FINAL JEE–MAIN EXAMINATION – APRIL, 2024

(Held On Friday 05 April, 2024) TIME : 3 : 00 PM to 6 : 00 PM

M ATHEM ATICS

TEST PAPER WITH SOLUTION

SECTION-A

१. Let $f : [-१, २] \to R$ be given by $f(x) = २x^{२} + x + [x] - [x]$, where $[t]$ denotes the greatest integer less than or equal to t. The number of points, where f is not continuous, is :

(१) ६ (२) ३

(३) ४ (४) ५

Ans. (३)

Sol. Doubtful points : $-१, ०, १, \sqrt{२}, \sqrt{३}, २$

at $x = \sqrt{२}, \sqrt{३}$

$f(x) = २x^{२} + x + [x]$ Cont. Cont. Discont.

at $x = -१$:

RHL $= \lim_{h \to ०} f(-१ + h) = \lim_{h \to ०} २(-१ + h)^{२} + (-१ + h) + [-१ + h] = २ - १ - ३ = -२$ Dis.

$f(-१) = २ - १ - ३ = -२$

at $x = २$:

LHL $= \lim_{h \to ०} f(२ - h) = ८ + २ - ३ - १२ = -५$ Cont.

$f(२) = ८ + २ - ४ - १२ = -५$

at $x = ०$:

LHL $= \lim_{h \to ०} f(०) = ०$ Dis.

$f(०) = ०$

at $x = १$

LHL $= \lim_{h \to ०} २(१ - h)^{२} + (१ - h) + [१ - h] = २ + १ - ३ = ०$

$f(१) = २ + १ - ३ = ०$ Cont.

RHL $= \lim_{h \to ०} २(१ + h)^{२} + (१ + h) + [१ + h] = २ + १ + १ = ४$

२. The differential equation of the family of circles passing the origin and having center at the line $y = x$ is :

(१) $(x^{२} - y^{२} + २xy)dx = (x - y^{२} + २xy)dy$

(२) $(x^{२} + y^{२} + २xy)dx = (x + y^{२} + २xy)dy$

(३) $(x^{२} - y^{२} + २xy)dx = (x - y^{२} + २xy)dy$

(४) $(x^{२} + y^{२} - २xy)dx = (x + y^{२} + २xy)dy$

Ans. (३)

Sol. $C : x^{२} + y^{२} + २gx + २gy = ०$ .....(१)

$२x + २yy' + २g + २gy' = ०$

$g = -\left(\frac{x + yy'}{१ + y'}\right)$

Put in (१)

$x^{२} + y^{२} - \left(\frac{x + yy'}{१ + y'}\right)२x + २yy' + (x + y) = ०$

$(x^{२} - y^{२} + २xy)y' = x - y^{२} + २xy$

३. Let $S_{१} = \{z \in C : |z| = ५\}$,

$S_{२} = \left\{z \in C : Im\left(\frac{z - १ + ३i}{१ + ३i}\right) = ०\right\}$ and

$S_{३} = \{z \in C : Re(z) \neq ०\}$. Then

(१) $\frac{१२५}{६}$ (२) $\frac{१२५}{२४}$

(३) $\frac{१२५}{४}$ (४) $\frac{१२५}{१२}$

Ans. (४)

Sol. $S_{१} : x^{२} + y^{२} = २५$ .....(१)

$S_{२} : Im \text{ of } \frac{z - १ + ३i}{१ + ३i} = ०$

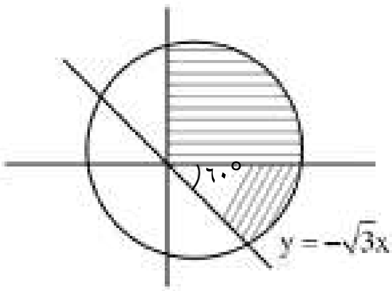
$Im \text{ of } \frac{x + iy - १ + ३i}{१ + ३i} = ०$

$Im \text{ of } \frac{(x - १) + i(y + ३)}{४} = ०$

$३x - y - ०$ .....

$S_{३} : x \neq ०$ (२)

$Area = \frac{५}{१२}\pi(५)^{२}$ ..... (३)



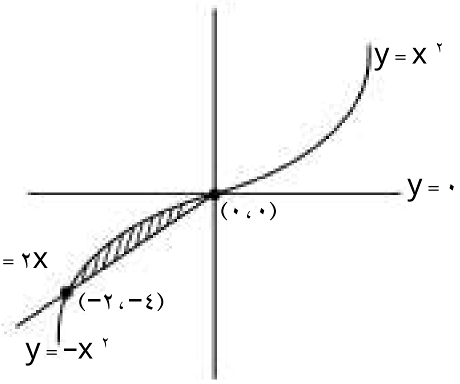
٤. The area enclosed between the curves y = x|x| and y = x − |x| is :

(١) ٨/٣ (٢) ٢/٣

(٣) ١ (٤) ٤/٣

Ans. (٤)

Sol.



A □ □ □ x2 □ 2x □ ٤ □2

٥. ٦٠ words can be made using all the letters of the word BHBJO، with or without meaning. If these words are written as in a dictionary، then the ٥٠th word is :

(١) OBBHJ (٢) HBBJO

(٣) OBBJH (٤) JBBOH

Ans. (٣)

Sol. B B H J O

B ______ ٤! □ ٢٤

H ______ ٤!/٢! □ ١٢

J ______ ٤!/٢! □ ١٢

O B B H J

O B B J H □ ٥٠ rankth

٦. Let a□٢i^□٥j^□k^، b□٢i^□٢j^□ ٢k^

and c be three vectors such that

□c□i^□□□a□b□i^□□a□□c□i^□ a.c□□□٢٩

then c.□□٢i^□j^□k^□ is equal to

(١) ١٠ (٢) ٥

(٣) ١٥ (٤) ١٢

Ans. (٢)

Sol. Let's assume v□a□b□i^

□٥i^□٣j^□k^

and c□i^□p

So،

p □ v □ a □ p

p □ v □ p □ a □ ٠

p □ □ v □ a □ □ 0

□ p □ □ □ v □ a □

c□i□□□٧i^□٨j^□

a.c□a.i^□□a.□٧i^□٨j^□

□٢٩□٢□□□١٤□٤٠□

□ □ □ ١/٢

c.□i^□j^□k^□ i.□٢i^□j^□k^□□□٧i^□٨j^□□□٢i^□j^□k^□

□ □ ١/٢ □ □ 14 □ 8 □ □ 2 □ 5 □

٧. Consider three vectors a،b،c. Let |a|□٢، |b|□٣

and a□b□c. If □□ □ □ □ □ □٠، π/3 □ is the angle between

the vectors b and c، then the minimum value of

٢٧|c□a|٢ is equal to :

(١) ١١٠ (٢) ١٠٥

(٣) ١٢٤ (٤) ١٢١

Ans. (٣)

Sol. $|c - a|^2 = |a|^2 - 2a \cdot c$

$|c|^2 - 4 = 0$

$\Rightarrow a \perp b \perp c$

$|a| |b| |c|$

$2 = 3|c|\sin\theta$

$|c| = \frac{2}{3}\csc\theta$ $\quad \theta \in \left(\frac{\pi}{3}, \frac{\pi}{2}\right]$

$|c|_{min} = \frac{2}{3} = \frac{2}{\sqrt{3}}$ $\quad \csc\theta = \left(\frac{2}{\sqrt{3}}, 1\right]$

$27|a \times c|^2_{min} = 27\left(\frac{16}{27}\right) = 4 \times 31$

٨.

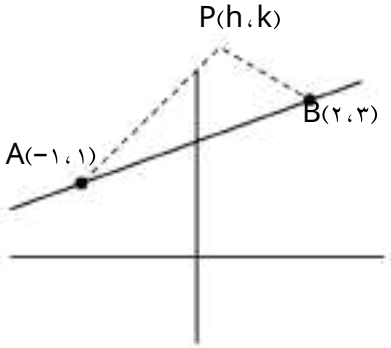
Let $A(-1, 1)$ and $B(2, 3)$ be two points and P be a variable point above the line AB such that the area of ΔPAB is ١٠. If the locus of P is $ax + by = 15$, then $5a + 2b$ is :

(١) $-\frac{12}{5}$ (٢) $-\frac{6}{5}$

(٣) ٤ (٤) ٦

Ans. (١)

Sol.



$\frac{1}{2}\begin{vmatrix} h & k & 1 \\ -1 & 1 & 1 \\ 2 & 3 & 1 \end{vmatrix} = 10$

$-2x + 3y = 25$

$\left(-\frac{6}{5}\right)x + \left(\frac{9}{5}\right)y = 15$

$a = -5, b = \frac{9}{5}$

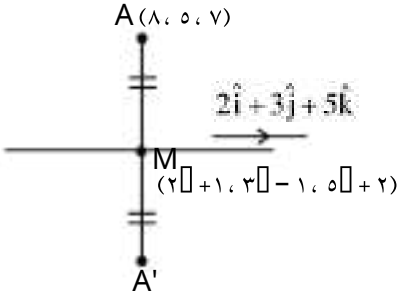
$5a = -6, 2b = \frac{18}{5}$

٩. Let (α, β, γ) be the point $(8, 5, 7)$ in the line $\frac{x-1}{2} = \frac{y-1}{3} = \frac{z-2}{5}$. Then $\alpha + \beta + \gamma$ is equal to

(١) ١٦ (٢) ١٨

(٣) ١٤ Ans. (٤) ٢٠

Sol.



