FINAL JEE–MAIN EXAMINATION – APRIL, 2024

(Held On Friday 05 April, 2024) TIME : 9 : 00 AM to 12 : 00 NOON

M ATHEM ATICS

SECTION-A

١. Let d be the distance of the point of intersection of the lines $\frac{x-٦}{٣}=\frac{y}{٢}=\frac{z-١}{١}$ and $\frac{x-٧}{٤}=\frac{y-٩}{٣}=\frac{z-٤}{٢}$ from the point (٧، ٨، ٩). Then $d^٢+٦$ is equalto :

(١) ٧٢ (٢) ٦٩

(٣) ٧٥ (٤) ٧٨

Ans. (٣)

Sol. $\frac{x-٦}{٣}=\frac{y}{٢}=\frac{z-١}{١}=\lambda$ ...(١)

$x=٣\lambda+٦،\ y=٢\lambda،\ z=\lambda+١$

$\frac{x-٧}{٤}=\frac{y-٩}{٣}=\frac{z-٤}{٢}=\mu$ ...(٢)

$x=٤\mu+٧،\ y=٣\mu+٩،\ z=٢\mu+٤$

$٣\lambda-٦=٤\mu+٧\ \Rightarrow\ ٣\lambda-٤\mu=١٣$ ...(٣) × ٢

$٢\lambda=٣\mu+٩\ \Rightarrow\ ٢\lambda-٣\mu=٩$ ...(٤) × ٣

$٦\lambda-٨\mu=٢٦$

$٦\lambda-٩\mu=٢٧$

$-\quad+\quad-$

$\mu=-١$

$\lambda٣-٤(-١)=١٣$

$٣\lambda=٩$

$\lambda=٣$

int. point (٣، ٦، ٢) ؛ (٧، ٨، ٩)

$d^٢=١٦+٤+٤٩=٦٩$

Ans. $d^٢+٦=٦٩+٦=٧٥$

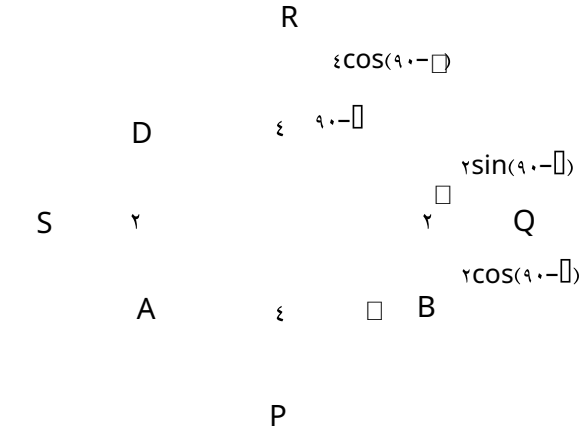
٢. Let a rectangle ABCD of sides ٢ and ٤ be inscribed in another rectangle PQRS such that the vertices of the rectangle ABCD lie on the sides of the rectangle PQRS. Let a and b be the sides of the rectangle PQRS when its area is maximum. Then $(a+b)^٢$ is equal to :

(١) ٧٢ (٢) ٦٠

(٣) ٨٠ (٤) ٦٤

Ans. (١)

Sol.



$Area=(٤\cos\theta+٢\sin\theta)(٢\cos\theta+٤\sin\theta)$

$=٨\cos^٢\theta+١٦\sin\theta\cos\theta+٤\sin\theta\cos\theta+٨\sin^٢\theta$

$=٨+٢٠\sin\theta\cos\theta$

$=٨+١٠\sin٢\theta$

Max Area = ٨ + ١٠ = ١٨ (sin٢θ = ١)

$(a+b)^٢=(٤\cos\theta+٢\sin\theta)^٢+(٢\cos\theta+٤\sin\theta)^٢$

$=(٦\cos\theta+٦\sin\theta)^٢$

$=٣٦(\sin\theta+\cos\theta)^٢$

$=٣٦(١+\sin٢\theta)$

θ = π/٤

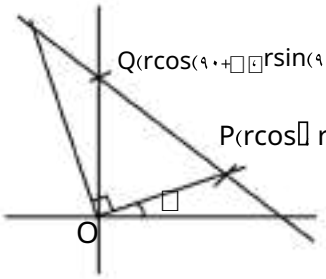
٣. Let two straight lines drawn from the origin O intersect the line $3x + 4y = 12$ at the points P and Q such that $\triangle OPQ$ is an isosceles triangle and $\angle POQ = 90^\circ$. If $l = OP + PQ + QO$, then the greatest integer less than or equal to l is :

(١) ٤٤ (٢) ٤٨

(٣) ٤٦ (٤) ٤٢

Ans. (٣)

Sol.



$3x + 4y = 12$

$3(r\cos\theta) + 4(r\sin\theta) = 12$

$r(3\cos\theta + 4\sin\theta) = 12 \ldots(1)$

$3(-r\sin\theta) + 4(r\cos\theta) = 12$

$r(-3\sin\theta + 4\cos\theta) = 12 \ldots(2)$

$\left(\frac{12}{r}\right)^2 + \left(\frac{12}{r}\right)^2 = (3\cos\theta + 4\sin\theta)^2 + (3\sin\theta - 4\cos\theta)^2$

$2\left(\frac{12}{r}\right)^2 = 9 + 16$

$\frac{2 \times 144}{r^2} = 25 \Rightarrow 288 = 25r^2$

$\Rightarrow r = \frac{288}{25}$

$\Rightarrow r = \sqrt{2}\left(\frac{12}{5}\right) = \sqrt{2}r$

$l = OP + PQ + QO$

$= r + r\sqrt{2} + r(\cos\theta + \sin\theta) + r(\sin\theta + \cos\theta)$

$= 2r + r(1 + \sin 2\theta + 1 - 2\sin^2\theta)$

$= 2r + 2r$

$= 4r$

$= 4\left(\frac{288}{25}\right) = \frac{1152}{25} = 46.08$

$\Rightarrow$ [illegible] = ٤٦

٤. If $y = y(x)$ is the solution of the differential equation $\frac{dy}{dx} + 2y = \sin(2x)$, $y(0) = \frac{3}{4}$, then $y\left(\frac{\pi}{8}\right)$ is equal to :

(١) $e^{-\pi/8}$ (٢) $e^{-\pi/4}$

Ans. (٢) (٣) e (٤) $e^{\pi/8}$

Sol. $\frac{dy}{dx} + 2y = \sin 2x, \quad y(0) = \frac{3}{4}$

$I.F = e^{\int 2dx} = e^{2x}$

$y \cdot e^{2x} = \int e^{2x}\sin 2x\, dx$

$y \cdot e^{2x} = \frac{e^{2x}(2\sin 2x - 2\cos 2x)}{4 + 4} + C$

$x = 0, y = \frac{3}{4} \Rightarrow \frac{3}{4} \cdot 1 = \frac{1(0 - 2)}{8} + C$

$\frac{3}{4} + \frac{1}{4} = C$

$1 = C$

$y = \frac{2\sin 2x - 2\cos 2x}{8} + 1 \cdot e^{-2x}$

$x = \frac{\pi}{8}, \quad y = \frac{1}{8} 2\sin\frac{\pi}{4} - 2\cos\frac{\pi}{4} + e^{-2\cdot\frac{\pi}{8}}$

$y = 0 + e^{-\frac{\pi}{4}}$

٥. For the function $f(x) = \sin x + 3x - \frac{2}{\pi}(x^2 + x)$, where $x \in \left[0, \frac{\pi}{2}\right]$, consider the following two statements :

(I) f is increasing in $\left[0, \frac{\pi}{2}\right]$.

(II) f' is decreasing in $\left[0, \frac{\pi}{2}\right]$.

Between the above two statements,

(١) only (I) is true.

(٢) only (II) is true.

(٣) neither (I) nor (II) is true.

(٤) both (I) and (II) are true.

Ans. (٤)

Sol. $f(x) = \sin x + 3x - \frac{2}{\square}(x^{\square} + x)$ $x \square \square$

$f'(x) = \cos x + 3 - \frac{2}{\square}(2x + 1) < 0$ $f(x)$

$f'(x) = -\sin x + 0 - \frac{\square}{\square}(2)$

$= -\sin x - \frac{4}{\square} > 0$ $f'(x)$

$0 > x > \frac{\square}{2}$

$\square \quad \square^{2} \square \square 0 \square \square 1 \square \square$

$\square \quad \square^{2} \quad \square^{2}$

$\square 3 \qquad \square 3 \qquad \square^{3}$

$3\square^{2} \quad \square 3\square^{2}(2x \square 1) \quad \square 3\square^{2}(\square \square 1)$

$(\square ve) \qquad\qquad (\square ve)$

٦. If the system of equations

$11x + y + \square z = -5$

$2x + 3y + 5z = 3$

$8x - 19y - 39z = \mu$

has infinitely many solutions, then $\square\mu$ is equal to :

(١) ٤٩ (٢) ٤٥

(٣) ٤٧ (٤) ٥١

Ans. (٣)

Sol. $11x + y + \square z = -5$

$2x + 3y + 5z = 3$

$8x - 19y - 39z = \mu$

for infinite sol.

$D \square \begin{vmatrix} 11 & 1 & \square \\ 2 & 3 & 5 \\ 8 & \square 19 & \square 39 \end{vmatrix} \square 0$

$\square 11(-117 + 95) - 1(-78 - 40) + \square(-38 - 24)$

$\square 11(-22) + 118 - \square(62) = 0$

$\square 62\square = 118 - 242$

$\square \quad \square\square\frac{\square 124}{62} \quad \square\square 2$

$D_{1} \square \begin{vmatrix} \square 5 & 1 & \square 2 \\ 3 & 3 & 5 \\ \mu & \square 19 & \square 39 \end{vmatrix} \square 0$

$\square -5(-117 + 95) - 1(-117 - 5\mu) - 2(-57 - 3\mu) = 0$

$\square -5(-22) + 117 + 5\mu + 114 + 6\mu = 0$

$\square 11\mu = -110 - 231 = -341$

$\square \mu = -31$

$\square \mu = (-2) - (-31) = 16 + 31 = 47$

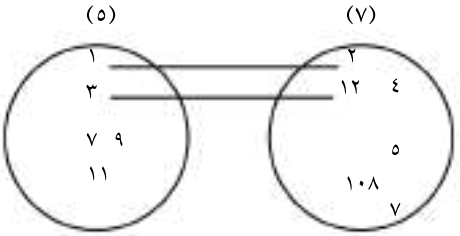
٧. Let $A = \{1, 3, 7, 9, 11\}$ and $B = \{2, 4, 5, 7, 8, 10, 12\}$. Then the total number of one-one maps $f : A \square B$, such that $f(1) + f(3) = 14$, is :

(١) ١٨٠ (٢) ١٢٠

(٣) ٤٨٠ (٤) ٢٤٠

Ans. (٤)

Sol.



