac{2x}{\sqrt{1 + 9x^2}}$. If the composition of f, $\underbrace{(f \circ f \circ f \circ \cdots \circ f)}_{10\ \text{times}}(x) = \frac{2^{10} x}{\sqrt{1 + 9\alpha x^2}}$, then the value of $\sqrt{3\alpha + 1}$ is equal to ... . .

Ans. (١٠٢٤)

Sol. $f(f(x)) = \dfrac{٢f(x)}{\sqrt{١+٩f^٢(x)}} = \dfrac{٤x}{\sqrt{١+٩x^٢+٩.٢^٢x^٢}}$

$f(f(f(x))) = \dfrac{٢^٣x/\sqrt{1+٩x^٢}}{\sqrt{١+٩(١+٢^٢+٢^٢)\dfrac{٢^٢x^٢}{١+٩x^٢}}} = \dfrac{٢^٣x}{\sqrt{١+٩x^٢(١+٢^٢+٢^٤)}}$

∴ By observation

$(١+٢^٢+٢^٤+....+٢^{١٨}) = \left(\dfrac{٢^{٢٠}-١}{٢^٢-١}\right) = \dfrac{٢^{٢٠}-١}{٣}$

$٣(١+٢^{٢٠}-١) = ٢^{٢٠} \Rightarrow \sqrt{٢^{٢٠}} = ١٠٢٤$

٢٥. Let A be a ٢ × ٢ symmetric matrix such that $A\begin{bmatrix}1\\1\end{bmatrix} = \begin{bmatrix}3\\7\end{bmatrix}$ and the determinant of A be ١.

If $A^{-١} = \alpha A + \beta I$, where I is an identity matrix of order ٢ × ٢, then $\alpha + \beta$ equals ... . .

Ans. (٥)

Sol. Let $A = \begin{bmatrix} a & b \\ b & d \end{bmatrix}$

$\begin{bmatrix} a & b \\ b & d \end{bmatrix}\begin{bmatrix}1\\1\end{bmatrix} = \begin{bmatrix}٣\\٧\end{bmatrix}$, $ad - b^٢ = ١$

$a + b = ٣$, $b + d = ٧$, $(٣ - b)(٧ - b) - b^٢ = ١$

$٢١ - ١٠b = ١ \Rightarrow b = ٢$, $a = ١$, $d = ٥$

$A = \begin{bmatrix} ١ & 2 \\ ٢ & 5 \end{bmatrix}$, $A^{-١} = \begin{bmatrix} ٥ & -٢ \\ -٢ & ١ \end{bmatrix}$

$A^{-١} = \alpha A + \beta I$

$\begin{bmatrix} ٥ & -٢ \\ -٢ & ١ \end{bmatrix} = \begin{bmatrix} \alpha+\beta & ٢\alpha \\ ٢\alpha & ٥\alpha+\beta \end{bmatrix}$

$\alpha = -١$, $\beta = ٦ \Rightarrow \alpha + \beta = ٥$

٢٦. There are ٤ men and ٥ women in Group A, and ٥ men and ٤ women in Group B. If ٤ persons are selected from each group, then the number of ways of selecting ٤ men and ٤ women is ... . .

Ans. (٥٦٢٦)

Sol.

From Group A	From Group B	Ways of selection
٤M	٤W	$^٤C_٤\ ^٤C_٤ = ١$
٣M ١W	١M ٣W	$^٤C_٣\ ^٥C_١\ ^٥C_١\ ^٤C_٣ = ٤٠٠$
٢M ٢W	٢M ٢W	$^٤C_٢\ ^٥C_٢\ ^٥C_٢\ ^٤C_٢ = ٣٦٠٠$
١M ٣W	٣M ١W	$^٤C_١\ ^٥C_٣\ ^٥C_٣\ ^٤C_١ = ١٦٠٠$
٤W	٤M	$^٥C_٤\ ^٥C_٤ = ٢٥$
Total		٥٦٢٦

Ans. ٥٦٢٦

٢٧. In a tournament, a team plays ١٠ matches with probabilities of winning and losing each match as $\dfrac{١}{٣}$ and $\dfrac{٢}{٣}$ respectively. Let x be the number of matches that the team wins, and y be the number of matches that team loses. If the probability $P(|x-y| \le ٢)$ is p, then $٣^٩p$ equals ... ...

Ans. (٨٢٨٨)

Sol. $P(W) = \dfrac{١}{٣}$ $P(L) = \dfrac{٢}{٣}$

x = number of matches that team wins

y = number of matches that team loses

$|x-y| \le ٢$ and $x + y = ١٠$

$|x-y| = ٠, ١, ٢$ $x, y \in N$

Case-I : $|x-y| = ٠ \Rightarrow x = y$

$\because x + y = ١٠ \Rightarrow x = ٥ = y$

$P(|x-y| = ٠) = {}^{١٠}C_٥ \left(\dfrac{1}{3}\right)^٥ \left(\dfrac{2}{3}\right)^٥$

Case-II : $|x-y| = ١ \Rightarrow x - y = \pm ١$

$x = y + ١$	$x = y - ١$
$\because x + y = ١٠ \Rightarrow$	$\because x + y = ١٠$
$٢y = ٩ \Rightarrow$	$٢y = ١١$
Not possible	Not possible

Case-III : |x−y|□□ x − y □□2

x−y□٢ OR x□y□−٢

∵ x□y□١٠ ∵ x□y□١٠

x□٦، y□٤ x□٤، y□٦

P□|x−y|□٢□ □ ¹⁰C₆ □١/٣□⁶ □٢/٣□⁴ □ ¹⁰C₄ □١/٣□⁴ □٢/٣□⁶

p □ ¹⁰C₅ ٢⁵/٣¹⁰ □ ¹⁰C₆ ٢⁴/٣¹⁰ □ C₄ ٢⁶/٣¹⁰

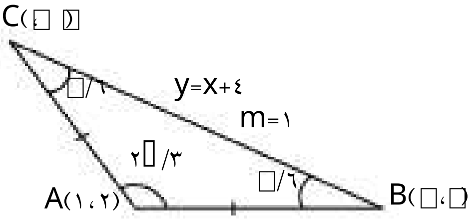
٣٩p □ ١/٣ □ ¹⁰C₅ ٢⁵ □ ¹⁰C₆ ٢⁴ □ ¹⁰C₄ ٢⁶ □

= ٨٢٨٨

٢٨. Consider a triangle ABC having the vertices A(١، ٢)، B(□□□) and C(□□□) and angles □ABC□ □/٦ and □BAC □ ٢□/٣. If the points B and C lie on the line y = x + ٤، then □ + □ is equal to ... . .

Ans. (١٤)

Sol.



Equation of line passes through point A(١، ٢) which makes angle □/٦ from y = x + ٤ is

y □٢ □ (١ □tan □/٦)/(١ ∓ tan □/٦) (x □١)

y □٢ □ (√٣ □1)/(√٣ ∓ ١) (x □١)

y□٢□(٢ □√٣)(x □١) | y□٢□(٢□ √٣)(x□١)

solve with y□x □٤ | solve with y□x□٤

x □٢□(٢ □√٣)x □٢ □√٣ | x □٢□(٢□ √٣)x□٢□ √٣

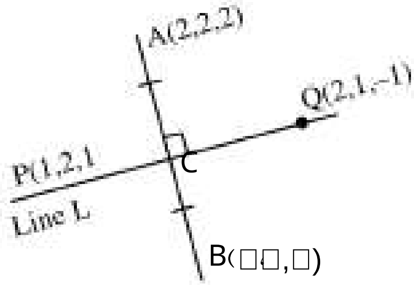
x □ (٤ □√٣)/(١ □√٣) | x □ (4 □√٣)/(1 □√٣)

□ □ □ □ (٤ □√٣)/(١ □√٣) □² □ (٤ □√٣)/(١ □√٣) □²

□٢□□٢□١٤

٢٩. Consider a line L passing through the points P(١، ٢، ١) and Q(٢، ١، −١). If the mirror image of the point A(٢، ٢، ٢) in the line L is (□□□□□)، then □ + □ + ٦□ is equal to ... . .

