FINAL JEE–MAIN EXAMINATION – APRIL, 2024

(Held On Monday 08 April, 2024) TIME : 9 : 00 AM to 12 : 00 NOON

M ATHEM ATICS

TEST PAPER WITH SOLUTION

SECTION-A

١. The value of k □□□ for which the integral

$I_n = \int_٠^١ (١ - x^k)^n dx$ ، n □□□ ، satisfies ١٤٧ $I_{٢٠}$ = ١٤٨ $I_{٢١}$

is : (١) ١٠

(٣) ١٤ (٢) ٨

Ans. (٤) (٤) ٧

Sol. $I_n = \int_٠^١ (١ □ x^k)^n . ١ dx$

I_n = (١ − x^k)^n . x − nk□ (١ □ x^k)$^{n□١}$. $x^{k□١}$. dx

I_n = nk □ (١ □ x^k □ ١) x^k)$^{n□١}$ dx

I_n = nkI_n − nkI_n

$\frac{I_n}{I_{n□١}}$ □ $\frac{nk}{nk □ ١}$

$\frac{I_{٢١}}{I_{٢٠}}$ □ $\frac{٢١k}{١ □ ٢١k}$

□ $\frac{١٤٧}{١٤٨}$ □□□ k = ٧

٢. The sum of all the solutions of the equation

(٨)^{٢x} − ١٦□ (٨)^x + ٤٨ = ٠ is :

(١) ١ + log٦(٨) (٢) log٨(٦)

(٣) ١ + log٨(٦) (٤) log٨(٤)

Ans. (٣)

Sol. (٨)^{٢x} − ١٦□ (٨)^x + ٤٨ = ٠

Put ٨^x = t

t^٢ − ١٦ + ٤٨ = ٠

□t = ٤ or t = ١٢

□٨ = ٤ ٨^x = ١٢

□x = log٨x x = log٨١٢

sum of solution = log٨٤ + log٨١٢

= log٨٤٨ = log٨(٦.٨)

= ١ + log٨٦

٣. Let the circles C١ : (x − □)^٢ + (y − □)^٢ = r$^٢_١$ and

C٢ : (x − ٨)^٢ + □y □ $\frac{١٥}{2}$□^{٢} = r٢^٢ touch each other

externally at the point (٦، ٦). If the point (٦، ٦)

divides the line segment joining the centres of the

circles C١ and C٢ internally in the ratio ٢ : ١، then

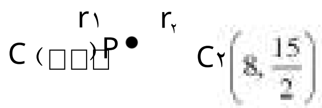
(□ + □) + ٤ □r١^2 □ r٢^2□ equals

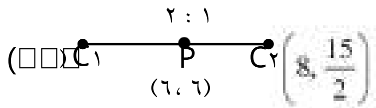
(١) ١١٠ (٢) ١٣٠

(٣) ١٢٥ (٤) ١٤٥

Ans. (٢)

Sol.





∴ $\frac{١٦ □ □}{٣}$ □ ٦ and $\frac{١٥ □ □}{٣}$ □ 6

□(□، □) □ (٢، ٣)

Also، C١C٢ = r١ + r٢

□□ $\sqrt{(٢ □ ٨)^٢ □ □٣ □ \frac{١٥}{٢}□^٢}$ □ ٢r٢ □ r٢

□r٢ = $\frac{٥}{٢}$ □r١ = ٢r٢ = ٥

□(□□□□□) + ٤(r٢١ □ r٢٢)

= ٥ + ٤ □$\frac{٢٥}{٤}$ □ ٢٥ □ = ١٣٠□

٤. Let P(x, y, z) be a point in the first octant, whose projection in the xy-plane is the point Q. Let OP = □; the angle between OQ and the positive x-axis be □; and the angle between OP and the positive z-axis be □, where O is the origin. Then the distance of P from the x-axis is :

(١) □ $\sqrt{١□\sin^٢□\cos^٢□}$ (٢) □ $\sqrt{١□\cos^٢□\sin^٢□}$

(٣) □ $\sqrt{١□\sin^٢□\cos^٢□}$ (٤) □ $\sqrt{١□\cos^٢□\sin^٢□}$

Ans. (١)

Sol. P(x, y, z), Q(x, y, O) ; $x^٢ + y^٢ + z^٢ = □^٢$

$\vec{OQ} □ x\hat{i} □ y\hat{j}$

$\cos□ = \dfrac{x}{\sqrt{x^٢ □ y^٢}}$

$\cos□ = \dfrac{z}{\sqrt{x^٢ □ y^٢ □ z^٢}}$

$□□\sin□□ = \dfrac{x^٢ □ y^٢}{x^٢ □ y^٢ □ z^٢}$

distance of P from x-axis $\sqrt{y^٢ □ z^٢}$

$□□□2□x2 \; □□ \; □1□\sqrt{\dfrac{x^٢}{□2}}$

$= □\sqrt{١□\cos^٢□\sin^٢□□}$

٥. The number of critical points of the function $f(x) = (x - ٢)^{٢/٣}(٢x + ١)$ is :

(١) ٢ (٢) ٠

(٣) ١ (٤) ٣

Ans. (١)

Sol. $f(x) = (x - ٢)^{٢/٣}(٢x + ١)$

$f'(x) = \dfrac{٢}{٣}(x - ٢)^{-١/٣}(٢x + ١) + (x - ٢)^{٢/٣}(٢)$

$f'(x) = ٢ \times \dfrac{(٢x□١)□(x □٢)}{٣(x□٢)^{١/٣}}$

$\dfrac{٣x□١}{(x□٢)^{١/٣}} □ ٠$

Critical points $x = \dfrac{١}{٣}$ and $x = ٢$

٦. Let f(x) be a positive function such that the area bounded by y = f(x), y = ٠ from x = ٠ to x = a is e^{-a} ٤a + a − ١. Then the differential equation, whose general solution is $y = c_١f(x) + c_٢$, where $c_١$ and $c_٢$ are arbitrary constants, is :

(١) $(٨e^x□١)\dfrac{d^٢y}{dx^٢} □ \dfrac{dy}{dx} □ ٠$

(٢) $(٨e^x□١)\dfrac{d^٢y}{dx^٢} □ \dfrac{dy}{dx} □ ٠$

(٣) $(٨e^x□١)\dfrac{d^٢y}{dx^٢} □ \dfrac{dy}{dx} □ ٠$

(٤) $(٨e^x□١)\dfrac{d^٢y}{dx^٢} □ \dfrac{dy}{dx} □ ٠$

Ans. (٣)

Sol. $\int_٠^a f(x)dx □ e^{-a} □ ٤a^٢ □ a □ 1$

$f(a) = -e^{-a} + ٨a + ١$; $f(x) = -e^{-x} + ٨x + ١$

Now $y = C_١f(x) + C_٢$

$\dfrac{dy}{dx} □ Cf'(x) = C_١(e^{-x} + ٨)$ ......(١)

$\dfrac{d^٢y}{dx^٢} = -Ce^{-x} □ -e^x\dfrac{d^٢y}{dx^٢}$

Put in equation (١)

$\dfrac{dy}{dx} □ □e^x\dfrac{d^٢y}{dx^٢}(e^{□x} □ ٨)$

$(٨e^x□١)\dfrac{d^٢y}{dx^٢} □ \dfrac{dy}{dx} □ ٠$

٧. Let $f(x) = ٤\cos^٣ x + ٣٣\sqrt{\;}\cos^٢ x - ١٠$. The number of points of local maxima of f in interval $(٠, ٢□)$ is:

(١) ١ (٢) ٢

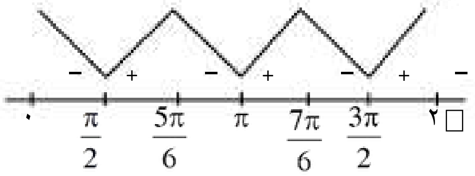
(٣) ٣ (٤) ٤

Ans. (٢)

Sol. $f(x) = ٤\cos^٣(x) + ٣٣\cos^٢(x) - ١٠, x □ (٠, ٢□)$

□□$f'(x) = ١٢\cos^٢ x(-\sin(x)) + ٣\sqrt{٣}(٢\cos(x))(-\sin(x))$

□□$f'(x) = -٦\sin(x)\cos(x)(٢\cos(x) + \sqrt{٣})$



local maxima at $x = \frac{5□}{٦}, \frac{7□}{٦}$

٨. Let $A = \begin{bmatrix} ٢ & a & 0 \\ ١ & ٣ & 1 \\ ٠ & ٥ & b \end{bmatrix}$. If $A^٣ = ٤A^٢ - A - ٢١I$, where b□□

I is the identity matrix of order ٣ × ٣, then ٢a + ٣b is equal to:

(١) −١٠ (٢) −١٣

(٣) −٩ (٤) −١٢

Ans. (٢)

Sol. $A^٣ - ٤A^٢ + A + ٢١I = ٠$

$tr(A) = ٤ = ٥ + ٦$ □□ $b = -١$

$|A| = -٢١$

$-١٦ + a = -٢١$ □ $a = -٥$

$٢a + ٣b = -١٣$

٩. If the shortest distance between the lines

$L_١ : r□(٢□□)i^□(١□٣□)j^□(٣□٤□)k^, □□$ ℝ

$L_٢ : r□٢(١□□)i^□٣(١□□)j^□(٥□□)k^, □□$ ℝ

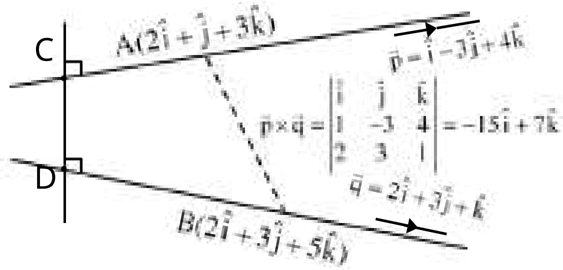
is $\frac{m}{\sqrt{n}}$, where gcd (m, n) = ١, then the value of m + n equals.

(١) ٣٨٤ (٢) ٣٨٧

(٣) ٣٧٧ (٤) ٣٩٠

Ans. (٢)

Sol.



Shortes distance (CD) $= \left| \frac{AB □ p □ q}{|p □ q|} \right|$

$= \left| \frac{(٠i^ □ ٢j^ □ ٢k^).(□١٥i^□٧j^□٩k^)}{\sqrt{٣٥٥}} \right|$

$= \frac{0 □١٤ □١٨}{\sqrt{٣٥٥}} = \frac{٣٢}{\sqrt{٣٥٥}}$

□ $m + n = ٣٢ + ٣٥٥ = ٣٨٧$

١٠. Let the sum of two positive integers be ٢٤. If the probability, that their product is not less than $\frac{٣}{٤}$ times their greatest positive product, is $\frac{m}{n}$, where gcd(m, n) = ١, then n − m equals:

(١) ٩ (٢) ١١

(٣) ٨ (٤) ١٠

Ans. (٤)

Sol. $x + y = ٢٤, x, y □□ N$

AM ≥ GM □□ $xy □ ١٤٤$

$xy □ ١٠٨$

Favorable pairs of (x, y) are

(١٣, ١١), (١٢, ١٢), (١٤, ١٠), (١٥, ٩), (١٦, ٨),

(١٧, ٧), (١٨, ٦), (٦, ١٨), (٧, ١٧), (٨, ١٦), (٩, ١٥),

(١٠, ١٤), (١١, ١٣)

i.e. ١٣ cases

Total choices for x + y = ٢٤ is ٢٣

Probability $= \frac{١٣}{٢٣} □ \frac{m}{n}$

$n - m = ١٠$

١١. If $\sin x = -\frac{٣}{٥}$ ، where $\pi > x > \frac{٣\pi}{٢}$ ،

then $٨٠(\tan^2 x - \cos x)$ is equal to :

(١) ١٠٩ (٢) ١٠٨

(٣) ١٨ (٤) ١٩

Ans. (١)