$A = \{1, 3, 7, 9, 11\}$

$B = \{2, 4, 5, 7, 8, 10, 12\}$

$f(1) + f(3) = 14$

(i) ٢ + ١٢

(ii) ٤ + ١٠

$2 \times (2 \times 5 \times 4 \times 3) = 240$

٨. If the function $f(x) \square \frac{\sin 3x \square\square \sin x \square\square \cos 3x}{x^{3}}$,

$x \square R$, is continuous at $x = 0$, then $f(0)$ is equal to :

(١) ٢ (٢) −٢

(٣) ٤ (٤) −٤

Ans. (٤)

Sol. $f(x) \square \frac{\sin 3x \square\square \sin x \square\square \cos 3x}{x^{3}}$

is continuous at $x = 0$

$\lim_{x \square 0} \frac{3x \square \frac{(3x)^{3}}{\lfloor 3} \square \cdots \square\square\square x \square \frac{x^{3}}{\lfloor 3} \cdots \square \square \square \square 1 \square \frac{(3x)^{2}}{\lfloor 2} \cdots \square}{x^{3}} \square f(0)$

$$\lim_{x\to 0}\frac{x(3+\cos x)-\frac{9x^2}{2!}+\frac{27}{3!}+\frac{x^3}{3}\ldots}{x^3}=f(0)$$

for exist

$\alpha=0,\ 3+\alpha=0,\quad \frac{27}{3!}+\frac{\alpha}{3}=f(0)$

$\alpha=-3,\ \frac{27}{6}+\frac{(-3)}{6}=f(0)$

$f(0)=\frac{27-3}{6}=4$

9. The integral $\int_{\pi/4}^{\pi/2}\frac{13\sin x}{3\sin x+5\cos x}dx$ is equal to :

(1) $3\pi-50\log_e 2+20\log_e 5$

(2) $3\pi-25\log_e 2+10\log_e 5$

(3) $3\pi-10\log_e 2+20\log_e 5$

(4) $3\pi-30\log_e 2+20\log_e 5$

Ans. (1)

Sol. $I=\int_{\pi/4}^{\pi/2}\frac{13\sin x}{3\sin x+5\cos x}dx$

$13\sin x=A(3\sin x+5\cos x)+B(3\cos x-5\sin x)$

$13=3A-5B \quad \ldots(1)$

$0=5A+3B \quad \ldots(2)$

$3B=-5A \Rightarrow B=-\frac{5}{3}A$

$13=3A+5\left(-\frac{5}{3}A\right)$

$13=3A-\frac{25}{3}A$

$13=\frac{34A}{3}$

$A=\frac{13\times 3}{34}=\frac{39}{34}$

$B=\frac{-5}{3}\left(\frac{39}{34}\right)=-\frac{65}{34}$

$I=\int_{\pi/4}^{\pi/2}\frac{A(3\sin x+5\cos x)}{3\sin x+5\cos x}dx+\int_{\pi/4}^{\pi/2}\frac{B(3\cos x-5\sin x)}{3\sin x+5\cos x}dx$

$=A\int_{\pi/4}^{\pi/2}dx+B\int_{\pi/4}^{\pi/2}\frac{3\cos x-5\sin x}{3\sin x+5\cos x}dx$

$=\frac{39}{34}\left(\frac{\pi}{2}-\frac{\pi}{4}\right)-\frac{65}{34}\left[\log_e(3\sin x+5\cos x)\right]_{\pi/4}^{\pi/2}$

$=\frac{39}{34}\left(\frac{\pi}{4}\right)-\frac{65}{34}\log_e\left(\frac{3}{\sqrt{2}}+\frac{5}{\sqrt{2}}\right)+\frac{65}{34}\log_e\left(\frac{8}{\sqrt{2}}\right)$

$=3\left(\frac{\pi}{4}\right)-\frac{65}{34}\log_e 4\sqrt{2}+\frac{65}{34}\log_e 5$

$=3\left(\frac{\pi}{4}\right)-\frac{65}{34}\log_e 2^{\frac{5}{2}}+\frac{65}{34}\log_e 5$

$=3\pi-\frac{325}{34}\log_e 2+\frac{65}{34}\log_e 5$

10. The coefficients a, b, c in the quadratic equation $ax^2+bx+c=0$ are chosen from the set $\{1,2,3,4,5,6,7,8\}$. The probability of this equation having repeated roots is :

(1) $\frac{3}{256}$ (2) $\frac{1}{128}$

(3) $\frac{1}{64}$ (4) $\frac{3}{128}$

Ans. (3)

Sol. $ax^2+bx+c=0$

$a, b, c \in \{1,2,3,4,5,6,7,8\}$

Repeated roots $D=0$

$b^2-4ac=0 \Rightarrow b^2=4ac$

$\text{Prob}=\frac{8}{8\times 8\times 8}=\frac{1}{64}$

(a, b, c)

$(1,2,1);(2,4,2);(1,4,4);(4,4,1);(3,6,3);$

$(2,8,8);(8,8,2);(4,8,4)$

8 case

١١. Let A and B be two square matrices of order ٣ such that $|A| = ٣$ and $|B| = ٢$.

Then $|A^T A(adj(٢A))(adj(٤B))(adj(AB))AA|^T$ is equal to :

(١) ٦٤ (٢) ٨١

(٣) ٣٢ (٤) ١٠٨

Ans. (١)

Sol. $|A| = ٣, |B| = ٢$

$|A^T A(adj(٢A))^{-١}(adj(٤B))(adj(AB))A^T A|$

$= ٣ \times ٣ \times |(adj(٢A)|^{-١}|adj(٤B)| \times |(adj(AB))| \times ٣ \times ٣$

$\frac{١}{|adj(٢A)|}$ ٢١٢×٢٢ $\frac{١}{|adj(AB)|}$

$\frac{١}{٢٦|adjA|}$ $\frac{١}{|adjB \cdot adjA|}$

$\frac{١}{٢٦ \cdot ٣٢}$ $\frac{١}{٢٢ \cdot ٣٢}$

$٣٠ \frac{١}{٢٠٣^{٢}} \cdot ٢^{١٢} \cdot ٢^{٢} \frac{١}{٢٠٣^{٢}} ٦٤$

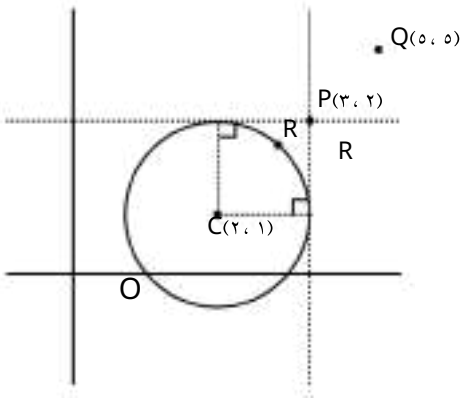
١٢. Let a circle C of radius ١ and closer to the origin be such that the lines passing through the point (٣, ٢) and parallel to the coordinate axes touch it. Then the shortest distance of the circle C from the point (٥, ٥) is :

(١) $٢\sqrt{}$ (٢) ٥

(٣) $٤\sqrt{}$ (٤) ٤

Ans. (٤)

Sol.



Coordinates of the centre will be (٢, ١)

Equation of circle will be

$(x - ٢)^{٢} + (y - ١)^{٢} = ١^{٢}$

$QC = \sqrt{(٥-٢)^{٢}+(٥-١)^{٢}}$

$QC = ٥$

shortest distance

$= RQ = CQ - CR$

$= ٥ - ١$

$= ٤$ Let the line $٢x + ٣y - k = ٠, k < ٠$, intersect the

١٣. x-axis and y-axis at the points A and B,

respectively. If the equation of the circle having the

line segment AB as a diameter is $x^{٢} + y^{٢} - ٣x - ٢y = ٠$ and the length of the latus rectum of the ellipse

$x^{٢} + ٩y^{٢} = k^{٢}$ is $\frac{m}{n}$, where m and n are coprime,

then $٢m + n$ is equal to

(١) ١٠ (٢) ١١

(٣) ١٣ (٤) ١٢

Ans. (٢)

Sol. Centre of the circle $= \left(\frac{٣}{٢}, ١\right)$

Equation of diameter $= ٢x + ٣y - k = ٠$

$2\left(\frac{3}{2}\right) + ٣(١) - k = ٠$

$k = ٦$

Now, Equation of ellipse becomes

$x^{٢} + ٩y^{٢} = ٣٦$

$\frac{x^{٢}}{٦^{٢}} + \frac{y^{٢}}{٢^{٢}} = ١$

٦ ٢

٢ ٢

length of LR $= \frac{2b^2}{a} = \frac{2 \cdot 22}{6} = \frac{8}{6} = \frac{4}{3} = \frac{m}{n}$

$\therefore 2m + n = 2(4) + 3 = 11$

14. Consider the following two statements :

Statement I : For any two non-zero complex numbers z_1, z_2

$|z_1| - |z_2| \left| \frac{z_1}{|z_1|} - \frac{z_2}{|z_2|} \right| \le 2 |z_1 - z_2|$ and

Statement II : If x, y, z are three distinct complex numbers and a, b, c are three positive real numbers such that $\frac{a}{|y-z|} = \frac{b}{|z-x|} = \frac{c}{|x-y|}$, then

$\frac{a^2}{y-z} + \frac{b^2}{z-x} + \frac{c^2}{x-y} = 0$.

Between the above two statements,

(1) both Statement I and Statement II are incorrect.

(2) Statement I is incorrect but Statement II is correct.

(3) Statement I is correct but Statement II is incorrect.

(4) both Statement I and Statement II are correct.

Ans. (3)

Sol. Statement I :

$|z_1| - |z_2| \left| \frac{z_1}{|z_1|} - \frac{z_2}{|z_2|} \right|$

Since $\left| \frac{z_1}{|z_1|} - \frac{z_2}{|z_2|} \right| \ge \left| \frac{z_1}{|z_1|} \right| - \left| \frac{z_2}{|z_2|} \right|$

$\left| \frac{z_1}{|z_1|} - \frac{z_2}{|z_2|} \right| \ge \frac{|z_1|}{|z_1|} - \frac{|z_2|}{|z_2|}$

$\left| \frac{z_1}{|z_1|} - \frac{z_2}{|z_2|} \right| \ge 2$

$|z_1| - |z_2| \left| \frac{z_1}{|z_1|} - \frac{z_2}{|z_2|} \right| \le 2 |z_1 - z_2|$

$\therefore$ statement I is correct

For Statement II :

$\frac{a}{|y-z|} = \frac{b}{|z-x|} = \frac{c}{|x-y|}$

$\frac{a^2}{|y-z|^2} = \frac{b^2}{|z-x|^2} = \frac{c^2}{|x-y|^2} = k$ (say)

$a^2 = k|y-z|^2 = k(y-z)(\bar{y}-\bar{z})$

$b^2 = k(z-x)(\bar{z}-\bar{x})$ and $c^2 = k(x-y)(\bar{x}-\bar{y})$

$\frac{a^2}{y-z} + \frac{b^2}{z-x} + \frac{c^2}{x-y} = k(\bar{y}-\bar{z} + \bar{z}-\bar{x} + \bar{x}-\bar{y}) = 0$.