$\overrightarrow{AM} \cdot (2\hat{i} + 3\hat{j} + 5\hat{k}) = 0$

$(2\lambda - 7)(2) + (3\lambda - 6)(3) + (5\lambda - 5)(5) = 0$

$38\lambda = 57$

$\lambda = \frac{3}{2}$

$M\left(4, \frac{7}{2}, 19\right)$

$A'(0, 2, 12)$

١٠. If the constant term in the expansion of $\left(\sqrt[5]{3}x + \frac{2}{\sqrt[3]{5}x}\right)^{12}$, $x > 0$, is $k \times 2 \times 53$, then $\sqrt{2k}$ is equal to :

(١) ٦٣٩ (٢) ٧٢٤

(٣) ٦٩٣ (٤) ٧٤٢

Ans. (٣)

Sol. $T_{r+1} = {}^{12}C_r \left(3^{1/5}\right)^{12-r} \left(\frac{2}{3^{1/2}}\right)^r x^{\frac{12-r}{5} - \frac{r}{2}}$

$r = 3$

$T_{r+1} = \frac{{}^{12}C_r \left(3^{\frac{12-r}{5}}\right)(2^r)}{5^{\frac{12-r}{2}}}$

$r = 6$

$T_7 = \frac{{}^{12}C_6 \left(3^{6/5}\right)(2^6)}{5^3} = \frac{924 \times 2.3^{1/5} \times 8}{25}$

$\sqrt{2k} = 693$

١١. Let $f, g : R \to R$ be defined as : $f(x) = |x - ١|$ and

$g(x) = \begin{cases} e^{x}, & x < ٠ \\ x - ١, & x \geq ٠ \end{cases}$. Then the function $f(g(x))$ is

(١) neither one-one nor onto.

(٢) one-one but not onto.

(٣) both one-one and onto.

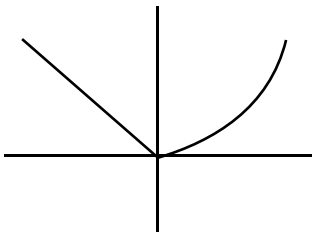
(٤) onto but not one-one.

Ans. (١)

Sol. $f(g(x)) = |g(x) - ١|$

$fog(x) = \begin{cases} |e^{x} - ١|, & x < ٠ \\ |x - ١|, & x \geq ٠ \end{cases}$

$fog(x) = \begin{cases} e^{x} - ١, & x < ٠ \\ x - ١, & x \geq ٠ \end{cases}$



١٢. Let the circle $C_١ : x^{٢} + y^{٢} - ٢(x + y) + ١ = ٠$ and $C_٢$ be a circle having centre at $(-١, ٠)$ and radius ٢. If the line of the common chord of $C_١$ and $C_٢$ intersects the y-axis at the point P, then the square of the distance of P from the centre of $C_١$ is :

(١) ٢ (٢) ١

(٣) ٦ (٤) ٤

Ans. (١)

Sol. $S_١ : x^{٢} + y^{٢} - ٢x - ٢y + ١ = ٠$

$S_٢ : x^{٢} + y^{٢} + ٢x - ٣ = ٠$

Common chord $= S_١ - S_٢ = ٠$

$-٤x - ٢y + ٤ = ٠$

$٢x + y = ٢ \Rightarrow P(٠, ٢)$

$d(C, P) = (١ - ٠)^{٢} + (٢ - ١) = ٣^{٢}$

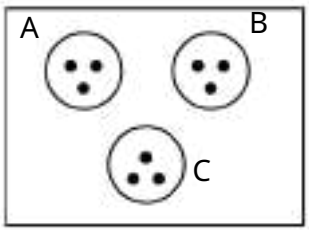
١٣. Let the set $S = \{٢, ٤, ٨, ١٦, \ldots\ldots, ٥١٢\}$ be partitioned into ٣ sets A, B, C with equal number of elements such that $A \cup B \cup C = S$ and $A \cap B = B \cap C = A \cap C = \phi$.
The maximum number of such possible partitions of S is equal to :

(١) ١٦٨٠ (٢) ١٥٢٠

(٣) ١٧١٠ (٤) ١٦٤٠

Ans. (١)

Sol.



$\frac{٩!}{٣!٣!٣!} \times \frac{١}{٣!}$

١٤. The values of m, n, for which the system of equations

$x + y + z = ٤,$

$٢x + ٥y + ٥z = ١٧,$

$x + ٢y + mz = n$

has infinitely many solutions, satisfy the equation :

(١) $m^{٢} + n - ٢m - n = ٤٦$

(٢) $m^{٢} + n + ٢m + n = ٦٤$

(٣) $m^{٢} + n + ٢mn = ٦٨$

(٤) $m^{٢} + n - ٢mn = ٣٩$

Ans. (٤)

Sol. $D = \begin{vmatrix} ١ & ١ & ١ \\ ٢ & ٥ & 5 \\ ١ & ٢ & m \end{vmatrix} = 0 \Rightarrow m = ٢$

$D_3 = \begin{vmatrix} ١ & ١ & ٤ \\ ٢ & ٥ & ١٧ \\ ١ & ٢ & n \end{vmatrix} = ٠ \Rightarrow n = ٧$

١٥. The coefficients a, b, c in the quadratic equation $ax^{٢} + bx + c = ٠$ are from the set $\{١, ٢, ٣, ٤, ٥, ٦\}$.
If the probability of this equation having one real root bigger than the other is p, then $٢١٦p$ equals :

(١) ٥٧ (٢) ٣٨

(٣) ١٩ (٤) ٧٦

Ans. (٢)

Sol. D < ٠

b٢ < ٤ac

b = ٣ : (a، c) = (١، ١)(١، ٢)(٢، ١)

b = ٤ : (a، c) = (١، ١)(١، ٢)(٢، ١)(١، ٣)(٣، ١)

b = ٥ : (a، c) = (١، ١)(١، ٢)(٢، ١) (١، ٣)(٣، ١)(١، ٤)(٤، ١)
(١، ٥)(٥، ١)(١، ٦)(٦، ١)(٢، ٣)(٣، ٢)(٢، ٢)

b = ٦ : (a، c) = (١، ١)(١، ٢)(٢، ١) (١، ٣)(٣، ١)(١، ٤)(٤، ١)
(١، ٥)(٥، ١)(١، ٦)(٦، ١)(٢، ٣)(٣، ٢)(٢، ٤)(٤، ٢)(٢، ٢)

fav. cases = ٣٨

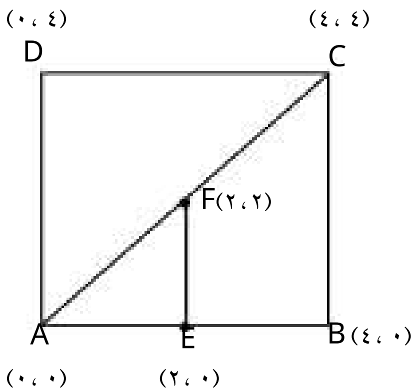
Prob. : ٣٨/□

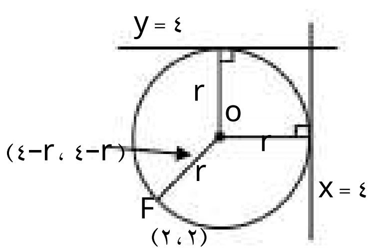
١٦. Let ABCD and AEFG be squares of side ٤ and ٢ units، respectively. The point E is on the line segment AB and the point F is on the diagonal AC. Then the radius r of the circle passing through the point F and touching the line segments BC and CD satisfies :

(١) r = ١
(٢) r٢ − ٨r + ٨ = ٠
(٣) ٢r٢ − ٤r + ١ = ٠
(٤) ٢r٢ − ٨r + ٧ = ٠

Ans. (٢)

Sol.





OF٢ = r٢

(٢ − r)٢ + (٢ − r)٢ = r٢

r٢ − ٨r + ٨ = ٠

١٧. Let β(m، n) = ∫₀¹ x^(m−١)(١ − x)^(n−١) dx، m، n > ٠. If ∫₀¹ (١ − x١٠)٢٠ dx = a β(b، c)، then ١٠٠(a + b + x) equals ________.

(١) ١٠٢١
(٢) ١١٢٠
(٣) ٢٠١٢
(٤) ٢١٢٠

Ans. (٤)

Sol. I = ∫₀¹ (١ − x١٠)٢٠ dx

x١٠ = t

x = t^(١/١٠)

dx = (١/١٠) t^(−٩/١٠) dt

I = ∫₀¹ (١ − t)٢٠ (١/١٠) t^(−٩/١٠) dt

I = (١/١٠) ∫₀¹ t^(١/١٠ − ١) (1 − t)^(٢١ − ١) dt

a = ١/١٠ b = ١/١٠ c = ٢١

١٨. Let □□ □، and A = □□□ ... 3 □ ... 2□□□. If B = □3□ □9 ... ٧ ... ٥ ... 3□□ □2□ □2□□□ is the matrix of cofactors of the elements of A، then det(AB) is equal to :

(١) ٣٤٣
(٢) ١٢٥
(٣) ٦٤
(٤) ٢١٦

Ans. (٤)

Sol. Equating co-factor fo A٢١

(٢□ − ٣□) = □

□ = ٠، ٢ (accept)

Now، ٢□٢ − □□ = ٣□

□ = ٢ □ = ١

|AB| = |A cof(A)| = |A|٣

A = | ١ ٢ ٣ ; ٢ ٢ ١ ; −١ ٢ ٤ | = ٦ − ٢(٩) + ٣(٦) = ٦

١٩. If $y(\theta) = \frac{2\cos\theta - \cos 2\theta}{\cos 4\theta - 4\cos 2\theta + 3}$,

then at $\theta = \frac{\pi}{4}$, $y'' + y' + y$ is equal to :

(١) $\frac{2}{2}$ (٢) ١

(٣) $\frac{1}{2}$ (٤) ٢

Ans. (٤)

Sol. $y = \frac{2\cos\theta - 2\cos^2\theta + 1}{4\cos^3\theta - 3\cos\theta - 4\cos^2\theta + 2 + 5\cos\theta - 2}$

$y = \frac{2\cos 2\theta - 2\cos\theta + 1}{2\cos 2\theta - 2\cos\theta + 1}$

$y = 1$

$y = \frac{1}{2}$ at $\theta = \frac{\pi}{4}$

$y' = 0$

$y = \frac{1}{2}$ at $\theta = \frac{\pi}{4}$

$y'' = 0$

$y = 1$ at $\theta = \frac{\pi}{4}$

٢٠. For $x \in [0, 1]$, the least value of K, for which 4^{x-1}, $\frac{K}{2}$, $16^x + 16^{-x}$ are three consecutive terms of an A.P. is equal to :

(١) ١٠ (٢) ٤

(٣) ٨ (٤) ١٦

Ans. (١)

Sol. $k = 4^x + \frac{1}{4^x}$ $\quad$ $4^{2x} + \frac{1}{4^{2x}}$

$k = 10$

SECTION-B

٢١. Let the mean and the standard deviation of the probability distribution

X		١	٠	−٣
P(X)	$\frac{1}{3}$	K	$\frac{1}{6}$	$\frac{1}{4}$

be α and β, respectively. If $\alpha - \beta = 2$, then $\alpha + \beta$ is equal to _____.