Sol. $\sin x = \frac{-٣}{٥}$ ، $\pi > x > \frac{٣\pi}{٢}$

$\tan x = \frac{٣}{٤}$ $\cos x = -\frac{٤}{٥}$

$٨٠(\tan^2 x - \cos x)$

$= ٨٠\left(\frac{9}{١٦} + \frac{٤}{٥}\right) = ٤٥ + ٦٤ = ١٠٩$

١٢. Let $I(x) = \int \frac{٦}{\sin^٢ x(١ - \cot x)^٢} dx$. If $I(٠) = ٣$ ، then

$I\left(\frac{\pi}{12}\right)$ is equal to :

(١) $\sqrt{٣}$ (٢) $٣\sqrt{٣}$

(٣) $٦\sqrt{٣}$ (٤) $٢\sqrt{٣}$

Ans. (٢)

Sol. $I(x) = \int \frac{٦dx}{\sin^٢ x(١ - \cot x)^٢} = \int \frac{٦\operatorname{cosec}^٢ x dx}{(١ - \cot x)^٢}$

Put $١ - \cot x = t$

$\operatorname{cosec}^2 x \, dx = dt$

$I = \int \frac{٦dt}{t^٢} = \frac{-6}{t} + c$

$I(x) = \frac{-٦}{١ - \cot x} + c$ ، $c = ٣$

$I(x) = ٣ - \frac{٦}{١ - \cot x}$ ، $I\left(\frac{\pi}{١٢}\right) = 3 - \frac{٦}{1 - (٢ + \sqrt{٣})}$

$I\left(\frac{\pi}{١٢}\right) = 3 + \frac{٦}{\sqrt{٣} + ١} = 3 + \frac{٦(\sqrt{٣} - ١)}{٢} = ٣\sqrt{٣}$

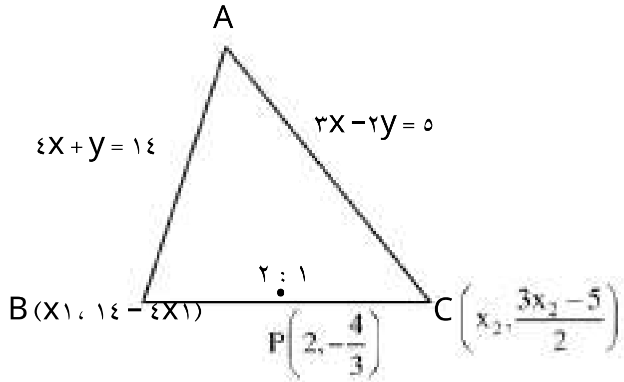
١٣. The equations of two sides AB and AC of a triangle ABC are $٤x + y = ١٤$ and $٣x - ٢y = ٥$ ، respectively. The point $\left(٢, -\frac{4}{3}\right)$ divides the third side BC internally in the ratio ٢ : ١. The equation of the side BC is :

(١) $x - ٦y - ١٠ = ٠$ (٢) $x - ٣y - ٦ = ٠$

(٣) $x + ٣y + ٢ = ٠$ (٤) $x + ٦y + ٦ = ٠$

Ans. (٣)

Sol.



$\frac{٢x_٢ + x_١}{٣} = ٢$ ، $\frac{2\left(\frac{٣x_2 - 5}{٢}\right) + (١٤ - ٤x_١)}{٣} = \frac{-4}{٣}$

$٢x_٢ + x_١ = ٦$ ، $٣x_٢ - ٤x_١ = -١٣$

$x_٢ = ١$ ، $x_١ = ٤$

So، $C(١ ، -١)$ ، $B(٤ ، -٢)$

$m = \frac{-١}{٣}$

Equation of BC : $y + ١ = \frac{-1}{٣}(x - ١)$

$٣y + ٣ = -x + ١$

$x + ٣y + ٢ = ٠$

١٤. Let $[t]$ be the greatest integer less than or equal to t. Let A be the set of al prime factors of ٢٣١٠ and $f : A \to \mathbb{N}$ be the function $f(x) = \left[\log_2\left(x^2 + \left[\frac{x^3}{5}\right]\right)\right]$.

The number of one-to-one functions from A to the range of f is :

(١) ٢٠
(٢) ١٢٠
(٣) ٢٥
(٤) ٢٤

Ans. (٢)

Sol. $N = ٢٣١٠ = ٢٣١ \times ١٠$

$= ٣ \times ١١ \times ٧ \times ٢ \times ٥$

$A = \{٢, ٣, ٥, ٧, ١١\}$

$f(x) = \left[\log_2\left(x^2 + \left[\frac{x^3}{5}\right]\right)\right]$

$f(٢) = [\log_2(٥)] = ٢$

$f(٣) = [\log_2(١٤)] = ٣$

$f(٥) = [\log_2(٢٥ + ٢٥)] = ٥$

$f(٧) = [\log_2(١١٧)] = ٦$

$f(١١) = [\log_2 ٣٨٧] = ٨$

Range of f : $B = \{٢, ٣, ٥, ٦, ٨\}$

No. of one-one functions $= ٥! = ١٢٠$

١٥. Let z be a complex number such that $|z + ٢| = ١$ and $\operatorname{Im}\left(\frac{z+١}{z+2}\right) = \frac{١}{٥}$. Then the value of $\operatorname{Re}\left(\overline{z+٢}\right)$ is :

(١) $\frac{\sqrt{٦}}{٥}$
(٢) $\frac{١+\sqrt{٦}}{٥}$
(٣) $\frac{٢٤}{٥}$
(٤) $\frac{٢\sqrt{٦}}{٥}$

Ans. (٤)

Sol. $|z + ٢| = ١$, $\operatorname{Im}\left(\frac{z+١}{z+٢}\right) = \frac{١}{٥}$

Let $z + ٢ = \cos\theta + i\sin\theta$

$\frac{١}{z+٢} = \cos\theta - i\sin\theta$

$\Rightarrow \frac{z+١}{z+٢} = 1 - \frac{١}{z+٢} = ١ - (\cos\theta - i\sin\theta)$

$= (١ - \cos\theta) + i\sin\theta$

$\operatorname{Im}\left(\frac{z+١}{z+٢}\right) = \sin\theta$, $\sin\theta = \frac{١}{٥}$

$\cos\theta = \pm\sqrt{١ - \frac{١}{٢٥}} = \pm\frac{٢\sqrt{٦}}{٥}$

$\left|\operatorname{Re}\left(\overline{z+٢}\right)\right| = \frac{٢\sqrt{٦}}{٥}$

١٦. If the set $R = \{(a, b) : a + ٥b = ٤٢, a, b \in \mathbb{N}\}$ has m elements and $\sum_{n=1}^{m}\left(1 - i^{n!}\right) = x + iy$, where $I = \sqrt{-١}$, then the value of $m + x + y$ is :

(١) ٨
(٢) ١٢
(٣) ٤
(٤) ٥

Ans. (٢)

Sol. $٥b = ٤٢ - a$, $a, b \in \mathbb{N}$

$b = ١, a = ٣٧$

$b = ٢, a = ٣٢$

$b = ٣, a = ٢٧$

$\vdots$

$b = ٨, a = ٢$

R has ٨ elements $\Rightarrow m = ٨$

$\sum_{n=1}^{8}\left(١ - i^{n!}\right) = x + iy$

for $n \geq ٤$, $i^{n!} = ١$

$\Rightarrow (١ - i) + (١ - i^{٢!}) + (١ - i^{٣!})$

$= ١ - I + ٢ + ١ + ١$

$= ٥ - I = x + iy$

$m + x + y = ٨ + ٥ - ١ = ١٢$

١٧. For the function $f(x) = (\cos x) - x + ١,\ x \in \mathbb{R}$, between the following two statements
$(S١)$ $f(x) = ٠$ for only one value of x is $[٠, □]$.
$(S٢)$ $f(x)$ is decreasing in $\left[٠, \frac{□}{2}\right]$ and increasing in $\left[\frac{□}{2}, □\right]$.

(١) Both $(S١)$ and $(S٢)$ are correct
(٢) Only $(S١)$ is correct
(٣) Both $(S١)$ and $(S٢)$ are incorrect
(٤) Only $(S٢)$ is correct

Ans. (٢)

Sol. $f(x) = \cos x - x + ١$
$f'(x) = -\sin x - ١$
f is decreasing $\forall x \in R$
$f(x) = ٠$
$f(٠) = ٢,\ f(□) = -□$
f is strictly decreasing in $[٠, □]$ and $f(٠) . f(□) < ٠$
$\therefore$ only one solution of $f(x) = ٠$
$S١$ is correct and $S٢$ is incorrect.

١٨. The set of all $□$, for which the vector $\vec{a} = □ t\hat{i} + ٦\hat{j} - ٣\hat{k}$ and $\vec{b} = t\hat{i} - ٢\hat{j} - ٢□ t\hat{k}$ are inclined at an obtuse angle for all $t \in R$ is:

(١) $[٠, ١)$ (٢) $(-٢, ٠]$

(٣) $\left[-\frac{٤}{٣}, ٠\right]$ (٤) $\left[-\frac{٤}{٣}, ١\right)$

Ans. (٣)

Sol. $\vec{a} = □ t\hat{i} + ٦\hat{j} - ٣\hat{k}$
$\vec{b} = t\hat{i} - ٢\hat{j} - ٢□ t\hat{k}$
so $\vec{a}.\vec{b} < ٠,\ \forall t \in R$
$\Rightarrow □ t^٢ - ١٢ + ٦□ t < ٠$
$\Rightarrow □ t^٢ + ٦□ t - ١٢ < ٠,\ \forall t \in R$
$□ < ٠$ and $D < ٠$
$٣٦□^٢ + ٤٨□ < ٠$
$١٢□(٣□ + ٤) < ٠$
$-\frac{٤}{٣} < □ < 0$
also for $a = ٠,\ \vec{a}.\vec{b} < ٠$
hence $a \in \left[-\frac{٤}{٣}, ٠\right]$

١٩. Let $y = y(x)$ be the solution of the differential equation $(١ + y^٢)e^{\tan x}dx + \cos^٢ x(١ + e^{٢\tan x})dy = ٠$, $y(٠) = ١$. Then $y\left(\frac{□}{٤}\right)$ is equal to:

(١) $\frac{٢}{e}$ (٢) $\frac{١}{e^٢}$

(٣) $\frac{١}{e}$ (٤) $\frac{٢}{e^٢}$

Ans. (٣)

Sol. $(١ + y^٢)e^{\tan x}dx + \cos^٢ x(١ + e^{٢\tan x})dy = ٠$

$\int \frac{\sec^٢ x\, e^{\tan x}}{١ + e^{٢\tan x}}dx + \int \frac{dy}{١ + y^٢} = C$

$\Rightarrow \tan^{-١}(e^{\tan x}) + \tan^{-١} y = C$

for $x = ٠,\ y = ١,\ \tan^{-١}(١) + \tan^{-١} ١ = C$

$C = \frac{□}{٢}$

$\tan^{-١}(e^{\tan x}) + \tan^{-١} y = \frac{□}{٢}$

Put $x = □,\ \tan^{-١} e + \tan^{-١} y = \frac{□}{٢}$

$\tan^{-١} y = \cot^{-١} e$

$y = \frac{١}{e}$

٢٠. Let $H: -\frac{x^٢}{a^٢} + \frac{y^٢}{b^٢} = ١$ be the hyperbola, whose eccentricity is $\sqrt{٣}$ and the length of the latus rectum is $٤\sqrt{٣}$. Suppose the point $(□, ٦), □ > ٠$ lies on H. If $□$ is the product of the focal distances of the point $(□, ٦)$, then $□ + □$ is equal to:

(١) ١٧٠ (٢) ١٧١
(٣) ١٦٩ (٤) ١٧٢

Ans. (٢)

Sol. H : $\frac{y^2}{b^2} - \frac{x^2}{a^2} = 1$, $e = \sqrt{3}$

$e = \sqrt{1 + \frac{a^2}{b^2}} = \sqrt{3} \Rightarrow \frac{a^2}{b^2} = 2$

$a^2 = 2b^2$

length of L.R. $= \frac{2a^2}{b} = 4\sqrt{3}$

$a = \sqrt{6}$

$P(\alpha, 6)$ lie on $\frac{y^2}{3} - \frac{x^2}{6} = 1$

$12 - \frac{\alpha^2}{6} = 1 \Rightarrow \alpha^2 = 66$

Foci $= (0, \pm be) = (0, 3)$ & $(0, -3)$

Let d_1 & d_2 be focal distances of $P(\alpha, 6)$

$d_1 = \sqrt{\alpha^2 + (6 - be)^2}$, $d_2 = \sqrt{\alpha^2 + (6 + be)^2}$

$d_1 = \sqrt{66 + 81}$, $d_2 = \sqrt{66 + 9}$

$\beta = d_1 d_2 = \sqrt{147 \times 75} = 105$

$\alpha^2 + \beta = 66 + 105 = 171$

SECTION-B

21. Let $A = \begin{bmatrix} 2 & -1 \\ 1 & 1 \end{bmatrix}$. If the sum of the diagonal elements of A^{13} is 3^n, then n is equal to ________.

