FINAL JEE–MAIN EXAMINATION – APRIL, 2024

(Held On Tuesday 09 April, 2024) TIME : 3 : 00 PM to 6 : 00 PM

MATHEMATICS TEST PAPER WITH SOLUTION

SECTION-A

1. $\lim\limits_{x\to 0}\frac{e-(1+2x)^{\frac{1}{2x}}}{x}$ is equal to :

(1) e (2) $\frac{-2}{e}$

(3) 0 (4) $e-e^2$

Ans. (1)

Sol. $\lim\limits_{x\to 0}\frac{e-e^{\frac{1}{2x}\ln(1+2x)}}{x}$

$= \lim\limits_{x\to 0}(-e)\frac{\left(e^{\frac{\ln(1+2x)}{2x}-1}-1\right)}{x}$

$= \lim\limits_{x\to 0}(-e)\frac{\ln(1+2x)-2x}{2x^2}$

$=(-e)\times(-1)\frac{4}{2\times 2}=e$

2. Consider the line L passing through the points (1, 2, 3) and (2, 3, 5). The distance of the point $\left(\frac{11}{3},\frac{11}{3},\frac{19}{3}\right)$ from the line L along the line $\frac{3x-11}{2}=\frac{3y-11}{1}=\frac{3z-19}{2}$ is equal to :

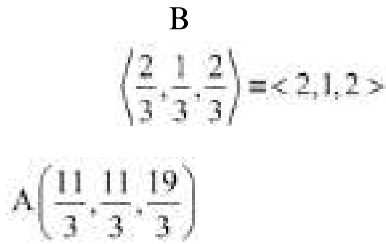
(1) 3 (2) 5

(3) 4 (4) 6

Ans. (1)

Sol. $\frac{x-1}{2-1}=\frac{y-2}{3-2}=\frac{z-3}{5-3}$

$\Rightarrow \frac{x-1}{1}=\frac{y-2}{1}=\frac{z-3}{2}=\lambda$



$B(1+\lambda, 2+\lambda, 3+2\lambda)$

D.R. of AB = $<\frac{3\lambda-8}{3},\frac{3\lambda-5}{3},\frac{6\lambda-10}{3}>$

$B\left(\frac{5}{3},\frac{8}{3},\frac{13}{3}\right)$ $\frac{3\lambda-8}{3\lambda-5}=\frac{2}{1}\Rightarrow 3\lambda-8=6\lambda-10$

$3\lambda=2$

$\lambda=\frac{2}{3}$

AB $=\frac{\sqrt{36+9+36}}{3}=\frac{9}{3}=3$

3. Let $\int_0^x\sqrt{1-(y'(t))^2}\,dt=\int_0^x y(t)\,dt$, $0\le x\le 3$, $y\ge 0$, $y(0)=0$. Then at $x=2$, $y''+y+1$ is equal to :

(1) 1 (2) 2

(3) $\sqrt{2}$ (4) 1/2

Ans. (1)

Sol. $\sqrt{1-(y'(x))^2}=y(x)$

$1-\left(\frac{dy}{dx}\right)^2=y^2$

$\left(\frac{dy}{dx}\right)^2=1-y^2$

$\frac{dy}{\sqrt{1-y^2}} = dx$ OR $\frac{dy}{\sqrt{1-y^2}} = -dx$

$\Rightarrow \sin^{-1} y = x + c$, $\sin^{-1} y = -x + c$

x = ٠ ، y = ٠ ⇒ c = ٠

$\sin^{-1} y = x$ ، as y ≥ ٠

$\sin x = y$

$\Rightarrow \frac{dy}{dx} = \cos x$

$\frac{d^2y}{dx^2} = -\sin x$

⇒ − sin x + sin x + ١ = ١

٤. Let z be a complex number such that the real part of $\frac{z-2i}{z+2i}$ is zero. Then، the maximum value of |z − (٦+٨i)| is equal to :

(١) ١٢ (٢) [illegible]

(٣) ١٠ (٤) ٨

Ans. (١)

Sol. $\frac{z-2i}{z+2i} + \frac{\bar{z}+2i}{\bar{z}-2i} = 0$

$\bar{z}z - 2i\bar{z} - 2iz + 4(-1)$

$+ z\bar{z} + 2zi + 2\bar{z}i + 4(-1) = 0$

⇒ ٢|z|² = ٨ ⇒ |z| = ٢

$|z-(6+8i)|_{maximum} = 10 + 2 = 12$

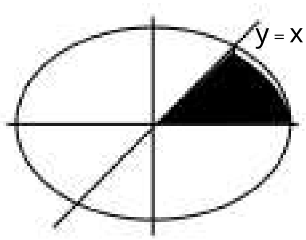
٥. The area (in square units) of the region enclosed by the ellipse x² + ٣y² = ١٨ in the first quadrant below the line y = x is :

