FINAL JEE–MAIN EXAMINATION – APRIL, 2024

(Held On Saturday 06 April, 2024) TIME : 9 : 00 AM to 12 : 00 NOON

MATHEMATICS

TEST PAPER WITH SOLUTION

SECTION-A

١. If $f(x)=\begin{cases} x^{٣}\sin\left(\frac{1}{x}\right), & x\neq ٠\\ ٠, & x=٠\end{cases}$, then

(١) $f''(٠)=١$ (٢) $f''\left(\frac{2}{\pi}\right)=\frac{٢٤-\pi^2}{٢\pi}$

(٣) $f''\left(\frac{٢}{\pi}\right)=\frac{١٢-\pi^2}{٢\pi}$ (٤) $f''(٠)=٠$

Ans. (٢)

Sol. $f'(x)=٣x^{٢}\sin\left(\frac{1}{x}\right)-x\cos\left(\frac{1}{x}\right)$

$f''(x)=٦x\sin\left(\frac{1}{x}\right)-٣\cos\left(\frac{1}{x}\right)-\cos\left(\frac{1}{x}\right)-\frac{\sin\left(\frac{1}{x}\right)}{x}$

$f''\left(\frac{٢}{\pi}\right)=\frac{١٢}{\pi}-\frac{\pi}{٢}=\frac{٢٤-\pi^2}{٢\pi}$

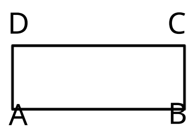
٢. If A(٣, ١, −١), $B\left(\frac{5}{3},\frac{٧}{٣},\frac{١}{٣}\right)$, C(٢, ٢, ١) and $D\left(\frac{١٠}{3},\frac{-٢}{٣},\frac{-١}{٣}\right)$ are the vertices of a quadrilateral ABCD, then its area is

(١) $\frac{٤\sqrt{٢}}{٣}$ (٢) $\frac{٥\sqrt{٢}}{٣}$

(٣) $٢\sqrt{٢}$ (٤) $\frac{٢\sqrt{٢}}{٣}$

Ans. (١)

Sol.



Area $=\frac{١}{٢}\left|\overrightarrow{BD}\times\overrightarrow{AC}\right|$

$\overrightarrow{BD}=\frac{٥}{٣}\hat{i}-\frac{٥}{٣}\hat{j}-\frac{٢}{٣}\hat{k}$

$\overrightarrow{AC}=-\hat{i}+\hat{j}+٢\hat{k}$

٣. $\int_0^{\pi/٤}\frac{\cos^{٢}x\sin^{٢}x}{\left(\cos^{٣}x+\sin^{٣}x\right)^2}dx$ is equal to

(١) ١/١٢ (٢) ١/٩

(٣) ١/٦ (٤) ١/٣

Ans. (٣)

Sol. Divide Nr & Dr by cosx

$\int_0^{\pi/٤}\frac{\tan^{٢}x\sec^{٢}x\,dx}{\left(١+\tan^{٣}x\right)^2}$

Let $١+\tan^{٣}x=t$

$\tan^{٢}x\sec^{٢}x\,dx=\frac{dt}{٣}$

$\frac{١}{3}\int_{١}^{٢}\frac{dt}{t^2}=\frac{١}{٦}$

٤. The mean and standard deviation of ٢٠ observations are found to be ١٠ and ٢, respectively. On respectively, it was found that an observation by mistake was taken ٨ instead of ١٢. The correct standard deviation is

(١) ٣.٨٦ (٢) ١.٨

(٣) ٣.٩٦ (٤) ١.٩٤

Ans. (٣)

Sol. Mean $(\bar{x})=١٠$

$\Rightarrow\frac{\sum x_i}{٢٠}=١٠$

$\sum x_i=١٠\times٢٠=٢٠٠$

If ٨ is replaced by ١٢, then $\sum x_i=٢٠٠-٨+١٢=٢٠٤$

∴ Correct mean $(\bar{x}) = \frac{\sum x_i}{٢٠}$

$= \frac{٢٠٤}{٢٠} = ١٠.٢$

∵ Standard deviation = ٢

∴ Variance = (S.D.)² = ٢² = ٤

$\therefore \frac{\sum x_i^2}{٢٠} - \left(\frac{\sum x_i}{٢٠}\right)^٢ = ٤$

$\therefore \frac{\sum x_i^٢}{٢٠} - (10)^٢ = ٤$

$\therefore \frac{\sum x_i^٢}{٢٠} = ١٠٤$

$\Rightarrow \sum x_i^٢ = ٢٠٨٠$

Now, replaced '٨' observations by '١٢'

Then, $\sum x_i^٢ = ٢٠٨٠ - ٨^٢ + ١٢^٢ = ٢١٦٠$

∴ Variance of removing observations

$= \frac{\sum x_i^2}{٢٠} - \left(\frac{\sum x_i}{20}\right)^٢$

$= \frac{٢١٦٠}{٢٠} - (١٠.٢)^٢$

$= ١٠٨ - ١٠٤.٠٤$

$= ٣.٩٦$

Correct standard deviation

$= \sqrt{٣.٩٦}$

٥. The function $f(x) = \frac{x^٢ + ٢x - ١٥}{x^٢ - ٤x + ٩}$, $x \in R$ is

(١) both one-one and onto.

(٢) onto but not one-one.

(٣) neither one-one nor onto.

(٤) one-one but not onto.

NTA Ans. (٣)

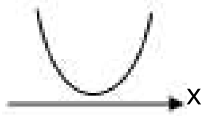
Ans. Bonus

Sol. $f(x) = \frac{(x+٥)(x-٣)}{x^٢ - ٤x + ٩}$

Let $g(x) = x^٢ - ٤x + ٩$

$D < ٠$

$g(x) > ٠$ for $x \in R$



$\because f(-٥) = ٠,\ f(٣) = ٠$

So, f(x) is many-one.

again,

$yx^٢ - ٤xy + ٩y = x^٢ + ٢x - ١٥$

$x^٢(y - ١) - ٢x(٢y + ١) + (٩y + ١٥) = ٠$

for $\forall x \in R \Rightarrow D \geq ٠$

$D = ٤(٢y + ١)^٢ - ٤(y - ١)(٩y + ١٥) \geq ٠$

$٥y^٢ + ٢y + ١٦ \leq ٠$

$(٥y - ٨)(y + ٢) \leq ٠$

-٢ $\quad$ $٨/٥$

$\boxed{y \in \left[-٢, \frac{8}{5}\right]}$ range

Note: If function is defined from $f: R \to R$ then only correct answer is option (٣)

∴ Bonus

٦. Let $A = \{n \in [١٠٠, ٧٠٠] \cap N : n$ is neither a multiple of ٣ nor a multiple of ٤$\}$. Then the number of elements in A is

(١) ٣٠٠ (٢) ٢٨٠

(٣) ٣١٠ (٤) ٢٩٠

Ans. (١)

Sol. n(٣) → multiple of ٣

١٠٢, ١٠٥, ١٠٨, ..., ٦٩٩

$T_n = ٦٩٩ = ١٠٢ + (n - ١)(٣)$

$n = ٢٠٠$

$n(٣) = ٢٠٠$

∴ n(٤) → multiple of ٤

١٠٠، ١٠٤، ١٠٨، . . . . ، ٧٠٠

$T_n = ٧٠٠ = ١٠٠ + (n - ١)(٤)$

$n = ١٥١$

$n(٤) = ١٥١$

$n(٣\square٤)$ □ multiple of ٣ & ٤ both

١٠٨، ١٢٠، ١٣٢، . . . . . . ، ٦٩٦

$T_n = ٦٩٦ = ١٠٨ + (n - ١)(١٢)$

$n = ٥٠$

$n(٣ \square\square ٤) = ٥٠$

$n(٣ \square\square ٤) = n(٣) + n(٤) - n(٣ \square\square ٤)$

$= ٢٠٠ + ١٥١ - ٥٠$

$= ٣٠١$

$n\square\cancel{٣\square٤}\square = \text{Total} - n(٣ \square\square ٤) =$ neither a multiple of ٣ nor a multiple of ٤

$= ٦٠١ - ٣٠١ = ٣٠٠$

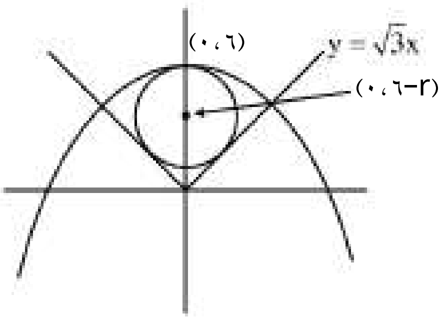
٧. Let C be the circle of minimum area touching the parabola $y = ٦ - x^{٢}$ and the lines $y = \sqrt{٣}|x|$. Then، which one of the following points lies on the circle C؟

(١) (٢، ٤) (٢) (١، ٢)

(٣) (٢، ٢) (٤) (١، ١)

Ans. (١)

Sol.



Equation of circle

$x^{٢} + (y - (٦ - r)) = r^{٢}$ ٢

touches $\sqrt{٣}x\square y\square ٠$

$p = r$

$$\frac{|٠\square(٦\square r)|}{٢} \square r$$

$|r - ٦| = ٢r$

$r = ٢$

□ Circle $x^{٢} + (y - ٤) = ٤^{٢}$

(٢، ٤) Satisfies this equation

٨. For □, □ □ R and a natural number n، let

$$A_r \square \begin{vmatrix} r & ١ & \frac{n^{٢}}{٢} \square\square \\ ٢r & ٢ & n^{٢} - \square \\ ٣r-٢ & ٣ & \frac{n(٣n-١)}{٢} \end{vmatrix}$$

. Then $٢A_{١٠} - A_{٨}$ is

(١) $٤\square + ٢\square$ (٢) $٢\square + ٤\square$

(٣) $٢n$ (٤) ٠

Ans. (١)

Sol. $$A_r \square \begin{vmatrix} r & ١ & \frac{n^{٢}}{٢} \square\square \\ ٢r & ٢ & n^{٢} - \square \\ ٣r-٢ & ٣ & \frac{n(٣n-١)}{٢} \end{vmatrix}$$

$$٢A_{١٠} - A_{٨} \square \begin{vmatrix} ٢٠ & ١ & \frac{n^{٢}}{٢} \square\square \\ ٤٠ & ٢ & n^{٢} - \square \\ ٥٦ & ٣ & \frac{n(٣n-١)}{٢} \end{vmatrix} - \begin{vmatrix} ٨ & ١ & \frac{n^{٢}}{٢} \square\square \\ ١٦ & ٢ & n^{٢} - \square \\ ٢٢ & ٣ & \frac{n\square٣n-١\square}{٢} \end{vmatrix}$$

$$\square \begin{vmatrix} ١٢ & ١ & \frac{n^{٢}}{٢} \square\square \\ ٢٤ & ٢ & n^{٢} - \square \\ ٣٤ & ٣ & \frac{n\square٣n-١\square}{٢} \end{vmatrix}$$

$$\square \begin{vmatrix} ٠ & ١ & \frac{n^{٢}}{٢} \square\square \\ ٠ & ٢ & n^{٢} - \square \\ -٢ & ٣ & \frac{n(٣n-١)}{٢} \end{vmatrix}$$

$\square\ -٢\square(n^{٢}) - \square n^{٢} \square 2\square\square\square$

$\square\ -٢(-\square-٢\ \square) \square 4\square\square 2\square$

٩. The shortest distance between the lines $\frac{x-٣}{٢}=\frac{y+١٥}{-٧}=\frac{z-٩}{٥}$ and $\frac{x+١}{٢}=\frac{y-١}{١}=\frac{z-٩}{-٣}$ is

(١) $٦\sqrt{٣}$ (٢) $٤\sqrt{٣}$

(٣) $٥\sqrt{٣}$ (٤) $٨\sqrt{٣}$

Ans. (٢)

Sol. $\frac{x-٣}{٢}=\frac{y+١٥}{-٧}=\frac{z-٩}{٥}$ & $\frac{x+١}{٢}=\frac{y-١}{١}=\frac{z-٩}{-٣}$

$$S.D=\frac{\left|(\vec{a}_٢-\vec{a}_١)\cdot(\vec{b}_١\times\vec{b}_٢)\right|}{\left|\vec{b}_١\times\vec{b}_٢\right|}$$

$a_١=٣,-١٥,٩$ $b_١=٢,-٧,٥$

$a_٢=-١,١,٩$ $b_٢=٢,١,-٣$

$a_٢-a_١=-٤,١٦,٠$

$$\vec{b}_١\times\vec{b}_٢=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ ٢ & -7 & ٥\\ ٢ & ١ & -٣\end{vmatrix}=\hat{i}(١٦)+\hat{j}(١٦)+\hat{k}(١٦)$$

$١٦(\hat{i}+\hat{j}+\hat{k})$

$\left|\vec{b}_١\times\vec{b}_٢\right|=١٦\sqrt{٣}$

$(\vec{a}_٢-\vec{a}_١)\cdot(\vec{b}_١\times\vec{b}_٢)=١٦(-4+١٦)=(١٦)(١٢)$

$$S.D.=\frac{(١٦)(١٢)}{١٦\sqrt{٣}}=٤\sqrt{٣}$$

١٠. A company has two plants A and B to manufacture motorcycles. ٦٠% motorcycles are manufactured at plant A and the remaining are manufactured at plant B. ٨٠% of the motorcycles manufactured at plant A are rated of the standard quality, while ٩٠% of the motorcycles manufactured at plant B are rated of the standard quality. A motorcycle picked up randomly from the total production is found to be of the standard quality. If p is the probability that it was manufactured at plant B, then ١٢٦p is

(١) ٥٤ (٢) ٦٤

(٣) ٦٦ (٤) ٥٦

Ans. (١)

Sol.