Ans. (7)

Sol. $A = \begin{bmatrix} 2 & -1 \\ 1 & 1 \end{bmatrix}$

$A^2 = \begin{bmatrix} 2 & -1 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} 2 & -1 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 3 & -3 \\ 3 & 0 \end{bmatrix}$

$A^3 = \begin{bmatrix} 3 & -3 \\ 3 & 0 \end{bmatrix}\begin{bmatrix} 2 & -1 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 3 & -6 \\ 6 & -3 \end{bmatrix}$

$A^4 = \begin{bmatrix} 3 & -6 \\ 6 & -3 \end{bmatrix}\begin{bmatrix} 2 & -1 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 0 & -9 \\ 9 & -9 \end{bmatrix}$

$A^5 = \begin{bmatrix} 0 & -9 \\ 9 & -9 \end{bmatrix}\begin{bmatrix} 2 & -1 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} -9 & -9 \\ 9 & -18 \end{bmatrix}$

$A^6 = \begin{bmatrix} -9 & -9 \\ 9 & -18 \end{bmatrix}\begin{bmatrix} 2 & -1 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} -27 & 0 \\ 0 & -27 \end{bmatrix}$

$A^7 = \begin{bmatrix} -27 & 0 \\ 0 & -27 \end{bmatrix}\begin{bmatrix} 2 & -1 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} -54 & 27 \\ -27 & -27 \end{bmatrix}$

$3^7 = 3^n \Rightarrow n = 7$

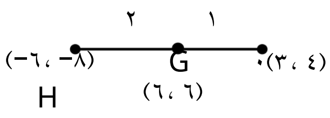
22. If the orthocentre of the triangle formed by the lines $2x + 3y - 1 = 0$, $x + 2y - 1 = 0$ and $ax + by - 1 = 0$, is the centroid of another triangle, whose circumecentre and orthocentre respectively are $(3, 4)$ and $(-6, -8)$, then the value of $|a - b|$ is ________.

Ans. (16)

Sol. $2x + 3y - 1 = 0$

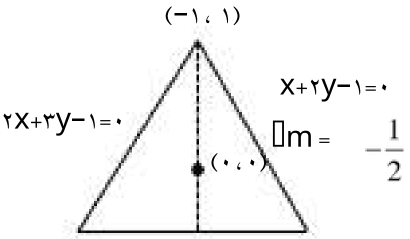
$x + 2y - 1 = 0$

$ax + by - 1 = 0$



$\left(\frac{6 - 6}{3}, \frac{8 - 8}{3}\right)$

$= (0, 0)$



$ax + by - ١ = ٠$

$\left(\frac{١ - ٠}{-١ - ٠}\right)\left(-\frac{a}{b}\right) = -1$

$\Rightarrow -a = b$

$ax - ay - ١ = ٠$

$ax - a\left(\frac{١ - ٢x}{3}\right) = ١$

$x\left(a + \frac{٢a}{3}\right) = ١ + \frac{a}{٣}$

$x = \frac{a + ٣}{٥a}$

$2\left(\frac{a + 3}{٥a}\right) + ٣y = 1 - 0$

$y = \frac{1 - \frac{٢a + ٦}{٥a}}{٣} = \frac{٣a - ٦}{٣ \times ٥a}$

$y = \frac{a - ٢}{٥a}$

$\frac{\left(\frac{a - ٢}{٥a}\right)}{\left(\frac{a + ٣}{٥a}\right)} = 2 \Rightarrow a - ٢ = ٢a + ٦$

$a = -٨$

$b = ٨$

$-٨x + ٨y - ١ = ٠$

$|a - b| = ١٦$

٢٣. Three balls are drawn at random from a bag containing ٥ blue and ٤ yellow balls. Let the random variables X and Y respectively denote the number of blue and Yellow balls. If $\bar{X}$ and $\bar{Y}$ are the means of X and Y respectively، then $٧\bar{X} + ٤\bar{Y}$ is equal to ________. Ans. (١٧)

Sol.

Blue balls	٠	١	٢	٣	٤	٥
Prob.	$\frac{^٥C_٠\,^٤C_٣}{^٩C_٣}$	$\frac{^٥C_١\,^٤C_٢}{^٩C_٣}$	$\frac{^٥C_٢\,^٤C_١}{^٩C_٣}$	$\frac{^٥C_٣\,^٤C_٠}{^٩C_٣}$	٠	٠

$٧\bar{X} = \frac{^٥C_١\,^٤C_٢ + ٢ \times {}^٥C_٢\,^٤C_١ + 3 \times {}^٥C_٣\,^٤C_٠}{^٩C_٣} \times 7$

$\frac{٣٠ + ٨٠ + ٣٠}{٨٤} \times 7$

$= \frac{١٤٠}{١٢} = \frac{٧٠}{٦} = \frac{٣٥}{٣}$

yellow	٠	١	٢	٣	٤
		$^٥C_٢\,^٤C_١$	$^٥C_١\,^٤C_٢$	$^٥C_٠\,^٤C_٣$	٠

$4\bar{y} = \frac{٤٠ + ٦٠ + ١٢}{٨٤} \times 4 = \frac{١١٢}{٢١} = \frac{١٦}{٣}$

٢٤. The number of ٣-digit numbers، formed using the digits ٢، ٣، ٤، ٥ and ٧، when the repetition of digits is not allowed، and which are not divisible by ٣، is equal to ________. Ans. (٣٦)

Sol. ٢، ٣، ٤، ٥، ٧

total number of three digit numbers not divisible by ٣ will be formed by using the digits

(٤، ٥، ٧)

(٣، ٤، ٧)

(٢، ٥، ٧)

(٢، ٤، ٧)

(٢، ٤، ٥)

(٢، ٣، ٥)

number of ways = ٦ × ٣! = ٣٦

٢٥. Let the positive integers be written in the form :

$$\begin{array}{ccccccc} & & & ١ & & & \\ & & ٢ & & ٣ & & \\ & ٤ & & ٥ & & ٦ & \\ ٧ & & ٨ & & ٩ & & ١٠ \\ \vdots & & & & & & \vdots \end{array}$$

If the k^{th} row contains exactly k numbers for every natural number k، then the row in which the number ٥٣١٠ will be، is __________.

Ans. (١٠٣)

Sol. $S = ١ + ٢ + ٤ + ٧ + \ldots + T_n$

$S = ١ + ٢ + ٤ + \ldots$

$T_n = ١ + ١ + ٢ + ٣ + \ldots + (T_n - T_{n-١})$

$T_n = ١ + \left(\frac{n-1}{٢}\right)\{٢ + (n-٢)١\}$

$T_n = ١ + ١ \times \frac{n(n-١)}{٢}$

$n = ١٠٠ \quad T_n = ١ + \frac{١٠٠ \times ٩٩}{٢} = ٤٩٥٠ + ١$

$n = ١٠١ \quad T_n = ١ + \frac{١٠١ \times ١٠٠}{٢} = ٥٠٥٠ + ١ = ٥٠٥١$

$n = ١٠٢ \quad T_n = ١ + \frac{١٠٢ \times ١٠١}{٢} = ٥١٥١ + ١ = ٥١٥٢$

$n = ١٠٣ \quad T_n = ١ + \frac{١٠٣ \times ١٠٢}{٢} = ٥٢٥٤$

$n = ١٠٤ \quad T_n = ١ + \frac{١٠٤ \times ١٠٣}{٢} = ٥٣٥٧$

٢٦. If the range of $f(\theta) = \frac{\sin^٤\theta + ٣\cos^٢\theta}{\sin^٤\theta + \cos^٢\theta}, \theta \in \mathbb{R}$ is $[\alpha, \beta]$، then the sum of the infinite G.P.، whose first term is ٦٤ and the common ratio is $\frac{\alpha}{\beta}$، is equal to __________.

Ans. (٩٦)

Sol. $f(\theta) = \frac{\sin^٤\theta + ٣\cos^٢\theta}{\sin^٤\theta + \cos^٢\theta}$

$f(\theta) = 1 + \frac{٢\cos^٢\theta}{\sin^٤\theta + \cos^٢\theta}$

$f(\theta) = \frac{٢\cos^٢\theta}{\cos^٤\theta - \cos^٢\theta + 1} + 1$

$f(\theta) = \frac{٢}{\cos^٢\theta + \sec^٢\theta - 1} + 1$

$f(\theta)\big|_{min.} = ١$

$f(\theta)_{max.} = ٣$

$S = \frac{٦٤}{١ - ١/٣} = ٩٦$

٢٧. Let $\alpha = \sum_{r=0}^{n} (٤r^٢ + ٢r + ١)\, {}^nC_r$ and $\beta = \left(\sum_{r=0}^{n} \frac{{}^nC_r}{r+١}\right) + \frac{١}{n+١}$. If $١٤٠ < \frac{٢\alpha}{\beta} < ٢٨١$، then the value of n is __________.

Ans. (٥)

Sol. $\alpha = \sum_{r=0} (٤r^٢ + ٢r + ١).{}^nC_r$

$= ٤\sum_{r=0} r^٢ . \frac{n}{r}\, {}^{n-١}C_{r-١} + ٢\sum_{r=0} r . \frac{n}{r}\, {}^{n-١}C_{r-١} + \sum_{r=0}^{n} {}^nC_r$

$= ٤n \sum_{r=1} (n-١)\, {}^{n-٢}C_{r-٢} + ٢n \sum_{r=0} {}^{n-١}C_{r-١} + \sum_{r=0} {}^nC_r$

$\alpha = ٤n(n-١).٢^{n-٢} + ٤n.٢^{n-١} + ٢n.٢^{n-1} + ٢^n$

$\alpha = ٢^{n-٢}\{٤n(n-١) + ٨n + ٤n + ٤\}$

$= ٢^{n-٢}\{٤n^٢ + ٨n + ٤\}$

$\alpha = ٢^n(n+١)^٢$

$\beta = \sum_{r=0}^{n} \frac{{}^nC_r}{r+١} + \frac{١}{n+١}$

$= \sum_{r=0} \frac{{}^{n+١}C_{r+١}}{n+1} + \frac{١}{n+١}$

$= \frac{١}{n+١}\left(1 + {}^{n+١}C_١ + \ldots + {}^{n+١}C_{n+١}\right)$

$= \frac{٢^{n+١}}{n+١}$

$\frac{2\alpha}{\beta} = \frac{٢ \cdot ٢^n(n+١)^٢}{٢^{n+١}}.(n+١) = (n+١)^٣$

$١٤٠ < (n+١)^٣ < ٢٨١$

$n = ٤ \Rightarrow (n+١)^٣ = ١٢٥$

$n = ٥ \Rightarrow (n+١)^٣ = ٢١٦$

$n = ٦ \Rightarrow (n+١)^٣ = ٣٤٣$

$\therefore n = ٥$

٢٨. Let $\vec a = ٩\hat i - ١٣\hat j + ٢٥\hat k$, $\vec b = ٣\hat i + ٧\hat j - ١٣\hat k$ and $\vec c = ١٧\hat i - ٢\hat j + \hat k$ be three given vectros. If $\vec r$ is a vector such that $\vec r \times \vec a = (\vec b + \vec c) \times \vec a$ and $\vec r \cdot (\vec b - \vec c) = ٠$, then $\frac{|٥٩٣\vec r + ٦٧\vec a|^٢}{(٥٩٣)^٢}$ is equal to __________.