Ans. (٦)



DR's of Line L □□−١ : ١ : ٢

DR's of AB □□□ − ٢ : □□− ٢ : □ − ٢

AB□□ar L □□٢ − □ + □ − ٢ + ٢ □ − ٤ = ٠

٢□ + □ − □ = ٤ . . .(١)

Let C is mid-point of AB

C □ (□ □ ٢)/٢ , (□ □ ٢)/٢ , (□ □ ٢)/٢ □

DR's of PC = □/٢ : (□ □ ٢)/٢ : □/٢

line L || PC □ □□/٢ □□٢/٢ □□/٤ □K(let)

□ = − ٢K

□ = ٢K + ٢

□ = ٤K

use in (١) □ K □ ١/٦

value of □ + □ + ٦□ = ٢٤K + ٢ = ٦

٣٠. Let $y = y(x)$ be the solution of the differential equation $(x + y + 2)^2 dx = dy$, $y(0) = -2$. Let the maximum and minimum values of the function $y = y(x)$ in $\left[0, \frac{\pi}{3}\right]$ be α and β, respectively. If $(3\alpha + \pi)^2 + \beta^2 = \gamma + \delta\sqrt{3}$, $\gamma, \delta \in \mathbb{Z}$, then $\gamma + \delta$ equals ......

Ans. (٣١)

Sol. $\frac{dy}{dx} = (x + y + 2)^2 \quad \ldots (1)$, $\quad y(0) = -2$

Let $x + y + 2 = v$

$1 + \frac{dy}{dx} = \frac{dv}{dx}$

from (1) $\frac{dv}{dx} = 1 + v^2$

$\Rightarrow \int \frac{dv}{1 + v^2} = \int dx$

$\tan^{-1}(v) = x + C$

$\tan^{-1}(x + y + 2) = x + C$

at $x = 0$, $y = -2 \Rightarrow C = 0$

$\therefore \tan^{-1}(x + y + 2) = x$

$y = \tan x - x - 2$

$f(x) = \tan x - x - 2$, $x \in \left[0, \frac{\pi}{3}\right]$

$\therefore f'(x) = \sec^2 x - 1 > 0 \Rightarrow f(x) \uparrow$

$f_{min} = f(0) = -2 = \beta$

$f_{max} = f\left(\frac{\pi}{3}\right) = \sqrt{3} - \frac{\pi}{3} - 2 = \alpha$

now $(3\alpha + \pi)^2 + \beta^2 = (3\sqrt{3} - 6)^2 + 4$

$\Rightarrow (3\alpha + \pi)^2 + \beta^2 = (3\sqrt{3} - 6)^2 + 4$

$= 67 - 36\sqrt{3}$

$\therefore \gamma = 67$ and $\delta = -36 \Rightarrow \gamma + \delta = 31$

PHYSICS TEST PAPER WITH SOLUTION

SECTION-A

٣١. The translational degrees of freedom (f_t) and rotational degrees of freedom (f_r) of $CH_٤$ molecule are :

(١) f_t = ٢ and f_r = ٢ (٢) f_t = ٣ and f_r = ٣

(٣) f_t = ٣ and f_r = ٢ (٤) f_t = ٢ and f_r = ٣

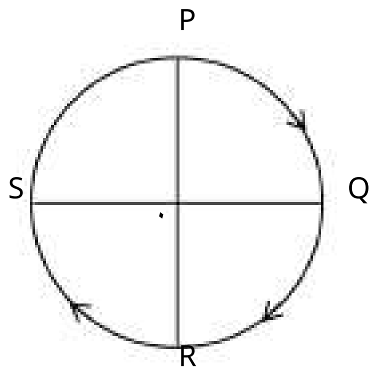
Ans. (٢)

Sol. Since $CH_٤$ is polyatomic Non-Linear

D.O.F of $CH_٤$

T. DOF = ٣ R DOF = ٣

٣٢. A cyclist starts from the point P of a circular ground of radius ٢ km and travels along its circumference to the point S. The displacement of a cyclist is :

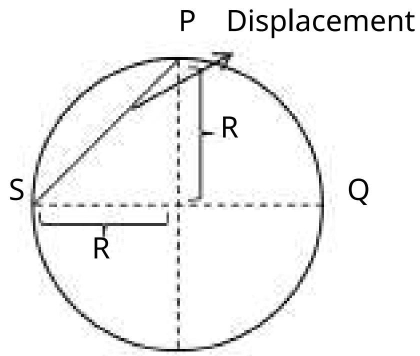


(١) ٦ km (٢) $\sqrt{٨}$ km

(٣) ٤ km (٤) ٨ km

Ans. (٢)

Sol.



□ Displacement = R$\sqrt{٢}$ = ٢$\sqrt{٢}$ □ $\sqrt{٨}$ km

٣٣. The magnetic moment of a bar magnet is ٠.٥ Am. It is suspended in a uniform magnetic field of ٨ × ١٠ T$^{-٢}$. The work done in rotating it from its most stable to most unstable position is :

(١) ١٦ × ١٠ J $^{-٢}$ (٢) ٨ × ١٠ J $^{-٢}$

(٣) ٤ × ١٠ J $^{-٢}$ (٤) Zero

Ans. (٢)

Sol. At stable equilibrium

U = –mB cos ٠° = –mB

At unstable equilibrium

U = –mB cos ١٨٠° = + mB

W = □U

W.D. = ٢ mB

= ٢ (٠.٥) ٨ × ١٠ $^{-٢}$ = ٨ × ١٠ J $^{-٢}$

٣٤. Which of the diode circuit shows correct biasing used for the measurement of dynamic resistance of p-n junction diode :

(١) $D_٤$ R

(٢) R $D_٢$ ٥V

(٣) R $D_٣$

(٤) R $D_١$ ٥V

Ans. (٢)

Sol. Diode should be in forward biased to calculate dynamic resistance
Hence correct answer would be ٢.

