FINAL JEE–MAIN EXAMINATION – APRIL, 2024

(Held On Tuesday 09 April, 2024) TIME : 9 : 00 AM to 12 : 00 NOON

MATHEMATICS

TEST PAPER WITH SOLUTION

SECTION-A

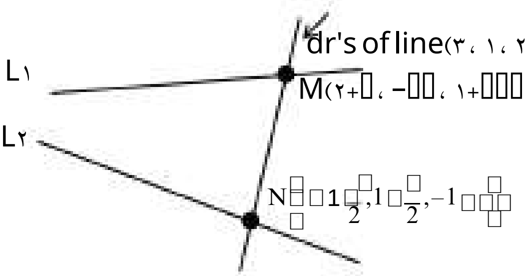
1. Let the line L intersect the lines $x-2=-y=z-1,\ 2(x+1)=2(y-1)=z+1$ and be parallel to the line $\frac{x-2}{3}=\frac{y-1}{1}=\frac{z-2}{2}$. Then which of the following points lies on L?

(1) $\left(-\frac{1}{3},1,1\right)$ (2) $\left(-\frac{1}{3},1,-1\right)$

(3) $\left(-\frac{1}{3},-1,-1\right)$ (4) $\left(-\frac{1}{3},-1,1\right)$

Ans. (2)

Sol.



$L_1 : \frac{x-2}{1}=\frac{y}{-1}=\frac{z-1}{1}=\lambda$

$L_2 : \frac{x+1}{\frac{1}{2}}=\frac{y-1}{\frac{1}{2}}=\frac{z+1}{1}=\mu$

dr of line MN will be

$\left(3+\lambda-\frac{\mu}{2},\ -1-\lambda-\frac{\mu}{2},\ 2+\lambda-\mu\right)$ & it will be proportional to (3, 1, 2)

$\Rightarrow \frac{3+\lambda-\frac{\mu}{2}}{3}=\frac{-1-\lambda-\frac{\mu}{2}}{1}=\frac{2+\lambda-\mu}{2}$

$4\lambda+\mu=-6$ $\quad$ $4+3\lambda=0$

$\Rightarrow \lambda=-\frac{4}{3}$ & $\mu=-\frac{2}{3}$

$\therefore$ Coordinate of M will be $\left(\frac{2}{3},\frac{4}{3},-\frac{1}{3}\right)$

and equation of required line will be.

$\frac{x-\frac{2}{3}}{3}=\frac{y-\frac{4}{3}}{1}=\frac{z+\frac{1}{3}}{2}=k$

So any point on this line will be

$\left(\frac{2}{3}+3k,\frac{4}{3}+k,-\frac{1}{3}+2k\right)$

$\because \frac{2}{3}+3k=-\frac{1}{3} \Rightarrow k=-\frac{1}{3}$

$\therefore$ Point lie on the line for

$k=-\frac{1}{3}$ is $\left(-\frac{1}{3},1,-1\right)$

2. The parabola $y^2=4x$ divides the area of the circle $x^2+y^2=5$ in two parts. The area of the smaller part is equal to :

(1) $\frac{2}{3}+5\sin^{-1}\left(\frac{2}{\sqrt{5}}\right)$ (2) $\frac{1}{3}+5\sin^{-1}\left(\frac{2}{\sqrt{5}}\right)$

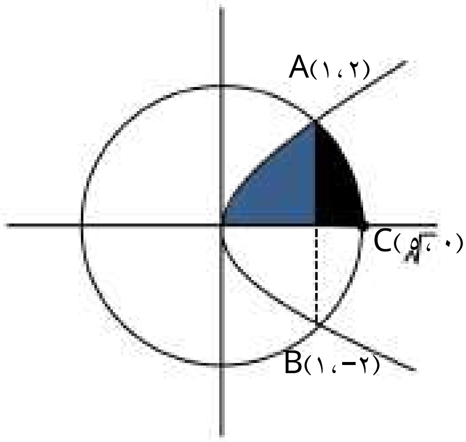
(3) $\frac{1}{3}+\sqrt{5}\sin^{-1}\left(\frac{2}{\sqrt{5}}\right)$ (4) $\frac{2}{3}+\sqrt{5}\sin^{-1}\left(\frac{2}{\sqrt{5}}\right)$

Ans. (1)

Sol. $y^2=4x$

$x^2+y^2=5$

$\therefore$ Area of shaded region as shown in the figure will be



$$A_1 = \int_0^1 \sqrt{4x}\,dx + \int_1^{\sqrt5} \sqrt{5-x^2}\,dx$$

$$= \frac{4}{3}\left[x^{\frac{3}{2}}\right]_0^1 + \left[\frac{x}{2}\sqrt{5-x^2} + \frac{5}{2}\sin^{-1}\frac{x}{\sqrt5}\right]_1^{\sqrt5}$$

$$= \frac{1}{3} + \frac{5\pi}{4} - \frac{5}{2}\sin^{-1}\left(\frac{1}{\sqrt5}\right)$$

∴ Required Area = ٢ A١

$$= \frac{2}{3} + \frac{5\pi}{2} - 5\sin^{-1}\left(\frac{1}{\sqrt5}\right)$$

$$= \frac{2}{3} + 5\left(\frac{\pi}{2} - \sin^{-1}\frac{1}{\sqrt5}\right)$$

$$= \frac{2}{3} + 5\cos^{-1}\frac{1}{\sqrt5}$$

$$= \frac{2}{3} + 5\sin^{-1}\left(\frac{2}{\sqrt5}\right)$$

٣. The solution curve, of the differential equation $2y\frac{dy}{dx} + 3 = 5\frac{dy}{dx}$, passing through the point (٠، ١) is a conic, whose vertex lies on the line :

(١) ٢x + ٣y = ٩ (٢) ٢x + ٣y = −٩

(٣) ٢x + ٣y = −٦ (٤) ٢x + ٣y = ٦

Ans. (١)

Sol. $(2y-5)\frac{dy}{dx} = -3$

$$(2y-5)dy = -3dx$$

$$2\frac{y^2}{2} - 5y = -3x + c$$

∵ Curve passes through (٠، ١)

⇒ c = −٤

∵ Curve will be

$$\left(y-\frac{5}{2}\right)^2 = -3\left(x-\frac{3}{4}\right)$$

∴ Vertex of parabola will be $\left(\frac{3}{4}, \frac{5}{2}\right)$

∵ ٢x + ٣y = ٩

٤. A ray of light coming from the point P (١، ٢) gets reflected from the point Q on the x-axis and then passes through the point R (٤، ٣). If the point S (h، k) is such that PQRS is a parallelogram، then hk is equal to :

(١) ٨٠ (٢) ٩٠

(٣) ٦٠ (٤) ٧٠

Ans. (٤)

Sol.

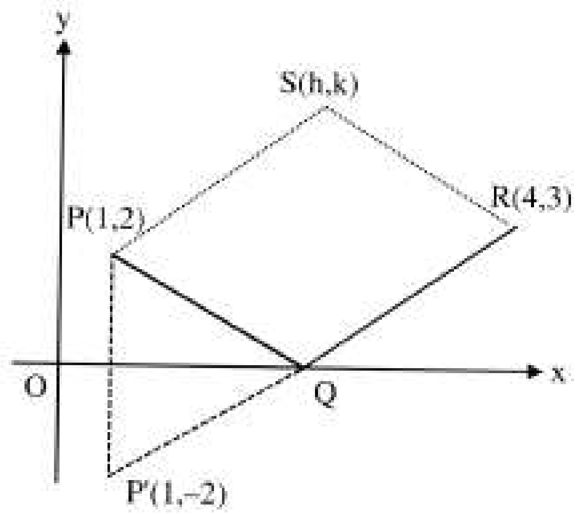


Image of P wrt x-axis will be P'(١، −٢) equation of line joining P'R will be

$$y - 3 = \frac{5}{3}(x-4)$$

Above line will meet x-axis at Q where

$$y = 0 \Rightarrow x = \frac{11}{5}$$

$$\therefore Q\left(\frac{11}{5}, 0\right)$$

∵ PQRS is parallelogram so their diagonals will bisects each other

$\frac{4-1}{2} = \frac{\frac{11}{5} - h}{\frac{14}{5}} = \frac{2-3}{2}$ & $\frac{k-0}{2} = \frac{2-3}{2}$

$h = \frac{5}{14}$ & $k = 5$

$hk2 = \frac{5}{14} \cdot 2 \cdot 5 = ... = \frac{52}{70}$

٥. Let □, □ ∈ R. If the system of equations

٣x + ٥y + □z = ٣

٧x + ١١y − ٩z = ٢

٩٧x + ١٥٥y − ١٨٩z = □

has infinitely many solutions، then □ + ٢□ is equal to :

(١) ٢٥ (٢) ٢٤

(٣) ٢٧ (٤) ٢٢

Ans. (١)

Sol.

٣x + ٥y + □z = ٣

٧x + ١١y − ٩z = ٢

٩٧x + ١٥٥y − ١٨٩z = □

□ ٩٣x + ١٥٥y + ٣١□z = ٩٣

٩٧x + ١٥٥y − ١٨٩z = □

□ −٤x + (٣١□ + ١٨٩)z = ٩٣ − □

١٠٨٥x + ١٧٠٥y − ١٣٩٥z = ٣١٠

١٠٦٧x + ١٧٠٥y − ٢٠٧٩z = ١١□

□ ١٨x + ٦٨٤z = ٣١٠ − ١١□

□ −٣٦x + ٩(٣١□ + ١٨٩)z = ٩(٩٣ − □)

٣٦x + ١٣٦٨z = ٢(٣١٠ − ١١□)

(٢٧٩□ + ٣٠٦٩)z = ١٤٥٧ − ٣١□

for infinite solutions −

□ $\frac{-3069}{279} = \frac{-341}{31}$, $\frac{1457}{31}$

$\frac{1457 - 682}{31} = \frac{775}{31} = 25$

٦. The coefficient of $x^{٧٠}$ in $x^{٢}(١ + x)^{٩٨} + x^{٣}(١ + x)^{٩٧} + x^{٤}(١ + x)^{٩٦} + ... + x^{٥٤}(١ + x)^{٤٦}$ is $^{٩٩}C_p - {}^{٤٦}C_q$.

Then a possible value to p + q is :

(١) ٥٥ (٢) ٦١

(٣) ٦٨ (٤) ٨٣

Ans. (٤)

Sol. $x^2(1+x)^{98} + x^3(1+x)^{97} + x^4(1+x)^{96} + ...$

$x^{54}(1+x)^{46}$

Coeff. of x^{70}: $^{98}C_{68} + {}^{97}C_{67} + {}^{96}C_{66} + ... + {}^{46}C_{16}$

$^{47}C_{17}$

$= {}^{46}C_{30} + {}^{47}C_{30} + ...{}^{98}C_{30}$

$= {}^{46}C_{30} + {}^{46}C_{31} + {}^{47}C_{30} + ...{}^{98}C_{30} - {}^{46}C_{31}$

$= {}^{47}C_{31} + {}^{47}C_{30} + ...{}^{98}C_{30} - {}^{46}C_{31}$

. . . . . .

$= {}^{99}C_{31} - {}^{46}C_{31} = {}^{99}C_p - {}^{46}C_q$

Possible values of (p + q) are ٦٢، ٨٣، ٩٩، ٤٦

□ p + q = ٨٣

٧. Let

$\int \frac{2 - \tan x}{3 + \tan x} dx = \frac{1}{2}\left(\alpha x + \log_e|\beta \sin x + \gamma \cos x|\right) + C$

، where C is the constant of integration.