Ans. (٥٦٩)

Sol. $\vec a = ٩\hat i - ١٣\hat j + ٢٥\hat k$

$\vec b = ٣\hat i + ٧\hat j - ١٣\hat k$

$\vec c = ١٧\hat i - ٢\hat j + \hat k$

$\vec b + \vec c = ٢٠\hat i + ٥\hat j - ١٢\hat k$

$\vec b - \vec c = -١٤\hat i + ٩\hat j - ١٤\hat k$

$\Rightarrow (\vec r - (\vec b + \vec c)) \times \vec a = 0$

$\vec r - (\vec b + \vec c) = \lambda \vec a$

$\vec r = \lambda \vec a + \vec b + \vec c$

But $\vec r \cdot (\vec b - \vec c) = ٠$

$\Rightarrow (\lambda \vec a + \vec b + \vec c) \cdot (\vec b - \vec c) = 0$

$\Rightarrow \lambda \vec a \cdot \vec b + \vec b \cdot \vec b - \vec b \cdot \vec c - \lambda \vec a \cdot \vec c + \vec b \cdot \vec c - \vec c \cdot \vec c = ٠$

$\Rightarrow \lambda = \frac{\vec c \cdot \vec c - \vec b \cdot \vec b}{\vec a \cdot \vec b - \vec a \cdot \vec c} = \frac{٢٩٤ - ٢٢٧}{-٣٨٩ - ٢٠٤} = \frac{-٦٧}{٥٩٣}$

$\Rightarrow \vec r - \vec b - \vec c = -\frac{٦٧}{٥٩٣}\vec a$

$\Rightarrow ٥٩٣\vec r + ٦٧\vec a = ٥٩٣(\vec b + \vec c)$

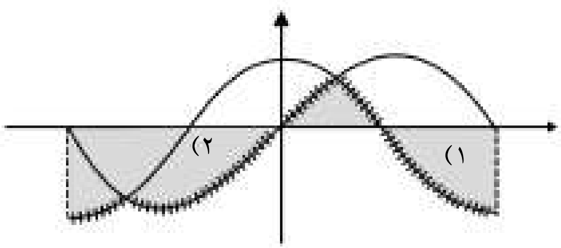
$\Rightarrow |\vec b + \vec c|^٢ = ٥٦٩$

٢٩. Let the area of the region enclosed by the curve $y = \min\{\sin x, \cos x\}$ and the x-axis between $x = -\pi$ to $x = \pi$ be A. Then $A^٢$ is equal to ___________.

Ans. (١٦)

Sol. $y = \min\{\sin x, \cos x\}$

x–axis $x = -\pi$, $x = \pi$



$\int^{\pi/٤} \sin x = (\cos x)^{\pi/٤} = ١ - \frac{١}{\sqrt{٢}}$

$\int^{3\pi/٤} (\sin x - \cos x) + (\cos x + \sin x)^{3\pi/٤}$

$(\cos x + \sin x)_{\pi/٤}^{\pi/٢}$

$= (1 - 0) - \left(\frac{١}{\sqrt{٢}} + \frac{١}{\sqrt{٢}}\right)$

$= ١ + \frac{١}{\sqrt{٢}} + \frac{١}{\sqrt{٢}}$

$\int_{\pi/٤}^{\pi/٢} \cos x\,dx = (\sin x)_{\pi/٤}^{\pi/٢} = 1 - \frac{١}{\sqrt{٢}}$

$A = ٤$

$A^٢ = ١٦$

٣٠. The value of

$$\lim_{x \to ٠} ٢\left(\frac{١ - \cos x \sqrt{\cos ٢x}\,\sqrt[٣]{\cos ٣x} \cdots \sqrt[١٠]{\cos ١٠x}}{x^٢}\right)$$ is

__________.

Ans. (٥٥)

Sol.

$$\lim_{x \to ٠} ٢\left(\frac{١ - \left(١ - \frac{x^٢}{٢!}\right)\left(1 - \frac{٤x^٢}{٢!}\right)^{١/٢}\left(١ - \frac{٩x^٢}{٢!}\right)^{١/٣} \cdots \left(١ - \frac{١٠٠x^٢}{٢!}\right)^{١/١٠}}{x^٢}\right)$$

By expansion

$$\lim_{x \to ٠} ٢\left(\frac{١ - \left(١ - \frac{x^٢}{٢}\right)\left(١ - \frac{١}{٢}\cdot\frac{٤x^٢}{٢}\right)\left(١ - \frac{١}{٣}\cdot\frac{٩x^٢}{٢}\right) \cdots \left(١ - \frac{١}{١٠}\cdot\frac{١٠٠x^٢}{٢}\right)}{x^٢}\right)$$

$$\lim_{x \to ٠} ٢\left(\frac{١ - \left(1 - \frac{x^٢}{٢}\right)\left(1 - \frac{٢x^٢}{٢}\right)\left(١ - \frac{٣x^٢}{٢}\right) \cdots \left(١ - \frac{١٠x^٢}{٢}\right)}{x^٢}\right)$$

$$\lim_{x \to ٠} \frac{٢\left(١ - \left(١ - x^٢\left(\frac{١}{٢} + \frac{٢}{٢} + \frac{٣}{٢} + \cdots + \frac{١٠}{٢}\right)\right)\right)}{x^٢}$$

$$2\left(\frac{1}{2} + \frac{٢}{٢} + \frac{٣}{٢} + \cdots + \frac{١٠}{٢}\right)$$

$$١ + ٢ + \ldots \ldots + ١٠ = \frac{١٠ \times ١١}{٢} = ٥٥$$

PHYSICS	TEST PAPER WITH SOLUTION

SECTION-A

٣١. Three bodies A، B and C have equal kinetic energies and their masses are ٤٠٠ g، ١.٢ kg and ١.٦ kg respectively. The ratio of their linear momenta is :

(١) $١ : \sqrt{٣} : ٢$ (٢) $١ : \sqrt{٣} : \sqrt{٢}$

(٣) $\sqrt{٢} : \sqrt{٣} : ١$ (٤) $\sqrt{٣} : \sqrt{٢} : ١$

Ans. (١)

Sol. $KE = \frac{P^٢}{٢m}$

$P \propto \sqrt{m}$

Hence، PA : PB : PC

$= \sqrt{٤٠٠} : \sqrt{١٢٠٠} : \sqrt{١٦٠٠} = ١ : \sqrt{٣} : ٢$

٣٢. Average force exerted on a non-reflecting surface at normal incidence is ٢.٤×١٠^{-٤} N. If ٣٦٠ W/cm² is the light energy flux during span of ١ hour ٣٠ minutes. Then the area of the surface is:

(١) ٠.٢ m² (٢) ٠.٠٢ m²

(٣) ٢٠ m² (٤) ٠.١ m²

Ans. (٢)

Sol. Pressure $= \frac{I}{C} = \frac{F}{A}$

$\Rightarrow \frac{٣٦٠}{10^{-4} \times ٣ \times ١٠^{٨}} = \frac{٢.٤ \times ١٠^{-4}}{A}$

$\Rightarrow A = ٢ \times ١٠^{-٢} m^٢ = ٠.٠٢ m^٢$

٣٣. A proton and an electron are associated with same de-Broglie wavelength. The ratio of their kinetic energies is:

(Assume $h = ٦.٦٣ \times ١٠^{-٣٤}$ J s، $m_e = ٩.٠ \times ١٠^{-٣١}$ kg and $m_p = ١٨٣٦$ times m_e)

(١) ١ : ١٨٣٦ (٢) $١ : \frac{١}{١٨٣٦}$

(٣) $١ : \frac{١}{\sqrt{١٨٣٦}}$ (٤) $١ : \sqrt{١٨٣٦}$

Ans. (١)

Sol. λ is same for both

$P = \frac{h}{\lambda}$ same for both

$P = \sqrt{٢mK}$

Hence،

$K \propto \frac{١}{m}$

$\Rightarrow \frac{KE_p}{KE_e} = \frac{m_e}{m_p} = \frac{١}{١٨٣٦}$

٣٤. A mixture of one mole of monoatomic gas and one mole of a diatomic gas (rigid) are kept at room temperature (٢٧°C). The ratio of specific heat of gases at constant volume respectively is:

(١) $\frac{٧}{٥}$ (٢) $\frac{٣}{٢}$

(٣) $\frac{٣}{٥}$ (٤) $\frac{٥}{٣}$

Ans. (٣)

Sol. $\frac{(C_v)_{mono}}{(C_v)_{dia}} = \frac{\frac{٣}{٢}R}{\frac{٥}{٢}R} = \frac{٣}{٥}$

٣٥. In an expression $a \times ١٠^{b}$:

(١) a is order of magnitude for b ≤ ٥

(٢) b is order of magnitude for a ≤ ٥

(٣) b is order of magnitude for ٥ < a ≤ ١٠

(٤) b is order of magnitude for a ≥ ٥

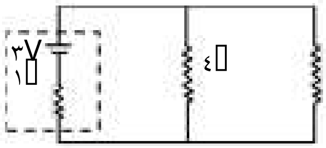
Ans. (٢)

Sol. $a \times ١٠^{b}$

if a ≤ ٥ order is b

a > ٥ order is b + ١

٣٦. In the given circuit، the terminal potential difference of the cell is :

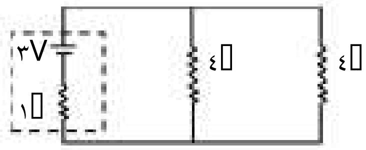


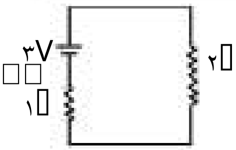
(١) ٢ V (٢) ٤ V

(٣) ١.٥ V (٤) ٣ V

Ans. (١)

Sol.





i = ٣/(١+٢) = ١A

v = E – ir

= ٣ – ١ × ١ = ٢V

٣٧. Binding energy of a certain nucleus is ١٨ × ١٠^٨ J. How much is the difference between total mass of all the nucleons and nuclear mass of the given nucleus:

(١) ٠.٢ g (٢) ٢٠ g

(٣) ٢ g (٤) ١٠ g

Ans. (٢)

Sol. mc^٢ = ١٨ × ١٠^٨

m × ٩ × ١٠ = ١٨ × ١٠^٨

m = ٢ × ١٠^-٨ kg = ٢٠ g.

٣٨. Paramagnetic substances:

A. align themselves along the directions of external magnetic field.

B. attract strongly towards external magnetic field.

C. has susceptibility little more than zero.

D. move from a region of strong magnetic field to weak magnetic field.

Choose the most appropriate answer from the options given below:

(١) A، B، C، D (٢) B، D Only

(٣) A، B، C Only (٤) A، C Only

Ans. (٤)

Sol. A، C only

٣٩. A clock has ٧٥ cm، ٦٠ cm long second hand and minute hand respectively. In ٣٠ minutes duration the tip of second hand will travel x distance more than the tip of minute hand. The value of x in meter is nearly (Take π = ٣.١٤):

(١) ١٣٩.٤ (٢) ١٤٠.٥

(٣) ٢٢٠.٠ (٤) ١١٨.٩

Ans. (١)

Sol. xmin = π × rmin

= π × ٦٠/١٠٠ m.

xsecond = ٣٠ × ٢π × rsecond

= ٣٠ × ٢π × ٧٥/١٠٠

x = x second – x min

= ١٣٩.٤ m

٤٠. Young's modulus is determined by the equation given by Y = ٤٩٠٠٠ (M/ℓ) mdyne/cm٢ where M is the mass and ℓ is the extension of wire used in the experiment. Now error in Young modules (Y) is estimated by taking data from M–ℓ plot in graph paper. The smallest scale divisions are ٥ g and ٠.٠٢ cm along load axis and extension axis respectively. If the value of M and ℓ are ٥٠٠ g and ٢ cm respectively then percentage error of Y is:

(١) ٠.٢ ٪ (٢) ٠.٠٢٪

(٣) ٢ ٪ (٤) ٠.٥٪

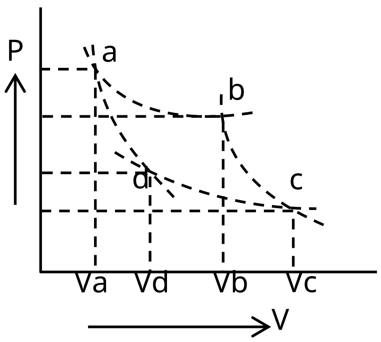
Ans. (٣)