Statement II is false

15. Suppose $\theta \in \left(0, \frac{\pi}{4}\right)$ is a solution of $4\cos\theta - 3\sin\theta = 1$.

Then $\cos\theta$ is equal to :

(1) $\frac{4}{3\sqrt{6} - 2}$ (2) $\frac{6 - \sqrt{6}}{3\sqrt{6} - 2}$

(3) $\frac{6 - \sqrt{6}}{3\sqrt{6} - 2}$ (4) $\frac{4}{3\sqrt{6} + 2}$

Ans. (1)

Sol. $4\left(\frac{1 - \tan^2 \theta/2}{1 + \tan^2 \theta/2}\right) - 3\left(\frac{2\tan\frac{\theta}{2}}{1 + \tan^2\frac{\theta}{2}}\right) = 1$

let $\tan\frac{\theta}{2} = t$

$\frac{4 - 4t^2 - 6t}{1 + t^2} = 1$

$4 - 4t^2 - 6t = 1 + t^2$

$5t^2 + 6t - 3 = 0$

$t = \frac{-6 \pm \sqrt{36 + 4(5)(3)}}{2(5)}$

$= \frac{-6 \pm \sqrt{96}}{10}$

$= \frac{-6 \pm 4\sqrt{6}}{10}$

$t = \frac{-3 \pm 2\sqrt{6}}{5}$

$\cos\theta = \frac{1 - t^2}{1 + t^2} = \frac{1 - \left(\frac{-3 + 2\sqrt{6}}{5}\right)^2}{1 + \left(\frac{-3 + 2\sqrt{6}}{5}\right)^2} = \frac{1 - \left(\frac{49 - 12\sqrt{6}}{25}\right)}{1 + \left(\frac{49 - 12\sqrt{6}}{25}\right)}$

$= \frac{25 - 49 + 12\sqrt{6}}{25 + 49 - 12\sqrt{6}} = \frac{-24 + 12\sqrt{6}}{74 - 12\sqrt{6}} = \frac{-12 + 6\sqrt{6}}{37 - 6\sqrt{6}}$

$= \frac{-100 + 150\sqrt{6}}{625} = \frac{-4 + 6\sqrt{6}}{25} = \frac{4 - 6\sqrt{6}}{4 - 6\sqrt{6}}$

$= \frac{25 - 4 - 6\sqrt{6}}{\sqrt{\quad}} = \frac{-8}{4 - 6\sqrt{6}} = \frac{2}{3\sqrt{6} - 2}$

١٦. If $\frac{1}{\sqrt{1} + \sqrt{2}} + \frac{1}{\sqrt{2} + \sqrt{3}} + \ldots + \frac{1}{\sqrt{99} + \sqrt{100}} = m$ and

$\frac{1}{1^2} + \frac{1}{2^2} + \ldots + \frac{1}{99^2} = n$, then the point (m, n)

lies on the line

(١) $11(x - 1) - 100(y - 2) = 0$

(٢) $11(x - 2) - 100(y - 1) = 0$

(٣) $11(x - 1) - 100y = 0$

(٤) $11x - 100y = 0$

Ans. (٤)

Sol. $\frac{1}{\sqrt{1} + \sqrt{2}} + \frac{1}{\sqrt{2} + \sqrt{3}} + \ldots + \frac{1}{\sqrt{99} + \sqrt{100}} = m$

$\frac{\sqrt{1} - \sqrt{2}}{-1} + \frac{\sqrt{2} - \sqrt{3}}{-1} + \ldots + \frac{\sqrt{99} - \sqrt{100}}{-1} = m$

$\sqrt{100} - 1 = m \Rightarrow m = 9$

$\frac{1}{1^2} + \frac{1}{2^2} + \ldots + \frac{1}{100^2} = n$

$\frac{1}{1} + \frac{1}{2} - \frac{1}{2} + \frac{1}{3} - \ldots - \frac{1}{99} + \frac{1}{100} = n$

$1 - \frac{1}{100} = n$

$\frac{99}{100} = n$

$(m, n) = \left(9, \frac{99}{100}\right)$

$11(9) - 100\left(\frac{99}{100}\right)$

$= 99 - 99 = 0$

Ans. option (٤) $11x - 100y = 0$

١٧. Let $f(x) = x^5 + 2x^3 + 3x + 1$, $x \in R$, and $g(x)$ be a function such that $g(f(x)) = x$ for all $x \in R$. Then $\frac{g'(7)}{g(7)}$ is equal to :

(١) ٧ (٢) ٤٢

(٣) ١ (٤) ١٤

Ans. (٤)

Sol. $f(x) = x^5 + 2x^3 + 3x + 1$

$f'(x) = 5x^4 + 6x + 3$

$f'(1) = 5 + 6 + 3 = 14$

$g(f(x)) = x$

$g'(f(x))f'(x) = 1$

for $f(x) = 7$

$x^5 + 2x^3 + 3x + 1 = 7$

$\Rightarrow x = 1$

$g'(7)\, f'(1) = 1 \Rightarrow g'(7) = \frac{1}{f'(1)} = \frac{1}{14}$

$x = ١, f(x) = ٧ \Rightarrow g(٧) = ١$

$$\frac{g(٧)}{g'(٧)} = \frac{١}{١/١٤} = ١٤$$

١٨. If $A(١, -١, ٢), B(٥, ٧, -٦), C(٣, ٤, -١٠)$ and $D(-١, -٤, -٢)$ are the vertices of a quadrilateral $ABCD$, then its area is :

(١) $١٢\sqrt{٢٩}$ (٢) $٢٤\sqrt{٢٩}$

(٣) $٢٤\sqrt{٧}$ (٤) $٤٨\sqrt{٧}$

Ans. (١)

Sol. $A(١, -١, ٢)$

$B(٥, ٧, -٦)$

$C(٣, ٤, -١٠)$

$D(-١, -٤, -٢)$

$$\text{Area} = \frac{١}{٢}|\overrightarrow{AC} \times \overrightarrow{BD}| = \frac{١}{٢}|٢\hat{i} + ٥\hat{j} + ١٢\hat{k} \times ٦\hat{i} + ١١\hat{j} + ٤\hat{k}|$$

$$= \frac{١}{٢}|١١٢\hat{i} + ٦٤\hat{j} + ٨\hat{k}|$$

$$= \frac{١}{٢}|٤١٤\hat{i} + ٨\hat{j} + \hat{k}|$$

$$= \frac{١}{٢}\sqrt{١٩٦ + ٦٤ + ١}$$

$$= \frac{١}{٢}\sqrt{٤٢٦١}$$

$$= \frac{١}{٢}\sqrt{١٢٢٩}$$

١٩. The value of $\int_{\pi/٤}^{\pi/٢} \frac{٢y(١ + \sin y)}{١ + \cos ٢y}\,dy$ is :

(١) $\pi^٢$ (٢) $\frac{\pi^٢}{٢}$

(٣) $\frac{\pi}{٢}$ (٤) $٢\pi^٢$

Ans. (١)

Sol. $\int_{\pi/٤}^{\pi/٢} \frac{٢y(١ + \sin y)}{١ + \cos ٢y}\,dy$

$$= \int_{\pi/٤}^{\pi/٢} \frac{٢y}{\cos ٢y}\,dy + \int_{\pi/٤}^{\pi/٢} \frac{٢y \sin y}{١ + \cos ٢y}\,dy$$

(Odd) (Even)

$$= ٠ + ٢.٢\int_{٠}^{\pi/٢} \frac{y \sin y}{\cos y}\,dy$$

$$I = ٤\int_{٠}^{\pi/٢} \frac{y \sin y}{١ + \cos ٢y}\,dy$$

$$I = ٤\int_{٠}^{\pi/٢} \frac{y \sin y}{٢\cos^٢ y}\,dy$$

$$٢I = ٤\int_{٠}^{\pi/٢} \frac{y \sin y}{\cos^٢ y}\,dy$$

$$I = ٢\int_{٠}^{\pi/٢} y \tan y \sec y\,dy$$

$$= ٢\left[y\sec y - \int \sec y\,dy\right]_{٠}^{\pi/٢}$$

$$= ٢\left[y\sec y - \ln|\sec y + \tan y|\right]_{٠}^{\pi/٢}$$

$$= ٢\left[\left(\frac{\pi}{٢}\sec\frac{\pi}{٢} - \ln\left|\sec\frac{\pi}{٢} + \tan\frac{\pi}{٢}\right|\right) - (٠ - \ln|١ + ٠|)\right]$$

$$= ٢\left[\infty - \ln|\infty| + \ln|١|\right]$$

$$= ٢\infty$$

٢٠. If the line $\frac{x-٢}{٣} = \frac{y-٣}{٤} = \frac{z-٤}{١}$ makes a right angle with the line $\frac{x-٣}{٣\mu} = \frac{y-١}{٦} = \frac{z-٥}{٧}$, then $٤\mu + ٩$ is equal to :

(١) ١٣ (٢) ٤

(٣) ٥ (٤) ٦

Ans. (٤)

Sol. $\frac{x-٢}{٣} = \frac{y-٣}{٤} = \frac{z-٤}{١}$ ...(١)

$$\frac{x-٢}{(٣)} = \frac{y-\frac{٣}{٢}}{\frac{٤}{٣}} = \frac{z-٤}{(١)}$$

$\frac{x-3}{3\mu} = \frac{1-2y}{6} = \frac{5-z}{7}$ ...(2)

$\frac{x-3}{3\mu} = \frac{y-\frac{1}{2}}{(-3)} = \frac{z-5}{(-7)}$

Right angle $\Rightarrow (-3)(3\mu) + 4(-1)(-3) + (-1)(-7) = 0$

$-9\mu - 4 - 1 + 7 = 0$

$4 + 9\mu = 6$

SECTION-B

21. From a lot of 10 items, which include 3 defective items, a sample of 5 items is drawn at random. Let the random variable X denote the number of defective items in the sample. If the variance of X is σ^2 then $96\sigma^2$ is equal to _____.

Ans. (56)

Sol. X = denotes number of defective

x	0	1	2	3
P(x)	$\frac{7}{15}$	$\frac{5}{12}$	$\frac{5}{12}$	$\frac{1}{12}$
x^2		1	4	9
		5	20	9
Pix^2	0	$\frac{}{12}$	$\frac{12}{10}$	$\frac{12}{3}$
pixi	0	$\frac{5}{12}$	$\frac{12}{}$	$\frac{12}{}$

$\mu = \sum pixi = \frac{18}{12}$

$\sum px^2 = \frac{34}{12}$

$\sigma^2 = \sum pixi^2 - (\mu)^2$

$= \frac{34}{12} - \left(\frac{18}{12}\right)^2 = \frac{17}{6} - \frac{9}{4}$

$\frac{34 - 27}{12} = \frac{7}{12}$

σ^2

$96\sigma^2 = \frac{7}{12} = 56$

22. If the constant term in the expansion of $(1 + 2x - 3x)^3 \left(\frac{3}{2}x^2 - \frac{1}{3x}\right)^9$ is p, then 108p is equal to

Ans. (54)

Sol. $(1 - 2x - 3x^2)^3 \left(\frac{3}{2}x^2 - \frac{1}{3x}\right)^9$

General term $m = \frac{3}{2}x^2 - \frac{1}{3x}$

$= {}^9C_r \frac{3^{9-2r}}{2^{9-r}} (-1)^r x^{18-3r}$

Put $r = 6$ to get coeff. of x^0 $= {}^9C_6 \frac{1}{3^3} x^0 = \frac{7}{18} x^0$

Put $r = 7$ to get coeff. of x^3 $= {}^9C_7 \frac{3}{2^2} (-1)^7 x^3$

$= -\frac{63}{4} x^3$

$1 - 2x - 3x^2 = \frac{7}{18} x^0 - \frac{63}{4} x^3$

$\frac{7}{18} - \frac{3}{27} - \frac{7}{18} \quad \frac{1}{9} = \frac{7-2}{18} = \frac{9}{18} = \frac{1}{2}$