Then □ + □/□ is equal to :

(١) ٣ (٢) ١

(٣) ٤ (٤) ٧

Ans. (٣)

Sol. $\int \frac{2 - \tan x}{3 + \tan x} dx = \int \frac{2\cos x - \sin x}{3\cos x + \sin x} dx$

٢ cosx − sinx = A(٣cosx + sinx) + B(cosx − ٣sinx)

٣A + B = ٢

A − ٣B = −١

$\Rightarrow A = \frac{1}{2}, B = \frac{1}{2}$

$\therefore \int \frac{2\cos x + \sin x}{3\cos x + \sin x}\,dx$

$= \frac{x}{2} + \frac{1}{2}\ln|3\cos x + \sin x| + C$

$= \frac{1}{2}\left(x + \ln|3\cos x + \sin x|\right) + C$

$= \frac{1}{2}\left(x + \ln|\sin x + 3\cos x|\right) + C$

$\lambda = 1,\ \mu = 1,\ \nu = 3$

$\therefore \lambda + \frac{\nu}{\mu} = 1 + \frac{3}{1} = 4$

٨. A variable line L passes through the point (٣، ٥) and intersects the positive coordinate axes at the points A and B. The minimum area of the triangle OAB, where O is the origin, is :

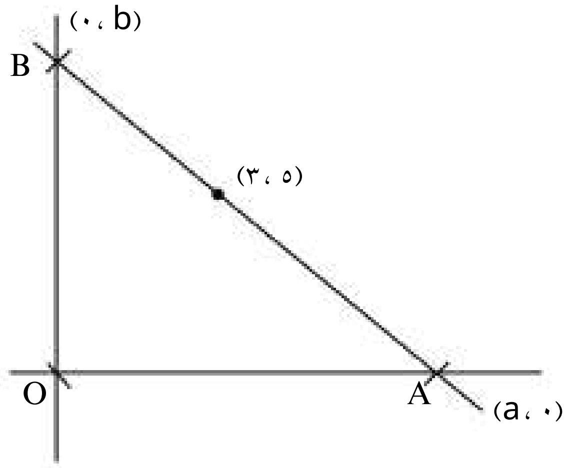
(٢) ۴٥

(۴) ٤٥

Ans. (١)

Sol. $\frac{x}{a} + \frac{y}{b} = 1$

$\frac{3}{a} + \frac{5}{b} = 1 \Rightarrow b = \frac{5a}{a-3},\ a > 3$



$A = \frac{1}{2}ab = \frac{1}{2}a\left(\frac{5a}{a-3}\right) = \frac{5}{2}\left(\frac{a^2}{a-3}\right)$

$= \frac{5}{2}\left[\frac{(a^2 - 9) + 9}{a-3}\right]$

$= \frac{5}{2}\left[(a-3) + \frac{9}{a-3} + 6\right]$

$\geq \frac{5}{2}\left[6 + 6\right] = 30$

٩. Let $|\cos\theta\cos(60-\theta)\cos(60+\theta)| \leq \frac{1}{8}, \theta \in [0, 2\pi]$. Then, the sum of all $\theta \in [0, 2\pi]$, whe attains its maximum value, is :

(١) 9π (٢) 18π

(٣) 6π (٤) 15π

Ans. (٣)

Sol. We know that

$(\cos\theta)(\cos(60° - \theta))(\cos(60° + \theta)) = \frac{1}{4}\cos 3\theta$

So equation reduces to $\left|\frac{1}{4}\cos 3\theta\right| \leq \frac{1}{8}$

$\Rightarrow |\cos 3\theta| \leq \frac{1}{2}$

$\Rightarrow -\frac{1}{2} \leq \cos 3\theta \leq \frac{1}{2}$

$\therefore$ maximum value of $\cos 3\theta = \frac{1}{2}$, here

$\Rightarrow 3\theta = 2n\pi \pm \frac{\pi}{3}$

$\Rightarrow \theta = \frac{2n\pi}{3} \pm \frac{\pi}{9}$

As $\theta \in [0, 2\pi]$ possible values are

$\theta = \left\{\frac{\pi}{9}, \frac{5\pi}{9}, \frac{7\pi}{9}, \frac{11\pi}{9}, \frac{13\pi}{9}, \frac{17\pi}{9}\right\}$

Whose sum is

$\frac{\pi}{9} + \frac{5\pi}{9} + \frac{7\pi}{9} + \frac{11\pi}{9} + \frac{13\pi}{9} + \frac{17\pi}{9} = \frac{54\pi}{9} = 6\pi$

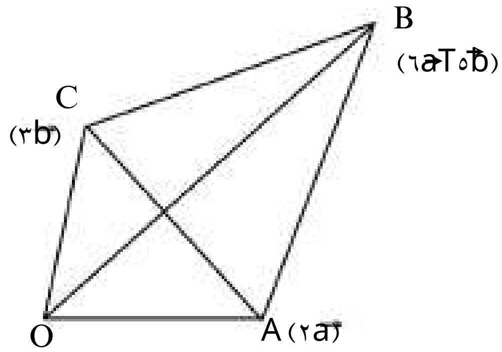
١٠. Let $\overrightarrow{OA} = 2\vec{a}, \overrightarrow{OB} = 6\vec{a} + 5\vec{b}$ and $\overrightarrow{OC} = 3\vec{b}$, where O is the origin. If the area of the parallelogram with adjacent sides OA and OC is ١٥ sq. units, then the area (in sq. units) of the quadrilateral OABC is equal to :

(١) ٣٨ (٢) ٤٠

(٣) ٣٢ (٤) ٣٥

Ans. (٤)

Sol.



Area of parallelogram having sides

$\overline{OA}$ & $\overline{OC} = \left|\overrightarrow{OA} \times \overrightarrow{OC}\right| = \left|2\vec{a} \times 3\vec{b}\right| = 15$

$6\left|\vec{a} \times \vec{b}\right| = 15$

$\Rightarrow \left|\vec{a} \times \vec{b}\right| = \frac{5}{2}$ ....(١)

Area of quadrilateral

$OABC = \frac{1}{2}\left|\vec{d_1} \times \vec{d_2}\right|$

$= \frac{1}{2}\left|\overrightarrow{AC} \times \overrightarrow{OB}\right| = \frac{1}{2}\left|(3\vec{b} - 2\vec{a}) \times (6\vec{a} + 5\vec{b})\right|$

$= \frac{1}{2}\left|18\,\vec{b} \times \vec{a} - 10\,\vec{a} \times \vec{b}\right| = 14\left|\vec{a} \times \vec{b}\right|$

$= 14 \times \frac{5}{2} = 35$

١١. If the domain of the function

$f(x) = \sin^{-1}\left(\frac{|x-1|}{2x+3}\right)$ is R − (α, β)

then ١٢αβ is equal to :

(١) ٣٦ (٢) ٢٤

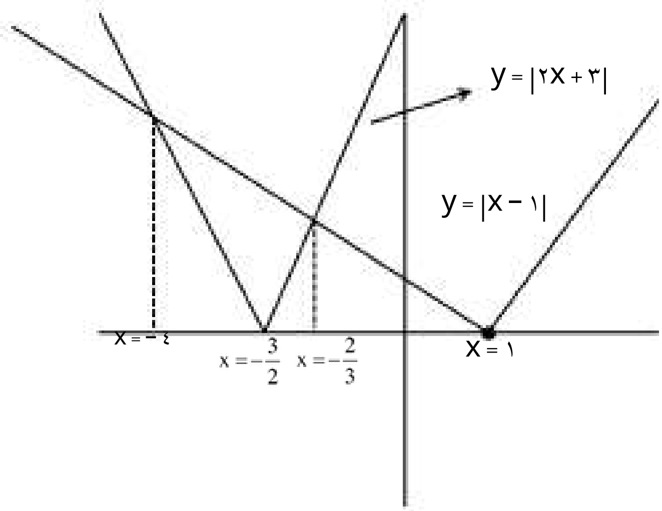
(٣) ٤٠ (٤) ٣٢

Ans. (٤)

Sol. Domain of $f(x) = \sin^{-1}\left(\frac{|x-1|}{2x+3}\right)$ is

$2x + 3 \neq 0$ & $x \neq -\frac{3}{2}$ and $\left|\frac{x-1}{2x+3}\right| \leq 1$

$|x-1| \leq |2x+3|$



For $|2x+3| \geq |x-1|$

$x \in (-\infty, -4] \cup \left[-\frac{2}{3}, \infty\right)$

$\alpha = -4$ & $\beta = -\frac{2}{3}$: $12\alpha\beta = 32$

١٢. If the sum of series

$\frac{1}{1(1+d)} + \frac{1}{(1+d)(1+2d)} + \dots + \frac{1}{(1+9d)(1+10d)}$

is equal to ٥, then ٥٠d is equal to :

(١) ٢٠ (٢) ٥

(٣) ١٥ (٤) ١٠

Ans. (٢)

Sol. $\frac{1}{1(1+d)} + \frac{1}{(1+d)(1+2d)} + \dots\dots\dots\dots + \frac{1}{(1+9d)(1+10d)} = 5$

$\frac{1}{d}\left[\frac{(1+d)-1}{1(1+d)}+\frac{(1+2d)-(1+d)}{(1+d)(1+2d)}+\dots\dots\dots\dots\right.$

$\left.\frac{(1+10d)-(1+9d)}{(1+9d)(1+10d)}\right]=5$

$\frac{1}{d}\left[\left(1-\frac{1}{1+d}\right)+\left(\frac{1}{1+d}-\frac{1}{1+2d}\right)+\dots\dots\dots\dots\right.$

$\left.\left(\frac{1}{1+9d}-\frac{1}{1+10d}\right)\right]=5$

$\frac{1}{d}\left[1-\frac{1}{(1+10d)}\right]=5$

$\frac{10d}{1+10d}=5d$

$50d=5$

١٣. Let $f(x)=ax^3+bx^2+cx+41$ be such that

f(١) = ٤٠, f'(١) = ٢ and f''(١) = ٤.

Then $a^2+b^2+c^2$ is equal to :

(١) ٦٢ (٢) ٧٣

(٣) ٥٤ (٤) ٥١

Ans. (٤)

Sol. $f(x)=ax^3+bx^2+cx+41$

$f'(x)=3ax^2+2bx+c$

$\therefore$ f'(١) = ٣a + ٢b + c = ٢ .... (١)

$f''(x)=6ax+2b$

$\therefore f''(1)=6a+2b=4$

٣a + b = ٢ .... (٢)

(١) − (٢)

b + c = ٠ .... (٣)

f(١) = ٤٠

a + b + c + ٤١ = ٤٠

use (٣)

a + ٤١ = ٤٠

by (٢)

−٣ + b = ٢ ⇒ b = ٥ & c = −٥

a² + b² + c² = ١ + ٢٥ + ٢٥ = ٥١

١٤. Let a circle passing through (٢, ٠) have its centre at the point (h, k). Let (x_c, y_c) be the point of intersection of the lines ٣x + ٥y = ١ and (٢ + c) x + ٥c²y = ١. If $h=\lim_{c\to1}x_c$ and $k=\lim_{c\to1}y_c$, then the equation of the circle is :

(١) ٢٥x² + ٢٥y² − ٢٠x + ٢y − ٦٠ = ٠

(٢) ٥x² + ٥y² − ٤x − ٢y − ١٢ = ٠

(٣) ٢٥x² + ٢٥y² − ٢x + ٢y − ٦٠ = ٠

(٤) ٥x² + ٥y² − ٤x + ٢y − ١٢ = ٠

Ans. (١)

Sol. $(2+c)x+5c^2\left(\frac{1-3x}{5}\right)=1$

$x=\frac{1-c^2}{2+c-3c^2},\ y=\frac{1-3x}{5}=\frac{c-1}{5(2+c-3c^2)}$

$h=\lim_{c\to1}\frac{(1-c)(1+c)}{(1-c)(2+3c)}=\frac{2}{5}$

$K=\lim_{c\to1}\frac{c-1}{5(1-c)(2+3c)}=-\frac{1}{25}$

Centre $\left(\frac{2}{5},-\frac{1}{25}\right)$,

$r=\sqrt{\left(2-\frac{2}{5}\right)^2+\left(0+\frac{1}{25}\right)^2}=\sqrt{\frac{64}{25}+\frac{1}{625}}$

$r=\sqrt{\frac{161}{25}}$

$\left(x-\frac{2}{5}\right)^2+\left(y+\frac{1}{25}\right)^2=\frac{161}{125}$

$\Rightarrow 25x^2+25y^2-20x+2y-60=0$

١٥. The shortest distance between the line

$\frac{x-3}{4}=\frac{y-7}{-11}=\frac{z-1}{5}$ and $\frac{x-5}{3}=\frac{y-9}{-6}=\frac{z+2}{1}$

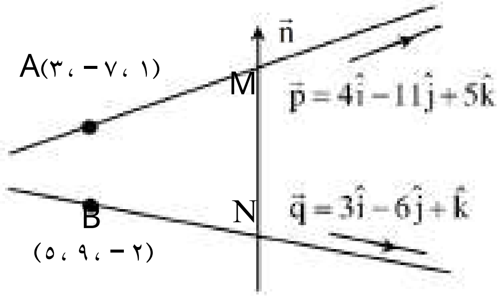
is :

(١) $\frac{187}{\sqrt{563}}$ (٢) $\frac{178}{\sqrt{563}}$

(٣) $\frac{185}{\sqrt{563}}$ (٤) $\frac{179}{\sqrt{563}}$

Ans. (١)

Sol.