Ans. (٥)

Sol. $\frac{13}{k} = \frac{1}{6} + \frac{1}{4} + 1$ $\quad$ $k = \frac{1}{4}$

$\frac{3}{4} - \frac{1}{4} = \frac{3}{4}$

$\alpha = \frac{1}{2}$

$\beta = \sqrt{\frac{1}{3} + \frac{1}{4} + 9\left(\frac{1}{4} - \frac{1}{2}\right)^2}$

$\beta = \sqrt{\frac{4 + 3 + 9}{9}}$

$\beta = 2$

$\alpha + \beta = 2 + 2$

$= 5$

٢٢. Let $y = y(x)$ be the solution of the differential equation $\frac{dy}{dx} = \frac{2x}{1+x^2} y + xe^{\tan^{-1}x}$, $y(0) = 0$.

Then the area enclosed by the curve $f(x) = y(x)e^{-\tan^{-1}x}$ and the line $y - x = 4$ is _____.

Ans. (١٨)

Sol. IF $= e^{\int \frac{2x}{1+x^2}dx} = e^{\ln(1+x^2)} = 1+x^2$

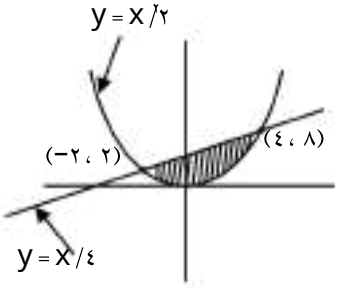
$y(1+x^2) = \int x(1+x^2)e^{-x^2}dx$

$y(1+x^2) = -\frac{1}{2}xe^{-x^2} + \frac{x^2}{2} + c$

$(0,0) \Rightarrow c = 0$

$y(x) = \frac{x^2}{2}e^{-x^2}$

$f(x) = \frac{x}{2}$



$A = \int_{-2}^{4}\left(x+4-\frac{x^2}{2}\right)dx = 18$

23. The number of solutions of $\sin^2 x + (2+2x-x)\sin x - 3(x-1) = 0$, where $-\pi \le x \le \pi$, is

Ans. (2)

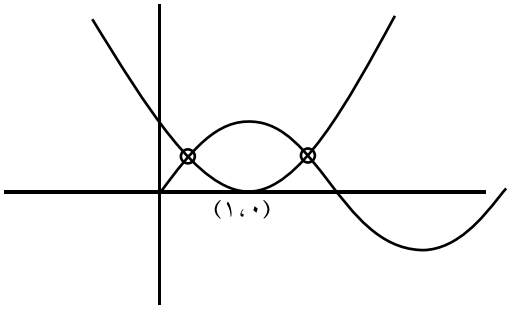
Sol. $\sin^2 x - (x^2-2x-2)\sin x - 3(x-1) = 0$

$\sin^2 x - (x-1)(\sin x - 3) = 0$

roots : -3, $(x-1)^2$

$\sin x = -3$ (reject) or $(x-1)^2$

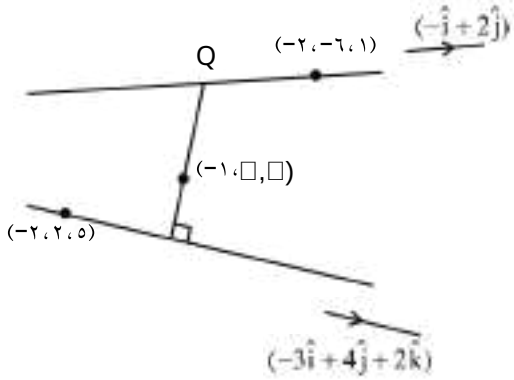
$\sin x = (x-1)^2$



24. Let the point $(-1, \alpha, \beta)$ lie on the line of the shortest distance between the lines $\frac{x+2}{3} = \frac{y+2}{4} = \frac{z-5}{2}$ and $\frac{x+2}{1} = \frac{y-6}{2} = \frac{z-1}{0}$. Then $(\alpha-\beta)^2$ is equal to _____.

Ans. (25)

Sol.



$P(-3\lambda-2, 4\lambda+2, 2\lambda+5)$

$Q(-\mu-2, 2\mu-6, 1)$

DRS of $PQ = (3\lambda-\mu, 2\lambda-4\mu-8, -2\lambda-4)$

$$\text{DRS of } PQ = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -1 & 2 & 0 \\ -3 & 4 & 2 \end{vmatrix}$$

$= 4\hat{i} + 2\hat{j} + 2\hat{k}$

OR

$(2, 1, 1)$

$\frac{\alpha-1}{2} = \frac{\beta-4}{1} = \frac{-2-4}{1}$

$\alpha = \lambda + 2$ & $\beta = \lambda - 8$

$\lambda = -1$ $\quad$ $\lambda = 1$

$Q : (-3, -4, 1)$

$LPQ = \frac{x+3}{2} = \frac{y+4}{1} = \frac{z-1}{1}$

$(-1, \alpha, \beta)$ $\quad$ $\frac{\alpha+4}{1} = \frac{\beta-1}{1}$

$\alpha = -3, \beta = 2$

$(\alpha-\beta)^2 = 25$

25. If

$$1+\frac{\sqrt{3}-\sqrt{2}}{2\sqrt{3}}+\frac{5-2\sqrt{6}}{18}+\frac{9\sqrt{3}-11\sqrt{2}}{36\sqrt{3}}+\frac{49-20\sqrt{6}}{180}+\dots$$

upto $\infty = 2+\left(\sqrt{\frac{b}{a}}+1\right)\log_e\left(\frac{a}{b}\right)$, where a and b are integers with gcd(a, b) = 1, then 11a + 18b is equal to ________.

Ans. (76)

Sol. $S = 1+\frac{x}{2\sqrt{3}}+\frac{x^2}{18}+\frac{x^3}{36\sqrt{3}}+\frac{x^4}{180}+\dots\infty$

Put $\frac{x}{\sqrt{3}} = t$, where $x = \sqrt{3}-\sqrt{2}$

$S = 1+\frac{t}{2}+\frac{t^2}{6}+\frac{t^3}{12}+\frac{t^4}{20}+\dots$

$S = 1+t\left(1-\frac{1}{2}\right)+t^2\left(\frac{1}{2}-\frac{1}{3}\right)+t^3\left(\frac{1}{3}-\frac{1}{4}\right)+t^4\left(\frac{1}{4}-\frac{1}{5}\right)+\dots$

$S = \left(1+t+\frac{t^2}{2}+\frac{t^3}{3}+\dots\right)-\left(\frac{t}{2}+\frac{t^2}{3}+\frac{t^3}{4}+\frac{t^4}{5}+\dots\right)$

$S = \left(t+\frac{t^2}{2}+\dots\right)-\frac{1}{t}\left(\frac{t^2}{2}+\frac{t^3}{3}+\dots\right)+2$

$S = 2+\left(1-\frac{1}{t}\right)(-\log(1-t))-\frac{1}{t}\cdot t\dots$ i.e. $S = 2+\left(\frac{1-t}{t}\right)\log(1-t)$

$S = 2+\left(\frac{\sqrt{3}}{\sqrt{3}-\sqrt{2}}-1\right)\log\left(1-\frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}}\right)$

$S = 2+\left(\frac{\sqrt{2}}{\sqrt{3}-\sqrt{2}}\right)\log_e\frac{\sqrt{2}}{\sqrt{3}}$

$S = 2+\frac{\sqrt{6}+2}{2}\log_e\frac{2}{3} = 2+\left(\sqrt{\frac{3}{2}}+1\right)\log_e\frac{2}{3}$

a = 2, b = 3

11a + 18b = 11 × 2 + 18 × 3 = 76

26. Let a > 0 be a root of the equation $2x^2+x-2=0$.

If $\lim_{x\to\frac{1}{a}}\frac{16\left(1-\cos\left(2+x-2x^2\right)\right)}{(1-ax)^2} = \alpha+\beta\sqrt{17}$, where $\alpha, \beta \in Z$ then $\alpha+\beta$ is equal to ________.

Ans. (170)

Sol. $2x^2+x-2=0$ → roots a, b

$2x^2-x-2=0$ → roots $\frac{1}{a}$, $\frac{1}{b}$

$\lim_{x\to\frac{1}{a}} 16\,\frac{1-\cos 2\left(x-\frac{1}{a}\right)\left(x-\frac{1}{b}\right)}{4\left(x-\frac{1}{a}\right)^2\left(x-\frac{1}{b}\right)^2}\times\frac{4\left(x-\frac{1}{a}\right)^2\left(x-\frac{1}{b}\right)^2}{a^2\left(x-\frac{1}{a}\right)^2}$

$= 16\times\frac{2}{a^2}\left(\frac{1}{a}-\frac{1}{b}\right)^2$

$= \frac{32\times17}{a^2\times4} = \frac{17\cdot8}{a^2} = \frac{17\times8\times16}{\left(-1+\sqrt{17}\right)^2}$

$= \frac{136\cdot16}{18-2\sqrt{17}}\times\frac{18+2\sqrt{17}}{18+2\sqrt{17}}$

$= \frac{136}{256}\left(18+2\sqrt{17}\right)16$

$= 153+17\sqrt{17} = \alpha+\beta\sqrt{17}$

$\alpha+\beta = 153+17 = 170$

27. If $f(t) = \int_0^{\pi}\frac{2x\,dx}{1-\cos^2 t\sin^2 x}$, $0 < t < \pi$, then the value of $\int_0^{\pi/2}\frac{\pi^2\,dt}{f(t)}$ equals ________.