	A	B
Manufactured	٦٠%	٤٠%
Standard quality	٨٠%	٩٠%

P(Manufactured at B / found standard quality) = ?

A : Found S.Q

B : Manufacture B

C : Manufacture A

$P(E_١)=\frac{٤٠}{١٠٠}$

$P(E_٢)=\frac{٦٠}{١٠٠}$

$P(A/E_١)=\frac{٩٠}{١٠٠}$

$P(A/E_٢)=\frac{٨٠}{١٠٠}$

$\therefore P(E_١/A)=\frac{P(A/E_١)P(E_١)}{P(A/E_١)P(E_١)+P(A/E_٢)P(E_٢)}=\frac{٣}{٧}$

$\Rightarrow ١٢٦P=٥٤$

١١. Let α, β be the distinct roots of the equation $x^٢-(t^٢-٥t+٦)x+١=٠$, $t\in R$ and $a_n=\alpha^n+\beta^n$. Then the minimum value of $\frac{a_{٢٠٢٣}+a_{٢٠٢٥}}{a_{٢٠٢٤}}$ is

(١) ١/٤ (٢) −١/٢

(٣) −١/٤ (٤) ١/٢

Ans. (٣)

Sol. by newton's theorem

$a_{n+٢}-(t^٢-٥t+٦)a_{n+١}+a_n=٠$

$\Rightarrow a_{٢٠٢٥}+a_{٢٠٢٣}=(t^٢-٥t+٦)a_{٢٠٢٤}$

$\Rightarrow \frac{a_{٢٠٢٥}+a_{٢٠٢٣}}{a_{٢٠٢٤}}=t^٢-٥t+٦$

$\because t^٢-٥t+٦=\left(t-\frac{٥}{٢}\right)^٢-\frac{١}{٤}$

$\therefore$ minimum value $=-\frac{١}{٤}$

١٢. Let the relations R١ and R٢ on the set X = {١، ٢، ٣، . . . . ، ٢٠} be given by

R١ = {(x، y) : ٢x − ٣y = ٢} and

R٢ = {(x، y) : −٥x + ٤y = ٠}. If M and N be the minimum number of elements required to be added

in R١ and R٢، respectively، in order to make the relations symmetric، then M + N equals

(١) ٨ (٢) ١٦

(٣) ١٢ (٤) ١٠

Ans. (٤)

Sol. X = {١، ٢، ٣، . . . . ٢٠}

R١ = {(x، y) : ٢x − ٣y = ٢}

R٢ = {(x، y) : −٥x + ٤y = ٠}

R١ = {(٤، ٢)، (٧، ٤)، (١٠، ٦)، (١٣، ٨)، (١٦، ١٠)، (١٩، ١٢)}

R٢ = {(٤، ٥)، (٨، ١٠)، (١٢، ١٥)، (١٦، ٢٠)}

in R١ ٦ element needed

in R٢ ٤ element needed

So، total ٦+٤ = ١٠ element

١٣. Let a variable line of slope m > ٠ passing through the point (٤، −٩) intersect the coordinate axes at the points A and B. the minimum value of the sum of the distances of A and B from the origin is

(١) ٢٥ (٢) ٣٠

(٣) ١٥ (٤) ١٠

Ans. (١)

Sol. equation of line is

y + ٩ = m (x − ٤)

$\Rightarrow A\left(\frac{9+4m}{m}, ٠\right)$

B = (٠، −٩ − ٤m)

$\Rightarrow OA + OB = \frac{٩+٤m}{m} + ٩ + ٤m$

∵ m > ٠

$= 13 + \frac{٩}{m} + ٤m$

∵ $\frac{٤m + \frac{٩}{m}}{٢} \geq \sqrt{٣٦} \Rightarrow ٤m + \frac{٩}{m} \geq ١٢$

$\Rightarrow OA + OB \geq ٢٥$

١٤. The interval in which the function $f(x) = x^x$، x > ٠، is strictly increasing is

(١) $\left(٠, \frac{١}{e}\right]$ (٢) $\left[\frac{١}{e^٢}, ١\right)$

(٣) (٠، ∞) (٤) $\left[\frac{١}{e}, \infty\right)$

Ans. (٤)

Sol. $f(x) = x^x$ x > ٠

$\ell ny = x\ell nx$

$\frac{١}{y}\frac{dy}{dx} = \frac{x}{x} + \ell nx$

$\frac{dy}{dx} = x^x(١ + \ell nx)$

for strictly increasing

$\frac{dy}{dx} \geq ٠ \Rightarrow x^x(١ + \ell nx) \geq ٠$

$\Rightarrow \ell nx \geq -١$

$x \geq e^{-١}$

$x \geq \frac{١}{e}$

$x \in \left[\frac{١}{e}, \infty\right)$

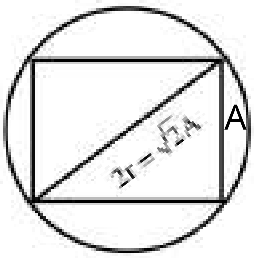
١٥. A circle in inscribed in an equilateral triangle of side of length ١٢. If the area and perimeter of any square inscribed in this circle are m and n، respectively، then $m + n$ is equal to

(١) ٣٩٦ (٢) ٤٠٨

(٣) ٣١٢ (٤) ٤١٤

Ans. (٢)

Sol. $\because r = \frac{\Delta}{s} = \frac{\frac{\sqrt{٣}a^{٢}}{٤}}{\frac{٣a}{٢}} = \frac{a}{٢\sqrt{٣}} = \frac{١٢}{٢\sqrt{٣}} = ٢\sqrt{٣}$



$\therefore A = r\sqrt{٢} = \sqrt{٢٦}$

$\therefore$ Area $= m = A^{٢} = ٢٤$

Perimeter $= n = ٤A = ٨\sqrt{٦}$

$\therefore m + n = ٢٤ + ٣٨٤$

$= ٤٠٨$

١٦. The number of triangles whose vertices are at the vertices of a regular octagon but none of whose sides is a side of the octagon is

(١) ٢٤ (٢) ٥٦

(٣) ١٦ (٤) ٤٨

Ans. (٣)

Sol. $\because$ no. of triangles having no side common with a n sided polygon $= \frac{^{n}C_{١} \cdot {}^{n-٤}C_{٢}}{٣}$

$= \frac{^{٨}C_{١} \cdot {}^{٤}C_{٢}}{٣} = ١٦$

١٧. Let $y = y(x)$ be the solution of the differential equation $(١+x^{٢})\frac{dy}{dx} + y = e^{\tan^{-١}x}$، $y(١) = ٠$. Then $y(٠)$ is

(١) $\frac{١}{٤}(e^{\pi/٢} - ١)$ (٢) $\frac{١}{٢}(١ - e^{\pi/٢})$

(٣) $\frac{١}{٤}(١ - e^{\pi/٢})$ (٤) $\frac{١}{٢}(e^{\pi/٢} - ١)$

Ans. (٢)

Sol. $\frac{dy}{dx} + \frac{y}{١+x^{٢}} = \frac{e^{\tan^{-1}x}}{1+x^{٢}}$

$I.F. = e^{\int \frac{١}{١+x^{٢}}dx} = e^{\tan^{-1}x}$

$y.e^{\tan^{-1}x} = \int \left(\frac{e^{\tan^{-1}x}}{1+x^{٢}}\right) e^{\tan^{-1}x}.dx$

Let $\tan^{-١}x = z$ $\qquad \Rightarrow \frac{dx}{١+x^{٢}} = dz$

$\therefore y.e^{z} = \int e^{٢z}dz = \frac{e^{٢z}}{٢} + C$

$y.e^{\tan^{-1}x} = \frac{e^{٢\tan^{-1}x}}{٢} + C$

$\Rightarrow y = \frac{e^{\tan^{-1}x}}{٢} + \frac{C}{e^{\tan^{-1}x}}$

$\because y(١) = ٠ \quad \Rightarrow 0 = \frac{e^{\pi/٤}}{٢} + \frac{C}{e^{\pi/٤}} \Rightarrow C = \frac{-e^{\pi/٢}}{٢}$

$\Rightarrow y = \frac{e^{\tan^{-1}x}}{٢} - \frac{e^{\pi/٢}}{٢e^{\tan^{-1}x}}$

$\boxed{\Rightarrow y(0) = \frac{١ - e^{\pi/٢}}{٢}}$

18. Let $y = y(x)$ be the solution of the differential equation $2x\log_e x \frac{dy}{dx} + 2y = \frac{3}{x}\log_e x$, $x > 0$ and $y(e^{-1}) = 0$. Then, $y(e)$ is equal to

(1) $-\frac{3}{2e}$ (2) $-\frac{2}{3e}$

(3) $-\frac{3}{e}$ (4) $-\frac{2}{e}$

Ans. (3)

Sol. $\frac{dy}{dx} + \frac{y}{x\ln x} = \frac{3}{2x^2}$

$\therefore$ I.F. $= e^{\int \frac{1}{x\ln x}dx} = e^{\ln(\ln(x))} = \ln x$

$\therefore y\ln x = \int \frac{3\ln x}{2x^2}dx$

$= \frac{3\ln x}{2}\int x^{-2}dx - \int\left(\frac{3}{2x}\int x^{-2}dx\right)dx$

$= \frac{3\ln x}{2}\left(-\frac{1}{x}\right) + \int \frac{3}{2x}\left(\frac{1}{x}\right)dx$

$y\ln x = \frac{-3\ln x}{2x} - \frac{3}{2x} + C$

$\because y(e^{-1}) = 0$

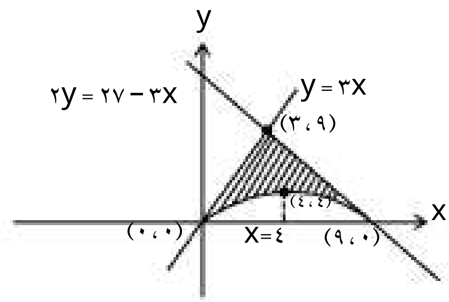
$\Rightarrow 0(-1) = \frac{3e}{2} - \frac{3e}{2} + C \Rightarrow C = 0$

$\therefore y = \frac{-3\ln x}{2x\ln x} - \frac{3}{2x\ln x}$

$$\boxed{\therefore y(e) = -\frac{3}{2e} - \frac{3}{2e} = -\frac{3}{e}}$$

19. Let the area of the region enclosed by the curves $y = 3x$, $2y = 27 - 3x$ and $y = 3x - x\sqrt{x}$ be A. Then 10 A is equal to (1) 184 (2) 154 (3) 172 (4) 162 Ans. (4)

Sol. $y = 3x$, $2y = 27 - 3x$ & $y = 3x - x\sqrt{x}$



$A = \int_0^3 \left(3x - \left(3x - x\sqrt{x}\right)\right)dx + \int_3^9 \left(\frac{27-3x}{2} - \left(3x - x\sqrt{x}\right)\right)dx$

$A = \int_0^3 x^{3/2}dx + \int_3^9 \left(\frac{27}{2} - \frac{9x}{2}\right) + x^{3/2}dx$

$A = \left[\frac{2x^{5/2}}{5}\right]_0^3 + \left[\frac{27}{2}x - \frac{9}{2}\frac{x^2}{2}\right]_3^9 + \left[\frac{2x^{5/2}}{5}\right]_3^9$

$A = \frac{2}{5}(3^{5/2}) + \frac{27}{2}(6) - \frac{9}{4}(72) + \frac{2}{5}\left(9^{5/2} - 3^{5/2}\right)$

$A = \frac{2}{5}(3^{5/2}) + (81 - 162) + \frac{2}{5}(243) - \frac{2}{5}(3^{5/2})$

$A = \frac{486}{5} - 81 = \frac{81}{5}$

$10A = 162$

Ans. = 4

20. Let $f : (-\infty, \infty) - \{0\} \to R$ be a differentiable function such that $f'(1) = \lim_{a\to\infty} a^2 f\left(\frac{1}{a}\right)$.