$\vec n = \vec p \times \vec q$

$\vec n = \begin{vmatrix} \hat i & \hat j & \hat k \\ 4 & -11 & 5 \\ 3 & -6 & 1 \end{vmatrix} = 19\hat i + 11\hat j + 9\hat k$

S.d. = projection of $\overrightarrow{AB}$ on $\vec n$

$= \left|\frac{\overrightarrow{AB}\cdot\vec n}{|\vec n|}\right| = \left|\frac{(2\hat i + 16\hat j - 3\hat k)\cdot(19\hat i + 11\hat j + 9\hat k)}{\sqrt{361+121+81}}\right|$

$= \frac{38+176-27}{\sqrt{563}}$

$S.d. = \frac{187}{\sqrt{563}}$

١٦. The frequency distribution of the age of students in a class of ٤٠ students is given below.

Age	١٥	١٦	١٧	١٨	١٩	٢٠
No. of Students	٥	٨	٥	١٢	x	y

If the mean deviation about the median is ١.٢٥, then ٤x + ٥y is equal to :

(١) ٤٣ (٢) ٤٤

(٣) ٤٧ (٤) ٤٦

Ans. (٢)

Sol. x + y = ١٠ ....(١)

Median = ١٨ = M

$M.D. = \frac{\sum f_i |x_i - M|}{\sum f_i}$

$1.25 = \frac{36 + x + 2y}{40}$

x + ٢y = ١٤ ....(١)

by (١) & (٢)

x = ٦، y = ٤

∴ ٤x + ٥y = ٢٤ + ٢٠ = ٤٤

Age(xi)	f	|xi − M|	fi|xi − M|
١٥	٥	٣	١٥
١٦	٨	٢	١٦
١٧	٥	١	٥
١٨	١٢	٠	٠
١٩	x	١	x
٢٠	y	٢	٢y

١٧. The solution of the differential equation $(x^2 + y^2)dx - 5xy\,dy = 0$, $y(1) = 0$, is :

(١) $|x^2 - 4y^2|^5 = x^2$ (٢) $|x^2 - 2y^2|^6 = x^2$

(٣) $|x^2 - y^2|^6 = x$ (٤) $|x^2 - 2y^2|^5$

Ans. (١)

Sol. $(x^2 + y^2)dx = 5xy\,dy$

$\Rightarrow \frac{dy}{dx} = \frac{x^2 + y^2}{5xy}$

Put y = Vx

$\Rightarrow V + x\frac{dv}{dx} = \frac{1 + V^2}{5V}$

$\Rightarrow x\frac{dv}{dx} = \frac{1 - 4V^2}{5V}$

$\Rightarrow \int \frac{5V}{1 - 4V^2}dV = \int \frac{dx}{x}$

Let $1 - 4V^2 = t$

$\Rightarrow -8V\,dV = dt$

$\Rightarrow \int \frac{dt}{(8-t)} = \int \frac{dx}{5x}$

$\Rightarrow \frac{-1}{8}\ln|t| = \frac{1}{5}\ln|x| + \ln C$

$\Rightarrow -5\ln|t| = 8\ln|x| + \ln K$

$\Rightarrow \ln x^8 + \ln|t^5| - \ln K = 0$

$\Rightarrow x^8|t^5| = C$

$\Rightarrow x^8|1-4V^2|^5 = C$

$\Rightarrow x^8\left|\frac{x^2-4y^2}{x^2}\right|^5 = C$

$\Rightarrow |x^2-4y^2|^5 = Cx^2$

given y(١) = ٠

$\Rightarrow |1|^5 = C \Rightarrow C = 1$

$\Rightarrow |x^2-4y^2|^5 = x^2$

١٨. Let three vectors $\vec a = \alpha\hat i + 4\hat j + 2\hat k$, $\vec b = 5\hat i + 3\hat j + 4\hat k$, $\vec c = x\hat i + y\hat j + z\hat k$ from a triangle such that $\vec c = \vec a - \vec b$ and the area of the triangle is $5\sqrt6$. If α is a positive real number, then $|\vec c|^2$ is:

(١) ١٦ (٢) ١٤

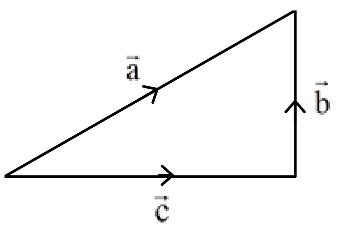
(٣) ١٢ (٤) ١٠

Ans. (٢)

Sol. $\vec c = \vec a - \vec b$

⇒ (x, y, z) = (α − ٥, ١, −٢)

⇒ x = α − ٥, y = ١, z = −٢ ....(١)



Area of Δ = ٥√6 (given)

$\frac{1}{2}|\vec a \times \vec c| = 5\sqrt6$

$\left|\begin{vmatrix} \hat i & \hat j & \hat k \\ \alpha & 4 & 2 \\ x & 1 & -2 \end{vmatrix}\right| = 10\sqrt6$

$\Rightarrow |-10\hat i - \hat j(-2\alpha - 2x) + \hat k(\alpha - 4x)| = 10\sqrt6$

⇒ (٢α + ٢α − ١٠)² + (α − ٤α + ٢٠)² = ٥٠٠

⇒ (٤α − ١٠)² + (٢٠ − ٣α)² = ٥٠٠

⇒ ٢٥α² − ٨٠α − ١٢٠α = ٠

⇒ α(٢٥α − ٢٠٠) = ٠

⇒ α = ٨ (given α is +ve number)

⇒ x = α − ٥ = ٣

$|\vec c|^2 = x^2 + y^2 + z^2$

= ٩ + ١ + ٤

= ١٤

١٩. Let α, β be the roots of the equation $x^2 + 2\sqrt2 x - 1 = 0$. The quadratic equation, whose roots are $\alpha^4 + \beta^4$ and $\frac{1}{10}(\alpha^6 + \beta^6)$, is:

(١) x² − ١٩٠x + ٩٤٦٦ = ٠

(٢) x² − ١٩٥x + ٩٤٦٦ = ٠

(٣) x² − ١٩٥x + ٩٥٠٦ = ٠

(٤) x² − ١٨٠x + ٩٥٠٦ = ٠

Ans. (٣)

Sol. x² + ٢√٢x − ١ = ٠

α + β = −٢√2

αβ = −١

α⁴ + β⁴ = (α² + β²)² − ٢α²β²

= ((α + β)² − ٢αβ)² − ٢(αβ)²

= (٨ + ٢)² − ٢(−١)²

= ١٠٠ − ٢ = ٩٨

α⁶ + β⁶ = (α³ + β³)² − ٢α³β³

= ((α + β)((α + β)² − ٣αβ))² − ٢(αβ)³

$= (-٢\sqrt{2}(٨ + ٣)) + ٢^٢$

$= (٨)(١٢١) + ٢ = ٩٧٠$

$\frac{1}{10}(\square + \square) = ٩٧$

$x^٢ - (٩٨ + ٩٧)x + (٩٨)(٩٧) = ٠$

$\square x^٢ - ١٩٥x + ٩٥٠٦ = ٠$

٢٠. Let $f(x) = x^2 - 9$, $g(x) = \frac{x}{x-9}$ and

$a = fog(١٠)$، $b = gof(٣)$. If e and l denote the eccentricity and the length of the latus rectum of the ellipse $\frac{x^2}{a} + \frac{y^2}{b} = 1$، then $٨e^٢ + l^٢$ is equal to.

(١) ١٦ (٢) ٨

(٣) ٦ (٤) ١٢

Ans. (٢)

Sol. $f(x) = x^٢ - ٩$ $g(x) = \frac{x}{x - 9}$

$a = f(g(١٠)) = f\left(\frac{10}{10-9}\right)$

$= f(١٠) = ١٠٩$

$b = g(f(٣)) = g(٩ + ٩)$

$= g(١٨) = \frac{18}{9} = ٢$

$E : \frac{x^2}{109} + \frac{y^2}{2} = 1$

$e^٢ = ١ - \frac{2}{109} = \frac{107}{109}$

$\ell = \frac{2 \times 2}{\sqrt{109}} = \frac{4}{\sqrt{109}}$

$8e^2 + \ell^2 = \frac{8 \times 107 + 16}{109}$

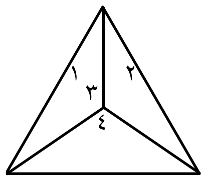
$= ٨$

SECTION-B

٢١. Let a، b and c denote the outcome of three independent rolls of a fair tetrahedral die، whose four faces are marked ١، ٢، ٣، ٤. If the probability that $ax^٢ + bx + c = ٠$ has all real roots is $\frac{m}{n}$، gcd(m، n) = ١، then m + n is equal to ____________.

Ans. (١٩)

Sol. a، b، c $\in$ {١، ٢، ٣، ٤}



Tetrahedral dice

$ax^٢ + bx + c = ٠$

has all real roots

$\Rightarrow D \geq 0$

$\Rightarrow b^2 - 4ac \geq 0$

Let b = ١ $\Rightarrow$ ١ $\geq$ ٤ac (Not feasible)

b = ٢ $\Rightarrow$ ٤ $\geq$ ٤ac $\Rightarrow$ ١ $\geq$ ac $\Rightarrow$ a = ١، c = ١،

b = 3 $\Rightarrow 9 - 4ac \geq 0$

$\frac{9}{4} \geq ac$

$\Rightarrow$ a = ١، c = ١

$\Rightarrow$ a = ١، c = ٢

$\Rightarrow$ a = ٢، c = ١

b = ٤ $\Rightarrow$ ١٦ $\geq$ ٤ac $\geq$ 0

$4 \geq ac$

$\Rightarrow$ a = ١، c = ١

$\Rightarrow$ a = ١، c = ٢ $\Rightarrow$ a = ٢، c = ١ a

$\Rightarrow$ a = ١، c = ٣ $\Rightarrow$ = ٣، c = ١ a =

$\Rightarrow$ a = ١، c = ٤ $\Rightarrow$ ٤، c = ١

$\Rightarrow$ a = ٢، c = ٢

Probability $= \frac{12}{4 \times 4 \times 4} = \frac{3}{16} = \frac{m}{n}$

m + n = ١٩

٢٢. The sum of the square of the modulus of the elements in the set

$\{z = a + ib : a, b \in Z, z \in C, |z - 1| \le 1, |z - 5| \le |z - 5i|\}$

is ___________.

Ans. (٩)

Sol. $|z - 1| \le 1$

$\Rightarrow |(x - 1) + iy| \le 1$

$\Rightarrow \sqrt{(x - 1)^2 + y^2} \le 1$

$\Rightarrow (x - 1)^2 + y^2 \le 1$ .... (١)

Also $|z - 5| \le |z - 5i|$

$(x - 5)^2 + y^2 \le x^2 + (y - 5)^2$

$-10x \le -10y$

$\Rightarrow x \ge y$ .... (٢)

Solving (١) and (٢)

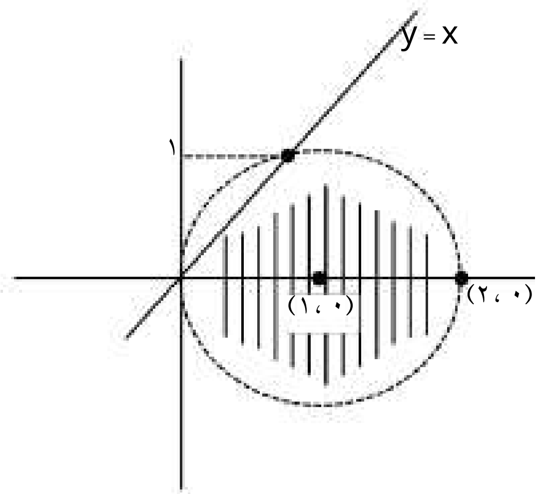
$\Rightarrow (x - 1)^2 + x^2 = 1$

$\Rightarrow 2x^2 - 2x = 0$

$\Rightarrow x(x - 1) = 0$

$\Rightarrow x = 0$ or $x = 1$

$y = 0$ or $y = 1$



Given x، y ∈ I

Points (٠، ٠)، (١، ٠)، (٢، ٠)، (١، ١)، (١، −١)

to find

$|z_1|^2 + |z_2|^2 + |z_3|^2 + |z_4|^2 + |z_5|^2$

= ٠ + ١ + ٤ + ١ + ١ + ١ + ١ = ٩

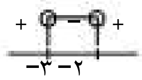
٢٣. Let the set of all positive values of λ, for which the point of local minimum of the function (١ + x (λ² − x²)) satisfies $\frac{x^2 + x + 2}{x^2 + 5x + 6} < 0$, be (α، β).

Then α² + β² is equal to ___________.