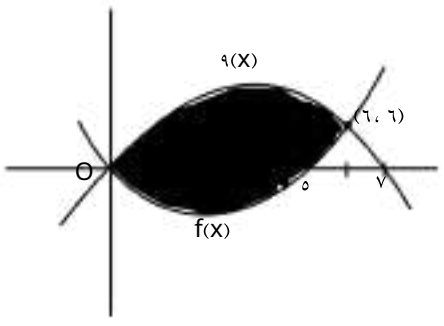
$= 108 \times \frac{1}{2} = 54$

23. The area of the region enclosed by the parabolas $y = x^2 - 5x$ and $y = 7x - x^2$ is _____.

Ans. (72)

NTA Ans. (198)

Sol. $y = x^2 - 5x$ and $y = 7x - x^2$



$\int_0^6 (g(x) - f(x))\,dx$

$\int_0^6 (7x - (x^2 - 5x))\,dx$

$\int_0^6 (12x - x^2)\,dx = \left[6x^2 - \frac{x^3}{3}\right]_0^6$

$= 6(6)^2 - \frac{1}{3}(6)^3$

$= 216 - 144 = 72$ unit^2

24. The number of ways of getting a sum 16 on throwing a dice four times is _____.

Ans. (125)

Sol. $(x^1 + x^2 + \ldots + x^6)^4$

$x^4 \cdot \left(\frac{1 - x^6}{1 - x}\right)^4$

$x^4 \cdot (1 - x^6)^4 (1 - x)^{-4}$

$x^4 (1 - 4x^6 + 6x^{12} - \ldots)(1 + {}^4C_1 x + {}^5C_2 x^2 + \ldots)^4$

$(x^4 - 4x^{10} + 6x^{12} - \ldots)(1 + {}^4C_1 x + {}^5C_2 x^2 + \ldots)^4$

$({}^7C_{12} - 4 \cdot {}^4C_6 + 6)x^{16}$

$({}^7C_3 - 4 \cdot {}^4C_6 + 6)$

$= 35 \times 13 - 6 \times 8 \times 7 + 6$

$= 455 - 336 + 6$

$= 125$

25. If $S = \{a \in R : |2a - 1| = 3[a] + 2\{a\}\}$, where $[t]$ denotes the greatest integer less than or equal to t and $\{t\}$ represents the fractional part of t, then $\sum_{a \in S} 72a$ is equal to _____.

Ans. (18)

Sol. $|2a - 1| = 3[a] + 2\{a\}$

$|2a - 1| = [a] + 2a$

Case-1 : $a < \frac{1}{2}$

$2a - 1 = [a] + 2a$

$[a] = -1 \Rightarrow a \in [-1, 0)$ Reject

Case-2 : $a \geq \frac{1}{2}$

$-2a + 1 = [a] + 2a$

$a = I + f$

$-2(I + f) + 1 = I + 2I + 2f$

$I = 0, f = \frac{1}{4} \Rightarrow a = \frac{1}{4}$

Hence $a = \frac{1}{4}$

$\sum_{a \in S} 72a = 72 \times \frac{1}{4} = 18$

26. Let f be a differentiable function in the interval $(0, \infty)$ such that $f(1) = 1$ and $\lim_{t \to x} \frac{t^2 f(x) - x^2 f(t)}{t - x} = 1$ for each $x > 0$. Then $2f(2) + 3f(3)$ is equal to _____.

Ans. (24)

Sol. $\lim_{t \to x} \frac{t^2 f(x) - x^2 f(t)}{t - x} = 1$

$\lim_{t \to x} \frac{2t \cdot f(x) - x^2 f'(x)}{1} = 1$

$2x \cdot f(x) - x^2 f'(x) = 1$

$\frac{d}{dx}\left(\frac{y}{x^2}\right) = \frac{1}{x^4}$

$I.F. = e^{\int -\frac{2}{x} dx} = \frac{1}{x^2}$

$\frac{y}{x^2} = \int \frac{1}{x^4} dx + C$

$\frac{y}{x^2} = \frac{1}{3x^3} + C$

Put $f(1) = 1$

$C \square \frac{2}{3}$

$y \square \frac{3x^{\frac{1}{2}}}{2x} \square \frac{2x^2}{3}$

$y \square \frac{3^{\square 1}}{3x}$

$f(2) \square \frac{17}{6}$

$f(3) \square \frac{55}{9}$

$2f(2) + 3f(3) = \frac{17}{3} \square \frac{55}{3} \square \frac{72}{3} \square 24$

27. Let $a_1, a_2, a_3, \ldots$ be in an arithmetic progression of positive terms.

Let $A_k = a_1^2 - a_2^2 + a_3^2 - a_4^2 + \ldots + a_{2k-1}^2 - a_{2k}^2$.

If $A_3 = -153$, $A_5 = -435$ and $a_1 + a_2 + a_3 = 66$, then $a_{17} - A_7$ is equal to _____.

Ans. (910)

Sol. d $\square$ common diff.

$A_k = -kd(2a + (2k-1)d)$

$A_3 = -153$

$\square 153 = 13d(2a + 5d)$

$51 = d(2a + 5d)$ ...(1)

$A_5 = -435$

$435 = 5d(2a + 9d)$

$87 = d(2a + 9d)$

(2) – (1)

$36 = 4d^2$

$d = 3, a = 1$

$a_{17} - A_7 = 49 - (-7.3)(2 + 39) = 910$

28. Let $\vec{a} \square i^ \square 3j^ \square 7k^$, $\vec{b} \square 2i^ \square j^ \square k^$ and $\vec{c}$ be a vector such that $\vec{a} \square 2\vec{b} \square \vec{c} \square 3(\vec{c} \square \vec{a})$.

$\vec{a} \square \vec{c} \square 130$, then $\vec{b} \square \vec{c}$ is equal to _____.

Ans. (30)

Sol. $\vec{a} \square 2\vec{b} \square \vec{c} \square 3\vec{c} \square 3\vec{a}$

$\square 2\vec{b} \square 4\vec{a} \square \vec{c} \square 0$

$\vec{c} \square \square 4\vec{a} \square 2\vec{b} \square \square \square 8i^ \square 14j^ \square 30k^ \square$

$\vec{a} \square \vec{c} \square 130$

$8 \square + 42 \square + 210 \square = 130$

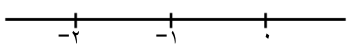
$\square \square \frac{1}{2}$

$\vec{c} \square 4i^ \square 7j^ \square 15k^$

$\vec{b} \square \vec{c} = 8 + 7 + 15 = 30$

29. The number of distinct real roots of the equation $|x||x+2| - 5|x+1| - 1 = 0$ is _____.

Ans. (3)



Sol.

Case–1

$x \square \square 0$

$x^2 + 2x - 5x - 5 - 1 = 0$

$x^2 - 3x - 6 = 0$

$x \square \frac{3 \square \sqrt{9 \square 24}}{2} \square \frac{3 \square \sqrt{33}}{2}$

One positive root

Case–2

$-1 \square \square x > 0$

$-x^2 - 2x - 5x - 5 - 1 = 0$

$x^2 + 7x + 6 = 0$

$(x+6)(x+1) = 0$

$x = -1$

one root in range

Case–3

$-2 \square \square x > -1$

$x^2 - 2x + 5x + 5 - 1 = 0$

$x^2 - 3x - 4 = 0$

$(x-4)(x+1) = 0$

No root in range

Case–٤

$x > -٢$

$x^{٢} + ٧x + ٤ = ٠$

$$x = \frac{-٧ - \sqrt{٤٩ - ١٦}}{٢} = \frac{-٧ - \sqrt{٣٣}}{٢}$$

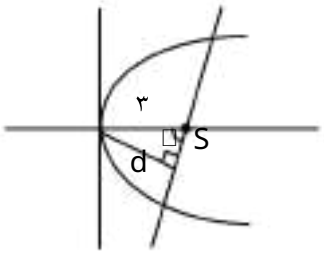
one root in range

Total number of distinct roots are ٣

٣٠. Suppose AB is a focal chord of the parabola $y^{٢} = ١٢x$ of length l and slope m, $m \in \sqrt{٣}$. If the distance of the chord AB from the origin is d, then $ld^{٢}$ is equal to _____.

Ans. (١٠٨)

Sol.



$\Delta = ٤a\,\mathrm{cosec}^{٢}\theta$

$$\ell = 12\sin^{٩}\theta$$

$$\frac{١٢}{d^{٢}}$$

$١٢d^{٢} = ١٠٨$

PHYSICS

SECTION-A

३१. Light emerges out of a convex lens when a source of light kept at its focus. The shape of wavefront of the light is:

(१) Both spherical and cylindrical

(२) Cylindrical

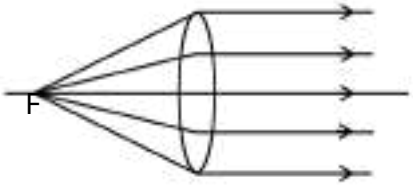
(३) Spherical

(४) Plane

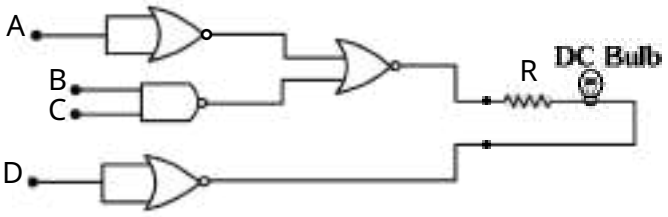
Ans. (४)

Sol. Light emerges parallel

planor wavefront



३२. Following gates section is connected in a complete suitable circuit.



For which of the following combination, bulb will glow (ON):

(१) A = ०, B = १, C = १, D = १

(२) A = १, B = ०, C = ०, D = ०

(३) A = ०, B = ०, C = ०, D = १

(४) A = १, B = १, C = १, D = ०

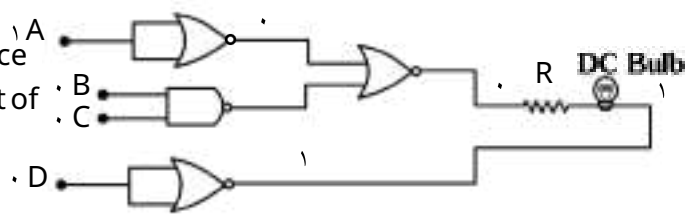
Ans. (२)

Sol. Bulb will glow if bulb have potential drop on it.

One end of bulb must be at high (१) and other must be at low (०).

Option (२) satisfy this condition

TEST PAPER WITH SOLUTION



३३. If G be the gravitational constant and u be the energy density then which of the following quantity have the dimension as that the G:

(१) Pressure gradient per unit mass

(२) Force per unit mass

(३) Gravitational potential

(४) Energy per unit mass

Ans. (२)

Sol. $[u] = [ML^{-1}T^{-2}]$

$[u] = [M^{1}L^{1}T^{-2}]$

$[u] = [LT^{-2}]$

Option (२) is correct

३४. Given below are two statements :

Statement-I : When a capillary tube is dipped into a liquid, the liquid neither rises nor falls in the capillary. The contact angle may be ०°.

Statement-II : The contact angle between a solid and a liquid is a property of the material of the solid and liquid as well :

In the light of above statement, choose the correct answer from the options given below.

(१) Statement-I is false but Statement-II is true.

(२) Both Statement-I and Statement-II are true.

(३) Both Statement-I and Statement-II are false.

(४) Statement-I is true and Statement-II is false.

Ans. (१)

Sol. Capillary rise

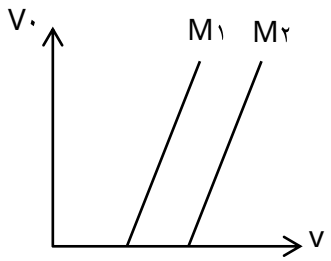
$$h = \frac{2T\cos\theta}{\rho g r}$$

If $\theta = ०°$ then rise is non-zero

Statement-१ is incorrect.