٣٥. Arrange the following in the ascending order of wavelength :

(A) Gamma rays (λ_1) (B) x-ray (λ_2)

(C) Infrared waves (λ_3) (D) Microwaves (λ_4)

Choose the most appropriate answer from the options given below :

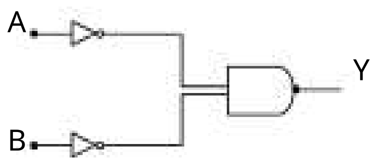
(١) □ > □ > □ > □ (٢) □ > □ > □ > □

(٣) □□ > □ > □ □ > □ (٤) □ > □ > □ > □

Ans. (٣)

Sol. □□ > □ > □ □ > □ ٤

٣٦. Identify the logic gate given in the circuit :



(١) NAND - gate (٢) OR - gate

(٣) AND gate (٤) NOR gate

Ans. (٢)

Sol. $Y = \overline{\bar{A}.\bar{B}}$

By De-Morgan Law

$Y = \overline{\bar{A}} + \overline{\bar{B}}$

$Y = A + B$

Hence OR gate

٣٧. The width of one of the two slits in a Young's double slit experiment is ٤ times that of the other slit. The ratio of the maximum of the minimum intensity in the interference pattern is :

(١) ٩ : ١ (٢) ١٦ : ١

(٣) ١ : ١ (٤) ٤ : ١

Ans. (١)

Sol. Since, Intensity ∝ width of slit (□)

so, $I_1 = I$, $I_2 = 4I$

$I_{min} = \left(\sqrt{I_1} - \sqrt{I_2}\right)^2 = I$

$I_{max} = \left(\sqrt{I_1} + \sqrt{I_2}\right)^2 = 9I$

$\frac{I_{max}}{I_{min}} = \frac{9I}{I} = \frac{9}{1}$

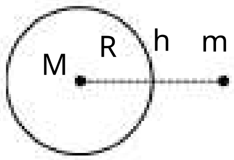
٣٨. Correct formula for height of a satellite from earths surface is :

(١) $\left(\frac{T^2R^2g}{4\pi}\right)^{1/2} - R$ (٢) $\left(\frac{T^2R^2g}{4\pi^2}\right)^{1/3} - R$

(٣) $\left(\frac{T^2R^2}{4\pi^2g}\right)^{1/3} - R$ (٤) $\left(\frac{T^2R^2}{4\pi^2}\right)^{-1/3} + R$

Ans. (٢)

Sol.



$\Rightarrow \frac{GMm}{(R+h)^2} = \frac{mv^2}{(R+h)}$

$\Rightarrow \frac{GM}{(R+h)} = v^2$ ....(١)

$\Rightarrow v = (R+h)\ \omega$

$\Rightarrow v = \frac{2\pi(R+h)}{T}$ ....(٢)

$\Rightarrow \frac{GM}{R^2} = g$

$\Rightarrow GM = gR^2$ ......(٣)

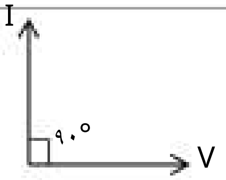
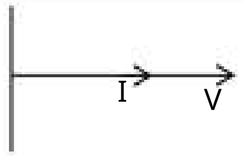
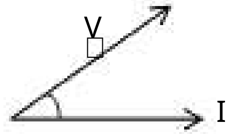
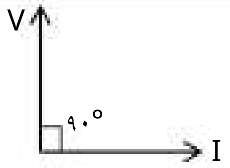
Put value from (٢) & (٣) in eq. (١)

$\Rightarrow \frac{gR^2}{(R+h)} = (R+h)^2\left(\frac{2\pi}{T}\right)^2$

$\Rightarrow \frac{T^2R^2g}{(2\pi)^2} = (R+h)^3$

$\Rightarrow \left(\frac{T^2R^2g}{4\pi^2}\right)^{1/3} - R = h$

٣٩. Match List I with List II

	List–I		List–II
A.	Purely capacitive circuit	I.	
B.	Purely inductive circuit	II.	
C.	LCR series at resonance	III.	
D.	LCR series circuit	IV.	

Choose the correct answer from the options given below :

(١) A–I، B–IV، C–III، D–II

(٢) A–IV، B–I، C–III، D–II

(٣) A–IV، B–I، C–II، D–III

(٤) A–I، B–IV، C–II، D–III

Ans. (٤)

Sol. A – V lags by ٩٠° from I hence option (I) is correct.

B – V lead by ٩٠° from I hence option (IV) is correct

C – In LCR resonance $X_L = X_C$. Hence circuit is purely resistive so option (II) is correct

D – In LCR series V is at some angle from I hence (III) is correct

Hence option (٤) is correct.

٤٠. Given below are two statements :

Statement I : The contact angle between a solid and a liquid is a property of the material of the solid and liquid as well.

Statement II : The rise of a liquid in a capillary tube does not depend on the inner radius of the tube.

In the light of the above statements، choose the correct answer from the options given below :

(١) Both Statement I and Statement II are false

(٢) Statement I is false but Statement II is true.

(٣) Statement I is true but Statement II is false.

(٤) Both Statement I and Statement II are true.

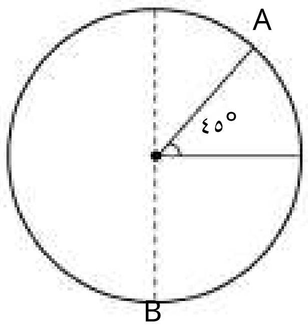
Ans. (٣)

Sol. Statement I is correct as we know contact angle depends on cohesine and adhesive forces.

Statement II is incorrect because height of liquid is given by $h = \frac{٢T\cos\theta}{\rho g r}$ where r is radius of Tube (assuming length of capillary is sufficient)

Hence option (٣) is correct.

٤١. A body of m kg slides from rest along the curve of vertical circle from point A to B in friction less path. The velocity of the body at B is :



(Vertical Circle)

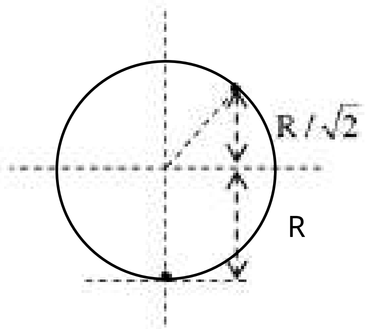
(given، R = ١٤ m، g = ١٠ m/s² and $\sqrt{٢} = ١.٤$)

(١) ١٩.٨ m/s (٢) ٢١.٩ m/s

(٣) ١٦.٧ m/s (٤) ١٠.٦ m/s

Ans. (٢)

Sol.



Apply W.E.T. from A to B

$\Rightarrow W_{mg} = K_B - K_A$

$\Rightarrow mg \times \left(\frac{R}{\sqrt{٢}} + R\right) = \frac{١}{٢} mv_B^{٢} - ٠ \quad (v_A = ٠,\ \text{rest})$

$\Rightarrow mgR\left(\frac{\sqrt{2}+1}{\sqrt{٢}}\right) = \frac{١}{٢} mv_B^{٢}$

$\Rightarrow \sqrt{gR\frac{2(\sqrt{2}+1)}{\sqrt{٢}}} = v_B$

$\Rightarrow \sqrt{\frac{١٠ \times ٤ \times ٢(١.٤+١)}{١.٤}} = v_B$

$\Rightarrow ٢١.٩ = v_B$

Hence option (٢) is correct

٤٢. An electric bulb rated ٥٠ W – ٢٠٠ V is connected across a ١٠٠ V supply. The power dissipation of the bulb is :

(١) ١٢.٥ W (٢) ٢٥ W

(٣) ٥٠ W (٤) ١٠٠ W

Ans. (١)

Sol. Rated power & voltage gives resistance

$R = \frac{V^{٢}}{P} = \frac{(٢٠٠)^{٢}}{٥٠} = \frac{٤٠٠٠٠}{٥٠}$

R = ٨٠٠

$P = \frac{(V_{applied})^{٢}}{R} = \frac{(١٠٠)^{٢}}{٨٠٠}$

P = ١٢.٥ watt

Hence option ١ is correct.

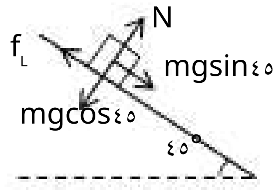
٤٣. A ٢ kg brick begins to slide over a surface which is inclined at an angle of ٤٥° with respect to horizontal axis. The co-efficient of static friction between their surfaces is :

(١) ١ (٢) $\frac{١}{\sqrt{٣}}$

(٣) ٠.٥ (٤) ١.٧

Ans. (١)

Sol.