(١) $\sqrt{3}\pi + \frac{3}{4}$ (٢) $\sqrt{3}\pi$

(٣) $\sqrt{3}\pi - \frac{3}{4}$ (٤) $\sqrt{3}\pi + 1$

Ans. (٢)

Sol. $\frac{x^2}{18} + \frac{y^2}{6} = 1$



$\frac{x^2}{18} + \frac{3x^2}{18} = 1 \Rightarrow 4x^2 = 18 \Rightarrow x^2 = \frac{9}{2}$

$\int_{3/\sqrt{2}}^{3\sqrt{2}} \frac{\sqrt{18-x^2}}{\sqrt{3}}\,dx$

$= \frac{1}{\sqrt{3}}\left[\frac{x\sqrt{18-x^2}}{2} + \frac{18}{2}\sin^{-1}\frac{x}{3\sqrt{2}}\right]_{3/\sqrt{2}}^{3\sqrt{2}}$

$= \frac{1}{\sqrt{3}}\left[9\frac{\pi}{2} - \left(\frac{3}{2\sqrt{2}}\cdot\frac{3\sqrt{3}}{\sqrt{2}} + 9\frac{\pi}{6}\right)\right]$

Required Area

$= \frac{1}{2}\cdot\frac{9}{2} + \left(\frac{18\pi}{6} - \frac{9\sqrt{3}}{4}\right)\frac{1}{\sqrt{3}}$

$= \sqrt{3}\pi$

٦. Let the foci of a hyperbola H coincide with the foci of the ellipse E : $\frac{(x-1)^2}{100} + \frac{(y-1)^2}{75} = 1$ and the eccentricity of the hyperbola H be the reciprocal of the eccentricity of the ellipse E. If the length of the transverse axis of H is α and the length of its conjugate axis is β، then ٣α² + ٢β² is equal to :

(١) ٢٤٢

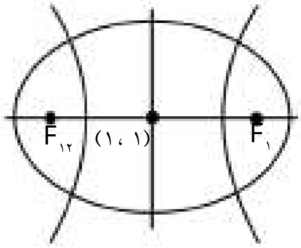
(٢) ٢٢٥

(٣) ٢٣٧

(٤) ٢٠٥

Ans. (٢)

Sol.



$e_١ = \sqrt{1 - \frac{75}{100}} = \frac{5}{10} = \frac{1}{2}$

$e_٢ = ٢$

F١ (٦، ١)، F٢ (−٤، ١)

٢ae٢ = ١٠ ⇒ a = $\frac{5}{2}$ ⇒ $2a = 5$

= ٥

٤ = ١ + $\frac{b^2}{a^2}$ ⇒ $b^2 = 3a^2$

b = $\sqrt{3} \times \frac{5}{2}$

= ٥ $\sqrt{3}$

٣ + ٢ = ٣ × ٢٥ + ٢ × ٢٥ × ٣

= ٢٢٥

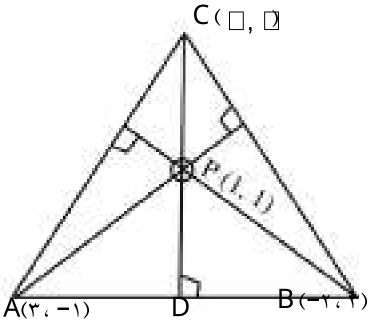
٧. Two vertices of a triangle ABC are A(٣، −١) and B (−٢، ٣)، and its orthocentre is P(١، ١). If the coordinates of the point C are (α، β) and the centre of the circle circumscribing the triangle PAB is (h، k)، then the value of (α + β) + ٢ (h + k) equals :

(١) ٥١ (٢) ٨١

(٣) ٥ (٤) ١٥

Ans. (٣)

Sol.



MAB = $\frac{4}{-5}$ ⇒ MDP = $\frac{5}{4}$

Equation of PC is y − ١ = $\frac{5}{4}(x - 1)$ . . . .(١)

MAP = $\frac{2}{-2}$ = −١ ⇒ MBC = + ١

Equation of BC is y − ٣ = (x + ٢) . . . .(٢)

On solving (١) and (٢)

x + ٤ = $\frac{5}{4}$(x − ١) ⇒ ٤x + ١٦ = ٥x − ٥ ⇒ α = ٢١

⇒ β = y = x + ٥ = ٢٦

α + β = ٤٧

Equation of ⊥ bisector of AP

y − ٠ = (x − ٢) . . . . (٣)

Equation of ⊥ bisector of AB

y − ١ = $\frac{5}{4}\left(x - \frac{1}{2}\right)$ . . . .(٤)

On solving (٣) & (٤)