Option (१) is correct

٣٥. Given below are two statements :



Statement-I : Figure shows the variation of stopping potential with frequency (ν) for the two photosensitive materials M١ and M٢. The slope gives value of $\frac{h}{e}$, where h is Planck's constant, e is the charge of electron.

Statement-II : M٢ will emit photoelectrons of greater kinetic energy for the incident radiation having same frequency.

In the light of the above statements, choose the most appropriate answer from the options given below.

(١) Statement-I is correct and Statement-II is incorrect.

(٢) Statement-I is incorrect but Statement-II is correct.

(٣) Both Statement-I and Statement-II are incorrect.

(٤) Both Statement-I and Statement-II are correct.

Ans. (١)

Sol. $eV_0 = h\nu - \phi$

$V_0 = \frac{h}{e}\nu - \frac{\phi}{e}$

M٢ material has higher work function, so statement-(II) is incorrect.

Option (١) is correct.

٣٦. The angle between vector Q and the resultant of $(2\vec{Q}-2\vec{P})$ and $(2\vec{Q}-7\vec{P})$ is :

(١) ٠°

(٢) $\tan^{-1}\left(\frac{(2Q-2P)}{2Q-2P}\right)$

(٣) $\tan^{-1}\left(\frac{1-\frac{P}{Q}}{}\right)$

(٤) $\tan^{-1}\left(\frac{2Q}{P}\right)$

Ans. (١)

Sol. $\vec{R} = (2\vec{Q}-2\vec{P}) + (2\vec{Q}-7\vec{P}) \rightarrow$

$\vec{R} = 4\vec{Q}$

Angle between $\vec{Q}$ and $\vec{R}$ is zero

Option (١) is correct

٣٧. In hydrogen like system the ratio of coulombian force and gravitational force between an electron and a proton is in the order of :

(١) 10^{39} (٢) 10^{19}

(٣) 10^{29} (٤) 10^{36}

Ans. (١)

Sol. $F_e = \frac{kQ_1Q_2}{r^2} = \frac{9\times10^9\times(1.6\times10^{-19})^2}{r^2}$

$F_g = \frac{Gm_1m_2}{r^2} = \frac{6.67\times10^{-11}\times(9.1\times10^{-31})\times(1.6\times10^{-27})}{r^2}$

$\frac{F_e}{F_g} = 0.23\times10^{40} = 2.3\times10^{39}$

Option (١)

٣٨. In a co-axial straight cable, the central conductor and the outer conductor carry equal currents in opposite directions. The magnetic field is zero.

(١) inside the outer conductor

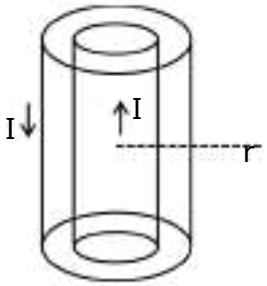
(٢) in between the two conductors

(٣) outside the cable

(٤) inside the inner conductor

Ans. (٣)

Sol.



$\oint \vec{B} \cdot d\vec{l} = \mu_0 i_{enc}$

$B = 0$ outside the cable

٣٩. An electron rotates in a circle around a nucleus having positive charge Ze. Correct relation between total energy (E) of electron to its potential energy (U) is :

(١) $E = 2U$ (٢) $2E = 3U$

(٣) $E = U$ (٤) $2E = U$

Ans. (٤)

Sol. $F = \frac{k(Ze)(e)}{r^2} = \frac{mv^2}{r}$

$KE = \frac{1}{2}mv^2 = \frac{1}{2}\frac{K(Ze)(e)}{r}$

$PE = -\frac{K(Ze)(e)}{r}$

$TE = -\frac{K(Ze)(e)}{2r} = -\frac{K(Ze)(e)}{r} = -\frac{K(Ze)(e)}{2r}$

$TE = \frac{PE}{2}$

$2TE = PE$

Option (٤)

٤٠. If the collision frequency of hydrogen molecules in a closed chamber at ٢٧°C is Z, then the collision frequency of the same system at ١٢٧° C is :

(١) $\frac{\sqrt{3}}{2}Z$ (٢) $\frac{4}{3}Z$

(٣) $\frac{Z}{\sqrt{3}}$ (٤) $\frac{4}{3}Z$

Ans. (٣)

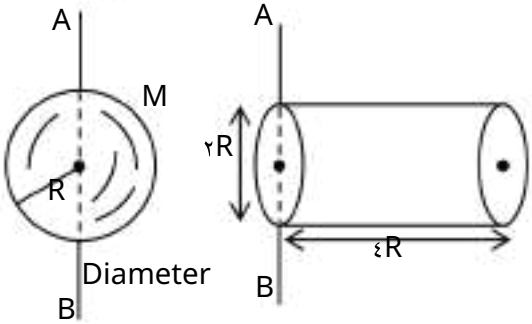
Sol. Assuming mean free path constant.

$f = v = \sqrt{T}$

$\frac{f_1}{f_2} = \sqrt{\frac{T_1}{T_2}} = \sqrt{\frac{300}{400}}$

$f_2 = \sqrt{\frac{4}{3}}f_1 = \frac{2}{\sqrt{3}}Z$

٤١. Ratio of radius of gyration of a hollow sphere to that of a solid cylinder of equal mass, for moment of Inertia about their diameter axis AB as shown in figure is $\sqrt{\frac{8}{x}}$. The value of x is :



(١) ٣٤ (٢) ١٧

(٣) ٦٧ (٤) ٥١

Ans. (٣)

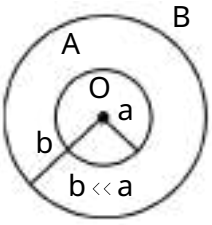
Sol. $I_{sphere} = \frac{2}{3}MR^2 = Mk^2$

$I_{cylinder} = \frac{1}{12}M(4R^2) = \frac{1}{4}MR^2 + M(2R)^2$

$= \frac{67}{12}MR^2 = Mk^2$

$\frac{k_1}{k_2} = \sqrt{\frac{2}{3}\cdot\frac{12}{67}} = \sqrt{\frac{8}{67}}$

٤٢. Two conducting circular loops A and B are placed in the same plane with their centres coinciding as shown in figure. The mutual inductance between them is :



(١) $\frac{0.8a^2}{2b}$ (٢) $\frac{0.8b^2}{2\pi}\cdot\frac{a}{a^2}$

(٣) $\frac{0.8b^2}{2a}$ (٤) $\frac{0.8}{2\pi}\cdot\frac{b}{}$

Ans. (١)

Sol. $\phi = Mi = BA$

$\therefore Mi = \frac{0.i}{2b}\pi a^2$

$\therefore M = \frac{0.8a^2}{2b}$

٤٣. Match list-I with list-II:

List-I	List-II
(A) Kinetic energy of planet	(I) $\frac{GMm}{a}$
(B) Gravitation Potential energy of Sun-planet system.	(II) $\frac{GMm}{2a}$
(C) Total mechanical energy of planet	(III) $\frac{Gm}{r}$
(D) Escape energy at the surface of planet for unit mass object	(IV) $\frac{GMm}{2a}$

(Where a = radius of planet orbit, r = radius of planet, M = mass of Sun, m = mass of planet)

Choose the correct answer from the options given below:

(١) (A) - II, (B) - I, (C) - IV, (D) - III

(٢) (A) - III, (B) - IV, (C) - I, (D) - II

(٣) (A) - I, (B) - IV, (C) - II, (D) - III

(٤) (A) - I, (B) - II, (C) - III, (D) - IV

Ans. (١)

Sol. $KE = \frac{1}{2}mv^2 = \frac{GMm}{2a}$

$PE = -2KE$

$TE = -KE$

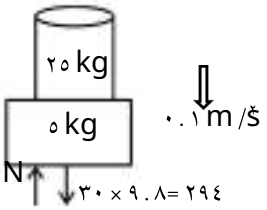
٤٤. A wooden block of mass ٥kg rests on soft horizontal floor. When an iron cylinder of mass ٢٥ kg is placed on the top of the block, the floor yields and the block and the cylinder together go down with an acceleration of ٠.١ ms^{-2}. The action force of the system on the floor is equal to:

(١) ٢٩٧ N (٢) ٢٩٤ N

(٣) ٢٩١ N (٤) ١٩٦ N

Ans. (٣)

Sol. Taking $g = ٩.٨ m/s^2$



$٢٩٤ - N = ٣٠ \times ٠.١$

$N = ٢٩١$

٤٥. A simple pendulum doing small oscillations at a place R height above earth surface has time period of $T_1 = ٤$ s. T_2 would be it's time period if it is brought to a point which is at a height ٢R from earth surface. Choose the correct relation

(٢) $٢T_1 = ٣T_2$

(٤) $٢T_1 = T_2$

Ans. (٣) R = radius of Earth

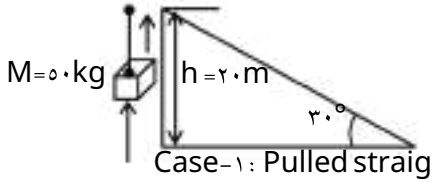
Sol. $T_1 = 2\pi\sqrt{\frac{\ell}{GM}(٢R)^2}$

(٣) $٣T_1 = ٢T_2$

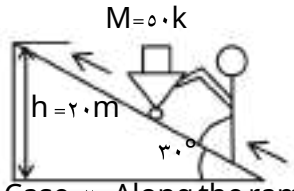
$T_2 = 2\pi\sqrt{\frac{\ell}{GM}(٣R)^2}$

$\frac{T_1}{T_2} = \frac{٢}{٣}$

٤٦. A body of mass ٥٠ kg is lifted to a height of ٢٠ m from the ground in the two different ways as shown in the figures. The ratio of work done against the gravity in both the respective cases, will be:



Case-١: Pulled straight up



Case-٢: Along the ramp

(١) ١ : ١ (٢) ٢ : ١

(٣) $\sqrt{٣}$: ٢ (٤) ١ : ٢

Ans. (١)

Sol. Work done by gravity is independent of path. It depends only on vertical displacement so work done in both cases will be same.

Option (١) is correct

٤٧. Time periods of oscillation of the same simple pendulum measured using four different measuring clocks were recorded as ٤.٦٢ s، ٤.٦٣٢ s، ٤.٦ s and ٤.٦٤ s. The arithmetic mean of these reading in correct significant figure is.

(١) ٤.٦٢٣ s (٢) ٤.٦٢ s

(٣) ٤.٦ s (٤) ٥ s

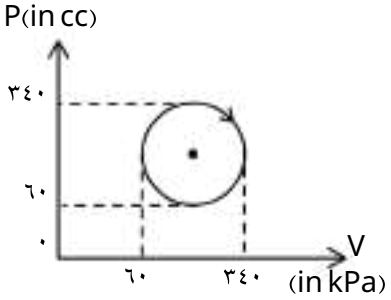
Ans. (٣)

Sol. Sum of number by considering significant digit

sum = ٤.٦ + ٤.٦ + ٤.٦ + ٤.٦ = ١٨.٤

Arithmetic Mean = $\frac{\text{sum}}{٤} = \frac{١٨.٤}{٤} = ٤.٦$

٤٨. The heat absorbed by a system in going through the given cyclic process is :



(١) ٦١.٦ J (٢) ٤٣١.٢ J

(٣) ٦١٦ J (٤) ١٩.٦ J

Ans. (١)

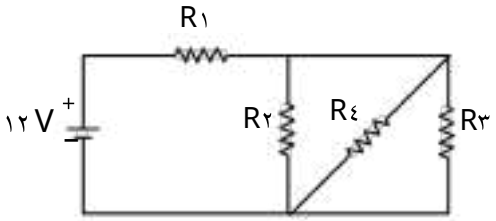
Sol. $\Delta U = ٠$ (Cyclic process)

$\Delta Q = W$ = area of P-V curve.