$mg \sin ٤٥ = f_L$

$mg \cos ٤٥ = N$

$f_L = \mu_s N$

$\mu_s = \tan ٤٥ = ١$

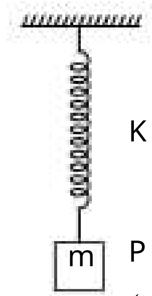
or

$\tan\theta = \mu_s$ (θ is angle of repose)

$\tan ٤٥ = \mu_s = ١$

correct option (١)

٤٤. In simple harmonic motion، the total mechanical energy of given system is E. If mass of oscillating particle P is doubled then the new energy of the system for same amplitude is :



(١) $\frac{E}{\sqrt{٢}}$ (٢) E

(٣) $E\sqrt{٢}$ (٤) ٢E

Ans. (٢)

Sol. T.E. = $\frac{١}{٢} kA^{٢}$

since A is same T.E. will be same

correct option (٢)

٤٥. Given below are two statements : one is labelled as Assertion A and the other is labelled as Reason R.

Assertion A : Number of photons increases with increase in frequency of light.

Reason R : Maximum kinetic energy of emitted electrons increases with the frequency of incident radiation.

In the light of the above statements, choose the most appropriate answer from the options given below :

(١) Both A and R are correct and R is NOT the correct explanation of A.

(٢) A is correct but R is not correct.

(٣) Both A and R are correct and R is the correct explanation of A.

(٤) A is not correct but R is correct.

Ans. (٤)

Sol. Intensity of light $I = \frac{nh□}{A}$

Here n is no. of photons per unit time.

$n □ \frac{IA}{h□}$ so on increasing frequency □, n decreases taking intensity constant.

$k_{max} = h□ - □$

So on increasing □, kinetic energy increases.

٤٦. According to Bohr's theory, the moment of momentum of an electron revolving in ٤[th] orbit of hydrogen atom is :

(١) ٨ $\frac{h}{□}$ (٢) $\frac{h}{□}$

(٣) ٢ $\frac{h}{□}$ (٤) $\frac{h}{٢□}$

Ans. (٣)

Sol. Moment of momentum is $\vec{P}r□$

$\vec{L} □ \vec{r} □ m\vec{v}$

$L = mvr = \frac{nh}{٢□} □ \frac{٤h}{٢□} □ \frac{٢h}{□}$

٤٧. A sample of gas at temperature T is adiabatically expanded to double its volume. Adiabatic constant for the gas is □ = ٣/٢. The work done by the gas in the process is : (μ = ١ mole)

(١) RT□٢□√٢□ (٢) RT□١□٢√٢□

(٣) RT□٢√٢□١□ (٤) RT□٢□√٢□

Ans. (٤)

Sol. $W□\frac{nR□T}{1□□}$

$TV^{□□1}□constant□T f□٢V□^{□□1}$

$T_f = T□\frac{1}{2}□^{١/٢} □ \frac{T}{\sqrt{٢}}$

$W□\frac{R□\frac{T}{\sqrt{2}} □T□}{1□\frac{٣}{٢}} □ ٢RT\frac{□2□1□}{\sqrt{٢}}$

$□RT□\sqrt{2}□2□$

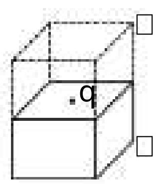
٤٨. A charge q is placed at the center of one of the surface of a cube. The flux linked with the cube is :-

(١) $\frac{q}{4□_0}$ (٢) $\frac{q}{2□_0}$

(٣) $\frac{q}{8□_0}$ (٤) Zero

Ans. (٢)

Sol. From



$2□□\frac{q}{□_0}$

$□□\frac{q}{2□_0}$

٤٩. Applying the principle of homogeneity of dimensions، determine which one is correct. where T is time period، G is gravitational constant، M is mass، r is radius of orbit.

(١) $T^{٢} = \frac{٤\pi^{٢} r}{GM^{٢}}$　　(٢) $T^{٢} = ٤\pi^{٢} r^{٣}$

(٣) $T^{٢} = \frac{٤\pi^{٢} r^{٣}}{GM}$　　(٤) $T^{٢} = \frac{٤\pi^{٢} r^{٢}}{GM}$

Ans. (٣)

Sol. According to principle of homogeneity dimension of LHS should be equal to dimensions of RHS so option (٣) is correct.

$$T^{٢} = \frac{٤\pi^{٢} r^{٣}}{GM}$$

$$\left[T^{٢}\right] = \frac{\left[L^{3}\right]}{\left[M^{-١}L^{٣}T^{-٢}\right][M]}$$

(Dimension of G is $\left[M^{-١}L^{٣}T^{-٢}\right]$)

$$\left[T^{٢}\right] = \left[\frac{L^{٣}}{L^{٣}T^{-2}}\right] = \left[T^{٢}\right]$$

٥٠. A ٩٠ kg body placed at ٢R distance from surface of earth experiences gravitational pull of :

(R = Radius of earth، g = ١٠ $ms^{-٢}$)

(١) ٣٠٠ N　　(٢) ٢٢٥ N

(٣) ١٢٠ N　　(٤) ١٠٠ N

Ans. (٤)

Sol. Value of $g = g_s \left(١ + \frac{h}{R}\right)^{-٢}$

$$= g_s (١ + ٢)^{-2} = \frac{g_s}{٩}$$

Here g_s = gravitational acceleration at surface

Force = mg = ٩٠ × $\frac{g_s}{٩}$ = ١٠٠ N

SECTION-B

٥١. The displacement of a particle executing SHM is given by $x = ١٠ \sin\left(\omega t + \frac{\pi}{3}\right)$ m. The time period of motion is ٣.١٤ s. The velocity of the particle at t = ٠ is ________ m/s.

Ans. (١٠)

Sol. Given

T = ٣.١٤ = $\frac{2\pi}{\omega}$

ω = ٢ rad/s

$$x = ١٠ \sin\left(\omega t + \frac{\pi}{٣}\right)$$

$$v = \frac{dx}{dt} = ١٠\,\omega \cos\left(\omega t + \frac{\pi}{3}\right)$$

at t = ٠

$$v = ١٠\,\omega \cos\left(\frac{\pi}{٣}\right) = ١٠ \times ٢ \times \frac{١}{٢} \quad (\text{using } \omega = ٢ \text{ rad/s})$$

v = ١٠ m/s

٥٢. A bus moving along a straight highway with speed of ٧٢ km/h is brought to halt within ٤s after applying the brakes. The distance travelled by the bus during this time (Assume the retardation is uniform) is ___________m.

Ans. (٤٠)

Sol. Initial velocity = u = ٧٢ km/h = ٢٠ m/s

v = u + at

⇒ ٠ = ٢٠ + a × ٤

a = −٥ $m/s^{٢}$

$v^{٢} - u^{٢} = ٢as$

⇒ $٠^{٢} - ٢٠^{٢} = ٢(-٥).s$

s = ٤٠ m

٥٣. A parallel plate capacitor of capacitance ١٢.٥ pF is charged by a battery connected between its plates to potential difference of ١٢.٠ V. The battery is now disconnected and a dielectric slab (ϵ_r = ٦) is inserted between the plates. The change in its potential energy after inserting the dielectric slab is __________ × $١٠^{-١٢}$ J.

Ans. (٧٥٠)

Sol. Before inserting dielectric capacitance is given $C_\circ = ١٢.٥$ pF and charge on the capacitor $Q = C_\circ V$

After inserting dielectric capacitance will become $\varepsilon_r C_\circ$.