Ans. (٣٩)

Sol. $\frac{x^2 + x + 2}{x^2 + 5x + 6} < 0$

$\Rightarrow \frac{1}{(x + 2)(x + 3)} < 0$



x ∈ (−٣، −٢) ....(١)

$f(x) = 1 + x(\lambda^2 - x^2)$

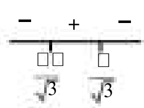
Finding local minima

$f'(x) = \lambda^2 - 3x^2$

Put f'(x) = ٠

$\lambda^2 = 3x^2$

$\Rightarrow x = \pm \frac{\lambda}{\sqrt{3}}$



Local min Local max

We want local min

$\Rightarrow x = \frac{-\lambda}{\sqrt{3}}$

∴ from (١)

∴ $x \in (-3, -2)$

$\left(3\sqrt{3}-\ldots\right)\left(\ldots-2\sqrt{3}\right)$

$3\sqrt{3} \qquad 2\sqrt{3}$

$\alpha = 2\sqrt{3},\ \beta = 3\sqrt{3}$

$\alpha^2+\beta^2 = 12+27 = 39$

٢٤. Let

$$\lim_{n\to\infty}\left(\frac{n}{\sqrt{n^4+1}}-\frac{2n}{(n^2+1)\sqrt{n^4+1}}+\frac{n}{\sqrt{n^4+16}}-\frac{8n}{(n^2+4)\sqrt{n^4+16}}+\ldots+\frac{n}{\sqrt{n^4+n^4}}-\frac{2n\cdot n^2}{(n^2+n^2)\sqrt{n^4+n^4}}\right)$$ be $\frac{\pi}{k}$,

using only the principal values of the inverse trigonometric functions. Then k^2 is equal to ________.

Ans. (٣٢)

Sol. $$\sum_{r=1}^{n}\left(\frac{n}{\sqrt{n^4+r^4}}-\frac{2nr^2}{(n^2+r^2)\sqrt{n^4+r^4}}\right)$$

$$=\sum_{r=1}^{n}\left(\frac{\frac{1}{n}}{\sqrt{1+\left(\frac{r}{n}\right)^4}}-\frac{2\frac{1}{n}\left(\frac{r}{n}\right)^2}{\left(1+\left(\frac{r}{n}\right)^2\right)\sqrt{1+\left(\frac{r}{n}\right)^4}}\right)$$

$$=\int_0^1\frac{dx}{\sqrt{1+x^4}}-\int_0^1\frac{2x^2\,dx}{(1+x^2)\sqrt{1+x^4}}$$

$$=\int_0^1\frac{1-x^2}{(1+x^2)\sqrt{1+x^4}}\,dx$$

$$=\int_0^1\frac{\frac{1}{x^2}-1}{\left(x+\frac{1}{x}\right)\sqrt{x^2+\frac{1}{x^2}}}\,dx$$

$$=-\int_0^1\frac{1-\frac{1}{x^2}}{\left(x+\frac{1}{x}\right)\sqrt{\left(x+\frac{1}{x}\right)^2-2}}\,dx$$

$x+\frac{1}{x}=t \qquad \left(1-\frac{1}{x^2}\right)dx = dt$

$$=-\int_{\infty}^{2}\frac{dt}{t\sqrt{t^2-2}}$$

$$=\int_{2}^{\infty}\frac{t\,dt}{t^2\sqrt{t^2-2}}$$

take $t^2-2=\theta^2$

$t\,dt=\theta\,d\theta$

$$=\int_{\sqrt{2}}^{\infty}\frac{\theta\,d\theta}{\theta(\theta^2+2)}$$

$$=\int_{\sqrt{2}}^{\infty}\frac{d\theta}{\theta^2+2}$$

$$=\frac{1}{\sqrt{2}}\left[\tan^{-1}\frac{\theta}{\sqrt{2}}\right]_{\sqrt{2}}^{\infty}$$

$$=\frac{1}{\sqrt{2}}\left(\tan^{-1}\infty-\tan^{-1}1\right)$$

$$=\frac{1}{\sqrt{2}}\left(\frac{\pi}{2}-\frac{\pi}{4}\right)$$

$$=\frac{\pi}{4\sqrt{2}}=\frac{\pi}{K}$$

So $K=4\sqrt{2}$

$K^2=32$

٢٥. The remainder when 428^{2024} is divided by 21 is ____________.

Ans. (١)

Sol. $(428)^{2024}=(420+8)^{2024}$

$=(21\times20+8)^{2024}$

$=21m+8^{2024}$

Now $8^{2024}=(8^2)^{1012}$

$=(64)^{1012}$

$=(63+1)^{1012}$

$=(21\times3+1)^{1012}$

$=21n+1$

$\therefore$ Remainder is 1.

٢٦. Let $f:(\cdot,\pi)\to R$ be a function given by

$$f(x)=\begin{cases}\left(8\right)^{\frac{\tan 8x}{\tan 7x}}, & 0<x<\frac{\pi}{2}\\ a-8, & x=\frac{\pi}{2}\\ \left(1+\left|\cot x\right|\right)^{\frac{b}{a}\left|\tan x\right|}, & \frac{\pi}{2}<x<\pi\end{cases}$$

Where a، b ∈ Z. If f is continuous at $x=\frac{\pi}{2}$، then

a٢ + b٢ is equal to _____________.

Ans. (٨١)

Sol. LHL at $x=\frac{\pi}{2}$

$$\lim_{x\to\frac{\pi}{2}^-}\left(\frac{8}{7}\right)^{\frac{\tan 8x}{\tan 7x}}=\left(\frac{8}{7}\right)^{0}=1$$

RHL at $x=\frac{\pi}{2}$

$$\lim_{x\to\frac{\pi}{2}^+}\left(1+\left|\cot x\right|\right)^{\frac{b}{a}\left|\tan x\right|}$$

$$=e^{\lim_{x\to\frac{\pi}{2}^+}\left|\cot x\right|\frac{b}{a}\left|\tan x\right|}=e^{\frac{b}{a}}$$

∴ ١ = a − ٨ = $e^{\frac{b}{a}}$

∴ a = ٩، b = ٠

∴ a٢ + b٢ = ٨١

٢٧. Let A be a non-singular matrix of order ٣. If det(٣adj(٢adj((detA)A))) = ٣$^{-١٣}$. ٢$^{-١٠}$ and det(٣adj(٢A)) = ٢^{m} . ٣^{n}، then |٣m + ٢n| is equal to _____________.

Ans. (١٤)

Sol. |٣ adj(٢adj(|A|A))| = |٣adj(٢|A|٣adj(A))|

= |٣ . ٢|A|٤ adj(adj(A))| = ٣٣ |A|١٢ |A|٤

= ٣٣ ٢٦ |A|١٦ = ٢$^{-١٠}$ ٣$^{-١٣}$

∴ |A|١٦ = ٢$^{-١٦}$ ٣$^{-١٦}$ ∴ |A| = ٢$^{-١}$ ٣$^{-١}$

Now |٣adj(٢A)| = |٣ . ٤ adj(A)|

= ٣٣ ٢٦ |A|٢ = ٢^{m} ٣^{n}

∴ ٢٤ ٣٣ ٣$^{-٢}$ ٢$^{-٢}$ = ٢^{m} ٣^{n}

∴ ٢^{m} ٣^{n} = ٢٤ ٣١

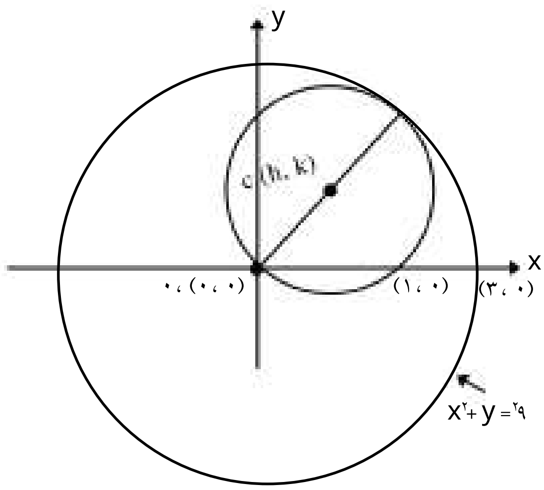
∴ m = ٤، n = ١

∴ |٣m + ٢n| = |١٢ + ٢| = ١٤

٢٨. Let the centre of a circle، passing through the point (٠، ٠)، (١، ٠) and touching the circle x٢ + y٢ = ٩، be (h، k). Then for all possible values of the coordinates of the centre (h، k)، ٤(h٢ + k٢) is equal to _____________.

Ans. (٩)

Sol.



(x − h)٢ + (y − k)٢ = h٢ + k٢

x٢ + y٢ − ٢hx − ٢ky = ٠

∵ passes through (١، ٠)

∴ ١ + ٠ − ٢h = ٠

∴ h = ١/٢

∴ $OC=\frac{OP}{2}$

$$\sqrt{\left(\frac{1}{2}\right)^2+k^2}=\frac{3}{2}$$

$\frac{1}{4} \le k^{2} \le \frac{9}{4}$

$k^{٢} = ٢$

$k = \pm\sqrt{2}$

∴ Possible coordinate of

$c(h, k) \equiv \left(\frac{1}{2}, \sqrt{2}\right), \left(\frac{1}{2}, -\sqrt{2}\right)$

$٤(h^{٢} + k)^{٢} = ٤\left(\frac{1}{4} + 2\right) = ٤\left(\frac{9}{4}\right) = ٩$

٢٩. If a function f satisfies f(m + n) = f(m) + f(n) for all m، n ∈ N and f(١) = ١، then the largest natural number □ such that $\sum_{k=1}^{2022} f(\square + k) \le (2022)^2$ is equal to ______________.

Ans. (١٠١٠)

Sol. f(m + n) = f(m) + f(n)

⇒ f(x) = kx ⇒ f

(١) = ١ ⇒ k = ١

f(x) = x

Now

$\sum_{k=1}^{2022} f(\square + k) \le (2022)2$

$\sum_{k=1}^{2022} (\square + k) \le (2022)2$

$\Rightarrow$ ٢٠٢٢□ + $\frac{2022 \times 2023}{2}$ ≤ (٢٠٢٢)^٢

□□□□□□ ٢٠٢٢ − $\frac{2023}{2}$

□□□□□□ ١٠١٠.٥

∴ largest natural no. □ is ١٠١٠.

٣٠. Let A = {٢، ٣، ٦، ٧} and B = {٤، ٥، ٦، ٨}. Let R be a relation defined on A × B by (a١، b١) R (a٢، b٢) is and only if a١ + a٢ = b١ + b٢. Then the number of elements in R is ______________.

Ans. (٢٥)

Sol. A = {٢، ٣، ٦، ٧}

B = {٢، ٥، ٦، ٨}

(a١، b١) R (a٢، b٢)

a١ + a٢ = b١ + b٢

١. (٢، ٤) R (٦، ٤)	٢. (٢، ٤) R (٧، ٥) ٤.
٣. (٢،٥) R (٧، ٤)	(٣، ٤) R (٦، ٥) ٦.
٥. (٣، ٥) R (٦، ٤)	(٣، ٥) R (٧، ٥) ٨.
٧. (٣،٦) R (٧، ٤)	(٣، ٤) R (٧، ٦) ١٠.
٩. (٦، ٥) R (٧، ٨)	(٦، ٨) R (٧، ٥) ١٢.
١١. (٧،٨) R (٧، ٦)	(٦، ٨) R (٦، ٤)
١٣. (٦،٦) R (٦، ٦)	

× ٢

Total ٢٤ + ١ = ٢٥

PHYSICS — TEST PAPER WITH SOLUTION

SECTION-A

٣١. A proton، an electron and an alpha particle have the same energies. Their de-Broglie wavelengths will be compared as:

(١) $\lambda_e < \lambda_p < \lambda_\alpha$ (٢) $\lambda_\alpha < \lambda_p < \lambda_e$

(٣) $\lambda_p < \lambda_e < \lambda_\alpha$ (٤) $\lambda_p < \lambda_\alpha < \lambda_e$

Ans. (٢)

Sol. $\lambda_{DB} = \frac{h}{p} = \frac{h}{\sqrt{٢mk}}$

$\lambda_{DB} \propto \frac{١}{\sqrt{m}}$

$\lambda_\alpha < \lambda_p < \lambda_e$

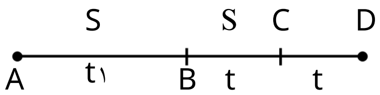
٣٢. A particle moving in a straight line covers half the distance with speed ٦ m/s. The other half is covered in two equal time intervals with speeds ٩ m/s and ١٥ m/s respectively. The average speed of the particle during the motion is:

(١) ٨.٨ m/s (٢) ١٠ m/s

(٣) ٩.٢ m/s (٤) ٨ m/s

Ans. (٤)

Sol.