Ans. (1)

Sol. $f(t) = \int_0^{\pi}\frac{2x}{1-\cos^2 t\sin^2 x}dx$ .....(1)

$$2\int_0^{\pi/2}\frac{(\pi-x)dx}{1+\cos^2 t\sin^2 x} \quad \ldots\ldots(2)$$

$$2f(t)=2\int_0^{\pi/2}\frac{\pi}{1+\cos^2 t\sin^2 x}dx$$

$$f(t)=\int_0^{\pi/2}\frac{\pi}{1+\cos^2 t\sin^2 x}dx$$

divide & by $\cos^2 x$

$$f(t)=\int_0^{\pi/2}\frac{\sec^2 x\,dx}{\sec^2 x+\cos^2 t\tan^2 x}$$

$$f(t)=2\int_0^{\pi/2}\frac{\sec^2 x\,dx}{\sec^2 x+\cos^2 t\tan^2 x}$$

$\tan x=z$

$\sec^2 x\,dx=dz$

$$f(t)=2\int_0^{\infty}\frac{dz}{1+\sin^2 t\,z^2}$$

$$=\frac{2\pi}{\sin t}$$

Then $\int_0^{\pi/2} f(t)\,dt$

$$=\int_0^{\pi/2}\pi\sin t\,dt$$

$=1$

28. Let the maximum and minimum values of $\sqrt{8x-x^2-12}$ $x^2+y^2=7,\ x\in R$ be M and m respectively. Then $M-m$ is equal to _____.

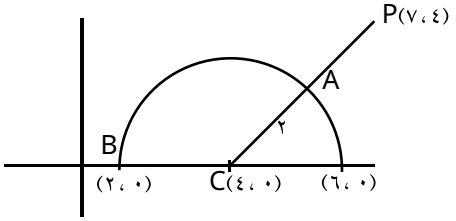
Ans. (1600)

Sol. $(x-7)^2+(y-4)^2$

$y=\sqrt{8x-x^2-12}$

$y^2=-(x-4)^2+16-12$

$(x-4)^2+y^2=4$



$m=9$

$M=41$

$M^2-m^2=41-9=1600$

29. Let a line perpendicular to the line $2x-y=10$ touch the parabola $y^2=4(x-9)$ at the point P. The distance of the point P from the centre of the circle $x^2+y^2-14x-8y+56=0$ is _____.

Ans. (10)

Sol. $y^2=4(x-9)$

slope of tangent $=\frac{1}{2}$

Point of contact $P\left(\frac{1}{\frac{1}{4}}+9, \frac{2}{-1}\right)$

$P(13, -4)$

center of circle $C(7, 4)$

distance $CP=\sqrt{(13-7)^2+(-4-4)^2}$

$=10$

30. The number of real solutions of the equation $x|x+5|+2|x+7|-2=0$ is _____.

Ans. (3)

30. The number of real solutions of the equation $x|x+5|+2|x+7|-2=0$ is _____.

Allen Ans. (3)

Sol. Case I : $x\ge -5$

$x^2+5x+2x+12=0$

$x^2+7x+12=0$

$x=-3, -4$

Case II : $-7<x<-5$

$-x^2-5x+2x+14-2=0$

$-x^{٢}-٣x+١٢=٠$

$x=\frac{-3\pm\sqrt{٩+٤٨}}{٢}$

$=\frac{-٣\pm\sqrt{٥٧}}{٢}$

$x=\frac{-٣-\sqrt{٥٧}}{٢}, \frac{-٣+\sqrt{٥٧}}{٢}$ (rejected)

Case III : $x<-٧$

$-x^{٢}-٥x-٢x-١٤-٢=٠$

$x^{٢}+٧x+١٦=٠$

$D=٤٩-٦٤<٠$

No solutions

No. of solutions $=٣$

SECTION-A

३१. Given below are two statements :

Statement I : When the white light passed through a prism, the red light bends lesser than yellow and violet.

Statement II : The refractive indices are different for different wavelengths in dispersive medium. In the light of the above statements, choose the correct answer from the options given below :

(१) Both Statement I and Statement II are true.

(२) Statement I is true but Statement II is false.

(३) Both Statement I and Statement II are false.

(४) Statement I is false but Statement II is true.

Ans. (१)

Sol. As $\lambda_{red} < \lambda_{yellow} < \lambda_{violet}$

Light ray with longer wavelength bends less.

३२. Which of the following statement is not true about stopping potential (V_0)?

(१) It depends on the nature of emitter material.

(२) It depends upon frequency of the incident light.

(३) It increases with increase in intensity of the incident light.

(४) It is १/e times the maximum kinetic energy of electrons emitted.

Ans. (३)

Sol. $KE_{max} = h\nu - \phi_0 = eV$

३३. The angular momentum of an electron in a hydrogen atom is proportional to : (Where r is the radius of orbit of electron)

(१) $\sqrt{r}$ (२) $\frac{1}{r}$

(३) r (४) $\frac{1}{\sqrt{r}}$

Ans. (१)

Sol. $F_C = \frac{mv^2}{r}$

$\frac{Kqq}{r^2} = \frac{mv^2}{r}$

$mv^2 r = Kq_1q_2r$

$\frac{L^2}{m} = Kq_1q_2r$

$L = r$

३४. A galvanometer of resistance १०० Ω when connected in series with ४०० Ω measures a voltage of upto १० V. The value of resistance required to convert the galvanometer into ammeter to read upto १० A is $x \times 10^{-2}$. The value of x is :

(१) २ (२) ८००

(३) २० (४) २००

Ans. (३)

Sol. $i_g = \frac{10}{400 + 100} = 20 \times 10^{-3}$ A

For ammeter

Let shunt resistance = S

$igR = (i - ig)S$

$20 \times 10^{-3} \times 100 = 10\,S$

$S = 20 \times 10^{-2}$

३५. The vehicles carrying inflammable fluids usually have metallic chains touching the ground :

(१) To conduct excess charge due to air friction to ground and prevent sparking.

(२) To alert other vehicles.

(३) To protect tyres from catching dirt from ground.

(४) It is a custom.

Ans. (१)

Sol. Static charge is developed due to air friction. This can result in combustion. So, metallic chains is used to discharge excess charge.

٣٦. If n is the number density and d is the diameter of the molecule, then the average distance covered by a molecule between two successive collisions (i.e. mean free path) is represented by :

(١) $\frac{1}{\sqrt{2n\pi d^2}}$ (٢) $\sqrt{2n\pi d^2}$

(٣) $\frac{1}{\sqrt{2n\pi d}}$ (٤) $\frac{1}{\sqrt{2n2\pi d^2}}$

Ans. (٣)

Sol. n = number of molecule per unit volume

d = diameter of the molecule

$\lambda = \frac{1}{\sqrt{2\pi d^2 n}}$ (By Theory)

٣٧. A particle moves in x-y plane under the influence of a force F such that its linear momentum is $\vec{P}(t) = \hat{i}\cos(kt) + \hat{j}\sin(kt)$. If k is constant, the angle between F and P will be :

(١) $\frac{\pi}{2}$ (٢) $\frac{\pi}{6}$

(٣) $\frac{\pi}{4}$ (٤) $\frac{\pi}{3}$

Ans. (١)

Sol. $\vec{P} = \cos(kt)\hat{i} + \sin(kt)\hat{j}$; $|\vec{P}| = 1$

$\because \vec{P} = m\vec{v}$

$\therefore \hat{P} = \hat{v}$

$\hat{v} = \cos(kt)\hat{i} + \sin(kt)\hat{j}$

$\hat{a} = \frac{-k\sin(kt)\hat{i} + k\cos(kt)\hat{j}}{k}$

$\hat{a} = -\sin kt\,\hat{i} + \cos kt\,\hat{j}$

$\because \hat{F} = \hat{a}$

$\therefore \hat{F} = -\sin kt\,\hat{i} + \cos kt\,\hat{j}$

$\cos\theta = \frac{\hat{F}\cdot\hat{P}}{|F||P|} = \frac{\sin kt\cos t - \sin kt\cos t}{1\times 1} = 0$

$\theta = \frac{\pi}{2}$

٣٨. The electrostatic force and magnetic force (F) acting on a charge q moving with velocity v can be written :

(١) $\vec{F}_1 = q\vec{V}.\vec{E},\ \vec{F}_2 = q(\vec{B}.\vec{V})$

(٢) $\vec{F}_1 = q\vec{B},\ \vec{F}_2 = q(\vec{B}\times\vec{V})$

(٣) $\vec{F}_1 = q\vec{E},\ \vec{F}_2 = q(\vec{V}\times\vec{B})$

(٤) $\vec{F}_1 = q\vec{E},\ \vec{F}_2 = q(\vec{B}\times\vec{V})$

Ans. (٣)

Sol. $\vec{F}_1 = q\vec{E}$ (Theory)

$\vec{F}_2 = q(\vec{V}\times\vec{B})$

٣٩. A man carrying a monkey on his shoulder does cycling smoothly on a circular track of radius ٩ m and completes ١٢٠ revolutions in ٣ minutes. The magnitude of centripetal acceleration of monkey is (in m/s²) :

(١) zero

(٢) ١٦ ms^{-2}

(٣) ٤ ms^{-2} (٤) ٥٧٦٠٠ ms^{-2}

Ans. (٢)

Sol. Given : R = ٩ m,

١٢٠ revolution in ٣ min

$\omega = \frac{120\ \text{Rev.}}{3\ \text{min.}} = \frac{120\times 2\pi\ \text{rad}}{3\times 60\ \text{sec}} = \frac{4\pi}{3}\ \text{rad/s}$

$a_{\text{centripetal}} = R\omega^2 = 9\left(\frac{4\pi}{3}\right)^2 = ١٦\pi^2\ \text{m/s}^2$

٤٠. A series LCR circuit is subjected to an AC signal of ٢٠٠ V, ٥٠ Hz. If the voltage across the inductor (L = ١٠ mH) is ٣١.٤ V, then the current in this circuit is _____ :

(١) ٦٨ A (٢) ٦٣ A

(٣) ١٠ A (٤) ١٠ mA

Ans. (٣)

Sol. Voltage across inductor $V_L = IX_L$

$٣١.٤ = I_{rms}L\omega_{rms}$

$٣١.٤ = I_{rms}L(2\pi f)_{rms}$

$٣١.٤ = I_{rms} \times ١٠ \times ١٠^{-3}(2 \times ٣.١٤) \times ٥٠$

$\therefore I = ١٠\ A$

٤١. What is the dimensional formula of ab in the equation $\left(P + \frac{a}{V^2}\right)(V - b) = RT$, where letters have their usual meaning.