Change in potential energy of the capacitor

$= E_i - E_f$

$$= \frac{Q^٢}{٢C_i} - \frac{Q^٢}{٢C_f} = \frac{Q^٢}{٢C_\circ}\left(١ - \frac{١}{\varepsilon_r}\right)$$

$$= \frac{(C_\circ V)^٢}{٢C_\circ}\left(١ - \frac{١}{\varepsilon_r}\right) = \frac{١}{٢} C_\circ V^٢\left(1 - \frac{1}{\varepsilon_r}\right)$$

Using $C_\circ = ١٢.٥$ pF, $V = ١٢$ V, $\varepsilon_r = ٦$

$$= \frac{١}{٢}(١٢.٥)(١٢)^٢\left(١ - \frac{١}{٦}\right) = \frac{١}{٢}(١٢.٥)(122)\left(\frac{٥}{٦}\right)$$

$= ٧٥٠$ pJ $= ٧٥٠ \times ١٠^{-١٢}$ J

٥٤. In a system two particles of masses $m_١ = ٣$kg and $m_٢ = ٢$kg are placed at certain distance from each other. The particle of mass $m_١$ is moved towards the center of mass of the system through a distance ٢cm. In order to keep the center of mass of the system at the original position, the particle of mass $m_٢$ should move towards the center of mass by the distance ______ cm.

Ans. (٣)

Sol. 

$$\Delta X_{C.O.M.} = \frac{m_١ \Delta x_١ + m_٢ \Delta x_٢}{m_١ + m_٢}$$

$$0 = \frac{٣(٢) + ٢(-x)}{٣ + ٢}$$

$\Rightarrow x = ٣$ cm

٥٥. The disintegration energy Q for the nuclear fission of $^{٢٣٥}U \rightarrow ^{١٤٠}Ce + ^{٩٤}Zr + n$ is ______MeV.

Given atomic masses of

$^{٢٣٥}U : ٢٣٥.٠٤٣٩u ; ^{١٤٠}Ce ; ١٣٩.٩٠٥٤u,$

$^{٩٤}Zr : ٩٣.٩٠٦٣u ; n : ١.٠٠٨٦u,$

Value of $c^٢ = ٩٣١$ MeV/u.

Ans. (٢٠٨)

Sol. $^{٢٣٥}U \rightarrow ^{١٤٠}Ce + ^{٩٤}Zr + n$

Disintegration energy

$Q = (m_R - m_P)c^٢$

$m_R = ٢٣٥.٠٤٣٩$ u

$m_P = ١٣٩.٩٠٥٤u + ٩٣.٩٠٦٣u + ١.٠٠٨٦$ u

$= ٢٣٤.٨٢٠٣u$

$Q = (٢٣٥.٠٤٣٩u - ٢٣٤.٨٢٠٣u)c^٢$

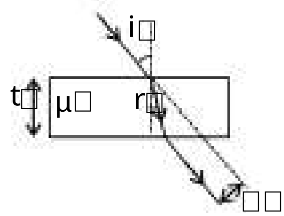
$= ٠.٢٢٣٦ c^٢$

$= ٠.٢٢٣٦ \times ٩٣١$

$Q = ٢٠٨.١٧١٦$

٥٦. A light ray is incident on a glass slab of thickness $٤\sqrt{٣}$ cm and refractive index $\sqrt{٢}$. The angle of incidence is equal to the critical angle for the glass slab with air. The lateral displacement of ray after passing through glass slab is ______cm. (Given sin ١٥° = ٠.٢٥)

Ans. (٢)

Sol. 

$i = \theta_C$

$\Rightarrow i = \sin^{-١}\left(\frac{١}{\mu}\right)$

$\Rightarrow i = ٤٥°$

and according to snell's law

$١ \sin ٤٥° = \sqrt{٢}\sin r$

$\Rightarrow r = ٣٠°$

Lateral displacement $= \frac{t\sin(i - r)}{\cos r}$

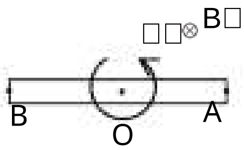
$$= \frac{٤\sqrt{٣}\sin ١٥°}{\cos ٣٠°}$$

$= ٢$ cm

٥٧. A rod of length ٦٠ cm rotates with a uniform angular velocity ٢٠ rad s^{-1} about its perpendicular bisector, in a uniform magnetic field ٠.٥ T. The direction of magnetic field is parallel to the axis of rotation. The potential difference between the two ends of the rod is ______V.

Ans. (٠)

Sol.



$\because V_O - V_A = \frac{B\omega\ell^2}{٢}$

$V_O - V_B = \frac{B\omega\ell^2}{٢}$

$\Rightarrow V_A - V_B = ٠$

٥٨. Two wires A and B are made up of the same material and have the same mass. Wire A has radius of ٢.٠ mm and wire B has radius of ٤.٠ mm. The resistance of wire B is ٢Ω. The resistance of wire A is _______Ω.

Ans. (٣٢)

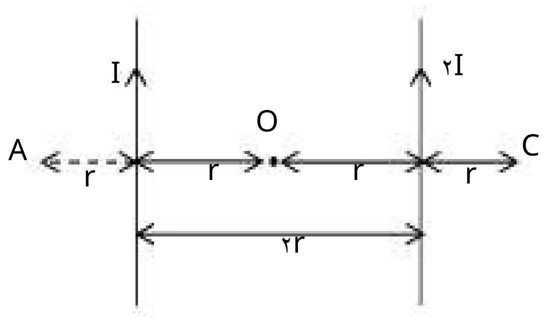
Sol. $\because R = \frac{\rho\ell}{A} = \frac{\rho V}{A^٢}$

$\Rightarrow \frac{R_A}{R_B} = \frac{A_B^٢}{A_A^٢} = \frac{r_B^٤}{r_A^٤}$

$\Rightarrow \frac{R_A}{٢} = \frac{(٤\times١٠^{-٣})^٤}{(٢\times١٠^{-٣})^٤}$

$\Rightarrow R_A = ٣٢\ \Omega.$

٥٩. Two parallel long current carrying wire separated by a distance ٢r are shown in the figure. The ratio of magnetic field at A to the magnetic field produced at C is $\frac{x}{٧}$. The value of x is _____.



Ans. (٥)

Sol. $B_A = \frac{\mu_0 i}{٢\pi r} + \frac{\mu_0 (٢i)}{٢\pi(٣r)} = \frac{٥\mu_0 i}{٦\pi r}$

$B_C = \frac{\mu_0 (٢i)}{٢\pi r} + \frac{\mu_0 i}{٢\pi(٣r)} = \frac{٧\mu_0 i}{٦\pi r}$

$\Rightarrow \frac{B_A}{B_C} = \frac{٥}{٧}$

$\Rightarrow x = ٥$

٦٠. Mercury is filled in a tube of radius ٢ cm up to a height of ٣٠ cm. The force exerted by mercury on the bottom of the tube is _____N.