Sol. $\frac{\Delta Y}{Y} = \frac{\Delta m}{m} + \frac{\Delta \ell}{\ell}$

$= \frac{٥}{٥٠٠} + \frac{٠.٠٢}{٢} = ٠.٠١ + ٠.٠١$

$\frac{\Delta Y}{Y} = ٠.٠٢ \Rightarrow \% \frac{\Delta Y}{Y} = ٢\%$

٤١. Two different adiabatic paths for the same gas intersect two isothermal curves as shown in P-V diagram. The relation between the ratio $\frac{V_a}{V_d}$ and the ratio $\frac{V_b}{V_c}$ is:



(١) $\frac{V_a}{V_d} = \left(\frac{V_b}{V_c}\right)^{-1}$ (٢) $\frac{V_a}{V_d} \neq \frac{V_b}{V_c}$

(٣) $\frac{V_a}{V_d} = \frac{V_b}{V_c}$ (٤) $\frac{V_a}{V_d} = \left(\frac{V_b}{V_c}\right)^{٢}$

Ans. (٣)

Sol. For adiabatic process

$TV^{\gamma-1}$ = constant

$T_a V_a^{\gamma-1} = T_d V_d^{\gamma-1}$

$\left(\frac{V_a}{V_d}\right)^{\gamma-1} = \frac{T_d}{T_a}$

$T_b V_b^{\gamma-1} = T_c V_c^{\gamma-1}$

$\left(\frac{V_b}{V_c}\right)^{\gamma-1} = \frac{T_c}{T_b}$

$\frac{V_a}{V_d} = \frac{V_b}{V_c}$ $\left(\because \begin{matrix} T_d = T_c \\ T_a = T_b \end{matrix}\right)$

٤٢. Two planets A and B having masses $m_١$ and $m_٢$ move around the sun in circular orbits of $r_١$ and $r_٢$ radii respectively. If angular momentum of A is L and that of B is ٣L, the ratio of time period is $\left(\frac{T_A}{T_B}\right)$:

(١) $\left(\frac{r_٢}{r_١}\right)^٣$ (٢) $\left(\frac{r1}{r2}\right)^٣$

(٣) $\frac{١}{٢٧}\left(\frac{m_٢}{m_١}\right)^٣$ (٤) $٢٧\left(\frac{m_١}{m_٢}\right)^٣$

Ans. (٣)

Sol. $\frac{\pi r_١^٢}{T_A} = \frac{L}{٢m_١}$ ......(١)

$\frac{\pi r_٢^٢}{T_B} = \frac{٣L}{٢m_٢}$ ......(٢)

$\Rightarrow \frac{T_A}{T_B} = ٣\frac{m_١}{m_٢}\left(\frac{r_١}{r_٢}\right)^٢$

$\left(\frac{T_A}{T_B}\right)^٢ = \left(\frac{r_١}{r_٢}\right)^٣ \Rightarrow \left(\frac{r_١}{r_٢}\right)^٢ = \left(\frac{T_A}{T_B}\right)^{٤/٣}$

$\Rightarrow \frac{١}{٢٧}\left(\frac{m_٢}{m_١}\right)^٣ = \frac{T_A}{T_B}$

٤٣. A LCR circuit is at resonance for a capacitor C, inductance L and resistance R. Now the value of resistance is halved keeping all other parameters same. The current amplitude at resonance will be now:

(١) Zero (٢) double

(٣) same (٤) halved

Ans. (٢)

Sol. In resonance Z = R

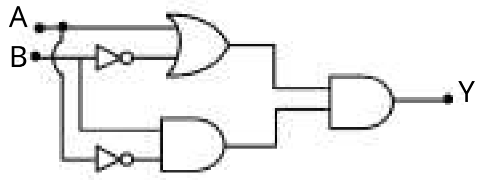
$I = \frac{V}{R}$

R → halved

$\Rightarrow I \rightarrow ٢I$

I becomes doubled.

٤٤. The output Y of following circuit for given inputs is :



(١) A•B(A + B) (٢) A • B

(٣) ٠ (٤) $\overline{A B}$

Ans. (٣)

Sol. By truth table

A	B	Y
٠	٠	٠
٠	١	٠
١	٠	٠
١	١	٠

٤٥. Two charged conducting spheres of radii a and b are connected to each other by a conducting wire. The ratio of charges of the two spheres respectively is :

(١) $\sqrt{ab}$ (٢) ab

(٣) $\frac{a}{b}$ (٤) $\frac{b}{a}$

Ans. (٣)

Sol. Potential at surface will be same

$$\frac{Kq_١}{a} = \frac{Kq_٢}{b}$$

$$\frac{q_١}{q_٢} = \frac{a}{b}$$

٤٦. Correct Bernoulli's equation is (symbols have their usual meaning) :

(١) $P + mgh + \frac{١}{٢} mv^٢ = \text{constant}$

(٢) $P + \rho gh + \frac{١}{٢} \rho v^٢ = \text{constant}$

(٣) $P + \rho gh + \rho v^٢ = \text{constant}$

(٤) $P + \frac{١}{٢} \rho gh + \frac{١}{٢} \rho v^٢ = \text{constant}$

Ans. (٢)

Sol. $P + \rho gh + \frac{١}{٢} \rho V^٢ = \text{constant}$

٤٧. A player caught a cricket ball of mass ١٥٠ g moving at a speed of ٢٠ m/s. If the catching process is completed in ٠.١ s, the magnitude of force exerted by the ball on the hand of the player is :

(١) ١٥٠ N (٢) ٣ N

(٣) ٣٠ N (٤) ٣٠٠ N

Ans. (٣)

Sol. $F = \frac{\Delta P}{\Delta t} = \frac{mv - ٠}{٠.١}$

$$= \frac{١٥٠ \times ١٠^{-3} \times ٢٠}{٠.١} = ٣٠ \text{ N}$$

٤٨. A stationary particle breaks into two parts of masses mA and mB which move with velocities vA and vB respectively. The ratio of their kinetic energies (KB : KA) is :

(١) vB : vA (٢) mB : mA

(٣) mB vB : mA vA (٤) ١ : ١

Ans. (١)

Sol. Initial momentum is zero.

Hence $|P_A| = |P_B|$

$\Rightarrow m_A v_A = m_B v_B$

$$\frac{(KE)_A}{(KE)_B} = \frac{\frac{١}{٢} m_A v_A^٢}{\frac{١}{٢} m_B v_B^٢} = \frac{v_A}{v_B}$$

$$\frac{(KE)_B}{(KE)_A} = \frac{v_B}{v_A}$$

٤٩. Critical angle of incidence for a pair of optical media is ٤٥°. The refractive indices of first and second media are in the ratio :

(١) $\sqrt{٢} : ١$ (٢) ١ : ٢

(٣) $١ : \sqrt{٢}$ (٤) ٢ : ١

Ans. (١)

Sol. $\sin \theta_c = \frac{\mu_R}{\mu_d} = \frac{\mu_2}{\mu_1}$

$\sin ٤٥° = \frac{\mu_٢}{\mu_١}$

$\Rightarrow \frac{١}{\sqrt{٢}} = \frac{\mu_٢}{\mu_١}$

$\Rightarrow \frac{\mu_١}{\mu_٢} = \frac{\sqrt{٢}}{١}$

٥٠. The diameter of a sphere is measured using a vernier caliper whose ٩ divisions of main scale are equal to ١٠ divisions of vernier scale. The shortest division on the main scale is equal to 1 mm. The main scale reading is ٢ cm and second division of vernier scale coincides with a division on main scale. If mass of the sphere is ٨.٦٣٥ g، the density of the sphere is:

(١) ٢.٥ g/cm³ (٢) ١.٧ g/cm³

(٣) ٢.٢ g/cm³ (٤) ٢.٠ g/cm³

Ans. (٤)

Sol. Given ٩MSD = ١٠VSD

mass = ٨.٦٣٥ g

LC = ١ MSD − ١ VSD

LC = ١ MSD − $\frac{٩}{١٠}$ MSD

LC = $\frac{١}{١٠}$ MSD

LC = ٠.٠١ cm

Reading of diameter = MSR + LC × VSR

= ٢ cm + (٠.٠١) × (٢)

= ٢.٠٢ cm

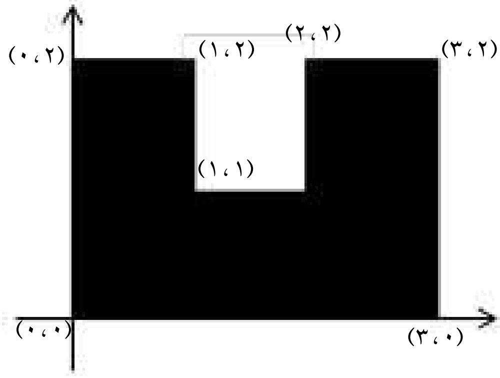
Volume of sphere $= \frac{4}{٣}\pi\left(\frac{d}{2}\right)^٣ = \frac{٤}{٣}\pi\left(\frac{٢.٠٢}{٢}\right)^٣$

= ٤.٣٢ cm³

Density = $\frac{\text{mass}}{\text{volume}} = \frac{٨.٦٣٥}{٤.٣٢} = ١.٩٩٨ \sim ٢.٠٠$ g

SECTION-B

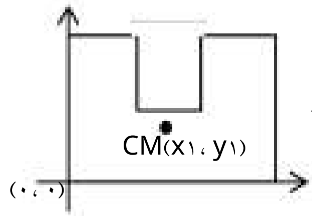
٥١. A uniform thin metal plate of mass ١٠ kg with dimensions is shown. The ratio of x and y coordinates of center of mass of plate in $\frac{n}{٩}$. The value of n is__________.



Ans. (١٥)

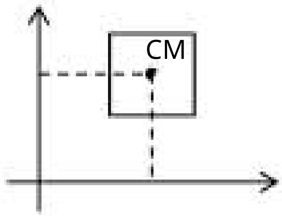
Sol. m١ = σ × ٥ = ١٠ Kg

m١ = σ × ٥ = ١٠Kg



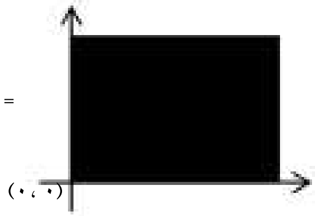
+

m٢ = σ × ١ = ٢Kg



m٣ = σ × ٦ = ١٢Kg

=



$\Rightarrow$ m١ x١ + m٢ x٢ = m٣ x٣

١٠x١ + ٢(١.٥) = ١٢(١.٥) $\Rightarrow$ x١ = ١.٥ cm

$\Rightarrow$ m١ y١ + m٢ y٢ = m٣ y٣

١٠y١ + ٢(١.٥) = ١٢ × ١ $\Rightarrow$ y١ = ٠.٩ cm

$\frac{x_١}{y} = \frac{١.٥}{٠.٩} = \frac{١٥}{٩}$

n = ١٥

٥٢. An electron with kinetic energy ٥ eV enters a region of uniform magnetic field of ٣ μT perpendicular to its direction. An electric field E is applied perpendicular to the direction of velocity and magnetic field. The value of E، so that electron moves along the same path، is______ NC⁻¹.

(Given، mass of electron = ٩ × ١٠⁻³¹ kg، electric charge = ١.٦ × ١٠⁻¹⁹ C)

Ans. (٤)

Sol. For the given condition of moving undeflected، net force should be zero.

qE = qVB

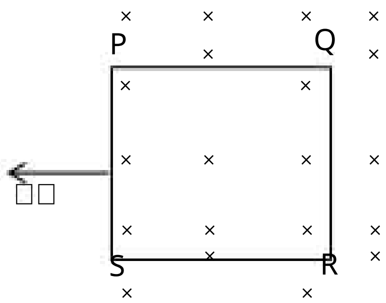
E = VB

$$= \sqrt{\frac{٢\,KE}{m}}\,B$$

$$= \sqrt{\frac{٢ \times ٥ \times ١.٦ \times ١٠^{-١٩}}{٩ \times ١٠^{-٣١}}} \times ٣ \times ١٠^{-٦}$$

= ٤ N/C

٥٣. A square loop PQRS having ١٠ turns، area ٣.٦ × ١٠⁻³ m² and resistance ١٠٠ Ω is slowly and uniformly being pulled out of a uniform magnetic field of magnitude B = ٠.٥ T as shown. Work done in pulling the loop out of the field in ١.٠ s is __________ × ١٠⁻⁶ J.