Then $\lim_{a\to\infty} \frac{a(a+1)}{2}\tan^{-1}\left(\frac{1}{a}\right) + a^2 - 2\log_e a$ is equal to

(1) $\frac{3}{2} + \frac{\pi}{4}$ (2) $\frac{3}{8} + \frac{\pi}{4}$

(3) $\frac{5}{2} + \frac{\pi}{8}$ (4) $\frac{3}{4} +$

Ans. (3)

Sol. $f : (-□, □) - \{٠\} □ R$

$f□(١) = \lim_{a□□} a^٢ f\left(□ \frac{١}{a} □\right)$

$\lim_{a□□} \frac{a□a□1□}{□ □1□} \tan□\left(\frac{١}{a}\right) □ a^٢ □ ٢\ell n□a□$

$\lim_{a□□}\left(\frac{\frac{١}{a}□}{٢}\right)\tan□\left(\frac{١}{a}\right)□1□\frac{٢}{a^٢}\ell n□a□$

$f(x) = \frac{١}{٢}(١ + x)\tan^{-١}(x) + ١ - ٢x□n(x)$

$f□(x) = \frac{1}{2}\left(□\frac{١□x}{١□x^٢}□\tan□x□□٤x\ell n□x□□٢x\right)$

$f□□1□ \frac{١}{٥}□□\frac{□}{4}□□2$

$f□□1□ \frac{٢}{□}□\frac{□}{٨}$

Ans. (٣)

SECTION-B

٢١. Let $□□□ = ٤٥$; $□, □□□□ □ R$. If $x(□, ١, ٢) + y(١, □, ٢) + z(٢, ٣, □) = (٠, ٠, ٠)$ for some $x, y, z □ R$, $xyz □ ٠$, then $٦□ + ٤□ + □$ is equal to__________

Ans. (٥٥)

Sol. $□□□ = ٤٥, □□□ □ R$

$x(□, ١, ٢) + y(١, □, ٢) + z(٢, ٣, □) = (٠, ٠, ٠)$

$x, y, z □ R, xyz □ ٠$

$□x + y + ٢z = ٠$

$x + □y + ٣z = ٠$

$٢x + ٢y + □z = ٠$

$xyz □ ٠ □$ non-trivial

$$\begin{vmatrix} □ & ١ & ٢ \\ ١ & □ & ٣ \\ ٢ & ٢ & □ \end{vmatrix} □ ٠$$

$□ □ (□□ - ٦) - ١(□ - ٦) + ٢(٢ - ٢□) = ٠$

$□□□□ - ٦□ - □ + ٦ + ٤ - ٤□ = ٠$

$□ ٦□ + ٤□ + □ = ٥٥$

٢٢. Let a conic C pass through the point $(٤, -٢)$ and $P(x, y)$, $x □ ٣$, be any point on C. Let the slope of the line touching the conic C only at a single point P be half the slope of the line joining the points P and $(٣, -٥)$. If the focal distance of the point $(٧, ١)$ on C is d, then ١٢d equals__________.

Ans. (٧٥)

Sol. $P(x, y)$ & $x □ ٣$

Slope of line at $P(x, y)$ will be $\frac{dy}{dx} □ \frac{١}{٢}□\frac{y□٥}{x□٣}□$

$□ ٢\frac{dy}{(y□٥)} □ \frac{١}{(x□٣)}dx$

$□ ٢□n(y + ٥) = □n(x - ٣) + C$

Passes through $(٤, -٢)$

$□ ٢□n(٣) = □n(١) + C$

$□ C = ٢□n(٣)$

$□ ٢□n(y + ٥) = □n(x - ٣) + ٢□n(٣)$

$□ ٢□□n□\frac{y□5}{٣}□□ □ \ell n(x□٣)$

$□ □\frac{y□5}{٣}□^٢ □ (x□٣)$

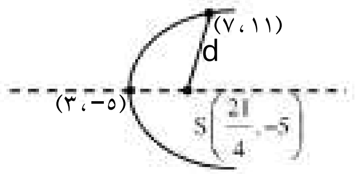
$□ (y + ٥)^٢ = ٩(x - ٣)$

□

Parabola

$٤a = ٩$

$a □ \frac{٩}{٤}$



$$d = \sqrt{\left(\frac{٧}{٤}\right)^{٢} + 6^{٢}}$$

$$d = \frac{\sqrt{٦٢٥}}{٤}$$

$$d = \frac{٢٥}{٤}$$

$$١٢d = ٧٥$$

٢٣. Let $r_k = \dfrac{\int_{٠}^{١}(١-x^{٧})^{k}dx}{\int_{٠}^{١}(١-x^{٧})^{k+١}dx}$ ، $k \in N$. Then the value of $\sum_{k=١}^{١٠} \frac{١}{٧(r_k-١)}$ is equal to ____________.

Ans. (٦٥)

Sol. $I_K = \int_{٠}^{١}(١-x^{٧})^{K}dx$

$$I_K = (١-x^{٧})^{K}x\Big|_{٠}^{١} + ٧K\int_{٠}^{١}(1-x^{٧})^{K-١}x^{٦}.x\,dx$$

$$I_K = -٧K\int_{٠}^{١}(1-x^{٧})^{K-١}((١-x^{٧})-١)dx$$

$$I_K = -٧K I_K + ٧K I_{K-١}$$

$$\Rightarrow \frac{I_K}{I_{K+١}} = \frac{٧K+٨}{٧K+٧}$$

$$r_K = \frac{٧K+٨}{٧K+٧}$$

$$r_K = 1 + \frac{١}{٧(K+١)}$$

$$\Rightarrow \frac{١}{٧(r_K-١)} = K+١$$

$$\sum_{K=١}^{١٠}(K+١) = ١١(٦) - 1 = ٦٥$$

٢٤. Let $x_١$ ، $x_٢$ ، $x_٣$ ، $x_٤$ be the solution of the equation $٤x^{٤} + ٨x^{٣} - ١٧x^{٢} - ١٢x + ٩ = ٠$ and $(4 + x_١^{٢})(٤ + x_٢^{٢})(٤ + x_٣^{٢})(4 + x_٤^{٢}) = \frac{١٢٥}{١٦}m$.

Then the value of m is ______________.

Ans. (٢٢١)

Sol. $٤x^{٤} + ٨x^{٣} - ١٧x^{٢} - ١٢x + ٩$

$= ٤(x - x_١)(x - x_٢)(x - x_٣)(x - x_٤)$

Put $x = ٢i$ & $-٢i$

$٦٤ - ٦٤i + ٦٨ - ٢٤i + ٩ = (٢i - x_١)(٢i - x_٢)(٢i - x_٣)(٢i - x_٤)$

$= ١٤١ - ٨٨i$ ......(١)

$٦٤ + ٦٤i + ٦٨ + ٢٤i + ٩ = ٤(-٢i - x_١)(-٢i - x_٢)(-٢i - x_٣)(-٢i - x_٤)$

$= ١٤١ + ٨٨i$ ....(٢)

$$\frac{١٢٥}{١٦}m = \frac{١٤١^{٢} + ٨٨^{٢}}{١٦}$$

$m = ٢٢١$

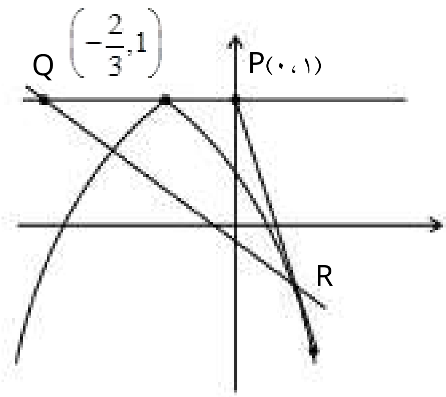
٢٥. Let $L_١$ ، $L_٢$ be the lines passing through the point $P(٠ ، ١)$ and touching the parabola $٩x^{٢} + ١٢x + ١٨y - ١٤ = ٠$. Let Q and R be the points on the lines $L_١$ and $L_٢$ such that the $\triangle PQR$ is an isosceles triangle with base QR. If the slopes of the lines QR are $m_١$ and $m_٢$ then [illegible] is equal to ____________.

Ans. (٦٨)

Sol. $٩x^{٢} + ١٢x + ٤ = -١٨(y - ١)$

$(٣x + ٢)^{٢} = -١٨(y - ١)$

$$\Rightarrow \left(x + \frac{٢}{٣}\right)^{٢} = -٢(y - ١)$$



(٠، ١)

$y = mx + ١$

$\left(x + \frac{٢}{٣}\right)^{٢} = -٢(y - ١)$

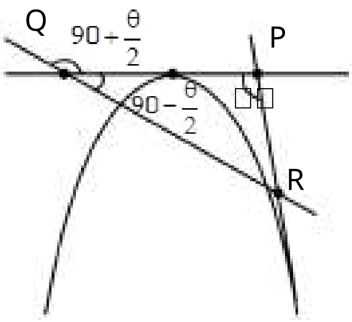
$(٣x + ٢)^{٢} = -١٨mx$

$٩x^{٢} + (١٢ + ١٨m)x + ٤ = ٠$

$٤(٦ + ٩m)^{٢} = ٤(٣٦)$

$٦ + ٩m = ٦، -٦$

$m = ٠، \frac{-٤}{٣}$



$\tan\theta = \frac{-٤}{٣}$

$$\frac{٢\tan\frac{\theta}{٢}}{١ - \tan^{٢}\frac{\theta}{٢}} = \frac{-٤}{٣}$$

$$\left(\tan\frac{\theta}{٢} - ٢\right)\left(٢\tan\frac{\theta}{٢} + 1\right) = 0$$

$\tan\frac{\theta}{٢} = ٢، \frac{-١}{٢}$

$m_{QR} = \tan\left(٩٠ + \frac{\theta}{2}\right)$

$= -\cot\frac{\theta}{٢}$

$m_1 = \frac{-١}{٢}$ $\qquad m_٢ = \frac{-١}{-١/٢} = ٢$

$١٦(m_١^{٢} + m_٢^{٢}) = 16\left(\frac{1}{4} + ٤\right)$

$= ٤ + ٦٤ = ٦٨$

٢٦. If the second, third and fourth terms in the expansion of $(x + y)^{n}$ are ١٣٥، ٣٠ and $\frac{١٠}{٣}$، respectively, then $٦(n^{٣} + x^{٢} + y^{٢})$ is equal to ______.