$= \frac{١}{٢} \times (١٤٠ \times ١٠^{٣}\ Pa) \times (١٤٠ \times ١٠^{-٦}\ m^{٣})$

$\Delta Q = ٦١.٦\ J$

٤٩. In the given figure $R_١ = ١٠\ \Omega$، $R_٢ = ٨\ \Omega$، $R_٣ = ٤\ \Omega$ and $R_٤ = ٨\ \Omega$. Battery is ideal with emf ١٢V. Equivalent resistant of the circuit and current supplied by battery are respectively.



(١) ١٢ Ω and ١١.٤ A (٢) ١٠.٥ Ω and ١.١٤ A

(٣) ١٠.٥ Ω and ١ A (٤) ١٢ Ω and ١ A

Ans. (٤)

Sol. Here $R_٢$، $R_٣$، $R_٤$ are in parallel

$\frac{١}{R_{٢٣٤}} = \frac{١}{R_٢} + \frac{١}{R_٣} + \frac{١}{R_٤}$

$R_{٢٣٤} = ٢\ \Omega$

$R_{٢٣٤}$ is in series with $R_١$ so

$R_{eq} = R_{٢٣٤} + R_١ = ٢ + ١٠ = ١٢\ \Omega$

$i = \frac{١٢}{١٢} = ١\ Amp$

٥٠. An alternating voltage of amplitude ٤٠ V and frequency ٤ kHz is applied directly across the capacitor of ١٢ μF. The maximum displacement current between the plates of the capacitor is nearly :

(١) ١٣ A (٢) ٨ A

(٣) ١٠ A (٤) ١٢ A

Ans. (٤)

Sol. Displacement current is same as conduction current in capacitor.

$X_C = \frac{١}{\omega C} = \frac{١}{٢\pi f C}$

$= \frac{١}{2 \times \frac{٢٢}{٧} \times ٤ \times ١٠^{٣} \times ١٢ \times ١٠^{-٦}} = ٣.٣١٧\ \Omega$

$$I = \frac{V}{X_C} = \frac{٤٠}{٣.٣١٧} = ١٢A$$

SECTION-B

٥١. In Young's double slit experiment, carried out with light of wavelength ٥٠٠٠Å, the distance between the slits is ٠.٣ mm and the screen is at ٢٠٠ cm from the slits. The central maximum is at x = ٠ cm. The value of x for third maxima is ..... mm.

Ans. (١٠)

Sol. $y = \frac{D}{d}\lambda = \frac{٢ \times ٥ \times ١٠^{-٧}}{٣ \times ١٠^{-٤}} = \frac{١٠ \times ١٠^{-٣}}{٣}$ m

٥٢. For ٣rd maxima $y_٣ = ٣ \times ١٠ \times ١٠^{-٣}$ m = ١٠ mm

A ٢A current carrying straight metal wire of resistance ١Ω, resistivity ٢×١٠^{-٦} Ωm, area of cross-section ١٠ mm^2 and mass ٥٠٠ g is suspended horizontally in mid air by applying a uniform magnetic field B. The magnitude of B is ..... $\times ١٠^{-١}$ T (given, g = ١٠ m/s^2)

Ans. (٥)

Sol. $R = \frac{\rho \ell}{A} = \frac{2 \times 10^{-6} \times 1}{10^{-5}} = 1 \times 5\Omega$

$mg = Bi\ell$

٥٣. $B = \frac{mg}{i\ell} = \frac{٥}{٢ \times ٥} \times ٠.٥ = ٥ \times ١٠^{-١}$ Tesla

The electric field between the two parallel plates of a capacitor of ١.٥ μF capacitance drops to one third of its initial value in ٦.٦ μs when the plates are connected by a thin wire. The resistance of this wire is ..... Ω. (Given, log ٣ = ١.١)

Ans. (٤)

Sol. $E = E_٠ e^{-\frac{t}{\tau}}$ $\quad V = V_٠ e^{-\frac{t}{\tau}}$

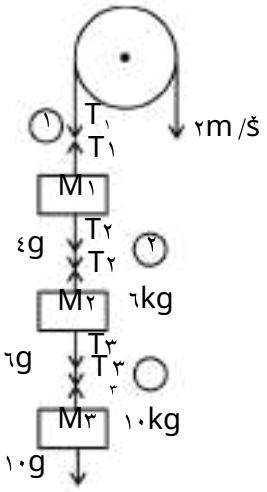
$\frac{V_٠}{٣} = V_٠ e^{-\frac{t}{\tau}}$

$t = \tau \ln ٣$

$٦.٦ \times ١٠^{-٦} = R(١.٥ \times ١٠^{-١٠})(١.١)$

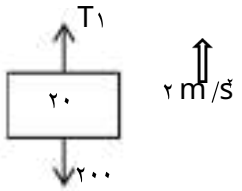
$R = \frac{٦}{١.٥} = ٤\Omega$

٥٤. Three blocks $M_١$, $M_٢$, $M_٣$ having masses ٤ kg, ٦ kg and ١٠ kg respectively are hanging from a smooth pully using rope ١, ٢ and ٣ as shown in figure. The tension in the rope ١, $T_١$, when they are moving upward with acceleration of ٢ m/s^2 is ..... N (If g = ١٢ m/s^2)



Ans. (٢٤٠)

Sol. FBD of $M_١$:



$T_١ - ٢٠٠ = (٤ + ٦ + ١٠) \times ٢$

$T_١ = ٢٤٠$

٥٥. The density and breaking stress of a wire are ٦ × ١٠ kg/m^3 and ١.٢ × ١٠ N/m^2 respectively. The wire is suspended from a rigid support on a planet where acceleration due to gravity is $\frac{١}{٣}$rd of the value on the surface of earth. The maximum length of the wire with breaking is ..... m (take, g = ١٠ m/s^2)

Ans. (٦٠٠)

Sol.

$T = mg$

$\sigma = \frac{T}{A} = \frac{mg}{A}$

$\frac{(\rho A \ell) g}{A}$

$\ell = \frac{\sigma}{\rho g} = \frac{1.2 \times 10^{8} \times 3}{6 \times 10^{4} \times 10} = 600$

٥٦. A body moves on a frictionless plane starting from rest. If Sn is distance moved between t = n − ١ and t = n and Sn − ١ is distance moved between t = n −٢ and t = n − ١، then the ratio $\frac{S_{n-1}}{S_n}$ is $\left(1 - \frac{2}{x}\right)$ for n = ١٠. The value of x is .....

Ans. (١٩)

Sol. $S_n = \frac{1}{2} a(2n-1) = \frac{19a}{2}$

$S_{n-1} = \frac{1}{2} a(2n-3) = \frac{17a}{2}$

$\frac{S_{n-1}}{S_n} = 1 - \frac{17}{19} = 1 - \frac{2}{x} \Rightarrow x = 19$

٥٧. If three helium nuclei combine to form a carbon nucleus then the energy released in this reaction is ..... × ١٠^-٢ MeV. (Given ١ u = ٩٣١ MeV/c^٢ atomic mass of helium = ٤.٠٠٢٦٠٣ u)

Ans. (٧٢٧)

Sol. Reaction :

$3\,{}_{2}^{4}He \rightarrow {}_{6}^{12}C + \gamma$ rays

Mass defect = $\Delta m = (3m_{He} - m_C)$

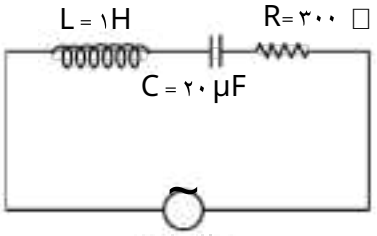
$= (3 \times 4.002603 - 12) = 0.007809$ u

Energy released

$= 931\,\Delta m$ MeV

$= 7.27$ MeV $= 727 \times 10^{-2}$ MeV

٥٨. An ac source is connected in given series LCR circuit. The rms potential difference across the capacitor of 20 μF is ..... V.



$V = 50\sqrt{2} \sin 100t$ volt

Ans. (٥٠)

Sol. $X_L = \omega L = 100 \times 1 = 100\ \Omega$

$X_C = \frac{1}{\omega C} = \frac{1}{100 \times 20 \times 10^{-6}} = 500\ \Omega$

$Z = \sqrt{(X_L - X_C)^2 + R^2}$

$\sqrt{(100 - 500)^2 + 300^2}$

$Z = 500\ \Omega$

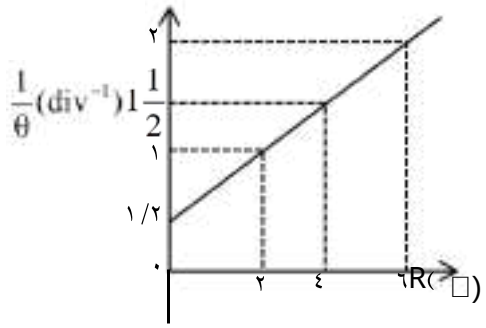
$i_{rms} = \frac{V_{rms}}{Z} = \frac{50}{500} = 0.1$ A

rms voltage across capacitor

$V_{rms} = X_C i_{rms}$

$= 500 \times 0.1 = 50$ V

٥٩. In the experiment to determine the galvanometer resistance by half-deflection method، the plot of $\frac{1}{\theta}$ vs the resistance (R) of the resistance box is shown in the figure. The figure of merit of the galvanometer is ..... ×١٠^-٦ A/division. The source has emf ٢V



Ans. (٥)

Sol. $i = K$

$$\frac{G \cdot R}{(G+R)^2} = K$$

$$\frac{1}{2} = \frac{K(G+R)}{2} = R = \frac{K}{2} = \frac{KG}{2}$$

$$\text{Slope} = \frac{K}{2} = \frac{1}{4} K = ٠.٥ \times ٥ \times ١٠ = ١A$$

٦٠. Three capacitors of capacitances ٢٥ μF, ٣٠ μF and ٤٥ μF are connected in parallel to a supply of ١٠٠ V. Energy stored in the above combination is E. When these capacitors are connected in series to the same supply, the stored energy is $\frac{9}{x}E$. The value of x is . . . . . .

Ans. (٨٦)

Sol. In parallel combination : Potential difference is same across all

$$\text{Energy} = \frac{1}{2}(C_1 + C_2 + C_3)V^2$$

$$= \frac{1}{2}(٢٥ + ٣٠ + ٤٥)(١٠٠)^2 \times ١٠^{-٦} = ٠.٥ = E$$

In series combination : Charge is same on all.

$$\frac{1}{C_{equ}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} = \frac{1}{٢} + \frac{1}{٥٣} + \frac{1}{٠٤٥}$$

$$\frac{1}{C_{equ}} = \frac{(١٨ + ١٥ + ١٠)}{٤٥٠} = \frac{٤٣}{٤٥٠} \quad C_{equ} = \frac{٤٥٠}{٤٣}$$

$$\text{Energy} = \frac{Q^2}{2C_1} + \frac{Q^2}{2C_2} + \frac{Q^2}{2C_3}$$

$$= \frac{Q^2}{2}\left[\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}\right]$$

$$\frac{(V \cdot C_{equ})^2}{2} = \frac{1}{C_{equ}} = \frac{V^2 C_{equ}}{2}$$

$$\frac{(١٠٠)^2}{2} = \frac{٤٥٠}{٤٣} \times ١٠^{-٦}$$

$$\frac{٤.٥}{٨٦} = \frac{9}{x}E = \frac{9}{x} \times ٠.٥ \quad x = ٨٦$$

CHEMISTRY

SECTION-A

٦١. The incorrect postulates of the Dalton's atomic theory are :

(A) Atoms of different elements differ in mass.