(١) ML^2T^{-2} (٢) ML^5T^{-2}

Ans. (٣) ML^5T^{-3} (٤) ML^4T^{-2}

Sol. $\because$ V = b

$\therefore$ Dimension of b = L^3

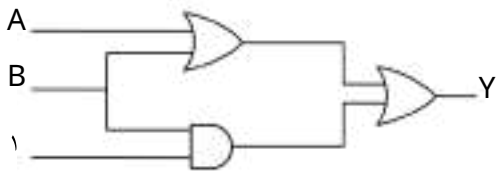
& $[P] = \left[\frac{a}{V^2}\right]$

$a = PV^2 = ML^{-1}T^{-2} \times L^6$

Dimension of a = ML^5T^{-2}

$ab^2 = [ML^5T^{-2}][L^3] = [ML^8T^{-2}]$

٤٢. The output (Y) of logic circuit given below is ० only when :

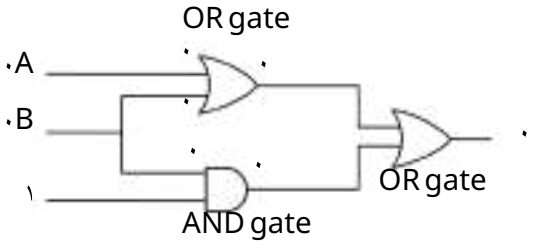


(१) A = १, B = ० (२) A = ०, B = ० (४)

(३) A = १, B = १ A = ०, B = १

Ans. (२)

Sol.



٤३. A body is moving unidirectionally under the influence of a constant power source. Its displacement in time t is proportional to :

(१) t^2 (२) $t^{2/3}$

(३) $t^{3/2}$

Ans. (३)

Sol. P = costant $\Rightarrow$ FV = constant

$\Rightarrow m\frac{dV}{dt}V = \text{constant}$

$\int_0^V VdV = (C)\int_0^t dt$

$\frac{V^2}{2} = Ct$

$V \propto t^{1/2}$

$\frac{ds}{dt} \propto t^{1/2}$

$\int_0^s ds = K\int_0^t t^{1/2}dt$

$S = K\frac{2}{3}t^{3/2}$

$S \propto t^{3/2}$

٤४. $\Rightarrow$ displacement is proportional to $t^{3/2}$

Match List-I with List-II :-

	List-I EM-Wave		List-II Wavelength Range
(A)	Infra-red	(I)	$> १० nm^{-3}$
(B)	Ultraviolet	(II)	४०० nm to १ nm
(C)	X-rays	(III)	१ mm to ७०० nm
(D)	Gamma rays	(IV)	१ nm to १० nm

Choose the correct answer from the options given below :

(१) (A)-(II), (B)-(I), (C)-(IV), (D)-(III)

(२) (A)-(III), (B)-(II), (C)-(IV), (D)-(I)

(३) (A)-(IV), (B)-(III), (C)-(II), (D)-(I)

(४) (A)-(I), (B)-(III), (C)-(II), (D)-(IV)

Ans. (२)

Sol. Infrared is the least energetic thus having biggest wavelength (λ) & gamma rays are most energetic thus having smallest wavelength (λ).

٤٥. During an adiabatic process، if the pressure of a gas is found to be proportional to the cube of its absolute temperature، then the ratio $\frac{C_P}{C_V}$ for the gas is :

(١) $\frac{5}{3}$ (٢) $\frac{9}{7}$

(٣) $\frac{3}{2}$ (٤) $\frac{7}{5}$

Ans. (٣)

Sol. $P \propto T^{٣}$

$PT^{-٣}$ = constant

$\because \frac{PV}{T} \propto nR$ = constant from ideal gas equation

$(P)(PV)^{-٣}$ constant

$P^{-٢}V^{-٣}$ = cosntant ...(١)

$\therefore$ Process equation for adiabatic process is

PV^{γ} = constant ...(٢)

Comparing equation (١) and (٢)

$\frac{C_P}{C_V} = \gamma = \frac{3}{2}$

٤٦. Match List-I with List-II :

	List-I		List-II
(A)	A force that restores an elastic body of unit area to its original state	(I)	Bulk modulus
(B)	Two equal and opposite forces parallel to opposite faces	(II)	Young's modulus
(C)	Forces perpendicular everywhere to the surface of unit area same everywhere	(III)	Stress
(D)	Two equal and opposite forces perpendicular to opposite faces	(IV)	Shear modulus

Choose the correct answer from the options given below : (١) (A)-(II)، (B)-(IV)، (C)-(I)، (D)-(III) (٢) (A)-(IV)، (B)-(II)، (C)-(III)، (D)-(I) (٣) (A)-(III)، (B)-(IV)، (C)-(I)، (D)-(II) (٤) (A)-(III)، (B)-(I)، (C)-(II)، (D)-(IV)

Ans. (٣)

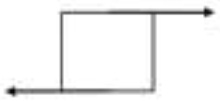
Sol. (A) stress $= \frac{F_{restoring}}{A}$

If A = ١

Stress = Frestoring

(A)-(III)

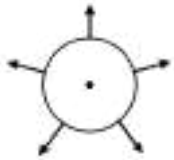
(B)



= Shear stress

(B)-(IV)

(C)



= Volumetric stress

(C)-(I)

(D)

= Longitudinal stress

(D)-(II)

٤٧. A vernier callipers has ٢٠ divisions on the vernier scale، which coincides with ١٩th division on the main scale. The least count of the instrument is ٠.١ mm. One main scale division is equal to _____mm.

(١) ١ (٢) ٠.٥

(٣) ٢ (٤) ٥

Ans. (٣)

Sol. ٢٠ VSD = ١٩ MSD

$1 \text{VSD} = \frac{19}{20} \text{MSD}$

L.C. = ١ MSD − ١ VSD

$٠.١ \text{ mm} = ١ \text{MSD} - \frac{19}{20} \text{MSD}$

$0.1 = \frac{1}{20} \text{MSD}$

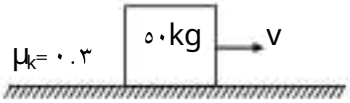
١ MSD = ٢mm

٤٨. A heavy box of mass ٥٠ kg is moving on a horizontal surface. If co-efficient of kinetic friction between the box and horizontal surface is ٠.٣ then force of kinetic friction is :

(١) ١٤.٧ N (٢) ١٤٧ N (٣) ١.٤٧ N (٤) ١٤٧٠ N

Ans. (٢)

Sol.



$F_k = \mu k N = ٠.٣ \times ٥٠ \times ٩.٨ = ١٤٧\ N$

٤٩. A satellite revolving around a planet in stationary orbit has time period ٦ hours. The mass of planet is one-fourth the mass of earth. The radius orbit of planet is : (Given = Radius of geo-stationary orbit for earth is ٤.٢ × ١٠⁴ km)

(١) ١.٤ × ١٠⁴ km

(٢) ٨.٤ × ١٠⁴ km

(٣) ١.٦٨ × ١٠⁴ km

(٤) ١.٠٥ × ١٠⁴ km

Ans. (٤)

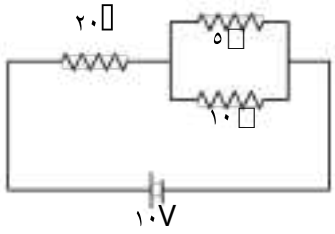
Sol. $T = \frac{2\pi r^{3/2}}{\sqrt{GM}}$

$$\frac{T_1}{T_2} = \left(\frac{r_1}{r_2}\right)^{3/2} \left(\frac{M_2}{M_T}\right)^{1/2}$$

$$\frac{6}{24} = \frac{(r_1)^{3/2}}{(4.2\times10^{4})^{3/2}} \left(\frac{M}{M/4}\right)^{1/2}$$

r١ = ١.٠٥ × ١٠⁴ km

٥٠. The ratio of heat dissipated per second through the resistance ٥ Ω and ١٠ Ω in the circuit given below is :

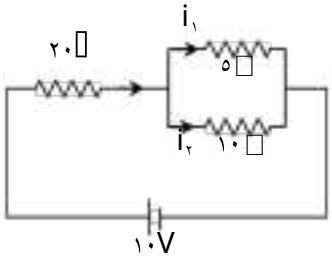


(١) ١ : ٢ (٢) ٢ : ١

(٣) ٤ : ١ (٤) ١ : ١

Ans. (٢)

Sol.



$$\frac{i_1}{i_2} = \frac{10}{5} = \frac{2}{1}$$

$$\frac{P_1}{P_2} = \frac{i_1^2 R_1}{i_2^2 R_2} = \left(\frac{2}{1}\right)^2 \left(\frac{5}{10}\right) = \frac{2}{1}$$

SECTION-B

٥١. A solenoid of length ٠.٥ m has a radius of ١ cm and is made up of 'm' number of turns. It carries a current of ٥A. If the magnitude of the magnetic field inside the solenoid is ٦.٢٨ × ١٠⁻³ T, then the value of m is :

Ans. (٥٠٠)

Sol. μ٠ni = B n = number of turns per unit length

$$\mu_0 \left(\frac{m}{\ell}\right) 1 = B$$

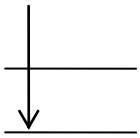
$$m = \frac{B.\ell}{\mu_0 i} = \frac{6.28\times10^{-3} \times 0.5}{12.56\times10^{-7} \times 5}$$

m = ٥٠٠

٥٢. The shortest wavelength of the spectral lines in the Lyman series of hydrogen spectrum is ٩١٥ Å. The longest wavelength of spectral lines in the Balmer series will be _____ Å.