Ans. (٨٠٦)

Sol. $^{n}C_{١}x^{n-١}y = ١٣٥$ $\quad$ ....(i)

$^{n}C_{٢}x^{n-٢}y^{٢} = ٣٠$ $\quad$ ....(ii)

$^{n}C_{٣}x^{n-٣}y^{٣} = \frac{١٠}{٣}$ $\quad$ ....(iii)

By $\frac{(i)}{(ii)}$

$\frac{^{n}C_{١}}{^{n}C_{٢}} \cdot \frac{x}{y} = \frac{٩}{٢}$ $\quad$ .....(iv)

By $\frac{(ii)}{(iii)}$

$\frac{^{n}C_{٢}}{^{n}C_{٣}} \cdot \frac{x}{y} = ٩$ $\quad$ .....(v)

By $\frac{(iv)}{(v)}$

$\frac{^{n}C_{١} \cdot {}^{n}C_{٣}}{^{n}C_{٢} \cdot {}^{n}C_{٢}} = \frac{١}{٢}$

$$\frac{٢n \cdot n(n-١)(n-٢)}{٦} = \frac{n(n-١)}{٢} \cdot \frac{n(n-١)}{٢}$$

$٤n - ٨ = ٣n - ٣$

$\Rightarrow \boxed{n = ٥}$

put in (v)

$\frac{x}{y}=٩$

$x=٩y$

put in (i)

$^{٥}C_{١}\,x^{٤}\left(\frac{x}{٩}\right)=١٣٥$

$x^{٥}=٢٧\times٩$

$\therefore x=٣,\quad y=\frac{١}{٣}$

$^{٦}C_{٣}\,(x^{٢})\,(y)$

$=\,^{٦}(٢٥)\,(9)\left(\frac{١}{٣}\right)$

$=٨٠٦$

٢٧. Let the first term of a series be $T_{١}=٦$ and its r^{th} term $T_r=٣T_{r-١}+٦^{r}$, $r=٢,٣,\ldots\ldots,n$. If the sum of the first n terms of this series is $\frac{١}{٥}\left(n^{٢}-١٢n+٣٩\right)\left(٤\cdot٦^{n}-٥\cdot٣^{n}+١\right)$. Then n is equal to ______________

Ans. (٦)

Sol. $T_r=٣T_{r-١}+٦^{r},\ r=٢,٣,٤,\ldots n$

$T_{٢}=٣\cdot T_{١}+٦^{٢}$

$T_{٢}=٣\cdot٦+٦^{٢}\qquad\ldots(١)$

$T_{٣}=٣T_{٢}+٦^{٣}$

$T_{٣}=٣T_{٢}+٦^{٣}$

$T_{٣}=٣(٣\cdot٦+٦^{٢})+٦^{٣}$

$T_{٣}=٣^{٢}\cdot٦+٣\cdot٦^{٢}+٦^{٣}\qquad\ldots(٢)$

$T_r=٣^{r-١}\cdot٦+٣^{r-٢}\cdot٦^{٢}+\ldots+٦^{r}$

$T_r=٣^{r-١}\cdot٦\left[١+\left(\frac{٦}{٣}\right)+\left(\frac{6}{٣}\right)^{٢}+\ldots+\left(\frac{6}{3}\right)^{r-١}\right]$

$T_r=٣^{r-١}\cdot٦(١+٢+٢^{٢}+\ldots+٢^{r-١})$

$T_r=٦\cdot٣^{r-١}\cdot\frac{١(٢^{r}-١)}{(٢-١)}$

$T_r=٦\cdot٣^{r-١}\cdot(٢^{r}-١)$

$T_r=\frac{٦\cdot٣^{r}}{٣}\cdot(٢^{r}-١)$

$T_r=٢\cdot(٦^{r}-٣^{r})$

$S_n=2\left(\sum ٦^{r}-\sum ٣^{r}\right)$

$S_n=٢\cdot\left[\frac{٦\cdot(٦^{n}-١)}{٥}-\frac{٣\cdot(٣^{n}-١)}{٢}\right]$

$S_n=2\left[\frac{١٢(٦^{n}-١)-١٥(٣^{n}-١)}{١٠}\right]$

$S_n=\frac{٣}{٥}\left[٤\cdot٦^{n}-٥\cdot٣^{n}+1\right]$

$\therefore n^{٢}-١٢n+٣٩=٣$

$n^{٢}-١٢n+٣٦=٠$

$n=٦$

٢٨. For $n\in N$, if $\cot^{-١}٣+\cot^{-١}٤+\cot^{-١}٥+\cot^{-١}n=\frac{\pi}{٤}$, then n is equal to ______________.

Ans. (٤٧)

Sol. $\cot^{-١}٣+\cot^{-١}٤+\cot^{-١}٥+\cot^{-١}n=\frac{\pi}{٤}$

$\tan^{-١}\frac{١}{٣}+\tan^{-١}\frac{١}{٤}+\tan^{-١}\frac{١}{٥}+\tan^{-١}\frac{١}{n}=\frac{\pi}{٤}$

$\tan^{-1}\left(\frac{٤٦}{٤٨}\right)+\tan^{-١}\frac{١}{n}=\frac{\pi}{٤}$

$\tan^{-1}\left(\frac{٢٣}{٢٤}\right)+\tan^{-١}\frac{١}{n}=\frac{\pi}{٤}$

$\tan^{-١}\frac{١}{n}=\tan^{-١}١-\tan^{-١}\frac{٢٣}{٢٤}$

$\tan^{-١}\frac{١}{n}=\tan^{-١}\left(\frac{١-\frac{٢٣}{٢٤}}{١+\frac{٢٣}{٢٤}}\right)$

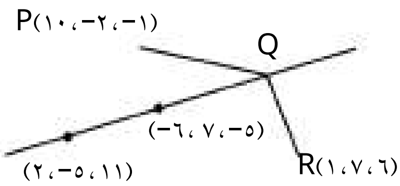
$\tan^{-١}\frac{١}{n}=\tan^{-١}\left(\frac{\frac{١}{٢٤}}{\frac{٤٧}{٢٤}}\right)$

$\tan^{-١}\frac{١}{n}=\tan^{-١}\frac{١}{٤٧}$

$n=٤٧$

29. Let P be the point $(10, -2, -1)$ and Q be the foot of the perpendicular drawn from the point $R(1, 7, 6)$ on the line passing through the points $(2, -5, 11)$ and $(-6, 7, -5)$. Then the length of the line segment PQ is equal to ____________.

Sol. Ans. (13)



Line : $\frac{x+6}{-8} = \frac{y-7}{12} = \frac{z+5}{-16}$

$\frac{x+6}{2} = \frac{y-7}{-3} = \frac{z+5}{4} = \lambda$

$Q(2\lambda-6,\ 7-3\lambda,\ 4\lambda-5)$

$\overrightarrow{QR} \langle 2\lambda-7,\ -3\lambda,\ 4\lambda-11 \rangle$

$\overrightarrow{QR} \cdot$ dr's of line $= 0$

$4\lambda - 14 + 9\lambda + 16\lambda - 44 = 0$

$29\lambda = 58 \Rightarrow \lambda = 2$

$Q(-2, 1, 3)$

$PQ = \sqrt{144 + 9 + 16} = \sqrt{169} = 13$

30. Let $\vec{a} = 2\hat{i} - 3\hat{j} + 4\hat{k}$, $\vec{b} = 3\hat{i} + 4\hat{j} - 5\hat{k}$, and a vector $\vec{c}$ be such that $\vec{a} \times (\vec{b} + \vec{c}) + \vec{b} \times \vec{c} = \hat{i} + 8\hat{j} + 13\hat{k}$. If $\vec{a} \cdot \vec{c} = 13$, then $(24 - \vec{b} \cdot \vec{c})$ is equal to __________.

Ans. (46)

Sol. $\vec{a} \times \vec{b} + \vec{a} \times \vec{c} + \vec{b} \times \vec{c} = (1, 8, 13)$

$\vec{a} \cdot (\vec{a} \times \vec{b}) + \vec{a} \cdot (\vec{a} \times \vec{c}) + \vec{a} \cdot (\vec{b} \times \vec{c}) = \vec{a} \cdot (\hat{i} + 8\hat{j} + 13\hat{k})$

$(\vec{a} \times \vec{b}) \times \vec{a} + (\vec{a} \times \vec{c}) \times \vec{a} + (\vec{b} \times \vec{c}) \times \vec{a} = (\hat{i} + 8\hat{j} + 13\hat{k}) \times \vec{a}$

$\Rightarrow -26\vec{a} + 29\vec{b} - 13\vec{a} + 29\vec{c} - 13\vec{b} - 26\vec{c} = \vec{a} \times (\hat{i} + 8\hat{j} + 13\hat{k})$

$\Rightarrow -13\vec{a} + 16\vec{b} + 3\vec{c} = \vec{a} \times (\hat{i} + 8\hat{j} + 13\hat{k})$

$\Rightarrow -13\vec{a} \cdot \vec{b} + 16\vec{b}^2 + 3\vec{b} \cdot \vec{c} = \left[\vec{a}\ (\hat{i} + 8\hat{j} + 13\hat{k})\ \vec{b}\right]$

$\Rightarrow -13(-26) + 16(50) + 3\vec{b} \cdot \vec{c} = \begin{vmatrix} 2 & -3 & 4 \\ 1 & 8 & 13 \\ 3 & 4 & -5 \end{vmatrix}$

$\Rightarrow 462 + 3\vec{b} \cdot \vec{c} = 396$

$\Rightarrow \vec{b} \cdot \vec{c} = -22$

$\Rightarrow$ Hence $24 - \vec{b} \cdot \vec{c} = 46$

PHYSICS

TEST PAPER WITH SOLUTION

SECTION-A

٣١. To find the spring constant (k) of a spring experimentally، a student commits ٢٪ positive error in the measurement of time and ١٪ negative error in measurement of mass. The percentage error in determining value of k is :

(١) ٣٪ (٢) ١٪

(٣) ٤٪ (٤) ٥٪

Ans. (٤)

Sol. $T□2□\sqrt{\frac{m}{k}}$

$T^{٢} □ \frac{m}{k}$

2□T

$\frac{}{T}\% □\frac{□m}{m}\% □\frac{□k}{k}٪$

$\frac{□k}{k}\% □\frac{□m}{m}\% □\frac{٢□T}{T}٪$

$\frac{□k}{k}٪ □ (□١)٪ □ ٢(٢)٪ | \quad □□٥٪|□٥٪$

٣٢. A bullet of mass ٥٠ g is fired with a speed ١٠٠ m/s on a plywood and emerges with ٤٠ m/s. The percentage loss of kinetic energy is :

(١) ٣٢٪ (٢) ٤٤٪

(٣) ١٦٪ (٤) ٨٤٪

Ans. (٤)

Sol. $K□ \frac{١}{٢} m(١٠٠)^{٢}$

$K_f □\frac{١}{٢} m(٤٠)^{٢}$

$٪loss □ \frac{|Kf □Ki|}{K_i} \times ١٠٠$

$= \frac{\left|\frac{١}{٢}m(٤٠)^{٢} □ \frac{١}{٢}m(١٠٠)^{٢}\right|}{\frac{١}{٢}m(١٠٠)^{٢}} □١٠٠$

$= \frac{|١٦٠٠□١٠٠□١٠٠|}{١٠٠} = ٨٤٪$

٣٣. The ratio of the shortest wavelength of Balmer series to the shortest wavelength of Lyman series for hydrogen atom is :

(١) ٤ : ١ (٢) ١ : ٢

(٣) ١ : ٤ (٤) ٢ : ١

Ans. (١)

Sol

n = □

Balmer n = ٢

Lyman n = ١

$\frac{١}{□} □Rz^{٢} □\frac{١}{n_١^{٢}} □ \frac{١}{n_٢^{٢}}□$

$\frac{١}{□_L} □Rz^{٢} □\frac{١}{١^{٢}}□$

$\frac{١}{□_B} □Rz^{٢} □\frac{١}{٢^{٢}}□$

$\frac{□_B}{□_L} □٤ : ١$

٣٤. To project a body of mass m from earth's surface to infinity، the required kinetic energy is (assume، the radius of earth is RE، g = acceleration due to gravity on the surface of earth) :

(١) ٢mgRE (٢) mgRE

(٣) $\frac{١}{٢}$ mgRE (٤) ٤mgRE

Ans. (٢)

Sol. $\frac{١}{٢} mv_e^{٢} □ \frac{GMm}{R_E}$

$g □\frac{GM}{R_E^{٢}}$

$K □ mgR_E$

٣٥. Electromagnetic waves travel in a medium with speed of ١.٥×١٠^{٨} ms$^{-١}$. The relative permeability of the medium is ٢.٠. The relative permittivity will be :

(١) ٥ (٢) ١

(٣) ٤ (٤) ٢

Ans. (٤)

Sol. $\frac{\mu_m \epsilon_m}{\mu_0 \epsilon_0} = \frac{\frac{١}{v^{٢}}}{\frac{١}{c^{٢}}}$

$\mu_r \epsilon_r = \frac{c^{٢}}{v^{٢}}$

$\epsilon_r \times 2 = \frac{(٣ \times ١٠^{٨})^{٢}}{(١.٥ \times ١٠^{٨})^{٢}}$

$\epsilon_r = \frac{٤}{٢}$

٣٦. Which of the following phenomena does not explain by wave nature of light.

(A) reflection (B) diffraction

(C) photoelectric effect (D) interference

(E) polarization

Choose the most appropriate answer from the options given below :

(١) E only (٢) C only

(٣) B, D only (٤) A, C only

Ans. (٢)

Sol. (Theory)

Photoelectric effect prove particle nature of light.