(Given, atmospheric pressure = $١٠^٥$ $Nm^{-٢}$, density of mercury = ١.٣٦×١٠^٤ kg $m^{-٣}$, g = ١٠ $ms^{-٢}$, $\pi = \frac{٢٢}{٧}$)

Ans. (١٧٧)

Sol. $F = P_0 A + \rho g h A$

$= ١٠^٥ \times \frac{٢٢}{٧} \times (2\times10^{-2})^2$

$+ ١.٣٦\times١٠^٤ \times 10 \times (٣٠\times١٠^{-٢}) \times \frac{٢٢}{٧} \times (٢\times١٠^{-٢})^٢$

F = ٥١.٢٩ + ١٢٥.٧١ = ١٧٧ N

CHEMISTRY

TEST PAPER WITH SOLUTION

SECTION-A

٦١. The equilibrium constant for the reaction

$$SO_3(g) \rightleftharpoons SO_2(g) + \frac{١}{٢}O_2(g)$$

is KC = ٤.٩ × ١٠. The value of KC for the reaction given below is

$$٢SO_2(g) + O_2(g) \rightleftharpoons ٢SO_3(g)$$ is

(١) ٤.٩ (٢) ٤١.٦

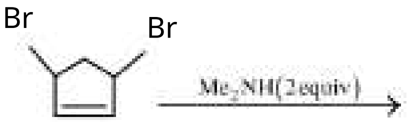
(٣) ٤٩ (٤) ٤١٦

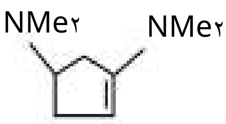
Ans. (٤)

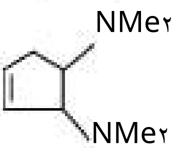
Sol. $K'_C = \left(\frac{1}{K_C}\right)^2 = \left(\frac{1}{4.9\times 10^{-2}}\right)^2$

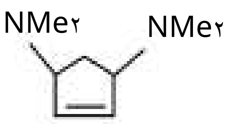
K'C = ٤١٦.٤٩

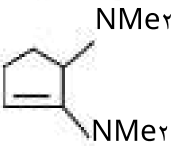
٦٢. Find out the major product formed from the following reaction. [Me : –CH٣]



(١) 

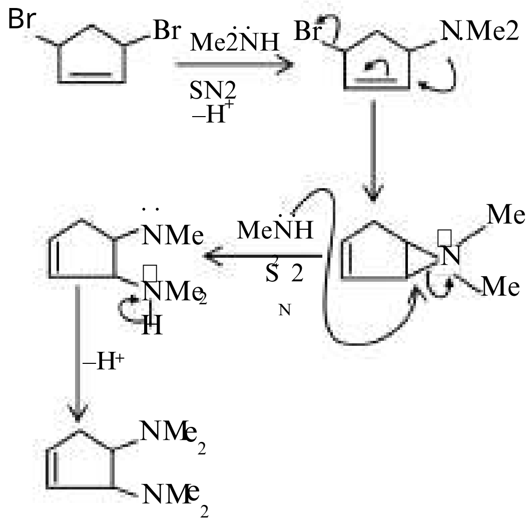
(٢) 

(٣) 

(٤) 

Ans. (٢)

Sol.



The above mechanism valid for both cis and trans isomers. So the products are same for both cis and trans isomers.

٦٣. When MnO_2 and H_2SO_4 is added to a salt (A), the greenish yellow gas liberated as salt (A) is :

(١) NaBr (٢) CaI₂

(٣) KNO₃ (٤) NH₄Cl

Ans. (٤)

Sol. $2NH_4Cl + MnO_2 + 2H_2SO_4 \rightarrow MnSO_4 + (NH_4)_2SO_4 + 2H_2O + \underset{\text{greenish yellow solution}}{Cl_2}$

٦٤. The correct statement/s about Hydrogen bonding is/are :

A. Hydrogen bonding exists when H is covalently bonded to the highly electro negative atom.

B. Intermolecular H bonding is present in o-nitro phenol

C. Intramolecular H bonding is present in HF.

D. The magnitude of H bonding depends on the physical state of the compound.

E. H-bonding has powerful effect on the structure and properties of compounds.

Choose the correct answer from the options given below :

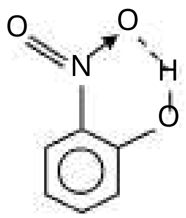
(١) A only (٢) A, D, E only

(٣) A, B, D only (٤) A, B, C only

Ans. (٢)

Sol. (A) Generally hydrogen bonding exists when H is covalently bonded to the highly electronegative atom like F, O, N.

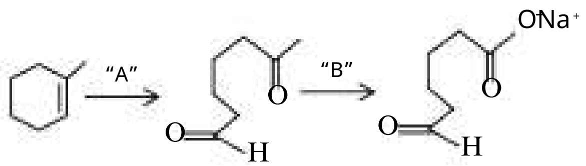
(B) Intramolecular H bonding is present in



(C) Intermolecular Hydrogen bonding is present in HF

(D) The magnitude has Hydrogen bonding in solid state is greater than liquid state.

(E) Hydrogen bonding has powerfull effect on the structure & properties of compound like melting point, boiling point, density etc.

٦٥. 

In the above chemical reaction sequence "A'' and "B" respectively are :

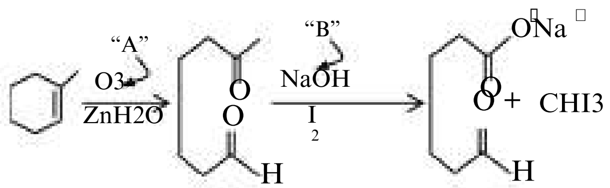
(١) $O_٣$, Zn/$H_٢O$ and NaOH(alc.) / $I_٢$

(٢) $H_٢O$, H^+ and NaOH(alc.) / $I_٢$

(٣) $H_٢O$, H^+ and $KMnO_٤$

(٤) $O_٣$, Zn/$H_٢O$ and $KMnO_٤$

Ans. (١)

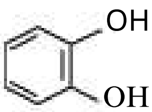
Sol. 

٦٦. Common name of Benzene-١,٢-diol is

(١) quinol (٢) resorcinol

(٣) catechol (٤) o-cresol

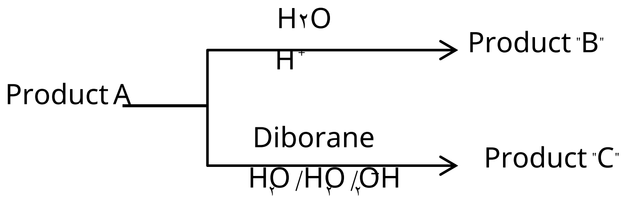
Ans. (٣)

Sol. 

IUPAC name : Benzene-١,٢-diol

Common name : catechol

٦٧. $CH_٣ - CH_٢ - CH_٢ - Br + NaOH$ C2H5OH Product 'A'



Consider the above reactions, identify product B and product C.

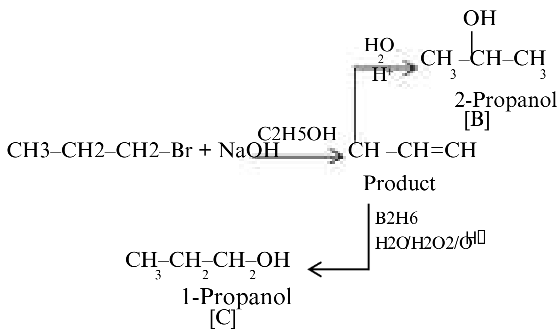
(١) B = C = ٢-Propanol

(٢) B = ٢-Propanol C = ١-Propanol

(٣) B = ١-Propanol C = ٢-Propanol

(٤) B = C = ١-Propanol

Ans. (٢)

Sol. 