BD ⇒ $S = ٩t + ١٥t = ٢٤t$

AB ⇒ $S = ٦t_١ = ٢٤t \Rightarrow t_١ = ٤t$

$\langle speed \rangle = \frac{dist.}{time} = \frac{٤٨t}{٢t + t_١} = \frac{٤٨t}{٢t + ٤t} = \frac{٤٨t}{٦t} = ٨$ m/s

٣٣. A plane EM wave is propagating along x direction. It has a wavelength of ٤ mm. If electric field is in y direction with the maximum magnitude of ٦٠ Vm^{-1}, the equation for magnetic field is:

(١) $B_z = ٦٠ \sin\left[\frac{\pi}{2}(x - ٣\times١٠^{٨}t)\right]\hat{k}$

(٢) $B_z = ٢\times١٠^{-٧} \sin\left[\frac{\pi}{2}\times١٠^{٣}(x - ٣\times١٠^{٨}t)\right]\hat{k}$ T

(٣) $B_x = ٦٠ \sin\left[\frac{\pi}{٢}(x - ٣\times١٠^{٨}t)\right]\hat{i}$ T

(٤) $B_z = ٢\times١٠^{-٧} \sin\left[\frac{\pi}{2}(x - ٣\times١٠^{٨}t)\right]\hat{k}$ T

Ans. (٢)

Sol. $E = BC \Rightarrow ٦٠ = B \times ٣ \times ١٠^{٨}$

$\Rightarrow B = ٢ \times ١٠^{-٧}$

Also $C = f\lambda$

$\Rightarrow ٣ \times ١٠^{٨} = f \times ٤ \times ١٠^{-٣}$

$\Rightarrow f = \frac{٣}{٤} \times ١٠^{١١}$

$\Rightarrow \omega = ٢\pi f = ٢\pi \times \frac{٣}{٤} \times ١٠^{١١}$

$\Rightarrow \omega = \frac{\pi}{٢} \times ١٠^{٣} C$

⇒ Electric field → y direction

Propagation → x direction

Magnetic field → z-direction

٣٤. Given below are two statements:

Statement (I): When an object is placed at the centre of curvature of a concave lens، image is formed at the centre of curvature of the lens on the other side.

Statement (II): Concave lens always forms a virtual and erect image.

In the light of the above statements، choose the correct answer from the options given below:

(١) Statement I is false but Statement II is true.

(٢) Both Statement I and Statement II are false.

(٣) Statement I is true but Statement II is false.

(٤) Both Statement I and Statement II are true.

NTA Ans. (١)

Allen Ans. (٢)

Sol. $\frac{١}{v} - \frac{١}{u} = \frac{١}{f}$

$\frac{١}{v} - \frac{١}{2f} = \frac{١}{f}$

$\frac{١}{v} = \frac{1}{٢f} \Rightarrow v = -٢f$

$\frac{١}{v} - \frac{١}{u} = \frac{١}{f}$ ⇒ Virtual image of Real object.

In statement II، it is not mentioned that object is real or virtual hence Statement II is false.

٣٥. A light emitting diode (LED) is fabricated using GaAs semiconducting material whose band gap is ١.٤٢ eV. The wavelength of light emitted from the LED is:

(١) ٦٥٠ nm
(٢) ١٢٤٣ nm
(٣) ٨٧٥ nm
(٤) ١٤٠٠ nm

Ans. (٣)

Sol. $\lambda = \frac{١٢٤٠}{١.٤٢} = ٨٧٥$ nm (Approx)

٣٦. A sphere of relative density σ and diameter D has concentric cavity of diameter d. The ratio of $\frac{D}{d}$ if it just floats on water in a tank is:

(١) $\left(\frac{\sigma}{\sigma-1}\right)^{\frac{١}{٣}}$
(٢) $\left(\frac{\sigma+١}{\sigma-١}\right)^{\frac{١}{٣}}$
(٣) $\left(\frac{\sigma-1}{\sigma}\right)^{\frac{١}{٣}}$
(٤) $\left(\frac{\sigma+٢}{\sigma-٢}\right)^{\frac{١}{٣}}$

Ans. (١)

Sol. weight (w) $= \frac{٤}{٣}\pi\left(\frac{D^٣-d^٣}{٨}\right)\sigma g$

Buoyant force $(F_b) = 1 \times \frac{٤}{٣}\pi\left(\frac{D^٣}{٨}\right) g$

For Just Float $\Rightarrow w = F_b$

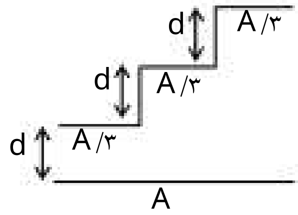
$\Rightarrow (D^٣ - d^٣)\sigma = D^٣$

$\Rightarrow 1 - \frac{d^٣}{D^٣} = \frac{١}{\sigma}$

$\Rightarrow \left(\frac{d}{D}\right)^٣ = 1 - \frac{١}{\sigma}$

$\Rightarrow \frac{D}{d} = \left(\frac{\sigma}{\sigma - 1}\right)^{\frac{١}{٣}}$

٣٧. A capacitor is made of a flat plate of area A and a second plate having a stair-like structure as shown in figure. If the area of each stair is $\frac{A}{٣}$ and the height is d, the capacitance of the arrangement is:



(١) $\frac{١١\varepsilon_0 A}{١٨d}$
(٢) $\frac{١٣\varepsilon_0 A}{١٧d}$
(٣) $\frac{١١\varepsilon_0 A}{٢٠d}$
(٤) $\frac{١٨\varepsilon_0 A}{١١d}$

Ans. (١)

Sol. All capacitor are in parallel combination.

Also effective area is common area only

$\therefore C_{eq} = C_١ + C_٢ + C_٣$

$\therefore C_{eq} = \frac{A\varepsilon_0}{٣d} + \frac{A\varepsilon_0}{٣(٢d)} + \frac{A\varepsilon_0}{٣(٣d)}$

$\Rightarrow C_{eq} = \frac{A\varepsilon_0}{٣}\left(\frac{١١}{٦}\right)$

$\Rightarrow C_{eq} = \frac{١١A\varepsilon_0}{١٨d}$

٣٨. A light unstretchable string passing over a smooth light pulley connects two blocks of masses $m_١$ and $m_٢$. If the acceleration of the system is $\frac{g}{٨}$ then the ratio of the masses $\frac{m_١}{m_٢}$ is:

(١) ٩ : ٧
(٢) ٤ : ٣
(٣) ٥ : ٣
(٤) ٨ : ١

Ans. (١)

Sol. $a_{sys} = \left(\frac{m_١ - m_٢}{m_١ + m_٢}\right) g = \frac{g}{٨}$

$\Rightarrow \frac{m_١}{m_٢} = \frac{٩}{٧}$

٣٩. The dimensional formula of latent heat is:

(١) $[MLT]$ (٢) $[MLT]$

(٣) $[MLT]$ (٤) $[MLT]$

Ans. (٣)

Sol. Latent heat is specific heat

$$\frac{MLT^{-2}}{M} = MLT^{٢\,-٢}$$

٤٠. The volume of an ideal gas (γ = ١.٥) is changed adiabatically from ٥ litres to ٤ litres. The ratio of initial pressure to final pressure is:

(١) $\frac{٤}{٥}$ (٢) $\frac{١٦}{٢٥}$

(٣) $\frac{٨}{٥\sqrt{٥}}$ (٤) $\frac{٢}{\sqrt{٥}}$

Ans. (٣)

Sol. For Adiabatic process

$P_iV_i^{\gamma} = P_fV_f^{\gamma}$

$P_i(٥)^{١.٥} = P_f(٤)^{١.٥}$

$$\frac{P_i}{P_f} = \left(\frac{٤}{٥}\right)^{\frac{٣}{٢}} = \frac{٤}{٥}\left(\frac{٤}{٥}\right)^{\frac{١}{٢}} = \frac{٨}{٥\sqrt{٥}}$$

٤١. The energy equivalent of ١g of substance is:

(١) ١١.٢ × ١٠ MeV (٢) ٥.٦ × ١٠ MeV

(٣) ٥.٦ eV (٤) ٥.٦ × ١٠ MeV

Ans. (٤)

Sol. $E = mC^{٢}$

$E = (١ \times ١٠) \times (٣ \times ١٠)^{٢}$ J

$E = (١٠)(٩ \times ١٠)(٦.٢٤١ \times ١٠^{١٨})$ eV

$E = ٥٦.١٦٩ \times ١٠$ eV

$E = ٥.٦ \times ١٠^{٢٦}$ MeV

٤٢. An astronaut takes a ball of mass m from earth to space. He throws the ball into a circular orbit about earth at an altitude of ٣١٨.٥ km. From earth's surface to the orbit, the change in total mechanical energy of the ball is $x\frac{GM_em}{٢١R_e}$. The value of x is (take R_e = ٦٣٧٠ km):

(١) ١١ (٢) ٩

(٣) ١٢ (٤) ١٠

Ans. (١)

Sol. $h = ٣١٨.٥ = \left(\frac{R_e}{٢٠}\right)$

$$T.E_i = \frac{-GM_em}{R_e}$$

$$T.E_f = \frac{-GM_em}{٢(R_e + h)} = \frac{-GM_em}{٢\left(R_e + \frac{R_e}{٢٠}\right)}$$

$$T.E_f = \frac{-١٠\,GM_em}{٢١R_e}$$

Change in total mechanical energy

$= TE_f - TE_i$

$$= \frac{GM_em}{R_e}\left(١ - \frac{١٠}{٢١}\right) = \frac{١١GM_em}{٢١R_e}$$

٤٣. Given below are two statements:

Statement (I): When currents vary with time, Newton's third law is valid only if momentum carried by the electromagnetic field is taken into account.

Statement (II): Ampere's circuital law does not depend on Biot-Savart's law.

In the light of the above statements, choose the correct answer from the options given below:

(١) Both Statement I and Statement II are false.

(٢) Statement I is true but Statement II is false.

(٣) Statement I is false but Statement II is true.

(٤) Both Statement I and Statement II are true.

Ans. (٢)

Sol. Conceptual.

٤٤. A particle of mass m moves on a straight line with its velocity increasing with distance according to $v = \alpha\sqrt{x}$, where α is a constant. The total work done by all the forces applied on the particle during its displacement from x = ٠ to x = d، will be:

(١) $\frac{m}{2\alpha^2 d}$ (٢) $\frac{md}{2\alpha^2}$

(٣) $\frac{m\alpha^2 d}{2}$ (٤) $2m\alpha^2 d$

Ans. (٣)

Sol. $v = \alpha\sqrt{x}$

at x = ٠ : v = ٠

& at x = d ; $v = \alpha\sqrt{d}$

$W.D = K_f - K_i$

$W.D = \frac{1}{2}m(\alpha\sqrt{d})^2 - \frac{1}{2}m(0)^2$

$\therefore W.D = \frac{m\alpha^2 d}{2}$

٤٥. A galvanmeter has a coil of resistance ٢٠٠ Ω with a full scale deflection at ٢٠ μA. The value of resistance to be added to use it as an ammeter of range (٠–٢٠) mA is:

(١) ٠.٤٠ Ω (٢) ٠.٢٠ Ω

(٣) ٠.٥٠ Ω (٤) ٠.١٠ Ω

Ans. (٢)

Sol. G = ٢٠٠ Ω

$i_g = 20\ \mu A$

$i = i_g\left(\frac{G}{S} + 1\right)$

$\Rightarrow 20 \times 10^{-3} = 20 \times 10^{-6}\left(\frac{200}{S} + 1\right)$

$\Rightarrow \frac{200}{S} = 999$

$\therefore S \approx 0.2\ \Omega$

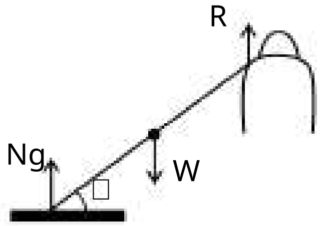
٤٦. A heavy iron bar، of weight W is having its one end on the ground and the other on the shoulder of a person. The bar makes an angle θ with the horizontal. The weight experienced by the person is:

(١) $\frac{W}{2}$ (٢) W

(٣) W cos θ (٤) W sin θ

Ans. (١)

Sol.



R = net reaction force by shoulder

Balancing torque about pt of contact on ground:

$W\left(\frac{L}{2}\cos\theta\right) = R(L\cos\theta)$

$\Rightarrow R = \frac{W}{2}$

٤٧. One main scale division of a vernier caliper is equal to m units. If n^{th} division of main scale coincides with $(n+1)^{th}$ division of vernier scale، the least count of the vernier caliper is:

(١) $\frac{n}{(n+1)}$ (٢) $\frac{m}{(n+1)}$

(٣) $\frac{1}{(n+1)}$ (٤) $\frac{m}{n(n+1)}$

Ans. (٢)

Sol. n MSD = (n + ١) VSD

$\Rightarrow 1\ VSD = \frac{n}{n+1}\ MSD$

L.C = ١ MSD – ١ VSD

$L.C = m - m\left(\frac{n}{n+1}\right)$

$L.C = m\left(\frac{n+1-n}{n+1}\right)$

$\therefore L.C = \left(\frac{m}{n+1}\right)$

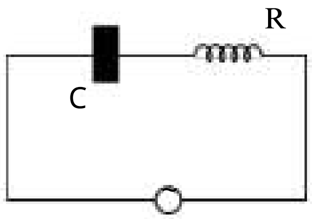
٤٨. A bulb and a capacitor are connected in series across an ac supply. A dielectric is then placed between the plates of the capacitor. The glow of the bulb:

(١) increases (٢) remains same

(٣) becomes zero (٤) decreases

Ans. (١)

Sol.