٣٧. While measuring diameter of wire using screw gauge the following readings were noted. Main scale reading is ١ mm and circular scale reading is equal to ٤٢ divisions. Pitch of screw gauge is ١ mm and it has ١٠٠ divisions on circular scale. The diameter of the wire is $\frac{x}{٥٠}$ mm. The value of x is :

(١) ١٤٢ (٢) ٧١

(٣) ٤٢ (٤) ٢١

Ans. (٢)

Sol. MSR = ١mm, CSR = ٤٢, pitch = ١ mm

$LC = \frac{\text{pitch}}{\text{No. of CSD}} = \left(\frac{1}{١٠٠}\right) = ٠.٠١ \text{mm}$

Diameter = MSR + LC × CSD

Diameter = ١ + (٠.٠١) × ٤٢ mm

Diameter = ١.٤٢ mm = $\frac{x}{٥٠}$

$\Rightarrow x = ٧١$

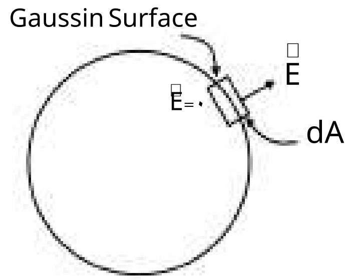
٣٨. σ is the uniform surface charge density of a thin spherical shell of radius R. The electric field at any point on the surface of the spherical shell is :

(١) $\sigma/\epsilon_0 R$ (٢) $\sigma/٢\epsilon_0$

(٣) σ/ϵ_0 (٤) $\sigma/٤\epsilon_0$

Ans. (٣)

Sol.

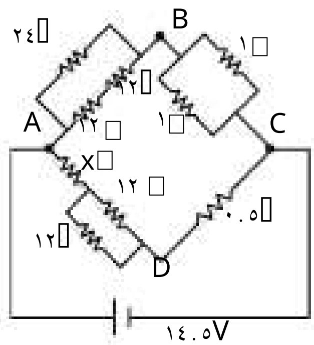


By Gauss Law $\oint \vec{E} \cdot d\vec{A} = \frac{q_{in}}{\epsilon_0}$

$EdA = \frac{\sigma dA}{\epsilon_0}$

$E = \frac{\sigma}{\epsilon_0}$

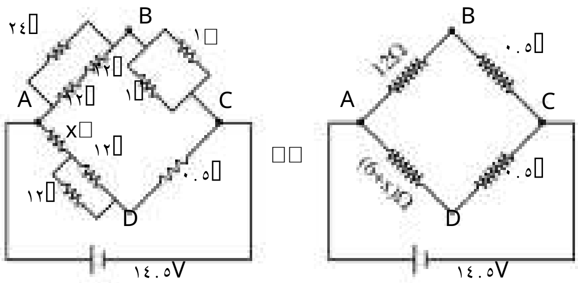
٣٩. The value of unknown resistance (x) for which the potential difference between B and D will be zero in the arrangement shown, is :



(١) ٣ Ω (٢) ٩ Ω

(٣) ٦ Ω (٤) ٤٢ Ω

Ans. (٣)

Sol. 

In case of balanced Wheatstone Bridge

$\frac{V_{AB}}{V_{AD}} = \frac{V_{BC}}{V_{CD}} \Rightarrow \frac{١٢}{٦ + x} = \frac{٠.٥}{٠.٥}$

$x = ٦\,\Omega$

٤٠. The specific heat at constant pressure of a real gas obeying $PV^٢ = RT$ equation is :

(١) $C_V + R$

(٢) $\frac{R}{٣} + C_V$

(٣) R

(٤) $C_V + \frac{R}{٢V}$

Ans. (٤)

Sol. $dQ = du + dW$

$CdT = C_VdT + PdV$ ... . .(١)

$PV^٢ = RT$

$P = \text{constant}$

$P(٢VdV) = RdT$

$PdV = \frac{RdT}{٢V}$

Put in equation (١)

$C = C_V + \frac{R}{٢V}$

٤١. Match List I with List II

LIST I		LIST II
A. Torque	I.	$[M^١L^١T^{-٢}A^{-٢}]$
B. Magnetic field	II.	$[L^٢A^١]$
C. Magnetic moment	III.	$[M^١T^{-٢}A^{-١}]$
D. Permeability of free space	IV.	$[M^١L^٢T^{-٢}]$

Choose the correct answer from the options given below :

(١) A-I, B-III, C-II, D-IV

(٢) A-IV, B-III, C-II, D-I

(٣) A-III, B-I, C-II, D-IV

(٤) A-IV, B-II, C-III, D-I

Ans. (٢)

Sol. $[\tau] = [r \times F] = [ML^٢T^{-٢}]$

$[F] = [qVB]$

$\Rightarrow B = \left[\frac{F}{qV}\right] = \left[\frac{MLT^{-٢}}{ATLT^{-١}}\right] = [MA^{-١}T^{-٢}]$

$[M] = [I \times A] = [AL^٢]$

$B = \frac{\mu_0}{٤\pi}\frac{Idl\sin\theta}{r^٢}$

$\Rightarrow [\mu_0] = \left[\frac{Br^٢}{Idl}\right] = \left[\frac{MT^{-٢}A^{-١}L^٢}{AL}\right]$

$= [MLT^{-٢}A^{-٢}]$

٤٢. Given below are two statements :

Statement I : In an LCR series circuit, current is maximum at resonance.

Statement II : Current in a purely resistive circuit can never be less than that in a series LCR circuit when connected to same voltage source.

In the light of the above statements, choose the correct from the options given below :

(١) Statement I is true but Statement II is false

(٢) Statement I is false but Statement II is true

(٣) Both Statement I and Statement II are true

(٤) Both Statement I and Statement II are false

Ans. (٣)

Sol. Statement-I

$I_m = \frac{V_m}{\sqrt{R^٢ + (X_L - X_C)^٢}}$ at resonance $X_L = X_C$

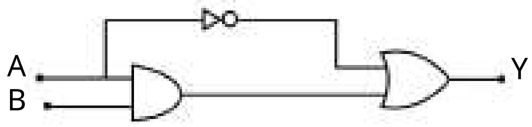
Thus, $I_m = \frac{V_m}{R}$

$\because$ Impendence is minimum therefore I is maximum at resonance.

Statement-II

$I = \left(\frac{V}{R}\right)$ in purely resistive circuit.

٤٣. The correct truth table for the following logic circuit is :



Options :

(١)

A	B	Y
٠	٠	٠
٠	١	١
١	٠	٠
١	١	١

(٢)

A	B	Y
٠	٠	١
٠	١	١
١	٠	٠
١	١	١

(٣)

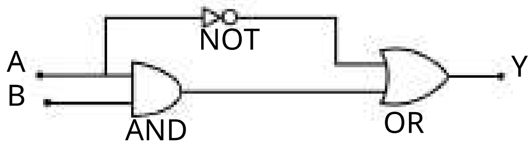
A	B	Y
٠	٠	١
٠	١	١
١	٠	٠
١	١	٠

(٤)

A	B	Y
٠	٠	٠
٠	١	٠
١	٠	٠
١	١	١

Ans. (٢)

Sol.



٤٤. A sample contains mixture of helium and oxygen gas. The ratio of root mean square speed of helium and oxygen in the sample, is :

(١) $\frac{١}{٣٢}$ (٢) $\frac{٢\sqrt{٢}}{١}$

(٣) $\frac{١}{٤}$ (٤) $\frac{١}{٢\sqrt{٢}}$

Ans. (٢)

Sol. $V_{rms} = \sqrt{\frac{٣RT}{M_w}}$

$$\Rightarrow \frac{V_{O_٢}}{V_{He}} = \sqrt{\frac{M_{w,He}}{M_{w,O_٢}}}$$

$$= \sqrt{\frac{٤}{٣٢}} = \frac{١}{٢\sqrt{٢}}$$

$$\frac{V_{He}}{V_{O_٢}} = \frac{٢\sqrt{٢}}{١}$$

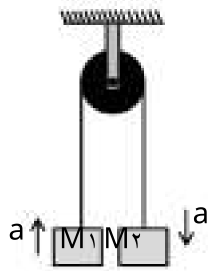
٤٥. A light string passing over a smooth light pulley connects two blocks of masses $m_١$ and $m_٢$ (where $m_٢ < m_١$). If the acceleration of the system is $\frac{g}{\sqrt{٨}}$, then the ratio of the masses is $\frac{m_١}{m_٢}$:

(١) $\frac{\sqrt{٢}+١}{\sqrt{٢}-١}$ (٢) $\frac{1+\sqrt{٥}}{\sqrt{٥}-١}$

(٣) $\frac{١+\sqrt{٥}}{\sqrt{٢}+١}$ (٤) $\frac{\sqrt{٣}+١}{\sqrt{٢}-١}$

Ans. (١)

Sol. $a = \left(\frac{M_١ - M_٢}{M_١ + M_٢}\right) g$



$$\frac{g}{٢\sqrt{٢}} = \left(\frac{M_١ - M_٢}{M_١ + M_٢}\right) g$$

$$M_1 + M_2 = ٢\sqrt{٢}M_١ - ٢\sqrt{٢}M_٢$$

$$\frac{M_١}{M_٢} = \frac{\sqrt{٢}+1}{\sqrt{٢}-1}$$

٤٦. Four particles A, B, C, D of mass $\frac{m}{٢}$, m, ٢m, ٤m, have same momentum, respectively. The particle with maximum kinetic energy is :

(١) D (٢) C

(٣) A (٤) B

Ans. (٣)

Sol. $KE = \frac{p^٢}{٢m}$

Same momentum, so less mass means more KE.

So $\frac{m}{٢}$ will have max. KE.

٤٧. A train starting from rest first accelerates uniformly up to a speed of ٨٠ km/h for time t, then it moves with a constant speed for time ٣t. The average speed of the train for this duration of journey will be (in km/h) :

(١) ٨٠ (٢) ٧٠

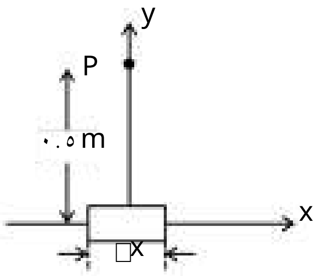
(٣) ٣٠ (٤) ٤٠

Ans. (٢)

Sol. Average speed = $\frac{\text{total distance}}{\text{time taken}}$

$$= \frac{\frac{٨٠ t}{٢} + ٨٠ \times ٣t}{٤t} = ٧٠\ km/hr.$$

٤٨. An element $\Delta l = \Delta x \hat{i}$ is placed at the origin and carries a large current I = ١٠A. The magnetic field on the y-axis at a distance of ٠.٥ m from the elements Δx of ١ cm length is :

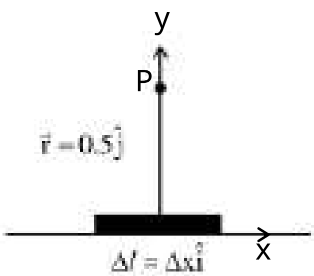


(١) ٤ × ١٠⁻⁸T (٢) ٨ × ١٠⁻⁸T

(٣) ١٢ × ١٠⁻⁸T (٤) ١٠ × ١٠⁻⁸T

Ans. (١)

Sol.