Ans. (٣)

Sol. ε = NBℓv

$$i = \frac{\varepsilon}{R} = \frac{NB\ell v}{R}$$

$$F = N(i\ell B) = \frac{N^{٢}B^{٢}\ell^{٢}v}{R}$$

$$W = F \cdot \ell = \frac{N^{٢}B^{٢}\ell^{٢}}{R}\left(\frac{\ell}{t}\right)\ell$$

A = ℓ²

$$W = \frac{(١٠)^{٢}(٠.٥)^{٢}(٣.٦ \times ١٠^{-٣})^{٣/٢}}{١٠٠ \times ١}$$

W = ٣.٢٤ × ١٠⁻⁶ J

٥٤. Resistance of a wire at ٠ °C، ١٠٠ °C and t °C is found to be ١٠ Ω، ١٠.٢ Ω and ١٠.٩٥ Ω respectively. The temperature t in Kelvin scale is__________.

Ans. (٧٤٨)

Sol. R = R₀(١ + αΔT)

$$\frac{\Delta R}{R_0} = \alpha\Delta T$$

Case-I

٠ °C → ١٠٠ °C

$$\frac{١٠.٢ - ١٠}{١٠} = \alpha(١٠٠ - ٠) \qquad \ldots(١)$$

Case-II

٠ °C → t °C

$$\frac{١٠.٩٥ - ١٠}{١٠} = \alpha(t - ٠) \qquad \ldots(٢)$$

$$\Rightarrow \frac{t}{١٠٠} = \frac{٠.٩٥}{٠.٢} \Rightarrow ٤٧٥\ °C$$

t = ٤٧٥ + ٢٧٣ = ٧٤٨ K

٥٥. An electric field، $\vec{E} = \frac{٢\hat{i} + ٦\hat{j} + ٨\hat{k}}{\sqrt{٦}}$ passes through the surface of ٤ m² area having unit vector $\hat{n} = \left(\frac{٢\hat{i} + \hat{j} + \hat{k}}{\sqrt{٦}}\right)$. The electric flux for that surface is ______________ V m.

Ans. (١٢)

Sol. $\phi = \vec{E} \cdot \vec{A}$

$$= \left(\frac{٢\hat{i} + ٦\hat{j} + ٨\hat{k}}{\sqrt{٦}}\right) \cdot ٤\left(\frac{2\hat{i} + \hat{j} + \hat{k}}{\sqrt{٦}}\right)$$

$$= \frac{٤}{6}(٤ + 6 + ٨) = ١٢\ \text{Vm}$$

٥٦. A liquid column of height ٠.٠٤ cm balances excess pressure of soap bubble of certain radius. If density of liquid is ٨×١٠^{٣} kg $m^{-٣}$ and surface tension of soap solution is ٠.٢٨ $Nm^{-١}$, then diameter of the soap bubble is ____________ cm.

(if $g = ١٠\ ms^{-٢}$)

Ans. (٧)

Sol. $\rho g h = \frac{٤S}{R}$

$\Rightarrow R = \frac{٤ \times ٠.٢٨}{٨ \times ١٠^{٣} \times ١٠ \times ٤ \times ١٠^{-٤}}$

$= \frac{٠.٢٨}{٨} m = \frac{٢٨}{٨} cm$

$\Rightarrow R = ٣.٥$ cm

Diameter = ٧ cm

٥٧. A closed and an open organ pipe have same lengths. If the ratio of frequencies of their seventh overtones is $\left(\frac{a-1}{a}\right)$ then the value of a is ________.

Ans. (١٦)

Sol. For closed organ pipe

$f_c = (٢n+١)\frac{v}{٤\ell} = \frac{١٥v}{٤\ell}$

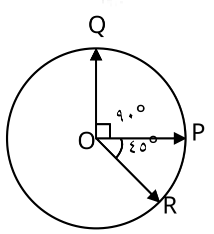
For open organ pipe

$f_o = (n+١)\frac{v}{٢\ell} = \frac{٨v}{٢\ell}$

$\frac{f_c}{f_o} = \frac{١٥}{١٦} = \frac{a-١}{a}$

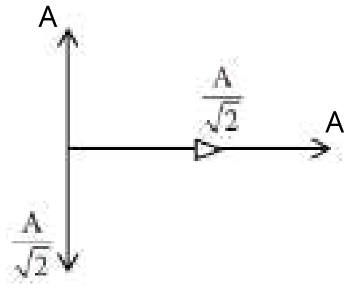
$\Rightarrow a = ١٦$

٥٨. Three vectors $\overline{OP}$, $\overline{OQ}$ and $\overline{OR}$ each of magnitude A are acting as shown in figure. The resultant of the three vectors is $A\sqrt{x}$. The value of x is _______.



Ans. (٣)

Sol.



$\vec{R} = \left(A + \frac{A}{\sqrt{٢}}\right)\hat{i} + \left(A - \frac{A}{\sqrt{٢}}\right)\hat{j}$

$|\vec{R}| = \sqrt{\left(A + \frac{A}{\sqrt{٢}}\right)^{٢} + \left(A - \frac{A}{\sqrt{٢}}\right)^{٢}} = \sqrt{٣}A$

٥٩. A parallel beam of monochromatic light of wavelength ٦٠٠ nm passes through single slit of ٠.٤ mm width. Angular divergence corresponding to second order minima would be ________ $\times ١٠^{-٣}$ rad.

Ans. (٦)

Sol. $\sin\theta \simeq \theta \simeq \frac{٢\lambda}{b}$

$= \frac{٢ \times ٦٠٠ \times ١٠^{-9}}{٤ \times ١٠^{-4}} = ٣ \times ١٠^{-٣}$ rad

Total divergence $= (٣+٣) \times ١٠^{-٣} = ٦ \times ١٠^{-٣}$ rad

٦٠. In an alpha particle scattering experiment distance of closest approach for the α particle is ٤.٥×١٠^{-١٤} m. If target nucleus has atomic number ٨٠, then maximum velocity of α-particle is ___________ $\times ١٠^{٥}$ m/s approximately.

($\frac{١}{4\pi\epsilon_0} = ٩ \times ١٠^{٩}$ SI unit, mass of α particle $= ٦.٧٢ \times ١٠^{-٢٧}$ kg)

Ans. (١٥٦)

Sol. $v = \sqrt{\frac{٤KZe^{٢}}{m\, r_{min}}}$

$= \sqrt{\frac{٤ \times ٩ \times ١٠^{٩} \times ٨٠}{٦.٧٢ \times ١٠^{-٢٧} \times ٤.٥ \times ١٠^{-١٤}}} \times ١.٦ \times ١٠^{-١٩}$

$= ٩.٧٥٩ \times ١٠^{٢٥} \times ١.٦ \times ١٠^{-١٩}$

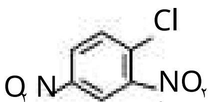
$= ١٥٦ \times ١٠^{٥}$ m/s

CHEMISTRY	TEST PAPER WITH SOLUTION

SECTION-A

٦١. Given below are two statements:

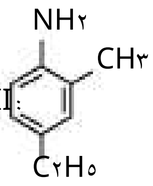
Statement I :



Compound-A

IUPAC name of Compound A is ٤-chloro-١، ٣-dinitrobenzene:

Statement II:



Compound-B

IUPAC name of Compound B is ٤-ethyl-٢-methylaniline.

In the light of the above statements، choose the most appropriate answer from the options given below:

(١) Both Statement I and Statement II are correct

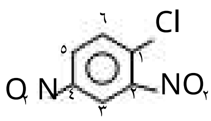
(٢) Statement I is incorrect but Statement II is correct

(٣) Statement I is correct but Statement II is incorrect

(٤) Both Statement I and Statement II are incorrect

Ans. (٢)

Sol. Statement I :

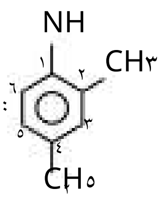


IUPAC name

□ ١-chloro-٢، ٤-dinitrobenzene

□ statement-I is incorrect

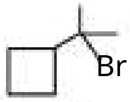
Statement-II :



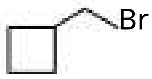
□ □ ٤-ethyl-٢-methylaniline

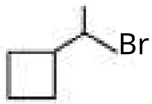
□ statement-II is correct

٦٢. Which among the following compounds will undergo fastest SN٢ reaction.

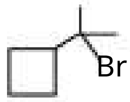
(١) 

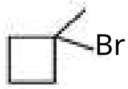
(٢) 

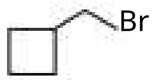
(٣) 

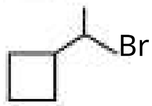
(٤) 

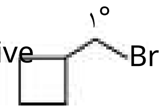
Ans. (٣)

Sol. ١ 

٢ 

٣ 

٤ 

fastest SN reaction give 

Rate of SN is Me – x < ١° – x < ٢° – x < ٣° – x

٦٣. Combustion of glucose (C٦H١٢O٦) produces CO٢ and water. The amount of oxygen (in g) required for the complete combustion of ٩٠٠ g of glucose is:

ﷺMolar mass of glucose in g mol = ١٨٠ﷺ

(١) ٤٨٠ (٢) ٩٦٠

(٣) ٨٠٠ (٤) ٣٢

Ans. (٢)

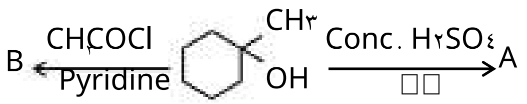
Sol. C٦H١٢O٦(s) + ٦O٢(g) □□ ٦CO٢(g) + ٦H٢O(□)

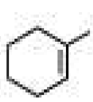
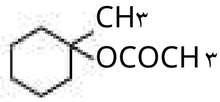
٩٠٠ / ١٨٠

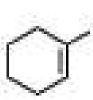
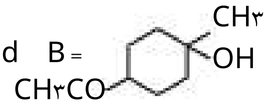
= ٥ mol ٣٠ mol

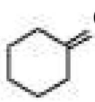
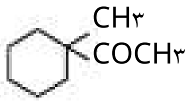
Mass of O٢ required = ٣٠ × ٣٢ = ٩٦٠ gm

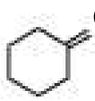
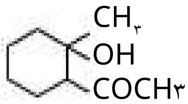
٦٤. Identify the major products A and B respectively in the following set of reactions.



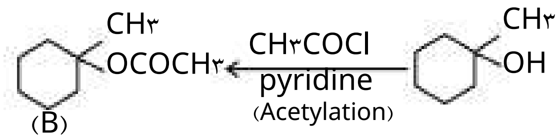
(١) A=  and B= 

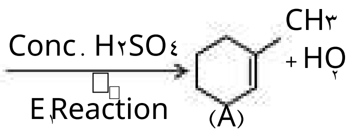
(٢) A=  and B = 

(٣) A=  and B = 

(٤) A=  and B = 

Ans. (١)

Sol. 



٦٥. Given below are two statements : One is labelled as Assertion A and the other is labelled as Reason R :

Assertion A : The stability order of +١ oxidation state of Ga, In and Tl is Ga > In > Tl.

Reason R : The inert pair effect stabilizes the lower oxidation state down the group.

In the light of the above statements, choose the correct answer from the options given below :

(١) Both A and R are true and R is the correct explanation of A.

(٢) A is true but R is false.

(٣) Both A and R are true but R is NOT the correct explanation of A.

(٤) A is false but R is true.

Ans. (١)

Sol. The relative stability of +١ oxidation state progressively increases for heavier elements due to inert pair effect.