(B) Matter consists of divisible atoms.

(C) Compounds are formed when atoms of different element combine in a fixed ratio.

(D) All the atoms of given element have different properties including mass.

(E) Chemical reactions involve reorganisation of atoms.

Choose the correct answer from the options given below :

(١) (B), (D), (E) only

(٢) (A), (B), (D) only

(٣) (C), (D), (E) only

(٤) (B), (D) only

Ans. (٤)

Sol. B, D

٦٢. The following reaction occurs in the Blast furnance where iron ore is reduced to iron metal

$$Fe_2O_3(s) + 3CO(g) \rightleftharpoons Fe(l) + 3CO_2(g)$$

Using the Le-chatelier's principle, predict which one of the following will not disturb the equilibrium.

(١) Addition of Fe_2O_3

(٢) Addition of CO_2

(٣) Removal of CO

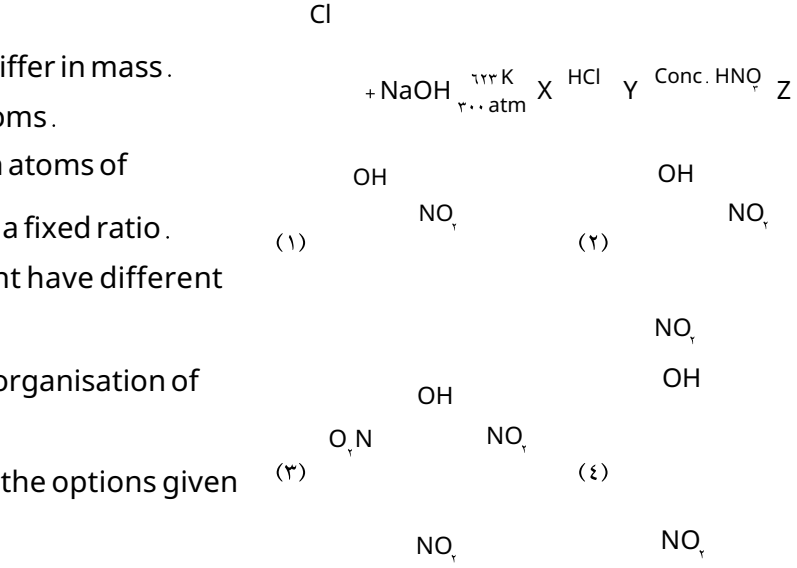
(٤) Removal of CO_2

Ans. (١)

Sol. When solid added no effect on equilibrium.

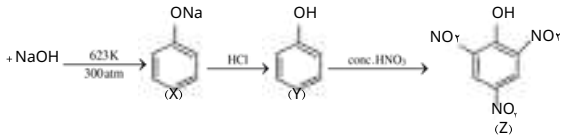
TEST PAPER WITH SOLUTION

٦٣. Identify compound (Z) in the following reaction sequence.



Ans. (٣)

Sol.



٦٤. Given below are two statements : One is labelled as Assertion (A) and the other is labelled as Reason (R)

Assertion (A) : Enthalpy of neutralisation of strong monobasic acid with strong monoacidic base is always $-٥٧\ kJ\ mol^{-١}$

Reason (R) : Enthalpy of neutralisation is the amount of heat liberated when one mole of H^+ ions furnished by acid combine with one mole of OH^- ions furnished by base to form one mole of water.

In the light of the above statements, choose the correct answer from the options given below.

(١) (A) is true but (R) is false

(٢) Both (A) and (R) are true and (R) is the correct explanation of (A)

(٣) (A) is false but (R) is true

(٤) Both (A) and (R) are true but (R) is not the correct explanation of (A)

Ans. (٢)

Sol. Enthalpy of neutralization of SA & SB is always -57 kJ / mol because strong monoacid gives one mole of H^+ and strong mono base gives one mole of OH^- which form one mole of water.

٦٥. The statement(s) that are correct about the species O^{2-}, F^-, Na^+ and Mg^{2+}.

(A) All are isoelectronic

(B) All have the same nuclear charge

(C) O^{2-} has the largest ionic radii

(D) Mg^{2+} has the smallest ionic radii

Choose the most appropriate answer from the options given below :

(١) (B), (C) and (D) only

(٢) (A), (B), (C) and (D)

(٣) (C) and (D) only

(٤) (A), (C) and (D) only

Ans. (٤)

Sol.

	O^{2-}	F^-	Na^+	Mg^{2+}
(No. of e^-)	١٠	١٠	١٠	١٠

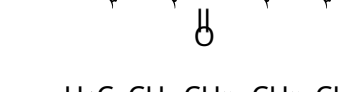
٦٦. (Ionic radius) $O^{2-} > F^- > Na^+ > Mg^{2+}$

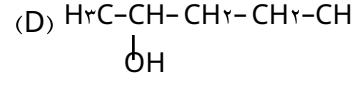
Zeff $O^{2-} < F^- < Na^+ < Mg^{2+}$

For the compounds :

(A) $H_3C-CH_2-O-CH_2-CH_2-CH_3$

(B) $H_3C-CH_2-CH_2-CH_2-CH_3$

(C) 

(D) 

The increasing order of boiling point is :

Choose the correct answer from the options given below :

(١) (A) > (B) > (C) > (D)

(٢) (B) > (A) > (C) > (D)

(٣) (D) > (C) > (A) > (B)

(٤) (B) > (A) > (D) > (C)

Ans. (٢)

Sol. Compounds having same number of carbon atoms follow the boiling point order as :

(Boiling point)Hydrogen bonding < (Boiling point)high polarity < (Boiling point)low polarity < (Boiling point)non polar

٦٧. Given below are two statements :

Statement I : In group ١٣, the stability of $+1$ oxidation state increases down the group.

Statement II : The atomic size of gallium is greater than that of aluminium.

In the light of the above statements, choose the most appropriate answer from the options given below :

(١) Statement I is incorrect but Statement II is correct

(٢) Both Statement I and Statement II are correct (٣) Both Statement I and Statement II areincorrect

(٤) Statement I is correct but Statement II is incorrect

Ans. (٤) Sol. Statement I : Number of d & f electrons, increases down the group and due to poor shielding of d & f e^-, stability of lower oxidation states increases down the group

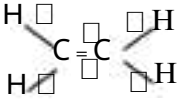
Statement II : The atomic size of aluminium is greater than that of gallium.

٦٨. Number of σ and π bonds present in ethylene molecule is respectively :

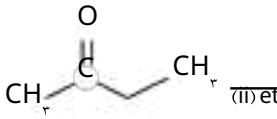
(١) ٣ and ١ (٢) ٥ and ٢

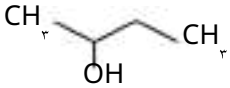
(٣) ٤ and ١ (٤) ٥ and ١

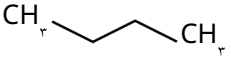
Ans. (٤)

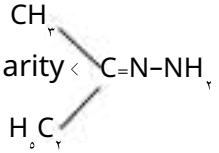
Sol. ethylene is  , it has ٥ σ bonds and ١ π bond.

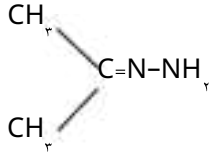
٦٩. Identify 'A' in the following reaction :



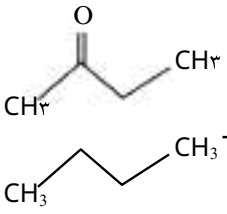
(١) 

(٢) 

(٣) 

(٤) 

Ans. (٢)

Sol. 

(ii) ethylene glycol / KOH – Wolf kishner reduction.

٧٠. The reaction at cathode in the cells commonly used in clocks involves.
(١) reduction of Mn from +٤ to +٣
(٢) oxidation of Mn from +٣ to +٤
(٣) reduction of Mn from +٧ to +٢
(٤) oxidation of Mn from +٢ to +٧

Ans. (١)

Sol. In the cathode reaction manganese (Mn) is reduced from the +٤ oxidation state to the +٣ state.

٧١. Which one of the following complexes will exhibit the least paramagnetic behaviour ?
(Atomic number, Cr = ٢٤, Mn = ٢٥, Fe = ٢٦, Co = ٢٧)
(١) $[Co(H_2O)_6]^{2+}$ (٢) $[Fe(H_2O)_6]^{2+}$
(٣) $[Mn(H_2O)_6]^{2+}$ (٤) $[Cr(H_2O)_6]^{2+}$

Ans. (١)

Sol.

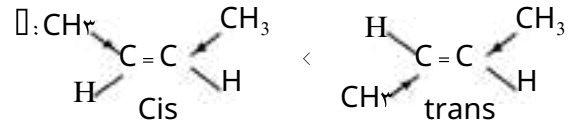
	Number of unpaired e⁻	$\sqrt{n(n+2)}$ B.M.
$[Co(H_2O)_6]^{2+}$		٣.٨٧ ٤.٩٥ ٤.٨٩
$[Fe(H_2O)_6]^{2+}$		
$[Mn(H_2O)_6]^{2+}$		
$[Cr(H_2O)_6]^{2+}$	٤	

Least paramagnetic behaviour = $[Co(H_2O)_6]^{2+}$

٧٢. Given below are two statements : one is labelled as Assertion (A) and the other is labelled as Reason (R).
Assertion (A) : Cis form of alkene is found to be more polar than the trans form
Reason (R) : Dipole moment of trans isomer of ٢-butene is zero.
In the light of the above statements, choose the correct answer from the options given below :
(١) Both (A) and (R) are true but (R) is NOT the correct explanation of (A)
(٢) (A) is true but (R) is false
(٣) Both (A) and (R) are true and (R) is the correct explanation of (A)
(٤) (A) is false but (R) is true

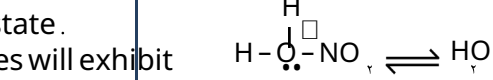
Ans. (٣)

Sol. Dipole moment is a vector quantity and for compound net dipole moment is the vector sum of all dipoles hence dipole moment of cis form is greater than trans form.