$d\vec{B} = \frac{\mu_0}{4\pi} \frac{I(d\vec{l} \times \vec{r})}{r^3}$ (Tesla)

$$= \frac{١٠^{-٧} \times ١٠ \times \left(\frac{١}{٢}\right) \times \frac{١}{١٠٠} (\hat{k})}{\left(\frac{١}{٢}\right)^٣} = ٤ \times ١٠^{-٨} T (\hat{k})$$

٤٩. A small ball of mass m and density ρ is dropped in a viscous liquid of density ρ_0. After sometime the ball falls with constant velocity. The viscous force on the ball is :

(١) mg $\left(\frac{\rho_0}{\rho} - ١\right)$ (٢) mg $\left(١ + \frac{\rho}{\rho_0}\right)$

(٣) mg $(١ - \rho\rho_0)$ (٤) mg $\left(١ - \frac{\rho_0}{\rho}\right)$

Ans. (٤)

Sol. mg − FB − Fv = ma

a = ٠ for constant velocity

mg − FB = Fv

$F_v = mg - v\rho_0 g = mg - \frac{m}{\rho}\rho_0 g = mg\left(١ - \frac{\rho_0}{\rho}\right)$

٥٠. In photoelectric experiment energy of ٢.٤٨ eV irradiates a photo sensitive material. The stopping potential was measured to be ٠.٥ V. Work function of the photo sensitive material is :

(١) ٠.٥ eV (٢) ١.٦٨ eV

(٣) ٢.٤٨ eV (٤) ١.٩٨ eV

Ans. (٤)

Sol. eVs = $h\nu - \phi$

٠.٥ V = ٢.٤٨ − ϕ

work function (ϕ) = ٢.٤٨ V − ٠.٥ V = ١.٩٨ V

SECTION-B

٥١. If the radius of earth is reduced to three-fourth of its present value without change in its mass then value of duration of the day of earth will be __________ hours ٣٠ minutes.

Ans. (١٣)

Sol. By conservation of angular momentum

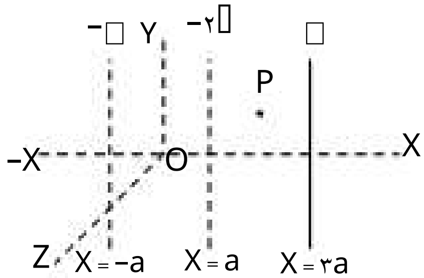
$I_١\omega_١ = I_٢\omega_٢$

$$\frac{٢}{٥} MR^٢ \left(\frac{2\pi}{T}\right) = \frac{٢}{٥} M \left(\frac{3}{4} R\right)^٢ \frac{٢\pi}{T_٢}$$

$$\frac{١}{T_١} = \frac{٩}{١٦ T_٢}$$

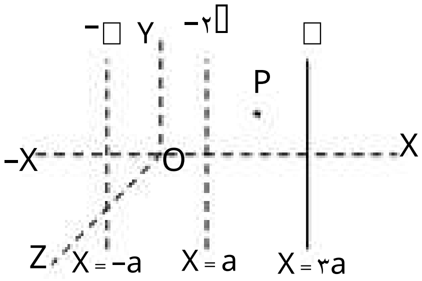
$$\frac{١}{T_٢} = \frac{٩}{١٦} \Rightarrow T1 = \frac{٩}{١٦} \times ٢٤ hr = \frac{٢٧}{٢} hr = ١٣\ hr\ ٣٠\ mins.$$

٥٢. Three infinitely long charged thin sheets are placed as shown in figure. The magnitude of electric field at the point P is $\frac{x\sigma}{\epsilon_0}$. The value of x is ____________ (all quantities are measured in SI units).



Ans. (٢)

Sol.



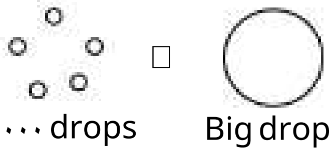
$E_p = \frac{\square}{2\square_0}(\square) + \frac{٢\square}{2\square_0}(\square) + \frac{\square}{2\square_0}(\hat{i})$

$= \frac{٢\square}{\square}\hat{i}$

٥٣. A big drop is formed by coalescing ١٠٠٠ small droplets of water. The ratio of surface energy of ١٠٠٠ droplets to that of energy of big drop is $\frac{١٠}{x}$. The value of x is ________________.

Ans. (١)

Sol.



$١٠٠٠ \cdot \frac{٤}{٣}\pi r^٣ = \frac{٤}{٣}\pi R^٣$

$١٠r = R$

$R = ١٠r$

$\frac{S.E.\ of\ ١٠٠٠\ drops}{S.E.\ of\ Big\ drop} = \frac{١٠٠٠(٤\pi r^٢)T}{٤\pi R^٢ T}$

$= \frac{١٠٠٠\,r^٢}{(١٠r)^٢} = ١٠ = \frac{١٠}{x}$

$x = 1$

٥٤. When a dc voltage of ١٠٠V is applied to an inductor, a dc current of ٥A flows through it. When an ac voltage of ٢٠٠V peak value is connected to inductor, its inductive reactance is found to be $٢٠\sqrt{٣}\,\Omega$. The power dissipated in the circuit is ________________W.

Ans. (٢٥٠)

Sol. For DC voltage

$R = \frac{V}{I} = \frac{١٠٠}{٥} = ٢٠\ \Omega$

for AC voltage

$X_L = ٢٠\sqrt{٣}\,\Omega$

$R = ٢٠\ \Omega$

$Z = \sqrt{X_L^٢ + R^٢} = \sqrt{٣ \times ٤٠٠ + ٤٠٠} = ٤٠\ \Omega$

$Power = I_{rms}^٢ R$

$= \left(\frac{V_{rms}}{Z}\right)^٢ R = \left(\frac{٢٠٠}{\sqrt{٢} \times ٤٠}\right)^٢ \times ٢٠ = ٢٥٠W$

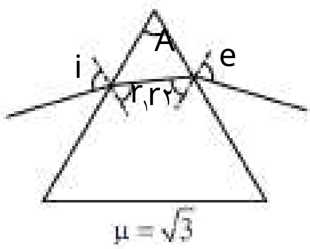
٥٥. The refractive index of prism is $\sqrt{٣}$ and the ratio of the angle of minimum deviation to the angle of prism is one. The value of angle of prism is ________________°.

Ans. (٦٠)

Sol. For δ_{min}

$i = e$

$r_١ = r_٢ = \frac{A}{٢}$



$\frac{\delta_{min}}{A} = ١$

$\frac{٢i - A}{A} = ١$

$٢i = ٢A$

$i = A$

Snell's law

$١ \times \sin i = \mu \sin r$

$\sin i = \mu \sin\left(\frac{A}{2}\right)$

$\sin A = \mu \sin\left(\frac{A}{2}\right)$

$٢\sin\frac{A}{٢}\cos\frac{A}{٢} = \sqrt{٣}\sin\left(\frac{A}{2}\right)$

$\cos\left(\frac{A}{2}\right) = \frac{\sqrt{٣}}{٢}$

$\Rightarrow \frac{A}{٢} = ٣٠°$

$\Rightarrow A = ٦٠°$

٥٦. A wire of resistance R and radius r is stretched till its radius became r/٢. If new resistance of the stretched wire is x R، then value of x is ___________.

Ans. (١٦)

Sol. We know $R = \frac{\rho l}{A}$، $R \propto \frac{l}{r^٢}$

As we starch the wire، its length will increase but its radius will decrease keeping the volume constant

Vi = Vf

$\pi r^2 l = \pi \frac{r^٢}{٤} l_f$

$l_f = ٤l$

$$\frac{R_{new}}{R_{old}} = \left(\frac{٤l}{\frac{r^٢}{٤}}\right)\left(\frac{r^٢}{l}\right) = ١٦$$

Rnew = ١٦R

∴ x = ١٦

٥٧. Radius of a certain orbit of hydrogen atom is ٨.٤٨ Å. If energy of electron in this orbit is E/x، then x = ___________. (Given a٠ = ٠.٥٢٩Å، E = energy of electron in ground state)

Ans. (١٦)

Sol. We know

$r = ٠.٥٢٩ \frac{n^٢}{Z}$ ⇒ ٨.٤٨ = ٠.٥٢٩ $\frac{n^٢}{١}$

n² = ١٦ ⇒ n = ٤

We know

$E \propto \frac{١}{n^٢}$

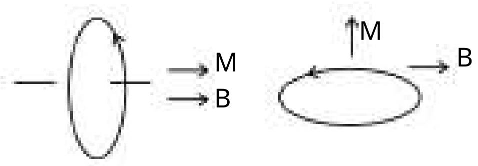
$E_{n^{th}} = \frac{E}{١٦}$

x = ١٦

٥٨. A circular coil having ٢٠٠ turns، ٢.٥ × ١٠⁻⁴ m² area and carrying ١٠٠ μA current is placed in a uniform magnetic field of ١ T. Initially the magnetic dipole moment (M) was directed along B. Amount of work، required to rotate the coil through ٩٠° from its initial orientation such that M becomes perpendicular to B، is ___________ μJ.

Ans. (٥)

Sol.



initial final

We know

Wext = ΔU + ΔKE

ΔP.E. = −(M·B)

= −(M·Bf − M·Bi) ٠

= −MB cos ٩٠ + MB cos ٠

= MB

= NIAB

= ٢٠٠ × ١٠٠ × ١٠⁻٦ × $\frac{٥}{٢}$ × ١٠⁻٤ × ١ = ٥ μJ

٥٩. A particle is doing simple harmonic motion of amplitude ٠.٠٦ m and time period ٣.١٤ s. The maximum velocity of the particle is ___________ cm

Ans. (١٢)

Sol. We know

vmax = ωA at mean position

$= \frac{2\pi}{T} A = \frac{2\pi}{\pi} \times ٠.٠٦ = ٠.١٢$ m/sec

vmax = ١٢ cm/sec

٦٠. For three vectors $\vec{A} = (-xî - ٦ĵ - ٢k̂)$، $\vec{B} = (-î + ٤ĵ + ٣k̂)$ and $\vec{C} = (-٨î - ĵ + ٣k̂)$، if $\vec{A}\cdot(\vec{B}\times\vec{C}) = ٠$، them value of x is ___________.

Ans. (٤)

Sol.
$$\vec{B}\times\vec{C} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -١ & ٤ & ٣ \\ -٨ & -1 & ٣ \end{vmatrix} = ١٥\hat{i} - ٢١\hat{j} + ٣٣\hat{k}$$

$\vec{A}\cdot(\vec{B}\times\vec{C}) = (-x\hat{i} - ٦\hat{j} - ٢\hat{k})\cdot(١٥\hat{i} - ٢١\hat{j} + ٣٣\hat{k})$

٠ = −١٥X + ١٢٦ − ٦٦

١٥X = ٦٠

X = ٤

CHEMISTRY

TEST PAPER WITH SOLUTION

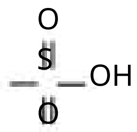
SECTION-A

٦١. Functional group present in sulphonic acid is :

(١) SO٤H (٢) SO٣H

(٣) –S–OH with =O (٤) –SO٢

Ans. (٢)

Sol. 

Group present in sulphonic acids

٦٢. Match List I with List II :

List I (Molecule / Species)		List II (Property / Shape)	
A.	SO٢Cl٢	I.	Paramagnetic
B.	NO	II.	Diamagnetic
C.	NO٢−	III.	Tetrahedral
		IV.	Linear
D.	I٣−		

Choose the correct answer from the options given below :

(١) A-IV, B-I, C-III, D-II

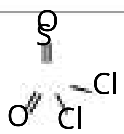
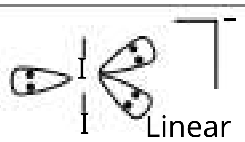
(٢) A-III, B-I, C-II, D-IV

(٣) A-II, B-III, C-I, D-IV

(٤) A-III, B-IV, C-II, D-I

Ans. (٢)

Sol.