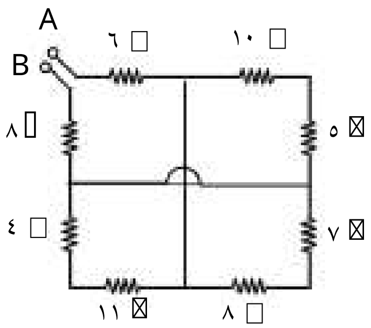
$Z = \sqrt{R^٢ + X_C^٢}$ & $X_C = \frac{١}{\omega C}$

due to dielectric

$C \uparrow X_C \downarrow Z \downarrow$

So, current increases & thus bulb will glow more brighter.

٤٩. The equivalent resistance between A and B is:

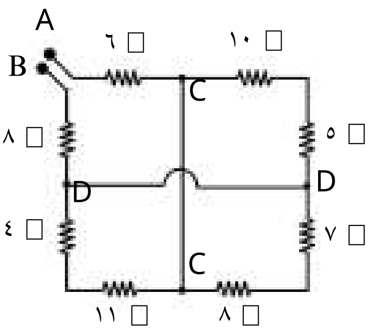


(١) ١٨ Ω (٢) ٢٥ Ω

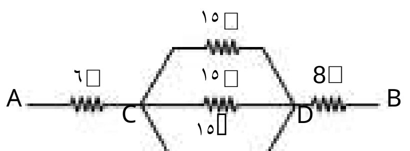
(٣) ٢٧ Ω (٤) ١٩ Ω

Ans. (٤)

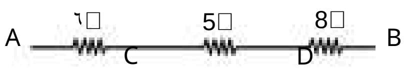
Sol.



⇒



⇒



$\Rightarrow R_{eq} = ٦\Omega + ٥\Omega + ٨\Omega = ١٩\ \Omega$

٥٠. A sample of ١ mole gas at temperature T is adiabatically expanded to double its volume. If adiabatic constant for the gas is $\gamma = \frac{٣}{٢}$, then the work done by the gas in the process is:

(١) $RT\left(٢ - \sqrt{2}\right)$ (٢) $\frac{R}{T}\left(2 - \sqrt{٢}\right)$

(٣) $RT\left(\sqrt{٢}\right)$ (٤) $\frac{T}{R}\left(2 - \sqrt{٢}\right)$

Ans. (١)

Sol. $TV^{\gamma - ١} = \text{constant}$

$\Rightarrow T(V)^{\frac{٣}{٢} - 1} = T_f (٢V)^{\frac{٣}{٢} - 1}$

$\Rightarrow TV^{\frac{١}{٢}} = T_f (٢)^{\frac{١}{٢}} (V)^{\frac{١}{٢}}$

$\Rightarrow T_f = \left(\frac{T}{\sqrt{٢}}\right)$

$\Rightarrow$ Now, W.D. $= \frac{nR\Delta T}{١ - \gamma} = \frac{١ \cdot R\left(T - \frac{T}{\sqrt{٢}}\right)}{1 - \frac{٣}{٢}}$

$\Rightarrow$ W.D. $= ٢RT\left(١ - \frac{١}{\sqrt{٢}}\right)$

$\Rightarrow$ W.D. $= RT\left(٢ - \sqrt{٢}\right)$

SECTION-B

٥١. If $\vec{a}$ and $\vec{b}$ makes an angle $\cos^{-1}\left(\frac{٥}{٩}\right)$ with each other, then $|\vec{a} + \vec{b}| = \sqrt{٢}\,|\vec{a} - \vec{b}|$ for $|\vec{a}| = n|\vec{b}|$

The integer value of n is ___________.

Ans. (٣)

Sol. $\cos\theta = \frac{٥}{٩}$

$\frac{\vec{a} \cdot \vec{b}}{ab} = \frac{٥}{٩}$ ....(١)

$|\vec{a} + \vec{b}| = \sqrt{٢}\,|\vec{a} - \vec{b}|$

$a^٢ + b^٢ + ٢\vec{a} \cdot \vec{b} = ٢a^٢ + ٢b^٢ - ٤\vec{a} \cdot \vec{b}$

$٦\vec{a} \cdot \vec{b} = a^٢ + b^٢$

$6□ \frac{٥}{٤} ab□a^٢□b^٢$

$\frac{١٠}{٣} ab□a^٢□b^٢$ & $a = nb$

$\frac{١٠}{٣} nb^2 □n^٢b^2 □b^٢$

$٣n^٢ - ١٠n + ٣ = ٠$

٥٢. $n□ \frac{١}{٣}$ and $n = ٣$

integer value $n = ٣$

At the centre of a half ring of radius $R = ١٠$ cm and linear charge density ٤n C m⁄ the potential is x □V. The value of x is ______________.

Ans. (٣٦)

Sol. Potential at centre of half ring

$V = \frac{KQ}{R}$

$V □\frac{K□□R}{R}$

$V = K□□ □V = ٩ \times ١٠ \times ٤ \times ٣٠□ \quad ^{-٩}$

$V = ٣٦□$

٥٣. A star has ١٠٠٪ helium composition. It starts to convert three ⁴He into one ¹²C via triple alpha process as $He_٤ + He_٤ + He_٤ □ C_{١٢} + Q$. The mass of the star is ٢.٠×١٠^٣kg and it generates energy at the rate of ٥.٨٠٨×١٠^٣W. The rate of converting these ⁴He to ¹²C is $n \times ١٠ s^٤$ where n is ____________.

Take، mass of He $= ٤.٠٠٢٦$ u، mass of C $= ١٢$ u

NTA Ans. (٥)

Sol. ٤ $He + He + He □ C + Q$

power generated $\frac{N}{t} = Q$

where، N □ No. of reaction /sec.

$Q = □٣m_{He}□m_C□C^٢$

$Q = (٣ \times ٤.٠٠٢٦ - ١٢)(٣ \times ١٠)^{٨\ ٢}$

$Q = ٧.٢٦٦$ MeV

$\frac{N}{t} □\frac{power}{Q} □\frac{٥.٨٠٨□١٠^{٣٠}}{٧.٢٦٦□١٠^٦□١.٦□١٠^{□١٩}}$

$\frac{N}{t} □٥□١٠^{٤٢}$

rate of conversion of He into $C = ١٥ \times ١٠^{٤٢}$

Hence، $n = ١٥$

٥٤. In a Young's double slit experiment، the intensity at a point is $□\frac{1}{4}□^{th}$ of the maximum intensity، the minimum distance of the point from the central maximum is ____________ □m.

(Given : □ = ٦٠٠ nm، d = ١.٠ mm، D = ١.٠ m)

Ans. (٢٠٠)

Sol. $I □I_٠ \cos^2□\frac{□}{2}□$

$\frac{I_٠}{٤}□\cos^٢ □\frac{□}{٢}□$

$□□□\frac{٢□}{٣}$

$\frac{2□}{□}□\frac{yd}{D}□□\frac{2□}{3}$

$y □\frac{□D}{٣d} □\frac{٦٠٠□١٠^{□٩}□1}{٣□١٠^{□٣}} = ٢ \times ١٠ m^٤$

٥٥. A string is wrapped around the rim of a wheel of moment of inertia ٠.٤٠ kgm and radius ١٠ cm. The wheel is free to rotate about its axis. Initially the wheel is at rest. The string is now pulled by a force of ٤٠ N. The angular velocity of the wheel after ١٠ s is x rad/s، where x is ___________.

Ans. (١٠٠)

Sol. $□ = FR = I□ □ ٤٠ \times ٠.١ = ٠.٤□$

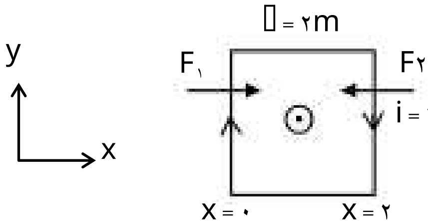
$□ = ١٠$ rad/s²

$W_f = ١٠ \times ١٠ = ١٠٠$ rad/s

٥٦. A square loop of edge length ٢ m carrying current of ٢ A is placed with its edges parallel to the x-y axis. A magnetic field is passing through the x-y plane and expressed as $\vec{B}□B_٠(١□٤x)\hat{k}$، where $B_٠ = ٥$ T. The net magnetic force experienced by the loop is ___________ N.

Ans. (١٦٠)

Sol.



B(x = ٠) = B٠ , B(x = ٢) = ٩B

Also, F = iℓB

F١ = iℓB & F٢ = ٩iℓB

F = F٢ − F١ = ٨iℓB = ٨ × ٢ × ٢ × ٥

F = ١٦٠ N

٥٧. Two persons pull a wire towards themselves. Each person exerts a force of ٢٠٠ N on the wire. Young's modulus of the material of wire is ١ × ١٠ N/m. Original length of the wire is ٢ m and the area of cross section is ٢cm. The wire will extend in length by __________ μm.

Ans. (٢٠)

Sol.

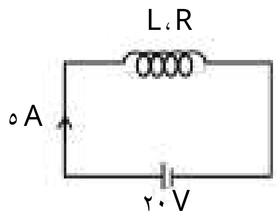
$\frac{F}{A} = Y\frac{\Delta\ell}{\ell} \Rightarrow \Delta\ell = \frac{F\ell}{AY}$

$\Delta\ell = \frac{٢٠٠ \times ٢}{٢ \times ١٠^{-٤} \times ١٠^{١١}} = ٢ \times ١٠^{-٥} = ٢٠ \mu m$

٥٨. When a coil is connected across a ٢٠ V dc supply, it draws a current of ٥ A. When it is connected across ٢٠ V, ٥٠ Hz ac supply, it draws a current of ٤ A. The self inductance of the coil is __________ mH. (Take π = ٣)

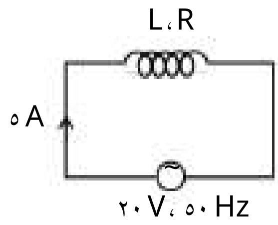
Ans. (١٠)

Sol. Case-I:



$i = \frac{٢٠}{R} \Rightarrow R = ٤\ \Omega$

Case-II:



$i = \frac{٢٠}{Z}$

$٤ = \frac{٢٠}{\sqrt{R^٢ + X_L^٢}} \Rightarrow \sqrt{R^٢ + X_L^٢} = ٥$

$R^٢ + X_L^٢ = ٢٥ \Rightarrow X_L = ٣\ \Omega$

$L = \frac{٣}{٢\pi f} = \frac{١}{٢ \times ٥٠} \times \frac{١٠٠٠}{١٠٠}$ mH

L = ١٠ mH

٥٩. The position, velocity and acceleration of a particle executing simple harmonic motion are found to have magnitudes of ٤ m, ٢ ms and ١٦ ms at a certain instant. The amplitude of the motion is $\sqrt{x}$ m where x is ____________.