(μ < ٠) (μ = ٠)

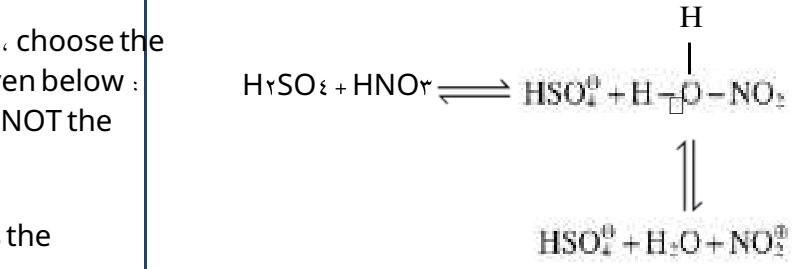
٧٣. Given below are two statements :
Statement I : Nitration of benzene involves the following step –



Statement II : Use of Lewis base promotes the electrophilic substitution of benzene. In the light of the above statements, choose the most appropriate answer from the options given below :
(١) Both Statement I and Statement II are incorrect
(٢) Statement I is correct but Statement II is incorrect
(٣) Both Statement I and Statement II are correct
(٤) Statement I is incorrect but Statement II is correct

Ans. (٢)

Sol. In nitration of benzene concentrated H_2SO_4 and HNO_3 is used as reagent which generates electrophile NO_2^+ in following steps :



Lewis acids can promote the formation of electrophiles not Lewis base

٧٤. The correct order of ligands arranged in increasing field strength.

(١) $Cl^- < OH^- < Br^- < CN^-$

(٢) $F^- < Br^- < I^- < NH_3$

(٣) $Br^- < F^- < H_2O < NH_3$

(٤) $H_2O < OH^- < CN^- < NH_3$

Ans. (٣)

Sol. Experimental order $Br^- < F^- < H_2O < NH_3$

٧٥. Which of the following gives a positive test with ninhydrin?

(١) Cellulose (٢) Starch

(٣) Polyvinyl chloride (٤) Egg albumin

Ans. (٤)

Sol. Ninhydrin test is a test of amino acids. Egg albumin contains protein which is a natural polymer of amino acids which will show positive ninhydrin test

٧٦. The metal that shows highest and maximum number of oxidation state is:

(١) Fe (٢) Mn

(٣) Ti (٤) Co

Ans. (٢)

Sol. Mn shows highest oxidation state (Mn^{+7}) in 3d series metals.

٧٧. Ail organic compound has ٤٢.١٪ carbon, ٦.٤٪ hydrogen and remainder is oxygen. If its molecular weight is ٣٤٢, then its molecular formula is:

(١) $C_{11}H_{18}O_{12}$ (٢) $C_{12}H_{20}O_{12}$

(٣) $C_{14}H_{20}O_{10}$ (٤) $C_{12}H_{22}O_{11}$

Ans. (٤)

Sol. only $C_{12}H_{22}O_{11}$ has ٤٢.١٪ carbon, ٦.٤٪ hydrogen & ٥١.٥ percent oxygen.

٧٨. Given below are two statement:

Statement I: Bromination of phenol in solvent with low polarity such as $CHCl_3$ or CS_2 requires Lewis acid catalyst.

Statement II: The lewis acid catalyst polarises the bromine to generate Br^+.

In the light of the above statements, choose the correct answer from the options given below:

(١) Statement I is true but Statement II is false.

(٢) Both Statement I and Statement II are true

(٣) Both Statement I and Statement II are false.

(٤) Statement I is false but Statement II is true.

Ans. (٤)

Sol. Phenol is a highly activated compound which can undergo bromination without any Lewis acid. Molar ionic conductivities of divalent cation and

٧٩. anion are ٥٧ $S\,cm^2\,mol^{-1}$ and ٧٣ $S\,cm^2\,mol^{-1}$ respectively. The molar conductivity of solution of an electrolyte with the above cation and anion will be:

(١) ٦٥ $S\,cm^2\,mol^{-1}$ (٢) ١٣٠ $S\,cm^2\,mol^{-1}$

(٣) ١٨٧ $S\,cm^2\,mol^{-1}$ (٤) ٢٦٠ $S\,cm^2\,mol^{-1}$

Ans. (٢)

Sol. $\lambda^\circ_{C^{2+}} = 57\,Scm^2mol^{-1}$

$\lambda^\circ_{A^{-}} = 73\,Scm^2mol^{-1}$

Solution

$\lambda^\circ = \lambda^\circ_{C^{2+}} + 2\lambda^\circ_{A^-}$

$= ٥٧ + ٧٣ = ١٣٠$

٨٠. The number of neutrons present in the more abundant isotope of boron is 'x'. Amorphous boron upon heating with air forms a product, in which the oxidation state of boron is 'y'. The value of x + y is ...

(١) ٤ (٢) ٦

(٣) ٣ (٤) ٩

Ans. (٤)

Sol. More abundant isotope $= {}^{11}B$

Number of neutrons $= 6$

$x = 6$

$B + O_2 \rightarrow B_2O_3$

Oxidation state of B in $B_2O_3 = +3$

So, $y = 3$

Hence $x + y = 9$

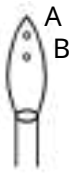
SECTION-B

٨١. The value of Rydberg constant (RH) is $2.18 \times 10^{-18}\,J$. The velocity of electron having mass $9.1 \times 10^{-31}\,kg$ in Bohr's first orbit of hydrogen atom $= \ldots\ldots\ldots \times 10^5\,ms^{-1}$ (nearest integer)

Ans. (٢٢)

Sol. $V = 2.18 \times 10^{-6} \times \frac{Z}{n}$

$= 21.8 \times 10^{5} \times \frac{1}{1} \approx 22 \times 10^5$ (nearest)

٨٢. 

In a borax bead test under hot condition، a metal salt (one from the given) is heated at point B of the flame، resulted in green colour salt bead. The spin-only magnetic moment value of the salt is ..... BM (Nearest integer)

Given atomic number of Cu = ٢٩، Ni = ٢٨، Mn = ٢٥، Fe = ٢٦

Ans. (٦)

Sol. Fe^{2+} will give green coloured bead when heated at point B.

Number of unpaired e in Fe^{2+} = ٥

$\mu = 5.92$

Nearest integer = ٦

٨٣. The heat of combustion of solid benzoic acid at constant volume is −٣٢١.٣٠ kJ at ٢٧°C. The heat of combustion at constant pressure is (−٣٢١.٣٠ − xR) kJ، the value of x is ..... .

Ans. (١٥٠)

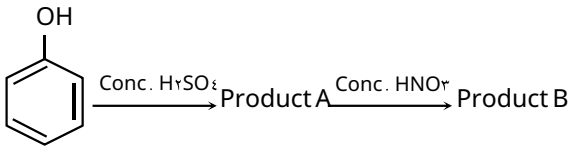
Sol. $C_6H_5COOH(S) + \frac{15}{2} O_2(g) \rightarrow 7CO_2(g) + 3H_2O(\ell)$

$\Delta H = \Delta U + \Delta ngRT$

$-321.30 \text{ kJ} = \frac{1}{2} R \times 300$

٨٤. $= (-321.30 - 150R)$ kJ

Consider the given chemical reaction sequence :



Total sum of oxygen atoms in Product A and Product B are ..... .

Ans. (١٤)

Sol. Picric acid is prepared by treating phenol first with concentrated sulphuric acid which converts it to phenol-٢،٤-disulphonic acid and then with concentrated nitric acid to get ٢،٤،٦ trinitrophenol.

٨٥. The spin only magnetic moment value of the ion among Ti^{2+}، V^{2+}، Co^{3+} and Cr^{2+}، that acts as strong oxidising agent in aqueous solution is ..... BM (Near integer).

(Given atomic numbers : Ti : ٢٢، V : ٢٣، Cr : ٢٤، Co : ٢٧)

Ans. (٥)

Sol. Strong oxidising agent = Co^{3+}

No. of unpaired e in Co^{3+} $3d^6$ = ٤

Hence $\mu = \sqrt{n(n+2)} = \sqrt{24}$ BM

Nearest integer = ٥

٨٦. During Kinetic study of reaction $2A + B \rightarrow C + D$، the following results were obtained :

	$[A]_0$ M	$[B]_0$ M	initial rate of formation of D
I	٠.١	٠.١	6.0×10^{-3}
II	٠.٣	٠.٢	7.2×10^{-2}
III	٠.٣	٠.٤	2.88×10^{-1}
IV	٠.٤	٠.١	2.40×10^{-2}

Based on above data، overall order of the reaction is ..... .

Ans. (٣)

Sol. $r = K[A]^x[B]^y$

(I) $6 \times 10^{-3} = K[0.1]^x[0.1]^y$

(IV) $2.4 \times 10^{-2} = K[0.4]^x[0.1]^y$

(IV)/(I)

$4 = (4)^x$

$x = 1$

$r = K[A]^x[B]^y$

(III) $2.88 \times 10^{-1} = K[0.3]^x[0.4]^y$

(II) $7.2 \times 10^{-2} = K[0.3]^x[0.2]^y$

(III)/(II)

$4 = 2^y$

$y = 2$

Overall order = $x + y = 1 + 2 = 3$

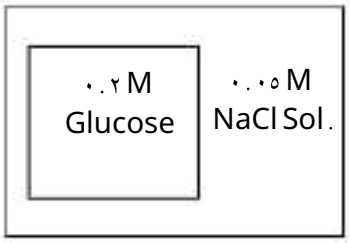
٨٧. An artificial cell is made by encapsulating ٠.٢ M glucose solution within a semipermeable membrane. The osmotic pressure developed when the artificial cell is placed within a ٠.٠٥ M solution of NaCl at ٣٠٠ K is _____ × ١٠ bar. (Nearest Integer)

Given : R = ٠.٠٨٣ L bar mol K

Assume complete dissociation of NaCl

Ans. (٢٥)

Sol.



$NaCl \rightarrow Na^+ + Cl^-$

٠.٠٥M ٠.٠٥M ٠.٠٥M

Total $C_1 = ٠.٠٥ + ٠.٠٥ = ٠.١$ M (NaCl)

$C_2 = ٠.٢$ M (glucose)

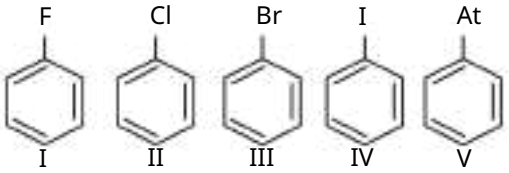
$\pi = (C_2 - C_1)RT$

$= (٠.٢ - ٠.١) \times ٠.٠٨٣ \times ٣٠٠$

$= ٢.٤٩$ bar

$= ٢٤.٩ \times ١٠^{-١}$ bar

٨٨. The number of halobenzenes from the following that can be prepared by Sandmeyer's reaction is . . . . . .



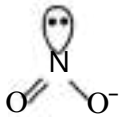
Ans. (٢)

Sol. In Sandmayer reaction only bromobenzene & chlorobenzene are prepared

٨٩. In the lewis dot structure for NO_2^-, total number of valence electrons around nitrogen is ... .

Ans. (٨)

Sol.

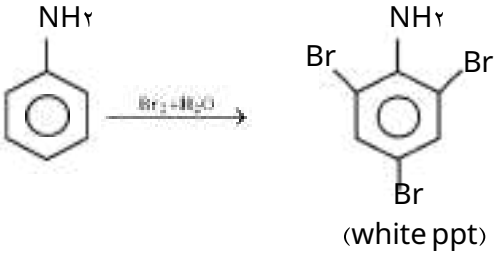


Number of valence e around N-atom = ٨

٩٠. ٩.٣ g of pure aniline is treated with bromine water at room temperature to give a white precipitate of the product 'P'. The mass of product 'P' obtained is ٢٦.٤ g. The percentage yield is ... ... %.

Ans. (٨٠)

Sol.



(white ppt)

٩٣ g of aniline produces ٣٣٠ g of ٢,٤,٦-tribromoaniline. Hence ٩.٣ g of aniline should produce ٣٣g of ٢,٤,٦-tribromoaniline. Hence percentage yield $= \frac{٢٦.٤}{٣٣} \times ١٠٠ = ٨٠\%$