Ans. (٦٥٨٨)

Sol. Lyman Series



n = L

Shortest: $\frac{hc}{\lambda} = 13.6\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$

$E = \frac{hc}{\lambda_0} = 13.6$ (1)

Balmer Series :

_____ n = ٣

_____ n = ٢

$\frac{hc}{\lambda} = 13.6\left(\frac{1}{2^2} - \frac{1}{3^2}\right)$

$\frac{hc}{\lambda_1} = 13.6\left(\frac{1}{4} - \frac{1}{9}\right)$

$\frac{hc}{\lambda_1} = 13.6\left(\frac{5}{36}\right)$

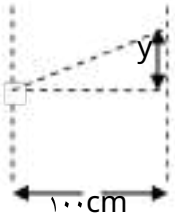
$\lambda_1 = \frac{13.6 \times 0 \times 13.6 \times \frac{5}{36}}{0}$

$\lambda_1 = \frac{0 \times 36}{5} = \frac{915 \times 36}{5} = 6588$

٥٣. In a single slit experiment, a parallel beam of green light of wavelength ٥٥٠ nm passes through a slit of width ٠.٢٠ mm. The transmitted light is collected on a screen ١٠٠ cm away. The distance of first order minima from the central maximum will be x × ١٠$^{-٣}$ m. The value of x is :

Ans. (٢٧٥)

Sol.



λ = ٥٥٠ nm

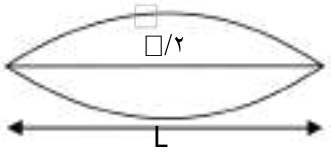
d = ٠.٢ mm

$y = \frac{\lambda D}{d} = \frac{550 \times 10 \times 9 \times 100 \times 10}{0.2 \times 10 \times 3} = ٢٧٥$

٥٤. A sonometer wire of resonating length ٩٠ cm has a fundamental frequency of ٤٠٠ Hz when kept under some tension. The resonating length of the wire with fundamental frequency of ٦٠٠ Hz under same tension

Ans. (٦٠) _____ cm.

Sol.



f_0 = ٤٠٠ Hz ; $v = \sqrt{\frac{T}{\mu}}$ = constant

$2L = \lambda$; $v = f_0\lambda$

$\frac{v}{2f_0} = L$; $v = 2Lf_0$

$L' = \frac{v}{2f'} = \frac{2Lf_0}{2f'}$

$= \frac{Lf_0}{f'} = \frac{90 \times 400}{600} = 60$

٥٥. A hollow sphere is rolling on a plane surface about its axis of symmetry. The ratio of rotational kinetic energy to its total kinetic energy is $\frac{x}{5}$. The value of x is _____.

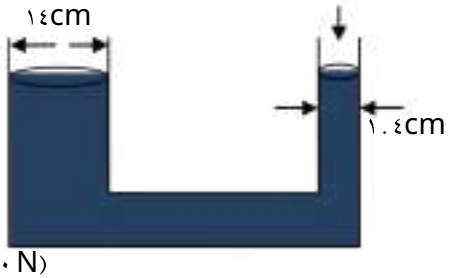
Ans. (٢)

Sol. $\frac{\frac{1}{2}I\omega^2}{\frac{1}{2}I\omega^2 + \frac{1}{2}mv^2} = \frac{\frac{1}{2}\left(\frac{2}{3}mR^2\right)\omega^2}{\frac{1}{2}\left(\frac{2}{3}mR^2\right)\omega^2 + \frac{1}{2}m(R\omega)^2}$

$= \frac{\frac{2}{3}}{\frac{2}{3}+1} = \frac{2}{5}$

x = ٢

٥٦. A hydraulic press containing water has two arms with diameters as mentioned in the figure. A force of ١٠ N is applied on the surface of water in the thinner arm. The force required to be applied on the surface of water in the thicker arm to maintain equilibrium of water is _____ N.



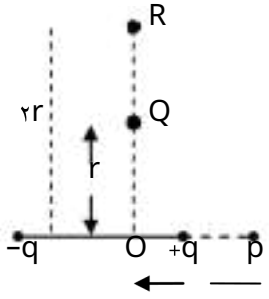
Ans. (١٠٠٠ N)

Sol. $\frac{F_1}{A_1} = \frac{F_2}{A_2}$

$\frac{F_1}{\pi(7)^2} = \frac{10}{\pi(0.7)^2}$

$F_1 = ١٠٠٠$ N

٥٧. The electric field at point p due to an electric dipole is E. The electric field at point R on equitorial line will be $\frac{E}{x}$. The value of x :



Ans. (١٦)

Sol. $E_P = \frac{2KP}{r^3} = E$

$E_R = \frac{KP}{(2r)^3} = \frac{E}{16}$

x = ١٦

٥٨. The maximum height reached by a projectile is ٦٤ m. If the initial velocity is halved, the new maximum height of the projectile is _____ m.

Ans. (١٦)

Sol. $H_{max} = \frac{u^2 \sin^2\theta}{2g}$

$\frac{H_{1max}}{H_{2max}} = \frac{u^2}{\frac{u^2}{2}}$

$\frac{64}{H_{2max}} = \frac{u^2}{(u/2)^2}$

$H_{2max} = ١٦$ m

٥٩. A wire of resistance ٢٠ Ω is divided into ١٠ equal parts. A combination of two parts are connected in parallel and so on. Now resulting pairs of parallel combination are connected in series. The equivalent resistance of final combination is _____ Ω.

Ans. (٥)

Sol.

٢٠ Ω
[ ] → ١٠ equal part

Each part has resistance = ٢ Ω

٢ parts are connected in parallel so, R = ١ Ω

Now, there will be ٥ parts each of resistance ١ Ω, they are connected in series.

Req = ٥R, Req = ٥ Ω

٦٠. The current in an inductor is given by $I = (٣t + ٨)$ where t is in second. The magnitude of induced emf produced in the inductor is ١٢ mV. The self-inductance of the inductor _____ mH.

Ans. (٤)

Sol. $I = ٣t + ٨$

$\varepsilon = ١٢$ mV

$|\varepsilon| = L\left|\frac{dI}{dt}\right|$

$١٢ = L \times ٣$

L = ٤ mH

SECTION-A

61. MatchList - I with List - II.

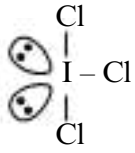
List - I	List - II
(A) ICI	(I) T -Shape
(B) ICI3	(II) Square pyramidal
(C) CIF5	(III) Pentagonal bipyramidal
(D) IF7	(IV) Linear

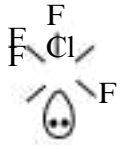
Choose the correct answer from the options given below:

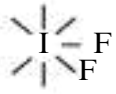
(1) (A)–(I), (B)–(IV), C–(III), D–(II)
(2) (A)–(I), (B)–(III), C–(II), D–(IV)
(3) (A)–(IV), (B)–(I), C–(II), D–(III)
(4) (A)–(IV), (B)–(III), C–(II), D–(I)

Ans. (3)

Sol. A. I – Cl (iv) linear

B.  (I) T-shape

C.  (II) Square pyramidal

D.  (III) Pentagonal bipyramidal

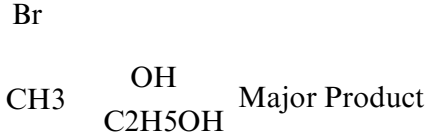
62. While preparing crystals of Mohr's salt, dil. H_2SO_4 is added to a mixture of ferrous sulphate and ammonium sulphate, before dissolving this mixture in water, dil. H_2SO_4 is added here to:

(1) prevent the hydrolysis of ferrous sulphate
(2) prevent the hydrolysis of ammonium sulphate
(3) make the medium strongly acidic
(4) increase the rate of formation of crystals

Ans. (1)

Sol. Fe^{+2} ions undergoes hydrolysis, therefore while preparing aqueous solution of ferrous sulphate and ammonium sulphate in water dilute sulphuric acid is added to prevent hydrolysis of ferrous sulphate.

63. Identify the major product in the following reaction.



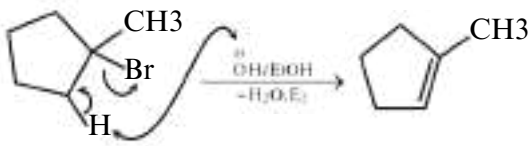
(1) 

(2) 

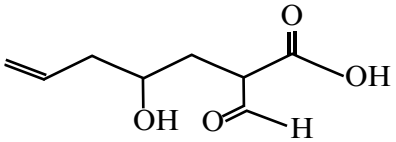
(3)

(4) 

Ans. (3)

Sol. 

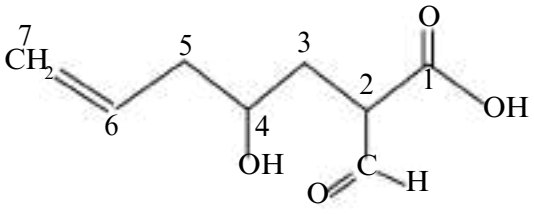
64. The correct nomenclature for the following compound is:



(1) 2–carboxy–4–hydroxyhept–6–enal (2) 2–carboxy–4–hydroxyhept–7–enal (3) 2–formyl–4–hydroxyhept–6–enoic acid (4) 2–formyl–4–hydroxyhept–7–enoic acid

Ans. (3)

Sol.



2-formly-4-hydroxyhept-6-enoic acid

65. Given below are two statements : one is labelled as Assertion (A) and the other is labelled as Reason (R).

Assertion (A) : NH3 and NF3 molecule have pyramidal shape with a lone pair of electrons on nitrogen atom. The resultant dipole moment of NH3 is greater than that of NF3.

Reason (R) : In NH3, the orbital dipole due to lone pair is in the same direction as the resultant dipole moment of the N–H bonds. F is the most electronegative element.

In the light of the above statements, choose the correct answer from the options given below:

(1) Both (A) and (R) are true and (R) is the correct explanation of (A)

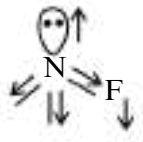
(2) (A) is false but (R) is true

(3) (A) is true but (R) is false

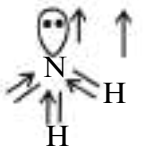
(4) Both (A) and (R) are true but (R) is NOT the correct explanation of (A)

Ans. (1)

Sol.



Resultant dipole moment = 0.80×10^{-30} Cm



Resultant dipole moment = 4.90×10^{-30} cm

66. Given below are two statements: Statement I : On passing HCl(g) through a saturated solution of BaCl2, at room temperature white turbidity appears.