Ans. (١٧)

Sol. x = ٤ m, V = ٢ m/s, a = ١٦ m/s٢

$|a| = \omega^٢ x$

$\Rightarrow ١٦ = \omega^٢ (٤)$

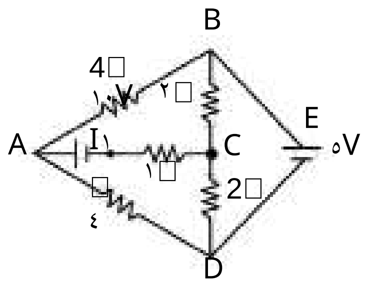
ω = ٢ rad/s

$v = \omega\sqrt{A^٢ - x^٢}$

$A = \sqrt{\frac{V^٢}{\omega^2} + x^٢} \Rightarrow A = \sqrt{\frac{٤}{٤} + ١٦}$

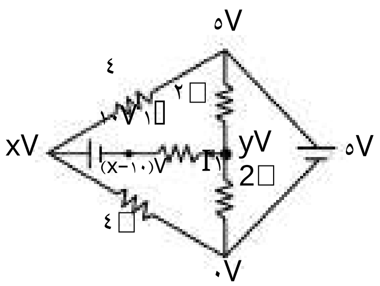
$A = \sqrt{١٧}$ m

٦٠. The current flowing through the ١ Ω resistor is $\frac{n}{١٠}$ A. The value of n is ______________.



Ans. (٢٥)

Sol.



$$\frac{y-٥}{٢}+\frac{y-٠}{٢}+\frac{y-x+١٠}{١}=٠$$

$$y-٥+y+٢y-٢x+٢٠=٠$$

$$٤y-٢x+١٥=٠ \quad ....(i)$$

$$\frac{x-٥}{٤}+\frac{x-٠}{٤}+\frac{x-١٠-y}{١}=٠$$

$$x-٥+x+٤x-٤٠-٤y=٠$$

$$٦x-٤y-٤٥=٠ \quad ..(i)$$

$$-٢x+٤y+١٥=٠ \quad ..(ii)$$

$$٤x-٣٠=٠$$

$$x=\frac{١٥}{٢} \quad \& \quad ٤y-١٥+١٥=٠$$

$$y=٠$$

$$i=\frac{y-x+١٠}{١}$$

$$i=\frac{٠-٧.٥+١٠}{١}$$

$$i=٢.٥A=\frac{n}{١٠}A$$

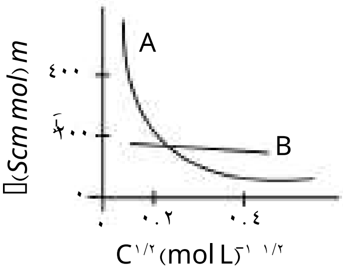
$$n=٢٥$$

CHEMISTRY

TEST PAPER WITH SOLUTION

SECTION–A

٦١. The molar conductivity for electrolytes A and B are plotted against $C^{1/2}$ as shown below. Electrolytes A and B respectively are :



A	B
(١) Weak electrolyte	weak electrolyte
(٢) Strong electrolyte	strong electrolyte
(٣) Weak electrolyte	strong electrolyte
(٤) Strong electrolyte	weak electrolyte

Ans. (٣)

Sol. A Weak electrolyte

B Strong electrolyte

٦٢. Methods used for purification of organic compounds are based on :

(١) neither on nature of compound nor on the impurity present.

(٢) nature of compound only.

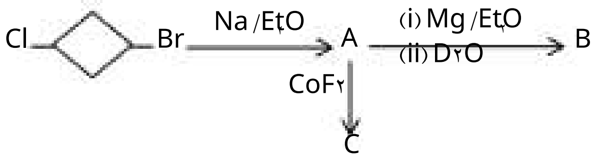
(٣) nature of compound and presence of impurity.

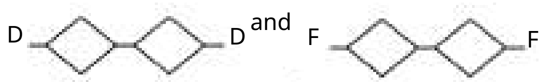
(٤) presence of impurity only.

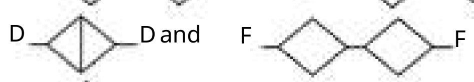
Ans. (٣)

Sol. Organic compounds are purified based on their nature and impruity present in it.

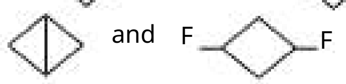
٦٣. In the following sequence of reaction, the major products B and C respectively are :



(١) 

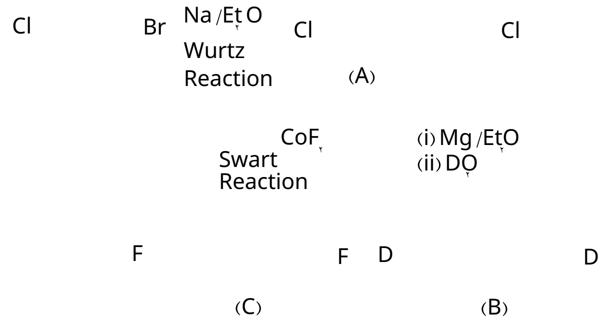
(٢) 

(٣) 

(٤) 

Ans. (١)

Sol.



٦٤. Correct order of basic strength of Pyrrole  ,

Pyridine  and Piperidine  is :

(١) Piperidine < Pyridine < Pyrrole

(٢) Pyrrole < Pyridine < Piperidine

(٣) Pyridine < Piperidine < Pyrrole

(٤) Pyrrole < Piperidine < Pyridine

Ans. (١)

Sol. Order of basic strength is

N(sp٣ localized lone pair) < N(sp٢ localized lone pair) < N(sp٢ delocalized lone pair, aromatic)

  Piperidine < Pyridine < Pyrrole

٦٥. In which one of the following pairs the central atoms exhibit sp^2 hybridization ?

(١) BF٣ and NO٢

(٢) NH٢ and H٢O

(٣) H٢O and NO٢

(٤) NH٢ and BF٣

Ans. (١)

Sol. BF٣ sp ٢

NO٢ sp ٢

H٢O sp ٣

NO٢ sp ٢

NH٢ sp ٣

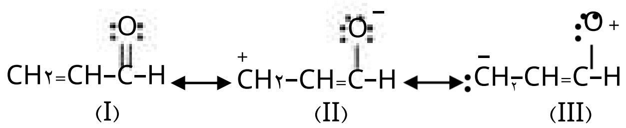
٦٦. The F⁻ ions make the enamel on teeth much harder by converting hydroxyapatite (the enamel on the surface of teeth) into much harder fluoroapatite having the formula.

(١) $[3(Ca_3(PO_4)_2).CaF_2]$
(٢) $[3(Ca_2(PO_4)_2).Ca(OH)_2]$
(٣) $[3(Ca_2(PO_4)_3).CaF_2]$
(٤) $[3(Ca_3(PO_4)_2).Ca(OH)_2]$

Ans. (١)

Sol. Fluoroapatite $[3Ca_3(PO_4)_2.CaF_2]$

٦٧. Relative stability of the contributing structures is :



(١) (I) < (III) < (II)
(٢) (I) < (II) < (III)
(٣) (II) < (I) < (III)
(٤) (III) < (II) < (I)

Ans. (٢)

Sol. (١) Neutral structures are more stable than charged ones. Therefore I is more stable than II and III.
(٢) +ve charge on less electronegative atom is more stable i.e. C is more stable than O
∴ Order is I < II < III

٦٨. Given below are two statements :

Statement (I) : The oxidation state of an element in a particular compound is the charge acquired by its atom on the basis of electron gain enthalpy consideration from other atoms in the molecule.

Statement (II) : pπ–pπ bond formation is more prevalent in second period elements over other periods.

In the light of the above statements, choose the most appropriate answer from the options given below :

(١) Both Statement I and Statement II are incorrect
(٢) Statement I is correct but Statement II is incorrect
(٣) Both Statement I and Statement II are correct
(٤) Statement I is incorrect but Statement II is correct

Ans. (٤)

Sol. Oxidation state of an element in a particular compound is defined by the charge acquired by its atom on the basis of electronegativity consideration from other atoms in molecule.

٦٩. Given below are two statements : one is labelled as Assertion (A) and the other is labelled as Reason (R) :

Assertion (A) : SN۲ reaction of $C_6H_5CH_2Br$ occurs more readily than the SN۲ reaction of CH_3CH_2Br.

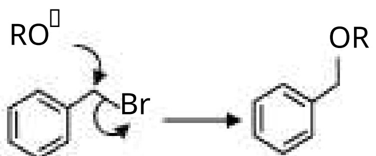
Reason (R) : The partially bonded unhybridized p-orbital that develops in the trigonal bipyramidal transition state is stabilized by conjugation with the phenyl ring.

In the light of the above statements, choose the most appropriate answer from the options given below :

(١) (A) is not correct but (R) is correct
(٢) Both (A) and (R) are correct but (R) is not the correct explanation of (A)
(٣) Both (A) and (R) are correct and (R) is the correct explanation of (A)
(٤) (A) is correct but (R) is not correct

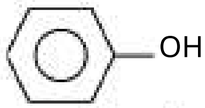
Ans. (٣)

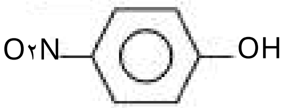
Sol. The benzyl group acts in much the same way using the π-system of the benzene ring for conjugation with the p-orbital in the transition state.

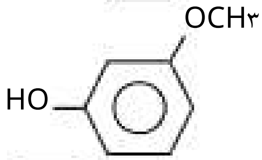


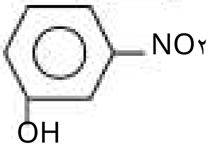
benzyl bromide

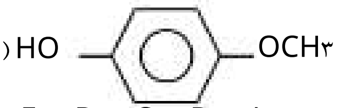
٧٠. For the given compounds، the correct order of increasing pKa value :

(A) 

(B) 

(C) 

(D) 

(E) 

(١) (E) > (D) > (C) > (B) > (A)
(٢) (D) > (E) > (C) > (B) > (A)
(٣) (E) > (D) > (B) > (A) > (C)
(٤) (B) > (D) > (A) > (C) > (E)

Ans. BONUS
NTA Ans. (٤)

Sol. Acidic strength order :-
B < D < C < A < E
Correct pKa Order :
B > D > C > A > E
All options are incorrect.

٧١. Given below are two statements : one is labelled as Assertion (A) : and the other is labelled as Reason (R).
Assertion (A) : Both rhombic and monoclinic sulphur exist as S٨ while oxygen exists as O٢.
Reason (R) : Oxygen forms p-p multiple bonds with itself and other elements having small size and high electronegativity like C، N، which is not possible for sulphur.
In the light of the above statements، choose the most appropriate answer from the options given below :
(١) Both (A) and (R) are correct and (R) is the correct explanation of (A).
(٢) Both (A) and (R) are correct but (R) is not the correct explanation of (A).
(٣) (A) is correct but (R) is not correct.
(٤) (A) is not correct but (R) is correct.

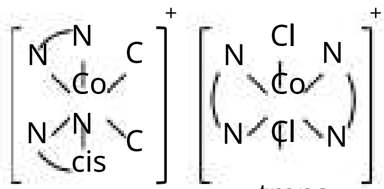
Ans. (٣)

Sol. Oxygen can form ٢p-٢p multiple bond with itself due to its small size while sulphur cannot form multiple bond with itself as ٣p-٣p bond will be unstable due to large size of sulphur، but sulphur can form multiple bond with small size atom like C and N. eg. S=C=S
-
S=C=N SCN

٧٢. Given below are two statements : one is labelled as Assertion (A) and the other is labelled as Reason (R).
Assertion (A): The total number of geometrical isomers shown by [Co(en)٢Cl٢] complex ion is thre
Reason (R): [Co(en)٢Cl٢] complex ion has an octahedral geometry.
In the light of the above statements، choose the mos appropriate answer from the options given below :
(١) Both (A) and (R) are correct and (R) is the correct explanation of (A).
(٢) (A) is correct but (R) is not correct.
(٣) (A) is not correct but (R) is correct.
(٤) Both (A) and (R) are correct but (R) is not the correct explanation of (A).

Ans. (٣)

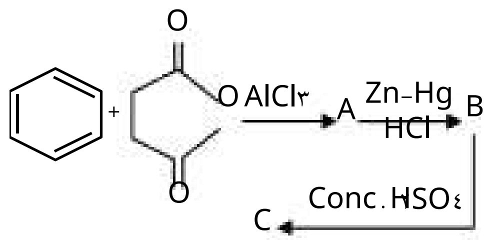
Sol. [Co(en)٢Cl٢] has octahedral geometry with two geometrical isomers.


cis trans

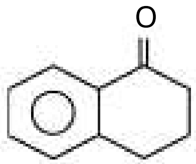
٧٣. The electronic configuration of Cu(II) is ٣d ٩ whereas that of Cu(I) is ٣d. Which of the following is correct ؟
(١) Cu(II) is less stable
(٢) Stability of Cu(I) and Cu(II) depends on nature of copper salts
(٣) Cu(II) is more stable
(٤) Cu(I) and Cu(II) are equally stable

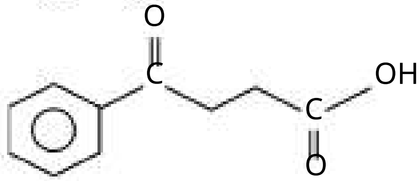
Ans. (٣)

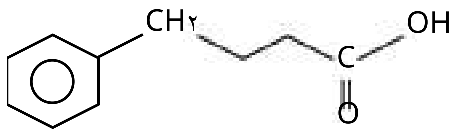
Sol. Cu(II) is more stable than Cu(I) because hydration energy of Cu ion compensate IE٢ of Cu.

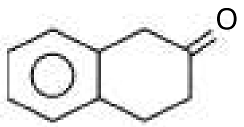
٧٤. 