∴ Stability of Al+١ < Ga+١ < In+١ < Tl+١

٦٦. Match List I with List-II

List-I (Name of the test)	List-II (Reaction sequence involved) (M is metal)
A Borax bead test	I. $MCO_{٣} \rightarrow MO \xrightarrow[\Delta]{Co(NO_{٣})_{٢}} CoO.MO$
B. Charcoal cavity test	II. $MCO_{٣} \rightarrow MCl_{٢} \rightarrow M^{٢+}$
C. Cobalt nitrate test	III $MSO_{٤} \xrightarrow[\Delta]{Na_{٢}B_{٤}O_{٧}} M(BO_{٢})_{٢} \rightarrow MBO_{٢} \rightarrow M$
D. Flame test	IV $MSO_{٤} \xrightarrow[\Delta]{Na_{٢}CO_{٣}} MCO_{٣} \rightarrow MO \rightarrow M$

Choose the correct answer from the option below :

(١) A-III, B-I, C-IV, D-II

(٢) A-III, B-II, C-IV, D-I

(٣) A-III, B-I, C-II, D-IV

(٤) A-III, B-IV, C-I, D-II

Ans. (٤)

Sol. Cobalt nitrate test

$MCO_{٣} \rightarrow MO \xrightarrow[\Delta]{Co(NO_{٣})_{٢}} CoO.MO$

Flame test

$MCO_{٣} \rightarrow MCl_{٢} \rightarrow M^{٢+}$

Borax Bead test

$MSO_{٤} \xrightarrow[\Delta]{Na_{٢}B_{٤}O_{٧}} M(BO_{٢})_{٢} \rightarrow MBO_{٢} \rightarrow M$

Charcoal cavity test

$MSO_{٤} \xrightarrow[\Delta]{Na_{٢}CO_{٣}} MCO_{٣} \rightarrow MO \rightarrow M$

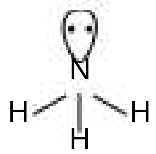
٦٧. Match List I and with List II

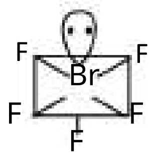
List-I (Molecule)		List-II (Shape)
A	$NH_٣$	I. Square pyramid
B.	$BrF_٥$	II. Tetrahedral
C.	$PCl_٥$	III Trigonal pyramidal
D.	$CH_٤$	IV Trigonal bipyramidal

Choose the correct answer from the option below : (١) A-IV, B-III, C-I, D-II (٢) A-II, B-IV, C-I, D-III (٣) A-III, B-I, C-IV, D-II (٤) A-III, B-IV, C-I, D-II

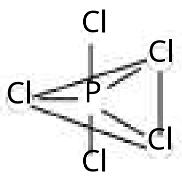
Ans. (٣)

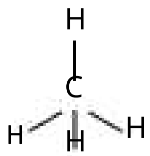
Sol.





Trigonal pyramidal   Square pyramidal





Trigonal bipyramidal   Tetrahedral

٦٨. For the given hypothetical reactions, the equilibrium constants are as follows:

$X \rightleftharpoons Y; K_١ = ١.٠$

$Y \rightleftharpoons Z; K_٢ = ٢.٠$

$Z \rightleftharpoons W; K_٣ = ٤.٠$

The equilibrium constant for the reaction

$X \rightleftharpoons W$ is

(١) ٦.٠   (٢) ١٢.٠

(٣) ٨.٠   (٤) ٧.٠

Ans. (٣)

Sol. $X \rightleftharpoons Y$ $k_١ = ١$

$Y \rightleftharpoons Z$ $k_٢ = ٢$

$Z \rightleftharpoons W$ $k_٣ = ٤$

$X \rightleftharpoons W$ $k_١ \times k_٢ \times k_٣ = k$

$= ١ \times ٢ \times ٤$ $k = ٨$

٦٩. Thiosulphate reacts differently with iodine and bromine in the reaction given below :

$٢S_٢O_٣^{٢-} + I_٢ \rightarrow S_٤O_٦^{٢-} + ٢I^-$

$S_٢O_٣^{٢-} + ٥Br_٢ + ٥H_٢O \rightarrow ٢SO_٤^{٢-} + ٤Br^- + ١٠H^+$

Which of the following statement justifies the above dual behaviour of thiosulphate?

(١) Bromine undergoes oxidation and iodine undergoes reduction by iodine in these reactions

(٢) Thiosulphate undergoes oxidation by bromine and reduction by iodine in these reaction

(٣) Bromine is a stronger oxidant than iodine

(٤) Bromine is a weaker oxidant than iodine

Ans. (٣)

Sol. In the reaction of $S_٢O_٣^{٢-}$ with $I_٢$, oxidation state of sulphur changes to +٢ to +٢.٥

In the reaction of $S_٢O_٣^{٢-}$ with $Br_٢$, oxidation state of sulphur changes from +٢ to +٦.

$\therefore$ Both $I_٢$ and $Br_٢$ are oxidant (oxidising agent) and $Br_٢$ is stronger oxidant than $I_٢$.

٧٠. An octahedral complex with the formula $CoCl_٣nNH_٣$ upon reaction with excess of $AgNO_٣$ solution given ٢ moles of AgCl. Consider the oxidation state of Co in the complex is 'x'. The value of "x + n" is ________.

(١) ٣   (٢) ٦

(٣) ٨   (٤) ٥

Ans. (٣)

Sol. $[Co(NH_٣)_٥Cl]Cl_٢ +$ excess $AgNO_٣ \rightarrow ٢A$

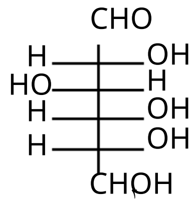
(٢ moles)

$x + ٠ - ١ - ٢ = ٠$

$x = +٣$

$n = ٥$

$\therefore x + n = ٨$

٧١. 

The incorrect statement regarding the given structure is

(١) Can be oxidized to a dicarboxylic acid with $Br_٢$ water

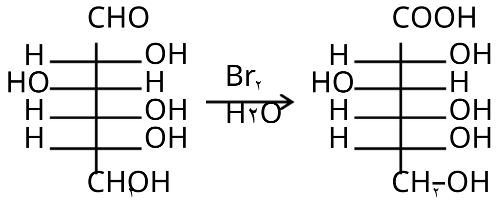
(٢) despite the presence of – CHO does not give Schiff's test

(٣) has ٤-asymmetric carbon atom

(٤) will coexist in equilibrium with ٢ other cyclic structure

Ans. (١)

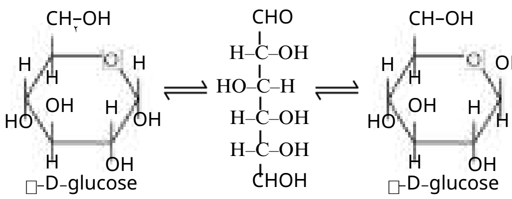
Sol.

(١) 

statement ١ is incorrect (monocarboxylic acid)

(٢) correct

(٣) c.c. is ٤ (correct)

(٤) 

(correct)

٧٢. In the given compound, the number of ٢° carbon atom/s is ________.

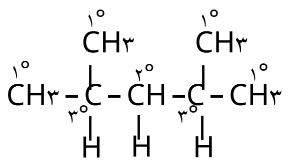
$CH_3-C(CH_3)-CH-C(CH_3)-CH_3$

H H H

(١) Three (٢) One

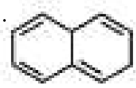
(٣) Two (٤) Four

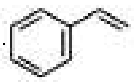
Ans. (٢)

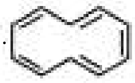
Sol. 

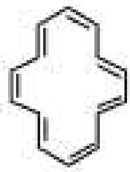
only one ٢° carbon is present in this compound.

٧٣. Which of the following are aromatics

A. 

B. 

C. 

D. 

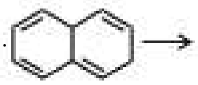
(١) B and D only

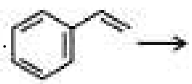
(٢) A and C only

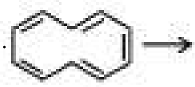
(٣) A and B only

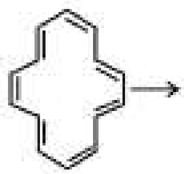
(٤) C and D only

Ans. (١)

Sol. A.  Non aromatic

B.  Aromatic

C.  Non aromatic

D.  Aromatic

٧٤. Among the following halogens

$F_٢$, $Cl_٢$, $Br_٢$ and $I_٢$

Which can undergo disproportionation reactions

(١) Only $I_٢$

(٢) $Cl_٢$, $Br_٢$ and $I_٢$

(٣) $F_٢$, $Cl_٢$ and $Br_٢$

(٤) $F_٢$ and $Cl_٢$

Ans. (٢)

Sol. fluorine do not disproportionate because do not exist in positive oxidation state however $Cl_٢$, $Br_٢$ & $I_٢$ undergoes disproportionation.

٧٥. Given below are two statements: Statement I : $N(CH_٣)_٣$ and $P(CH_٣)_٣$ can act as ligands to form transition metal complexes.

Statement II: As N and P are from same group, the nature of bonding of $N(CH_٣)_٣$ and $P(CH_٣)_٣$ is always same with transition metals. In the light of the above statements, choose the most appropriate answer from the options given below:

(١) Statement I is incorrect but Statement II is correct

(٢) Both Statement I and Statement II are correct

(٣) Statement I is correct but Statement II is incorrect

(٤) Both Statement I and Statement II are incorrect

Ans. (٣)

Sol. $N(CH_٣)_٣$ and $P(CH_٣)_٣$ both are Lewis base and acts as ligand, However, $P(CH_٣)_٣$ has a π-acceptor character.

٧٦. Match List I with List II

List-I (Elements)		List-II (Properties in their respective groups)
A	Cl, S	I. Elements with highest electronegativity
B.	Ge, As	II. Elements with largest atomic size
C.	Fr, Ra	III Elements which show properties of both metals and non metal
D.	F, O	IV Elements with highest negative electron gain enthalpy

Choose the correct answer from the options given below:

(١) A-II, B-III, C-IV, D-I

(٢) A-III, B-II, C-I, D-IV

(٣) A-IV, B-III, C-II, D-I

(٤) A-II, B-I, C-IV, D-III

Ans. (٣)

Sol. Elements with highest electronegativity → F, O

Elements with largest atomic size → Fr, Ra

Elements which shows properties of both metal and non-metals i.e. metalloids → Ge, As

Elements with highest negative electron gain enthalpy → Cl, S

٧٧. Iron (III) catalyses the reaction between iodide and persulphate ions, in which

A. $Fe^{٣+}$ oxidises the iodide ion

B. $Fe^{٣+}$ oxidises the persulphate ion

C. $Fe^{٢+}$ reduces the iodide ion

D. $Fe^{٢+}$ reduces the persulphate ion

Choose the most appropriate answer from the options given below:

(١) B and C only (٢) B only

(٣) A only (٤) A and D only

Ans. (٤)

Sol. $٢Fe^{٣+} + ٢I^- \rightarrow ٢Fe^{٢+} + I_٢$

$٢Fe^{٢+} + S_٢O_٨^{٢-} \rightarrow ٢Fe^{٣+} + ٢SO_٤^{٢-}$

$Fe^{+٣}$ oxidises I^- to $I_٢$ and convert itself into $Fe^{+٢}$.

This $Fe^{+٢}$ reduces $S_٢O_٨^{٢-}$ to $SO_٤^{٢-}$ and converts itself into $Fe^{+٣}$.