٦٨. The adsorbent used in adsorption chromatography is/are

A. silica gel B. alumina

C. quick lime D. magnesia

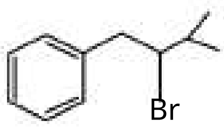
Choose the most appropriate answer from the options given below :

(١) B only (٢) C and D only

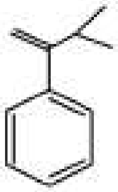
(٣) A and B only (٤) A only

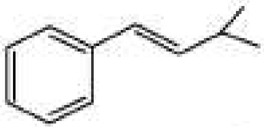
Ans. (٣)

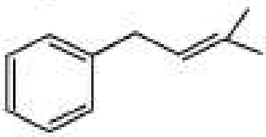
Sol. The most common polar and acidic support used is adsorption chromatography is silica. The surface silanol groups on their supported to adsorb polar compound and work particularly well for basic substances. Alumina is the example of polar and basic adsorbentthat is used in adsorption chromatography.

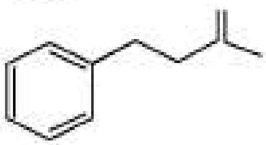
٦٩.  KOH(alc) major product "P"

Product P is

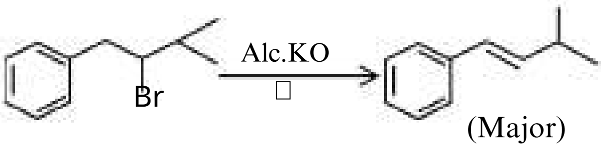
(١) 

(٢) 

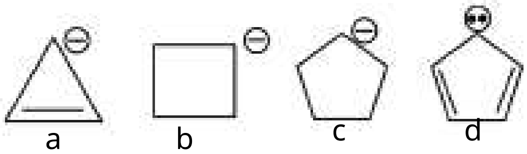
(٣) 

(٤) 

Ans. (٢)

Sol. 

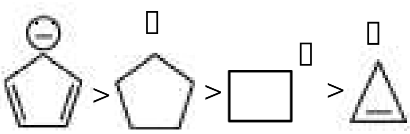
٧٠. Correct order of stability of carbanion is



(١) c < b < d < a　　(٢) a < b < c < d
(٣) d < a < c < b　　(٤) d < c < b < a

Ans. (٤)

Sol. As we know compound (d) is aromatic and the compound (a) is anti-aromatic. Hence compound (d) is most stable and compound (a) is least stable among these in compound (b) and (c) carbon atom of that positive charge is sp hybridised they on the basis of angle strain theory compound (c) is more stable than compound (b).



٧١. The correct order of the first ionization enthalpy is

(١) Al < Ga < Tl　　(٢) Ga < Al < B
(٣) B < Al < Ga　　(٤) Tl < Ga < Al

Ans. (٤)

Sol. (i) due to lanthanide contraction Tl has more I.E. as compared to Ga and Al.

(ii) due to scandide contraction Ga has more I.E. as compared to Al.

٧٢. If an iron (III) complex with the formula $[Fe(NH_3)_x(CN)_y]^-$ has no electron in its e_g orbital, then the value of x + y is

(١) ٥　　(٢) ٦
(٣) ٣　　(٤) ٤

Ans. (٢)

Sol. Complex is $[\overset{III}{Fe}(NH_3)_2(CN)_4]^-$

x = ٢
y = ٤
so x + y = ٦

٧٣. Fuel cell, using hydrogen and oxygen as fuels,

A. has been used in spaceship
B. has as efficiency of ٤٠٪ to produce electricity
C. uses aluminium as catalysts
D. is eco-friendly
E. is actually a type of Galvanic cell only

(١) A, B, C only　　(٢) A, B, D only
(٣) A, B, D, E only　　(٤) A, D, E only

Ans. (٤)

Sol. Fuel cell is used in spaceship and it is type of galvanic cell.

٧٤. Choose the Incorrect Statement about Dalton's Atomic Theory

(١) Compounds are formed when atoms of different elements combine in any ratio
(٢) All the atoms of a given element have identical properties including identical mass
(٣) Matter consists of indivisible atoms
(٤) Chemical reactions involve recorganization of atoms

Ans. (١)

Sol. In compound atoms of different elements combine in fixed ratio by mass.

٧٥. Match List I with List II

	LIST I		LIST II
A	a - Glucose and a-Galactose	I.	Functional isomers
.	a - Glucose and b-Glucose	II.	Homologous
B	a - Glucose and a-Fructose	III.	Anomers
.	a - Glucose and a-Ribose	IV.	Epimers

Choose the correct answer from the options given below: (١) A-III, B-IV, C-II, D-I (٢) A-III, B-IV, C-I, D-II (٣) A-IV, B-III, C-I, D-II (٤) A-IV, B-III, C-II, D-I

Ans. (٣)

Sol. Based on biomolecules theory and structure of these named compounds –

(A) a-Glucose and a-Galactose (IV) Epimers.

(B) a-Glucose and b-Glucose (III) Anomers (C) a-Glucose and a-Fructose (I) Functional isomers (D) a-Glucose and a-Ribose (II) Homologous

٧٦. Given below are two statements: Statement I : The correct order of first ionization enthalpy values of Li, Na, F and Cl is Na > Li > Cl > F. Statement II : The correct order of negative electron gain enthalpy values of Li, Na, F and Cl is Na > Li > F > Cl In the light of the above statements, choose the correct answer from the options given below :

(١) Both Statement I and Statement II are true

(٢) Both Statement I and Statement II are false

(٣) Statement I is false but Statement II is true

(٤) Statement I is true but Statement II is false

Ans. (١)

Sol.. (i) Na < Li < Cl < F

I.E1 in kJ/mol 496 520 1256 1681

(ii) Na < Li < F < Cl

$\Delta_{eg}H$ in kJ/mol –53 –60 –328 –349

٧٧. For a strong electrolyte, a plot of molar conductivity against (concentration)$^{1/2}$ is a straight line, with a negative slope, the correct unit for the slope is

(١) $S\ cm^2\ mol^{-3/2}\ L^{1/2}$ (٢) $S\ cm^2\ mol^{-1}\ L^{1/2}$

(٣) $S\ cm^2\ mol^{-3/2}$ (٤) $S\ cm^2\ mol^{-3/2}\ L^{-1/2}$

Ans. (١)

Sol. $\Lambda_m = \Lambda^{o}_m - A\sqrt{C}$

Units of $A\sqrt{C}$ = $S\ cm^2\ mole^{-1}$

Uits of A = $S\ cm^2\ mole^{-3/2}\ L^{1/2}$

٧٨. A first row transition metal in its +٢ oxidation state has a spin-only magnetic moment value of ٣.٨٦ BM. The atomic number of the metal is

(١) ٢٥ (٢) ٢٦

(٣) ٢٢ (٤) ٢٣

Ans. (٤)

Sol. ٢٢Ti^{2+} [Ar]٣d٢

٢٣V^{2+} [Ar]٣d٣

٢٥Mn^{2+} [Ar]٣d٥

٢٦Fe^{2+} [Ar]٣d٦

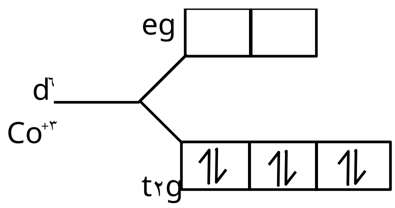
٧٩. The number of unpaired d-electrons in [Co(H٢O)٦]$^{3+}$ is__________

(١) ٤ (٢) ٢

(٣) ٠ (٤) ١

Ans. (٣)

Sol. [Co(H٢O)٦]$^{3+}$



No unpaired electrons

٨٠. The number of species from the following that have pyramidal geometry around the central atom is_____________

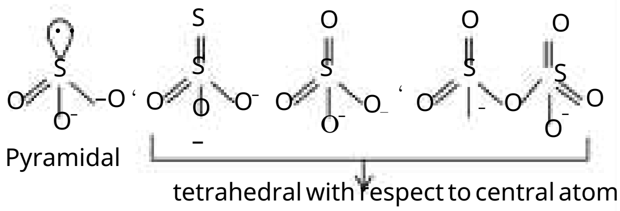
SO_3^{2-}, SO_4^{2-}, SO_3, SO_2^{-} (printed: SO٢ ، SO٢ ، SO٢ SO٢ ; ٢ ٣ ٤ ٣ ٢ ٧)

(١) ٤ (٢) ٣

(٣) ١ (٤) ٢

Ans. (٣)

Sol.