What is the structure of C ؟

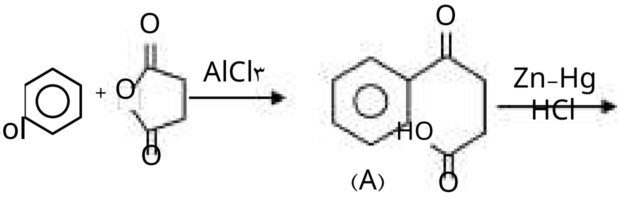
(١) 

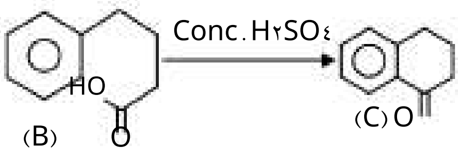
(٢) 

(٣) 

(٤) 

Ans. (١)

Sol 



٧٥. Compare the energies of following sets of quantum numbers for multielectron system.

(A) n = ٤, l = ١ (B) n = ٤, l = ٢ (C) n = ٣, l = ١ (D) n = ٣, l = ٢ (E) n = ٤, l = ٠ Choose the correct answer from the options given below : (١) (B) < (A) < (C) < (E) < (D) (٢) (E) < (C) > (D) > (A) > (B) (٣) (E) < (C) < (A) < (D) < (B) (٤) (C) > (E) > (D) > (A) > (B)

Ans. (٤)

Sol. Energy level can be determined by comparing (n + l) values

(A) n = ٤, l = ١ (n + l) = ٥

(B) n = ٤, l = ٢ (n + l) = ٦

(C) n = ٣, l = ١ (n + l) = ٤

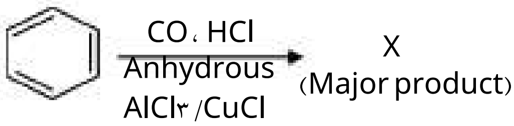
(D) n = ٣, l = ٢ (n + l) = ٥

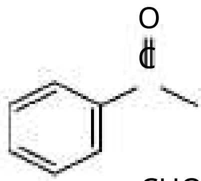
(E) n = ٤, l = ٠ (n + l) = ٤

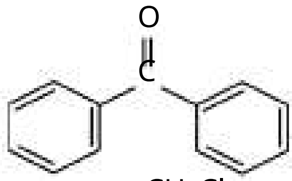
For same value of (n + l), orbital having higher value of n, will have more energy.

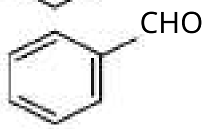
(B) < (A) < (D) < (E) < (C)

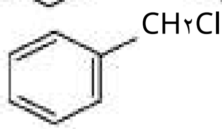
٧٦. Identify major product "X" formed in the following reaction :



(١) 

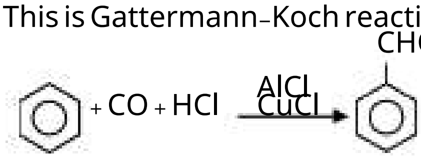
(٢) 

(٣) 

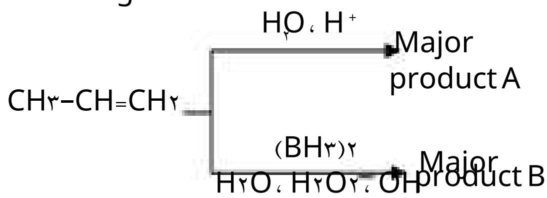
(٤) 

Ans. (٣)

Sol. This is Gattermann-Koch reaction



٧٧. Identify the product A and product B in the following set of reactions.



(١) A-CH٣CH٢CH٢-OH, B-CH٣CH٢CH٢-OH

(٢) A-CH٣CH٢CH٢-OH, B-CH٣-CH(OH)-CH٣

(٣) A-CH٣-CH(OH)-CH٣, B-CH٣CH٢CH٢-OH

(٤) A-CH٣CH٢CH٣, B-CH٣CH٢CH٣

Ans. (٣)

Sol. (١) Hydration Reaction :

CH٣ CH CH٢ H CH٣ CH CH٣
(More stable)

CH٣–CH–CH٣ + H٢O ⟶ (CH٣–CH–CH٣) + H⁺
(+ on central C; OH on central C of product)
(A)

(٢) Hydroboration Oxidation Reaction :

٣CH٣–CH=CH٢ + BH٣ T HF

٢(CH٣CH٢CH٢)٣B

(CH٣CH٢CH٢)٣B + ٣H٢O٢ OH⁻

٣CH٣CH٢CH٢OH + H٣BO٣
(B)

٧٨. On reaction of Lead Sulphide with dilute nitric acid which of the following is not formed ?

(١) Lead nitrate (٢) Sulphur

(٣) Nitric oxide (٤) Nitrous oxide

Ans. (٤)

Sol. PbS + HNO٣ → Pb(NO٣)٢ + NO + S + H٢O

Nitrous oxide (N٢O) is not formed during the reaction.

٧٩. Identify the incorrect statements regarding primary standard of titrimetric analysis

(A) It should be purely available in dry form.

(B) It should not undergo chemical change in air.

(C) It should be hygroscopic and should react with another chemical instantaneously and stoichiometrically.

(D) It should be readily soluble in water.

(E) KMnO٤ & NaOH can be used as primary standard.

Choose the correct answer from the options given below :

(١) (C) and (D) only (٢) (B) and (E) only

(٣) (A) and (B) only (٤) (C) and (E) only

Ans. (٤)

Sol. KMnO٤ & NaOH → Secondary standard.

Primary standard should not be Hygroscopic.

٨٠. ٠.٠٥M CuSO٤ when treated with ٠.٠١M K٢Cr٢O٧ gives green colour solution of Cu٢Cr٢O٧. The (SPM : Semi Permeable Membrane)

K٢Cr٢O٧	CuSO٤

Side X SPM Side Y

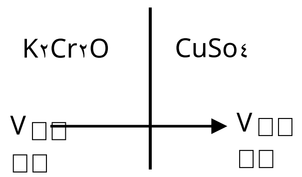
Due to osmosis : (١) Green colour formation observed on side Y. (٢) Green colour formation observed on side X.

(٣) Molarity of K٢Cr٢O٧ solution is lowered.

(٤) Molarity of CuSO٤ solution is lowered.

Ans. (٤)

Sol. Only solvent Molecules are allowed to pass through the SPM.



SECTION-B

٨١. The heat of solution of anhydrous CuSO٤ and CuSO٤·٥H٢O are –٧٠ kJ mol and +١٢ kJ mol⁻¹ respectively.

The heat of hydration of CuSO٤ to CuSO٤·٥H٢O is –x kJ. The value of x is________.

Ans. (٨٢)

Sol.

(١) CuSO٤(s) + ٥H٢O → CuSO٤·٥H٢O –x
(٢) CuSO٤·٥H٢O + H٢O → CuSO٤(aq) 12
CuSO٤ + H٢O → CuSO٤(aq) –70

from (١) & (٢)

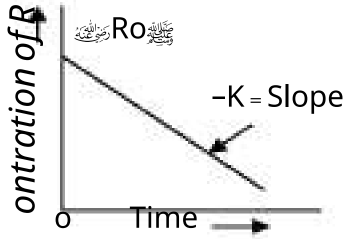
–٧٠ = x + ١٢

x = –٨٢

٨٢. Given below are two statements :

Statement I : The rate law for the reaction A + B C is rate (r) = kAB. When the concentration of both A and B is doubled، the reaction rate is increased "x" times.

Statement II :



The figure is showing "the variation concentration against time plot" for a "y" order reaction.

The value of x + y is ____________________.

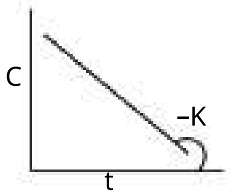
Ans. (٨)

Sol. r = KA|B|

if conc. are doubled

r' = K٢A٢B

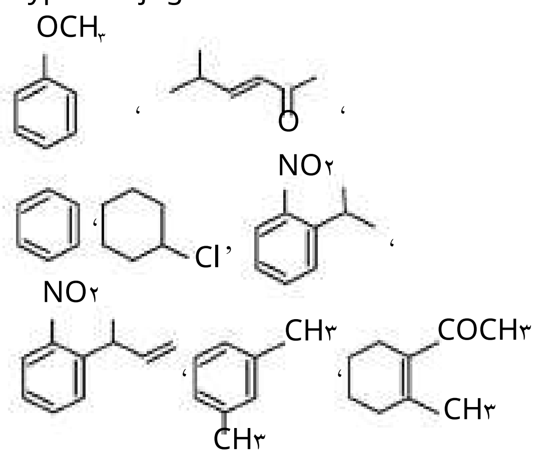
r' = ٨r x = ٨



 Zero order، y = ٠

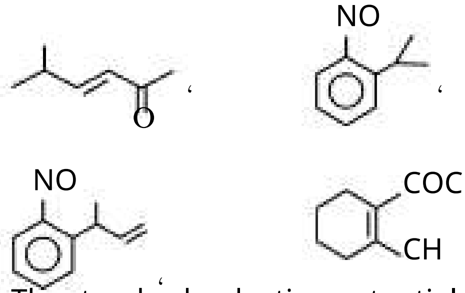
x + y = ٨

٨٣. How many compounds among the following compounds show inductive، mesomeric as well as hyperconjugation effects؟



Ans. (٤)

Sol.



٨٤. The standard reduction potentials at ٢٩٨ K for the following half cells are given below :

$Cr_2O_7^{2-}$ + ١٤H + ٦e ٢Cr + ٧H٢O، E° = ١.٣٣V

Fe(aq) + ٢e Fe E° = –٠.٤٤V

Ni(aq) + ٢e Ni E° = –٠.٢٥V

Ag(aq) + e Ag E° = ٠.٨٠V

Au(aq) + ٣e Au E° = ١.٤٠V

Consider the given electrochemical reactions، The number of metal(s) which will be oxidized be $Cr_2O_7^{2-}$، in aqueous solution is ________________.

Ans. (٣)

Sol. Fe، Ni، Ag will be oxidized due to lower S.R.P.

٨٥. When equal volume of ١M HCl and ١M H_2SO_4 are separately neutralised by excess volume of ١M NaOH solution. X and y kJ of heat is liberated respectively. The value of y/x is _______________.

Ans. (٢)

Sol. H + OH H٢O x

٢H + ٢OH ٢H٢O ٢x = y

y/x = ٢

٨٦. Molarity (M) of an aqueous solution containing x g of anhyd. $CuSO_4$ in ٥٠٠ mL solution at ٣٢ °C is ٢ × ١٠ M. Its molality will be __________ × ١٠ m. (nearest integer).

[Given density of the solution = ١.٢٥ g/mL.]

NTA Ans. (٨١) BONUS

Sol.

M_{soln} v_{soln} d_{soln}

٥٠٠١.٢٥٦٢٥g

Mass of solute (x) = ٠.٢ × ٠.٥ × ١٥٩.٥

= ١٥.٩٥

nsolute = ٠.١،

Mass of solvent = Mass of solution – Mass of solute

= ٦٢٥ – ١٥.٩٥

= ٦٠٩.٠٥

m ٠.١ / (٦٠٩.٠٥ / ١٠٠٠)

m ٠.١٦٤ ١٦٤ ١٠٣

87. The total number of species from the following in which one unpaired electron is present is N_2, O_2, C_2^-, O_2^-, O_2^{2-}, H_2^+, CN^-, He_2^+

Ans. (4)

Sol. One unpaired e^- is present in C_2^-, O_2^-, H_2^+, He_2^+

88. Number of ambidentate ligands among the following is ___________.

NO_2^-, SCN^-, $C_2O_4^{2-}$, NH_3, CN^-, SO_4^{2-}, H_2O.

Ans. (3)

Sol. Ligands which have two different donor sites but at a time connects with only one donor site to central metal are ambidentate ligands.

Ambidentate ligands are NO_2^-, SCN^-, CN^-

89. Total number of essential amino acid among the given list of amino acids is __________.

Arginine, Phenylalanine, Aspartic acid, Cysteine, Histidine, Valine, Proline

Ans. (4)

Sol. Essential Amino acids are :-

Arginine, Phenylalanine, Histidine, Valine

90. Number of colourless lanthanoid ions among the following is _____________.

Eu^{3+}, Lu^{3+}, Nd^{3+}, La^{3+}, Sm^{3+}

Ans. (2)

Sol. La^{+3} – $[Xe]4f^0$

Nd^{+3} – $[Xe]4f^3$

Sm^{+3} – $[Xe]4f^5$

Eu^{+3} – $[Xe]4f^6$

Lu^{+3} – $[Xe]4f^{14}$

La^{+3} and Lu^{+3} do not show any colour because no unpaired electron is present.