(A)	SO٢Cl٢	sp٣	 Tetrahedral
(B)	NO		Paramagnetic
(C)	NO٢−		Diamagnetic
(D)	I٣−	sp٣d	 Linear

٦٣. Given below are two statements :

Statement I : Picric acid is ٢, ٤, ٦-trinitrotoluene.

Statement II : Phenol-٢, ٤-disulphuric acid is treated with conc. HNO٣ to get picric acid.

In the light of the above statement, choose the most appropriate answer from the options given below :

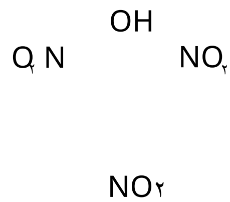
(١) Statement I is incorrect but Statement II is correct.

(٢) Both Statement I and Statement II are incorrect.

(٣) Statement I is correct but Statement II is incorrect.

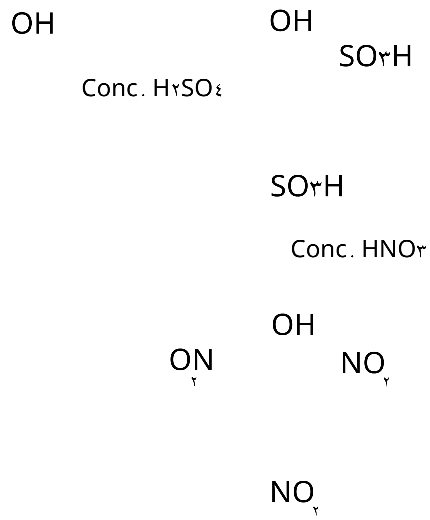
(٤) Both Statement I and Statement II are correct.

Ans. (١)

Sol. 

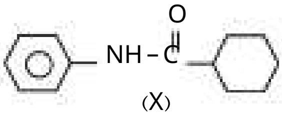
picric acid

(٢, ٤, ٦ – trinitrophenol)

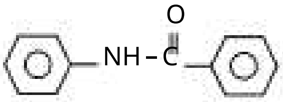


Picric acid

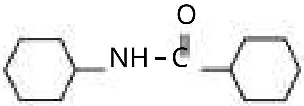
٦٤. Which of the following is metamer of the given compound (X)؟

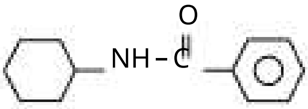


(X)

(١) 

(٢) 

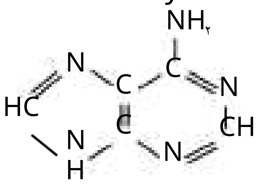
(٣) 

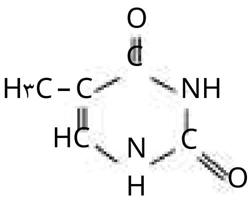
(٤) 

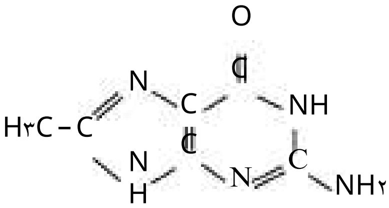
Ans. (٤)

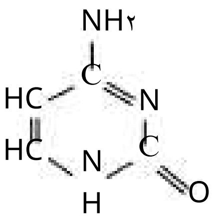
Sol. Metamer □□Isomer having same molecular formula، same functional group but different alkyl /aryl groups on either side of functional group.

٦٥. DNA molecule contains ٤ bases whoes structure are shown below. One of the structure is not correct، identify the incorrect base structure.

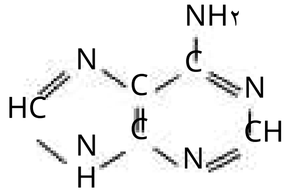
(١) 

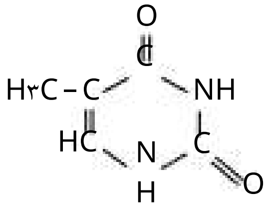
(٢) 

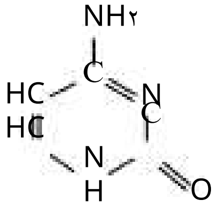
(٣) 

(٤) 

Ans. (٣)

Sol.  □ Adenine

 □ Thymine

 □Cytosine

Are bases of DNA molecule. As DNA contain four bases، which are adenine، guanine، cytosine and thymine.

٦٦. Match List I with List II :

LIST I (Hybridization)		LIST II (Orientation in Space)	
A.	sp^3	I.	Trigonal bipyramidal
B.	dsp^2	II.	Octahedral
C.	sp^3d	III.	Tetrahedral
D.	sp^3d^2	IV.	Square planar

Choose the correct answer from the options given below :

(١) A-III، B-I، C-IV، D-II

(٢) A-II، B-I، C-IV، D-III

(٣) A-IV، B-III، C-I، D-II

(٤) A-III، B-IV، C-I، D-II

Ans. (٤)

Sol. sp^3 □ Tetrahedral

dsp^2 □ Square planar

sp^3d □ Trigonal Bipyramidal

sp^3d^2 □ Octahedral

٦٧. Given below are two statements :

Statement I : Gallium is used in the manufacturing of thermometers.

Statement II : A thermometer containing gallium is useful for measuring the freezing point (٢٥٦ K) of brine solution.

In the light of the above statement, choose the correct answer from the options given below :

(١) Both Statement I and Statement II are false.

(٢) Statement I is false but Statement II is true.

(٣) Both Statement I and Statement II are true.

(٤) Statement I is true but Statement II is false.

Ans. (٤)

Sol. Statement - I → Correct

Statement - II → False

Ga is used to measure high temperature

٦٨. Which of the following statements are correct?

A. Glycerol is purified by vacuum distillation because it decomposes at its normal boiling point.

B. Aniline can be purified by steam distillation as aniline is miscible in water.

C. Ethanol can be separated from ethanol water mixture by azeotropic distillation because it forms azeotrope.

D. An organic compound is pure, if mixed M.P. is remained same.

Choose the most appropriate answer from the options given below : (١) A, B, C only (٢) A, C, D only (٣) B, C, D only (٤) A, B, D only

Ans. (٢)

Sol. Option (B) is incorrect because aniline is immisible in water.

٦٩. Match List I with List II :

LIST I (Compound / Species)		LIST II (Shape / Geometry)	
A.	SF٤	I.	Tetrahedral
B.	BrF٣	II. III.	Pyramidal See saw
C.	BrO٣⁻	IV.	Bent T-shape
D.	NH٤⁺		

Choose the correct answer from the options given below :

(١) A-II, B-III, C-I, D-IV

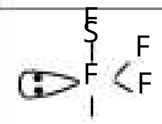
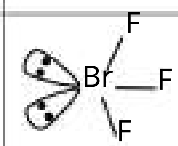
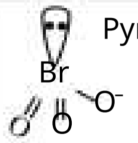
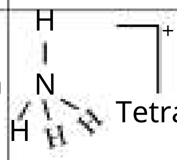
(٢) A-III, B-IV, C-II, D-I

(٣) A-II, B-IV, C-III, D-I

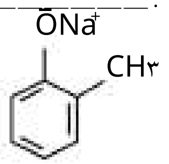
(٤) A-III, B-II, C-IV, D-I

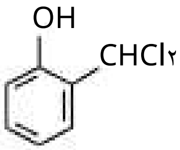
Ans. (٢)

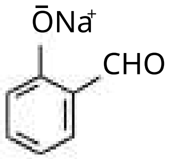
Sol.

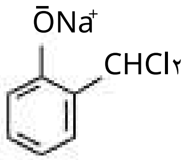
(A)	SF٤	$sp^{3}d$ hybridisation	
(B)	BrF٣	$sp^{3}d$ hybridisation	 Bent T-Shape
(C)	BrO٣⁻	sp^{3} hybridisation	 Pyramidal
(D)	NH٤⁺	sp^{3} hybridisation	 Tetrahedral

70. In Reimer – Tiemann reaction, phenol is converted into salicylaldehyde through an intermediate. The structure of intermediate is __________.

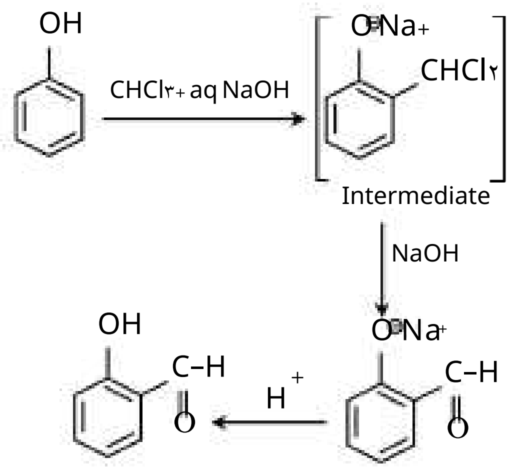
(1) 

(2) 

(3) 

(4) 

Ans. (4)

Sol. 

71. Which of the following material is not a semiconductor.

(1) Germanium

(2) Graphite

(3) Silicon

(4) Copper oxide

Ans. (2)

Sol. Graphite is conductor

72. Consider the following complexes.

$[CoCl(NH_3)_5]^{2+}$, $[Co(CN)_6]^{3-}$,

(A) (B)

$[Co(NH_3)_5(H_2O)]^{3+}$, $[Cu(H_2O)_4]^{2+}$

(C) (D)

The correct order of A, B, C and D in terms of wavenumber of light absorbed is :

(1) C > D > A > B

(2) D > A > C > B

(3) A > C > B > D

(4) B > C > A > D

Ans. (2)

Sol. As ligand field increases, light of more energy is absorbed

Energy ∝ wave number

73. Match List I with List II :

	LIST I (Precipitating reagent and conditions)		LIST II (Cation)
A.	$NH_4Cl + NH_4OH$	I.	Mn^{2+}
B.	$NH_4OH + Na_2CO_3$	II.	Pb^{2+}
C.	$NH_4OH + NH_4Cl + H_2S$ gas	III.	Al^{3+}
D.	dilute HCl	IV.	Sr^{2+}

Choose the correct answer from the options given below :

(1) A-IV, B-III, C-II, D-I

(2) A-IV, B-III, C-I, D-II

(3) A-III, B-IV, C-I, D-II

(4) A-III, B-IV, C-II, D-I

Ans. (3)

Sol. Theory based question

٧٤. The electron affinity value are negative for :

A. Be □ Be^-

B. N □ N^-

C. O □ $O^{٢-}$

D. Na □ Na^-

E. Al □ Al^-

Choose the most appropriate answer from the options given below :

(١) D and E only (٢) A، B، D and E only

(٣) A and D only (٤) A، B and C only

Allen Ans. (٤)

NTA Ans. (١)

Sol. (A) Be + e □ Be^- ، E.A = –ive

(B) N + e^- □ N^- E.A = –ive

(C) O + e^- □ O^- E.A = –ive

O^- + e □ $O^{-٢}$ E.A = +ive

(D) Na + e^- □ Na^- E.A = +ive

(E) A□ + e^- □ A □–

Ans. A، B and C only

٧٥. The number of element from the following that do not belong to lanthanoids is :

Eu، Cm، Er، Tb، Yb and Lu

(١) ٣ (٢) ٤

(٣) ١ (٤) ٥

Ans. (٣)

Sol. Cm is Actinide

٧٦. The density of 'x' M solution ('x' molar) of NaOH is ١.١٢ g $mL^{-١}$ while in molality، the concentration of the solution is ٣ m (٣ molal). Then x is

(Given : Molar mass of NaOH is ٤٠ g /mol)

(١) ٣.٥ (٢) ٣.٠

(٣) ٣.٨ (٤) ٢.٨

Ans. (٢)

Sol. Molality = $\dfrac{١٠٠٠ □ M}{١٠٠٠ □ d □ M □ (Mw)_{solute}}$

$٣ = \dfrac{١٠٠٠ □ x}{١٠٠٠ □ ١.١٢ □ (x □ ٤٠)}$

x = ٣

٧٧. Which among the following aldehydes is most reactive towards nucleophilic addition reactions؟

(١) $H-\overset{\overset{O}{\|}}{C}-H$ (٢) $C_٢H_٥-\overset{\overset{O}{\|}}{C}-H$

(٣) $CH_٣-\overset{\overset{O}{\|}}{C}-H$ (٤) $C_٣H_٧-\overset{\overset{O}{\|}}{C}-H$

Ans. (١)

Sol. $H-\overset{\overset{O}{\|}}{C}-H$ has low steric hindrance at carbonyl carbon and high partial positive charge at carbonyl carbon.