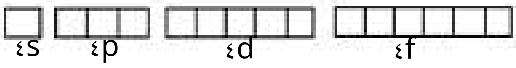


SECTION-B

٨١. The maximum number of orbitals which can be identified with n = ٤ and ml = ٠ is_______

Ans. (٤)

Sol. n = ٤ ،



m ١ ١ ١ ١

So answer is ٤.

٨٢. Number of compounds/species from the following with non-zero dipole moment is________

BeCl٢، BCl٣، NF٣، XeF٤، CCl٤، H٢O H٢S، HBr، CO٢، H٢، HCl

Ans. (٥)

Sol. Polar molecule: NF3,H2O,H2S,HBr,HCl
$(\mu \neq 0)$

NonPolar molecule: BeCl2 , BCl3 , XeF4 , CCl4 , CO2 , H 2
$(\mu = 0)$

So answer is ٥.

٨٣. Three moles of an ideal gas are compressed isothermally from ٦٠ L to ٢٠ L using constant pressure of ٥ atm. Heat exchange Q for the compression is - ______ Lit. atm.

Ans. (٢٠٠)

Sol. As isothermal $\Delta U = ٠$

and process is irreversible

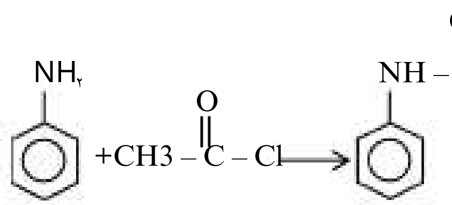
Q = -W = -[- Pext (V٢ - V١)]

Q = ٥ (٢٠ - ٦٠) = -٢٠٠ atm-L

٨٤. From ٦.٥٥ g of aniline، the maximum amount of acetanilide that can be prepared will be_____×١٠$^{-١}$ g.

Ans. (٩٥)

Sol.



٩٣ g aniline form ١٣٥ gm acetanlide

so ٦.٥٥ g anilne form $\frac{١٣٥}{٩٣}$ × ٦.٥٥ = ٩.٥

٩٥ × ١٠ $^{-١}$

٨٥. Consider the following reaction، the rate expression of which is given below

A + B → C

rate = k[A]$^{١/٢}$[B]$^{١/٢}$

The reaction is initiated by taking ١M concentration A and B each. If the rate constant (k) is ٤.٦ × ١٠$^{-٢}$ s$^{-١}$، then the time taken for A to become ٠.١ M is_______sec. (nearest integer)

Ans. (٥٠)

Sol. $K = \frac{2.303}{t}\log\frac{1}{0.1}$

٤.٦ × ١٠ $^{-٢}$ = $\frac{2.303}{t}$

t = ٥٠ sec.

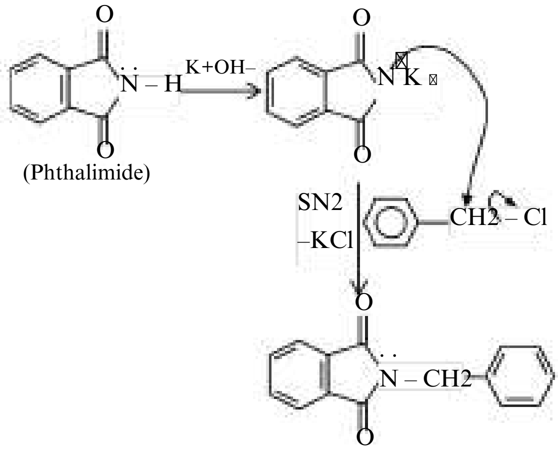
٨٦. Phthalimide is made to undergo following sequence of reactions.

Phthalimide $\xrightarrow[\text{(ii)Benzylchloride}]{\text{(i)KOH}}$ 'P'

Total number of π bonds present in product 'P' is/are

Ans. (٨)

Sol.



(P)

Total number of π-bonds present in product P is ٨

٨٧. The total number of 'sigma' and 'Pi' bonds in ٢-oxohex-٤-ynoic acid is______.

Ans. (١٨)

Sol.

HO–C(=O)–C(=O)–CH₂–C≡C–CH₃

2-Oxohex-4-ynoic acid

Number of σ-bonds = ١٤

Number of π-bonds = ٤

= ١٨

٨٨. A first row transition metal with highest enthalpy of atomisation, upon reaction with oxygen at high temperature forms oxides of formula M_2O_n (where n = ٣,٤,٥). The 'spin-only' magnetic moment value of the amphoteric oxide from the above oxides is____ BM (near integer)

(Given atomic number : Sc : ٢١, Ti : ٢٢, V : ٢٣, Cr : ٢٤, Mn : ٢٥, Fe : ٢٦, Co : ٢٧, Ni : ٢٨, Cu : ٢٩, Zn : ٣٠)

Ans. (٠)

Sol. 'V' has highest enthalpy of atomisation (٥١٥ kJ/mol) among first row transition elements.

V_2O_5

Here 'V' is in +٥ oxidation state

V^{+5} : $1s^2 2s^2 2p^6 3s^2 3p^6$ (no unpaired electrons)

٨٩. ٢.٧ Kg of each of water and acetic acid are mixed, The freezing point of the solution will be –x °C. Consider the acetic acid does not dimerise in water, nor dissociates in water x = __________(nearest integer)

Given : Molar mass of water = ١٨ g mol^{-1}, acetic acid = ٦٠ g mol^{-1}

K_f H_2O : ١.٨٦ K kg mol^{-1}

K_f acetic acid : ٣.٩٠ K kg mol^{-1}

freezing point : H_2O = ٢٧٣ K, acetic acid = ٢٩٠ K

Ans. (٣١)

Sol. As moles of water < moles of CH_3COOH

water is solvent.

$T°F - (T_F)S = K_F \times M$

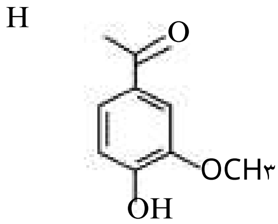
$٠ - (T_F)S = ١.٨٦ \times \frac{2700/60}{2700/1000}$

$(T_F)S = -٣١°C$.

٩٠. Vanillin compound obtained from vanilla beans, has total sum of oxygen atoms and π electrons is___

Ans. (١١)

Sol. Vanillin compound is an organic compound molecular formula $C_8H_8O_3$. It is a phenolic aldehyde. Its functional compounds include aldehyde, hydroxyl and ether. It is the primary component of the extract of the vanilla beans.



Total sum of oxygen atoms and π-electrons is ٣ + ٨ = ١١

Total number of oxygen atoms = ٣

Total number of π-bonds = ٤

∴ Total number of π-electrons = ٨