Statement II : When HCl gas is passed through a saturated solution of NaCl, sodium chloride is precipitated due to common ion effect.

In the light of the above statements, choose the most appropriate answer from the options given below:

(1) Statement I is correct but Statement II is incorrect

(2) Both Statement I and Statement II are incorrect

(3) Statement I is incorrect but Statement II is correct

(4) Both Statement I and Statement II are correct

Ans. (1)

Sol. BaCl2, NaCl are soluble but on adding HCl(g) to BaCl2, NaCl solutions, Sodium or Barium chlorides may precipitate out, as a consequence of the law of mass action.

67. The metal atom present in the complex MABXL (where A, B, X and L are unidentate ligands and M is metal) involves sp3 hybridization. The number of geometrical isomers exhibited by the complex is:

(1) 4 (2) 0

(3) 2 (4) 3

Ans. (2)

Sol. Tetrahedral complex does not show geometrical isomerism.

68. MatchList - I with List - II.

List -I (Pair of Compounds)	List - II (Isomerism)
(A) n-propanol and Isopropanol	(I) Metamerism
(B) Methoxypropane and ethoxyethane	(II) Chain Isomerism
(C) Propanone and propanal	(III) Position Isomerism
(D) Neopentane and Isopentane	(IV) Functional Isomerism

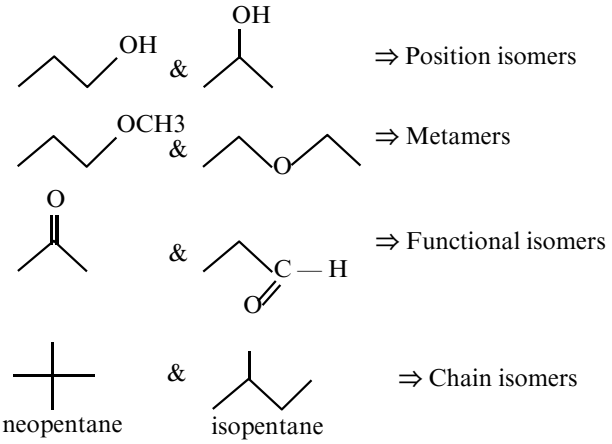
(1) (A)–(II), (B)–(I), (C)–(IV), (D)–(III)

(2) (A)–(III), (B)–(I), (C)–(II), (D)–(IV)

(3) (A)–(I), (B)–(III), (C)–(IV), (D)–(II)

(4) (A)–(III), (B)–(I), (C)–(IV), (D)–(II)

Ans. (4)

Sol. 

69. The quantity of silver deposited when one coulomb charge is passed through AgNO3 solution:
(1) 0.1 g atom of silver
(2) 1 chemical equivalent of silver
(3) 1 g of silver
(4) 1 electrochemical equivalent of silver

Ans. (4)

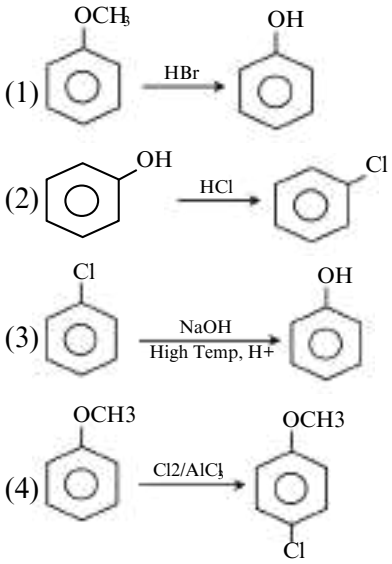
Sol. W = ZIt

W = ZQ

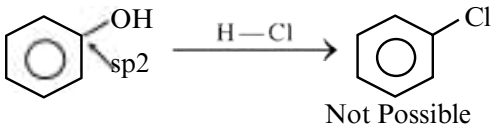
$Q = \frac{W}{Z}$

W = ZQ = (electrochemical equivalent)

70. Which one of the following reactions is NOT possible?



Ans. (2)

Sol. 

71. Given below are two statements :
Statement I : The metallic radius of Na is 1.86 A° and the ionic radius of Na+ is lesser than 1.86 A°.
Statement II : Ions are always smaller in size than the corresponding elements.
In the light of the above statements, choose the correct answer from the options given below :
(1) Statement I is correct but Statement II is false
(2) Both Statement I and Statement II are true (3) Both Statement I and Statement II are false (4) Statement I is incorrect but Statement II is true

Ans. (1)

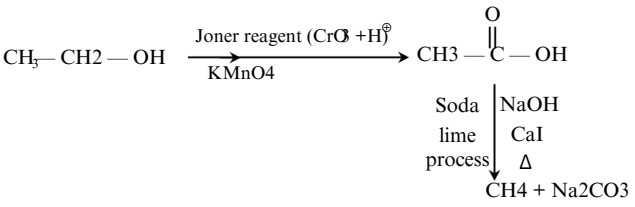
Sol. rNa > r_{Na^+}

So, Statement (I) is correct but size of anions are greater than size of neutral atoms.
So statement (II) is incorrect.

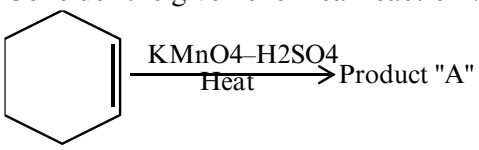
72. CH3CH2–OH $\xrightarrow[\text{(ii) KMnOP4}]{\text{(i) Jone's Reagent}}$ (iii)NaOH, CaO,Δ

Consider the above reaction sequence and identify the major product P.
(1) Methane (2) Methanal
(3) Methoxymethane (4) Methanoic acid

Ans. (1)

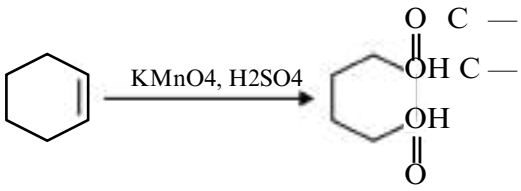
Sol. 

73. Consider the given chemical reaction :



Product "A" is :
(1) picric acid (2) oxalic acid
(3) acetic acid (4) adipic acid

Ans. (4)

Sol. 

74. For the electro chemical cell
M|M2+||X|X2–
If $E^{0}_{(M^{2+}/M)} = 0.46\,V$ and $E^{0}_{(X/X^{2-})} = 0.34\,V$.
Which of the following is correct ?
(1) Ecell = –0.80 V
(2) M + X → M2 + X2– is a spontaneous reaction
(3) M2+ + X2– → M + X is a spontaneous reaction
(4) Ecell = 0.80 V

Ans. (3)

Sol. M | M+2 || X / X2–

$E^{o}_{cell} = E^{o}_{M/M^{+2}} + E^{o}_{X/X^{-2}}$

= –0.46 + 0.34 = – 0.12V

As Eocell is negative so anode becomes cathode and cathode become anode. Spontaneous reaction will be
M+2 + X2– → M + X

75. The number of moles of methane required to produce 11g CO2(g) after complete combustion is:
(Given molar mass of methane in g mol–1 : 16)
(1) 0.75 (2) 0.25
(3) 0.35 (4) 0.5

Ans. (2)

Sol. $C_nH_{2n+2} + \frac{3n+1}{2}O_2 \rightarrow nCO_2 + (n+1)H_2O$

$CH_4 + 2O_2 \rightarrow CO_2 + 2H_2O$

4gm 11gm
0.25 mole 0.25 mole
0.25 mole CH4 gives 0.25 mole (or 11gm) CO2

76. The number of complexes from the following with no electrons in the t2 orbital is _____.
$TiCl_4$, $[MnO_4]^-$, $[FeO_4]^{2-}$, $[FeCl_4]^-$, $[CoCl_4]^{2-}$
(1) 3 (2) 1
(3) 4 (4) 4

Ans. (1)

Sol. $TiCl_4$ Ti+4 $e^0t_2^0$

MnO_4^- Mn+7 $e^0t_2^0$

FeO_4^{2-} Fe+6 $e^2t_2^0$

$FeCl_4^-$ Fe+2 $e^3t_2^3$

$CoCl_4^{2-}$ Co+2 $e^4t_2^3$

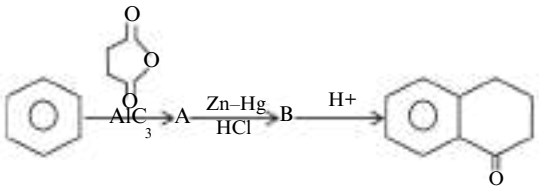
77. The number of ions from the following that have the ability to liberate hydrogen from a dilute acid is _____. Ti2+, Cr2+ and V2+
(1) 0 (2) 2
(3) 3 (4) 1

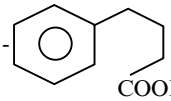
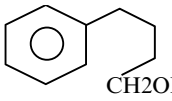
Ans. (3)

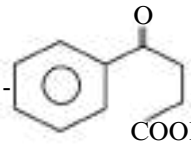
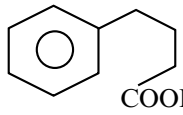
Sol. The ions Ti+2, V+2 Cr+2 are strong reducing agents and will liberate hydrogen from a dilute acid, eg.

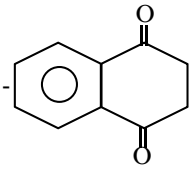
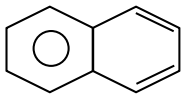
$2Cr^{+2}_{(aq.)} + 2H^+_{(aq.)} \rightarrow 2Cr^{+3}_{(aq.)} + H_2(g)$

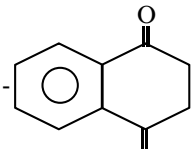
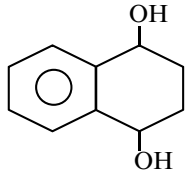
78. Identify A and B in the given chemical reaction sequence : -



(1) A -  , B - 

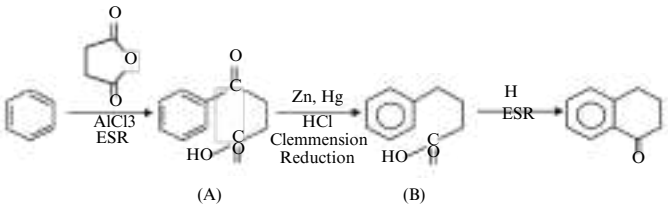
(2) A -  , B - 

(3) A -  , B - 

(4) A -  , B - 

Ans. (2)

Sol.