٧٨. At –٢٠ °C and ١ atm pressure، a cylinder is filled with equal number of $H_٢$. $I_٢$ and HI molecules for the reaction

$H_٢(g) + I_٢(g) \rightleftharpoons ٢HI(g)$، the KP for the process is x × $١٠^{-١}$. x = ______________.

﴾Given : R = ٠.٠٨٢ L atm $K^{-١}$ mol﴿

(١) ٢ (٢) ١

(٣) ١٠ (٤) ٠.٠١

Ans. (٣)

Sol. □n_g = ٠ Kp = $\dfrac{(n_{HI})^٢}{n_{H_٢} \, n_{I_٢}} □ \dfrac{P_T}{n_T}□^{□ng}$

$n_{HI} □ n_{H_٢} □ n_{I_٢}$ so KP = ١

١ = x × $١٠^{-١}$ x = ١٠

٧٩. Match List I with List II :

LIST I (Compound)		LIST II (Uses)	
A.	Iodoform	I.	Fire extinguisher
B.	Carbon tetrachloride	II.	Insecticide
C.	CFC	III.	Antiseptic
D.	DDT	IV.	Refrigerants

Choose the correct answer from the options given below :

(١) A-I، B-II، C-III، D-IV

(٢) A-III، B-II، C-IV، D-I

(٣) A-III، B-I، C-IV، D-II

(٤) A-II، B-IV، C-I، D-III

Ans. (٣)

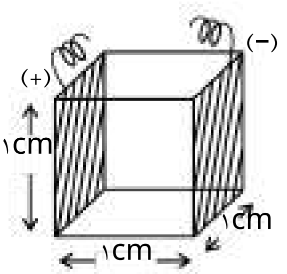
Sol. Iodoform – Antiseptic

$CCl_٤$ – Fire extinguisher

CFC – Refrigerants

DDT – Insecticide

٨٠. A conductivity cell with two electrodes (dark side) are half filled with infinitely dilute aqueous solution of a weak electrolyte. If volume is doubled by adding more water at constant temperature, the molar conductivity of the cell will –



(١) increase sharply

(٢) remain same or can not be measured accurately

(٣) decrease sharply

(٤) depend upon type of electrolyte

Ans. (٢)

Sol. Solution is already infinitely dilute, hence no change in molar conductivity upon addition of water

SECTION-B

٨١. Consider the dissociation of the weak acid HX as given below

$HX(aq) \rightleftharpoons H^+(aq) + X^-(aq)$, $K_a = ١.٢ \times ١٠^{-٥}$

[K_a: dissociation constant]

The osmotic pressure of ٠.٠٣ M aqueous solution of HX at ٣٠٠ K is ________ $\times ١٠^{-٢}$ bar (nearest integer).

[Given: R = ٠.٠٨٣ L bar $Mol^{-1} K^{-1}$]

Ans. (٧٦)

Sol. $HX \rightleftharpoons H^+ + X^-$ $K_a = ١.٢ \times ١٠^{-٥}$

٠.٠٣M

٠.٠٣ − x   x   x

$K_a = ١.٢ \times ١٠^{-٥} = \frac{x^٢}{٠.٠٣ - x}$

$٠.٠٣ - x \approx ٠.٠٣$ (Ka is very small)

$\frac{x^٢}{٠.٠٣} = ١.٢ \times ١٠^{-٥}$

$x = ٦ \times ١٠^{-٤}$

Final solution: $٠.٠٣ - x + x + x$

$= ٠.٠٣ + x = ٠.٠٣ + ٦ \times ١٠^{-٤}$

$\pi = (٠.٠٣ + (٦ \times ١٠^{-٤})) \times ٠.٠٨٣ \times ٣٠٠$

$= ٧٦.١٩ \times ١٠^{-٢} \approx ٧٦ \times ١٠^{-٢}$

٨٢. The difference in the 'spin-only' magnetic moment values of $KMnO_٤$ and the manganese product formed during titration of $KMnO_٤$ against oxalic acid in acidic medium is ________ BM. (nearest integer)

Ans. (٦)

Sol. Spin only magnetic moment of Mn in $KMnO_٤$ = ٠

Spin only value of manganese product fromed during titration of $KMnO_٤$ aganist oxalic acid in acidic medium is = ٦

Ans. ٦

٨٣. Time required for ٩٩.٩% completion of a first order reaction is ________ time the time required f completion of ٩٠% reaction. (nearest integer).

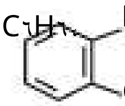
Ans. (٣)

Sol. $K = \frac{١}{t_{٩٩.٩\%}} \ell n\left(\frac{١٠٠}{٠.١}\right) = \frac{١}{t_{٩٠\%}} \ell n\left(\frac{١٠٠}{١٠}\right)$

$t_{٩٩.٩\%} = t_{٩٠\%} \frac{\ell n(١٠^٣)}{\ell n ١٠}$

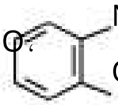
$t_{٩٩.٩\%} = t_{٩٠\%} \times ٣$

٨٤. Number of molecules from the following which can exhibit hydrogen bonding is __________. (near integer)

$CH_٣OH$, $H_٢O$, $C_٢H_٦$, $C_٦H_٦$, 

HF, $NH_٣$

Ans. (٥)

Sol. $CH_٣OH$, $H_٢O$, 

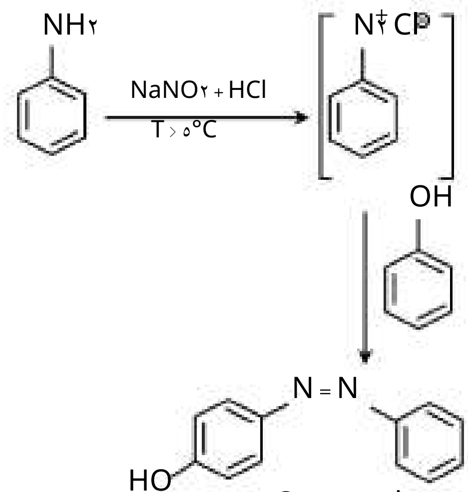
HF, $NH_٣$

Can show H-bonding.

٨٥. ٩.٣ g of pure aniline upon diazotisation followed by coupling with phenol gives an orange dye. The mass of orange dye produced (assume ١٠٠% yield/ conversion) is ________ g. (nearest integer)

Ans. (٢٠)

Sol.



Orange dye

Reaction suggests that ١ mole of aniline give ١ mole of orange dye.

so (mol)aniline = (mole)orange dye

$$\frac{9.3\text{g}}{93\text{g mol}^{-1}} = \frac{\text{mass of orange dye}}{199\text{g mol}^{-1}}$$

mass of orange dye = ١٩.٩ g ≈ ٢٠ g

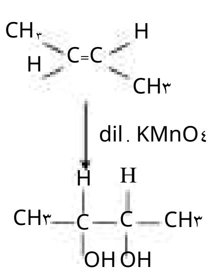
٨٦. The major product of the following reaction is P.

CH٣C ≡ C – CH٣ $\xrightarrow[\text{(ii)dil.KMnO}_4\ 273\text{K}]{\text{(i)Na/liq.NH}_3}$ 'P'

Number of oxygen atoms present in product 'P' is ________ (nearest integer).

Ans. (٢)

Sol. CH٣ – C≡C–CH٣ $\xrightarrow{\text{Na/liq.NH}_3}$



(Product P)

٨٧. Frequency of the de-Broglie wave of election in Bohr's first orbit of hydrogen atom is ________ × ١٠^١٣ Hz (nearest integer).

[Given : RH (Rydberg constant) = ٢.١٨ × ١٠^-١٨ J.

h (Plank's constant) = ٦.٦ × ١٠^-٣٤ J.s.]

Allen Ans. (٦٦١)

NTA Ans. (٦٥٨)

Sol. $\lambda = \frac{h}{mv}$

$\nu = \frac{hv}{mv^2}$

$\frac{mv^2}{h} = \frac{v}{\lambda} = \nu$ (frequency)

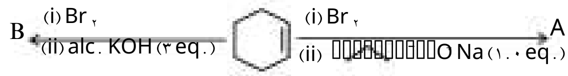
Given $\frac{1}{2}mv^2 = 2.18 \times 10^{-18}$ J

$h = 6.6 \times 10^{-34}$

$\nu = \frac{4.36 \times 10^{-18}}{6.6 \times 10^{-34}} = 660.60 \times 10^{13}$ Hz

$\simeq 661 \times 10^{13}$ Hz

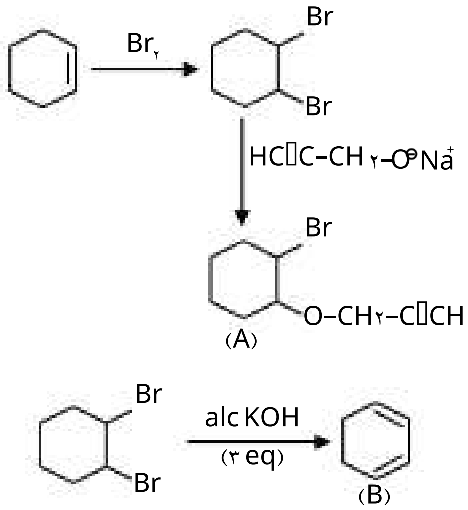
٨٨. The major products from the following reaction sequence are product A and product B.



The total sum of π electrons in product A and product B are ________ (nearest integer)

Ans. (٨)

Sol.



٨٩. Among CrO، $Cr_٢O_٣$ and $CrO_٣$، the sum of spin-only magnetic moment values of basic and amphoteric oxides is ________ $\times ١٠^{-٢}$ BM (nearest integer).
(Given atomic number of Cr is ٢٤)

Ans. (٨٧٧)

Sol. CrO Basic oxide

$Cr_٢O_٣$ Amphoteric oxide

In CrO، Cr exist as $Cr^{+٢}$ and have μ only = ٤.٩٠

In $Cr_٢O_٣$، Cr exist as $Cr^{+٣}$ and have μ only = ٣.٨٧

Sum of spin only magnetic moment

= ٤.٩٠ + ٣.٨٧ = ٨.٧٧

μonly = ٨٧٧ × $١٠^{-٢}$

Ans. ٨٧٧

٩٠. An ideal gas، $\left(C_V = \frac{٥}{٢}R\right)$، is expanded adiabatically against a constant pressure of ١ atm untill it doubles in volume. If the initial temperature and pressure is ٢٩٨ K and ٥ atm، respectively then the final temperature is __________ K (nearest integer).
[C_V is the molar heat capacity at constant volume]

Ans. (٢٧٤)

Sol. $\Delta U = q + w\ (q = ٠)$

$nC_V\Delta T = -P_{ext}(V_٢ - V_١)$

$V_٢ = ٢V_١$

$$\frac{nRT_٢}{P_٢} = \frac{٢nRT_١}{P_١}$$

$P_١ = ٥،\ T_١ = ٢٩٨$

$$P_٢ = \frac{٥T_٢}{٢ \times ٢٩٨}$$

$$n\frac{٥}{٢}R(T_٢ - T_١) = -١\left(\frac{nRT_٢}{P_٢} - \frac{nRT_١}{P_١}\right)$$

Put $T_١ = ٢٩٨$

and $P_٢ = \frac{٥T_٢}{٢ \times ٢٩٨}$

Solve and we get $T_٢ = ٢٧٤.١٦$ K

$T_٢ \approx ٢٧٤$ K