٧٨. Match List I with List II

List-I (Compound)		List-II (Colour)
A	$Fe_٤[Fe(CN)_٦]_٣.xH_٢O$	I. Violet
B.	$[Fe(CN)_٥NOS]^{٤-}$	II. Blood Red
C.	$[Fe(SCN)]^{٢+}$	III. Prussian Blue
D.	$(NH_٤)_٣PO_٤.١٢MoO_٣$	IV. Yellow

Choose the correct answer from the options given below:

(١) A-III, B-I, C-II, D-IV

(٢) A-IV, B-I, C-II, D-III

(٣) A-II, B-III, C-IV, D-I

(٤) A-I, B-II, C-III, D-IV

Ans. (١)

Sol. $Fe_٤[Fe(CN)_٦]_٣.xH_٢O$ → Prussian Blue

$[Fe(CN)_٥NOS]^{٤-}$ → Violet

$[Fe(SCN)]^{٢+}$ → Blood Red

$(NH_٤)_٣PO_٤.١٢MoO_٣$ → Yellow

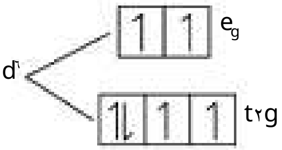
٧٩. Number of complexes with even number of electrons in $t_٢g$ orbitals is -

$[Fe(H_٢O)_٦]^{٢+}$, $[Co(H_٢O)_٦]^{٢+}$, $[Co(H_٢O)_٦]^{٣+}$, $[Cu(H_٢O)_٦]^{٢+}$, $[Cr(H_٢O)_٦]^{٢+}$

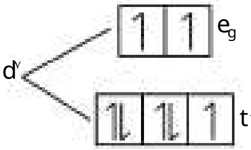
(١) ١ (٢) ٣

(٣) ٢ (٤) ٥

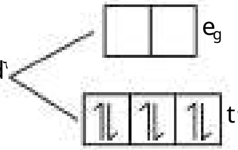
Ans. (٢)

Sol. [Fe(H۲O)۶]۲+ Fe+۲ □ d۶ 

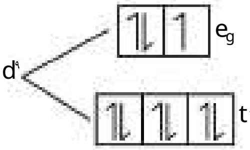
Electron in t۲g = ٤(even)

[Co(H۲O)۶]۲+ Co+۲ □ d۷ 

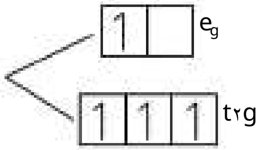
Electron in t۲g = ٥(odd)

[Co(H۲O)۶]۳+ Co+۳ □ d۶ 

Electron in t۲g = ٦(even)

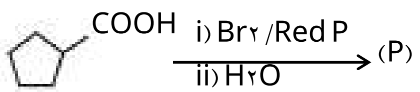
[Cu(H۲O)۶]۲+ Cu+۲ □ d۹ 

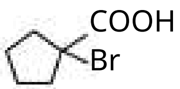
Electron in t۲g = ٦(even)

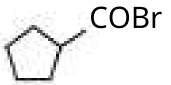
[Cr(H۲O)۶]۳+ Cr+۳ □ d۳ 

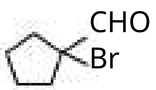
Electron in t۲g = ۳(odd)

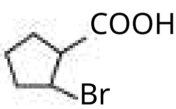
۸۰. Identify the product (P) in the following reaction :



(۱) 

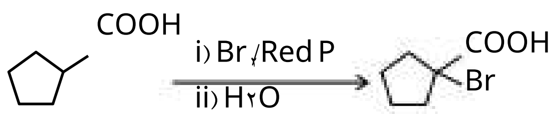
(۲) 

(۳) 

(٤) 

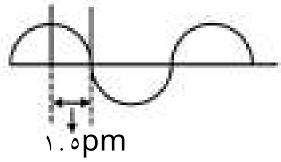
Ans. (۱)

Sol. HVZ Reaction



SECTION-B

۸۱. A hypothetical electromagnetic wave is show below.



The frequency of the wave is x × ۱۰^۱۹ Hz.

x = __________ (nearest integer)

Ans. (٥)

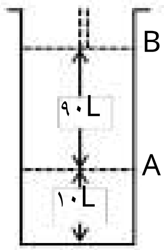
Sol. λ = ۱.٥ × ٤ pm

= ٦ × ۱۰^−۱۲ meter

λν = C

٦ × ۱۰^−۱۲ × ν = ۳ × ۱۰^۸

ν = ٥ × ۱۰^۱۹ Hz

۸۲. 

Consider the figure provided.

۱ mol of an ideal gas is kept in a cylinder, fitted with a piston, at the position A, at ۱۸°C. If the piston is moved to position B, keeping the temperature unchanged, then 'x' L atm work is done in this reversible process.

x = _________ L atm. (nearest integer)

[Given : Absolute temperature = °C + ۲۷۳.۱٥,

R = ۰.۰۸۲۰٦ L atm mol^−۱ K^−۱]

Ans. (٥٥)

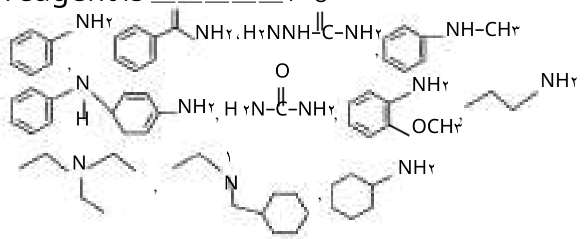
Sol. W = −nRT ln (V۲ / V۱)

= −۱ × ۰.۰۸۲۰٦ × ۲۹۱.۱٥ ln (۱۰۰ / ۱۰)

= −٥٥.۰۱۲۸

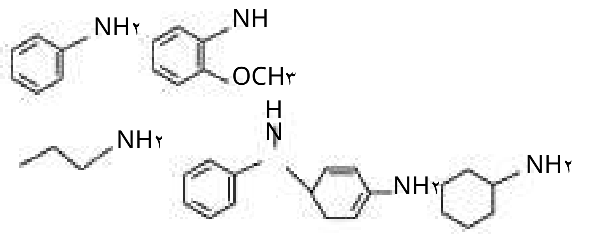
Work done by system □ ٥٥ atm lit.

٨٣. Number of amine compounds from the following giving solids which are soluble in NaOH upon reaction with Hinsberg's reagent is ______.

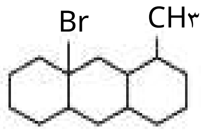


Ans. (٥)

Sol. Primary amine give an ionic solid upon reaction with Hinsberg reagent which is soluble in NaOH.

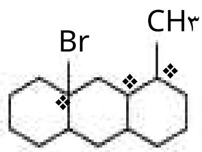


٨٤. The number of optical isomers in following compound is : ________.



Ans. (٣٢)

Sol.



Total chiral centre = ٥

No. of optical isomers = ٢٥ = ٣٢.

٨٥. The 'spin only' magnetic moment value of MO٤²⁻ is ______ BM. (Where M is a metal having least metallic radii. among Sc, Ti, V, Cr, Mn and Zn). (Given atomic number : Sc = ٢١, Ti = ٢٢, V = ٢٣, Cr = ٢٤, Mn = ٢٥ and Zn = ٣٠)

Ans. (٠)

Sol. Metal having least metallic radii among Sc, Ti, V, Cr, Mn & Zn is Cr.

Spin only magnetic moment of CrO٤²⁻.

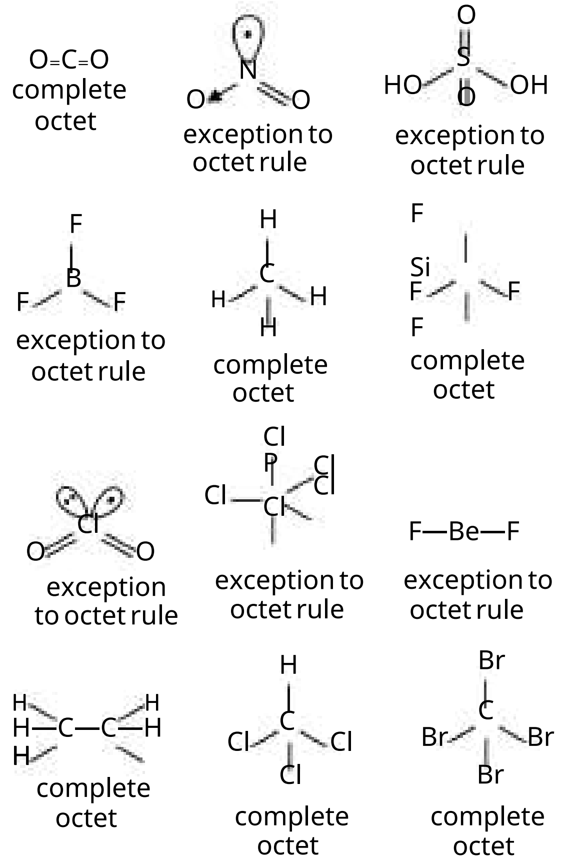
Here Cr⁶⁺ is in d⁰ configuration (diamagnetic).

٨٦. Number of molecules from the following which are exceptions to octet rule is ________.

CO٢, NO٢, H٢SO٤, BF٣, CH٤, SiF٤, ClO٢, PCl٥, BeF٢, C٢H٦, CHCl٣, CBr٤

Ans. (٦)

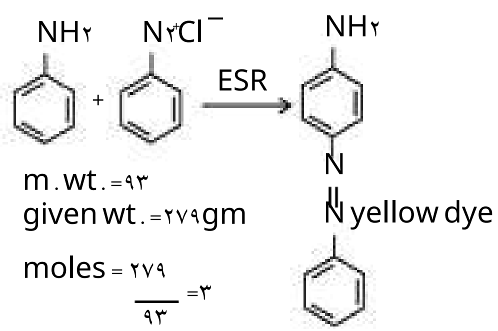
Sol.



٨٧. If ٢٧٩ g of aniline is reacted with one equivalent of benzenediazonium chloride, the mximum amount of aniline yellow formed will be ______ g. (nearest integer) (consider complete conversion)

Ans. (٥٩١)

Sol.



m.wt. = ٩٣

given wt. = ٢٧٩gm

moles = ٢٧٩ / ٩٣ = ٣

moles formed = ٣

m.wt. = ١٩٧

amount formed

= ١٩٧ × ٣ = ٥٩١ gm

٨٨. Consider the following reaction

A + B □ C

The time taken for A to become ٣th/٤ of its initial concentration is twice the time taken to become ١/٢ of the same. Also، when the change of concentration of B is plotted against time، the resulting graph gives a straight line with a negative slope and a positive intercept on the concentration axis.

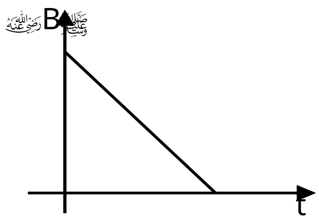
The overall order of the reaction is ______.

Ans. (١)

Sol. For ١st order reaction

٧٥٪ life = ٢ × ٥٠٪ life

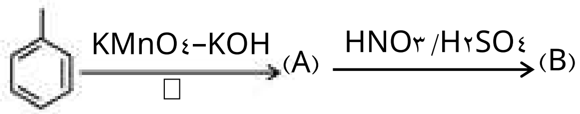
So order with respect to A will be first order.



So order with respect to B will be zero.

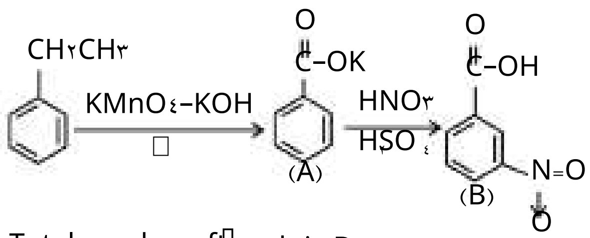
Overall order of reaction = ١ + ٠ = ١

٨٩. Major product B of the following reaction has ______ □-bond.



Ans. (٥)

Sol. Major product B is□



Total number of □bonds in B are ٥

٩٠. A solution containing ١٠g of an electrolyte AB٢ in ١٠٠g of water boils at ١٠٠.٥٢°C. The degree of ionization of the electrolyte (□) is ______ × ١٠⁻¹ (nearest integer)

[Given: Molar mass of AB٢ = ٢٠٠g mol.⁻¹ Kb (molal boiling point elevation const. of water) = ٠.٥٢ K kg mol⁻¹، boiling point of water = ١٠٠°C؛ AB٢ ionises as AB٢ □ A²⁺ + ٢B⁻]

Ans. (٥)

Sol. AB٢ □ A + ٢B□

i = ١ + (٣ − ١)□

i = ١ + ٢□

□Tb = kb im

$$٠.٥٢ = ٠.٥٢ (١ + ٢□) \frac{\frac{١٠}{٢٠٠}}{\frac{١٠٠}{١٠٠٠}}$$

$$١ = (١ + ٢□) \frac{١٠}{٢٠}$$

٢ = ١ + ٢□

□ = ٠.٥

Ans. □ = ٥ × ١٠⁻¹