79. The correct statements from the following are :

(A) The decreasing order of atomic radii of group 13 elements is Tl > In > Ga > Al > B.

(B) Down the group 13 electronegativity decreases from top to bottom.

(C) Al dissolves in dil. HCl and liberate H2 but conc. HNO3 renders Al passive by forming a protective oxide layer on the surface.

(D) All elements of group 13 exhibits highly stable +1 oxidation state.

(E) Hybridisation of Al in $[Al(HO)_6]^{3+}$ ion is sp3d2.

Choose the correct answer from the options given below :

(1) (C) and (E) only

(2) (A), (C) and (E) only

(3) (A), (B), (C) and (E) only

(4) (A) and (C) only

Ans. (1)

Sol. A. size order T□ > In > Al > Ga > B

B. Electronegativity order B > Al < Ga < In < T□

D. B, Al are more stable in +3 oxidation state

So, only C, E statements are correct.

80. Coagulation of egg, on heating is because of :

(1) Denaturation of protein occurs

(2) The secondary structure of protein remains unchanged

(3) Breaking of the peptide linkage in the primary structure of protein occurs

(4) Biological property of protein remains unchanged

Ans. (1)

Sol. Coagulation of egg give primary structure of protein, which is known as denaturation of protein

SECTION-B

81. Combustion of 1 mole of benzene is expressed at

$$C_6H_6(l) + \frac{15}{2}O_2(g) \rightarrow 6CO_2(g) + 3H_2O(l)$$

The standard enthalpy of combustion of 2 mol of benzene is – 'x' kJ.

x = _____.

(1) standard Enthalpy of formation of 1 mol of C6H6(1), for the reaction

6C(graphite) + 3H2(g) → C6H6(1) is 48.5 kJ mol^{-1}.

(2) Standard Enthalpy of formation of 1 mol of CO2(g), for the reaction

C(graphite) + O2(g) → CO2(g) is –393.5 kJ mol^{-1}.

(3) Standard and Enthalpy of formation of 1 mol of H2O(1), for the reaction

$H_2(g) + \frac{1}{2}O_2(g) \rightarrow H_2O(l)$ is –286 kJ mol^{-1}.

Ans. (6535)

Sol. 6C(graphite)+3H2(g) → C6H6(□); ΔH = 48.5 kJ/mol

C(graphite)+O2(g) → CO2(g); ΔH = –393.5 kJ/mol

$H_2(g) + \frac{1}{2}O_2(g) \rightarrow H_2O(l)$; ΔH = – 286 kJ/mol

equation –(1) × 1 + (2) × 6 + (3) × 3

– 48.5 –6 × 393.5 – 3 × 286

= – 3267.5 kJ for 1 mol

= – 6535 kJ for 2 mol

Ans. 6535 kJ

82. The fusion of chromite ore with sodium carbonate in the presence of air leads to the formation of products A and B along with the evolution of CO2. The sum of spin-only magnetic moment values of A and B is ___ B.M. (Nearest integer)

(Given atomic number : C : 6, Na : 11, O : 8, Fe : 26, Cr : 24]

Ans. (6)

Sol. $4FeCr_2O_4 + 8Na_2CO_3 + 7O_2 \rightarrow 8Na_2CrO_4 + 2Fe_2O_3 + 8CO_2$

A B

Spin only magnetic moment

For Na2CrO4 μB = 0

For Fe2O3 μB = 5.9

sum = 5.9

83. X of enthanamine was subjected to reaction with NaNO2/HCl followed by hydrolysis to liberate N2
and HCl. The HCl generated was completely neutralised by 0.2 moles of NaOH. X is ____ g.

Ans. (9)

Sol. $CH_3—CH_2—NH_2 \xrightarrow{NaNO_2 + HCl} CH_3—CH_2—\overset{\oplus}{N_2}Cl \xrightarrow{HOH} CH_3—CH_2—OH + N_2 + HCl$

0.2 mole
MW of ethanamine = 45
45 × 0.2 = 9 gm

(g)
0.2 mole

84. In an atom, total number of electrons having quantum numbers n = 4, |ml| = 1 and $m_s = -\frac{1}{2}$ is

Ans. (6)

Sol. n = 4

	m
0	0
1	–1, 0, +1
2	–2, –1, 0, +1, +2, +3

So number of orbital associated with
n = 4, |ml| = 1 are 6

Now each orbital contain one e– with $m_s = -\frac{1}{2}$

85. Using the given figure, the ratio of Rf values of sample A and sample C is x × 10–2. Value of x is _____.

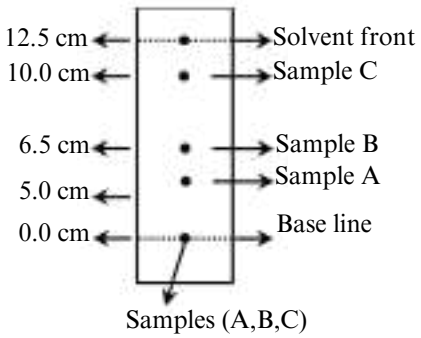


Fig : Paper chromatography of Samples

Ans. (50)

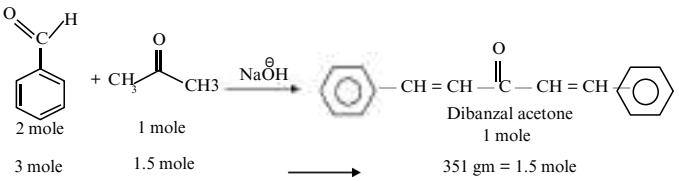
Sol. R_f of A = $\frac{5}{12.5}$ Rf of C = $\frac{10}{12.5}$

Ratio = $\frac{R_{f(A)}}{R_{f(C)}} = \frac{1}{2}$ = 0.5 or 50 × 10–2

86. In the Claisen-Schmidt reaction to prepare 351 g of dibenzalacetone using 87 g of acetone, the amount of benzaldehyde required is _____g. (Nearest integer)

Ans. (318)

Sol. Claisen Schmidt reaction



mw of benzaldehyde = 106

106 × 3 = 318 gm. Benzaldehyde is required to give 1.5 mole (or 351 gm) product

87. Consider the following single step reaction in gas

phase at constant temperature.

2A(g) + B(g) C(g) The initial rate of the reaction is recorded as r1
when the reaction starts with 1.5 atm pressure of A

and 0.7 atm pressure of B. After some time, the 1 2 .
(Nearest integer)
rate r2 is recorded when the pressure of C becomes 0.5 atm. The ratio r : r is _____ ×

Ans. (315)

Sol. 2A(g) + B(g) C(g)
10–1

r1 1.5 atm 0.7 atm

r2 0.5 atm 0.2 atm 0.5 atm

r = K 2

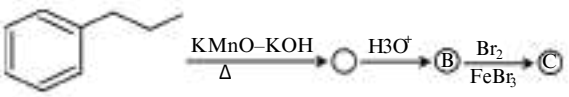
$r_1 = K[1.5]^2[0.7]$

$r_2 = K[0.5]^2[0.2]$

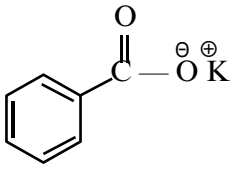
$\frac{r_1}{r_2} = 9 \times \frac{7}{2} = 31.5 = 315 \times 10^{-1}$

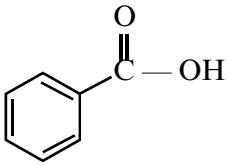
Ans. 315

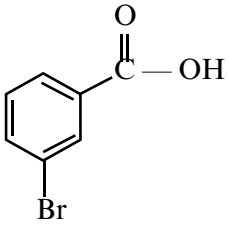
88. The product © in the following sequence of reactions has _____ π bonds.



Ans. (4)

Sol. A = 

B = 

C = 

π bonds = 4

89. Considering acetic acid dissociates in water, its dissociation constant is 6.25 × 10–5. If 5 mL of acetic acid is dissolved in 1 litre water, the solution will freeze at –x × 10–2 °C, provided pure water freezes at 0 °C.

x = _____. (Nearest integer)

Given : $(K)_{fwater}$ = 1.86 K kg mol^{-1}.

density of acetic acid is 1.2 g mol–1

molar mass of water = 18 g mol–1.

molar mass of acetic acid = 60 g mol–1.

density of water = 1 g cm–3

Acetic acid dissociates as

$$CH_3COOH \rightleftharpoons CH_3COO^{\ominus} + H^{\oplus}$$

Ans. (19)

Sol. Mass of CH3COOH = V × d

= 5 ml × 1.2 g/ml

= 6 gm

$$n_{CH_3COOH} = \frac{6}{60} = 0.1 \text{ mol}$$

$$m_{CH_3COOH} \approx M$$

$$CH_3COOH = \frac{0.1}{1} = 0.1\,M$$

$$CH_3COOH \rightleftharpoons CH_3COO^{-} + H^{+}$$

C

C – Cα Cα Cα

$$K_a = \frac{C\alpha^2}{1-\alpha}$$

$1 - \alpha \approx 1 \Rightarrow K_a = C\alpha^2$

$$\alpha = \sqrt{\frac{K_a}{C}} = \sqrt{\frac{6.25\times10^{-5}}{0.1}} = 25 \times 10^{-3}$$

V.f. (i) = 1 + α(n – 1) = 1 + α(2 – 1) = 1 + α

= 1 + 25 × 10–3 = 1.025

$\Delta T_f = iK_f m$

= (1.025)(1.86)(0.1)

= 0.19

= 19 × 10–2

90. Number of compounds from the following with zero dipole moment is _____.

HF, H2, H2S, CO2, NH3, BF3, CH4, CHCl3, SiF4, H_2O, BeF2

Ans. (6)

Sol. H_2, CO2, BF3, CH4, SiF4 , BeF2

are symm. molecule so dipole moment is zero