(x − ٣)٤ = ٥x − $\frac{5}{2}$

x = $\frac{-19}{2}$ = h

y = $\frac{-23}{2}$ = k

⇒ ٢(h + k) = −٤٢

٨. If the variance of the frequency distribution is ١٦٠، then the value of c ∈ N is

x	c	٢c	٣c	٤c	٥c	٦c
f	٢	١	١	١	١	١

(١) ٥ (٢) ٨

(٣) ٧ (٤) ٦

Ans. (٣)

Sol.

x	C	٢C	٣C	٤C	٥C	٦C
f	٢	١	١	١	١	١

$\bar{x} = \frac{(2 + 2 + 3 + 4 + 5 + 6)C}{7} = \frac{22C}{7}$

$$Var(x) = \frac{c^2(2+2^2+3^2+4^2+5^2+6^2)}{7} - \left(\frac{22c}{7}\right)^2$$

$$= \frac{92c^2}{7} - c^2 \times \frac{484}{49}$$

$$= \frac{(644-484)c^2}{49} = \frac{160c^2}{49}$$

$$160 = \frac{160c^2}{49} \Rightarrow c = 7$$

٩. Let the range of the function

$f(x) = \frac{1}{2 - \sin 3x + \cos 3x}$, $x \in IR$ be $[a, b]$.

If α and β are respectively the A.M. and the G.M. of a and b, then $\frac{\alpha}{\beta}$ is equal to :

(١) $\sqrt{2}$ (٢) ٢

(٣) $\sqrt{\pi}$ (٤) π

Ans. (١)

Sol. $f(x) = \frac{1}{2 - \sin 3x + \cos 3x}$

$$\in \left[\frac{1}{2+\sqrt{2}}, \frac{1}{2-\sqrt{2}}\right]$$

$$\Rightarrow \frac{\alpha}{\beta} = \frac{a+b}{2\sqrt{ab}} = \frac{1}{2}\left(\sqrt{\frac{a}{b}} + \sqrt{\frac{b}{a}}\right)$$

$$= \frac{1}{2}\left(\sqrt{\frac{2-\sqrt{2}}{2+\sqrt{2}}} + \sqrt{\frac{2+\sqrt{2}}{2-\sqrt{2}}}\right)$$

$$= \frac{(2-\sqrt{2})+(2+\sqrt{2})}{2\sqrt{2}} = \sqrt{2}$$

١٠. Between the following two statements :

Statement-I : Let $\vec{a} = \hat{i} + 2\hat{j} - 3\hat{k}$ and $\vec{b} = 2\hat{i} + \hat{j} - \hat{k}$. Then the vector $\vec{r}$ satisfying $\vec{a} \times \vec{r} = \vec{a} \times \vec{b}$ and $\vec{a} \cdot \vec{r} = 0$ is of magnitude $\sqrt{10}$.

Statement-II : In a triangle ABC, $\cos 2A + \cos 2B + \cos 2C \geq -\frac{3}{2}$.

(١) Both Statement-I and Statement-II are incorrect

(٢) Statement-I is incorrect but Statement-II is correct

(٣) Both Statement-I and Statement-II are correct

(٤) Statement-I is correct but Statement-II is incorrect

Ans. (٢)

Sol. $\vec{a} = \hat{i} + 2\hat{j} - 3\hat{k}$

$\vec{b} = 2\hat{i} + \hat{j} - \hat{k}$

$\vec{a} \times \vec{r} = \vec{a} \times \vec{b}$; $\vec{a} \cdot \vec{r} = 0$

$\Rightarrow \vec{a} \times (\vec{r} - \vec{b}) = 0$

$\Rightarrow \vec{a} = \lambda(\vec{r} - \vec{b})$

$\vec{a} \cdot \vec{a} = \lambda(\vec{a} \cdot \vec{r} - \vec{a} \cdot \vec{b})$

$14\lambda = -7 \Rightarrow \lambda = -\frac{1}{2}$

$\Rightarrow -\frac{\vec{a}}{2} = \vec{r} - \vec{b} \Rightarrow \vec{r} = \vec{b} - \frac{\vec{a}}{2}$

$$= \frac{2\vec{b} - \vec{a}}{2} = \frac{3\hat{i} + \hat{k}}{2}$$

Statement (I) is incorrect

$\cos 2A + \cos 2B + \cos 2C \geq -\frac{3}{2}$

$2A + 2B + 2C = 2\pi$

$\cos 2A + \cos 2B + \cos 2C$

$= -1 - 4 \cos A \cdot \cos B \cdot \cos C$

$$\geq -1 - 4 \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2}$$

$$= -\frac{3}{2}$$

Statement (II) is correct.

11. $\lim_{x\to\frac{\pi}{2}}\left[\frac{\int_{x^3}^{(\pi/2)^3}\left(\sin\left(2t^{1/3}\right)+\cos\left(t^{1/3}\right)\right)dt}{\left(x-\frac{\pi}{2}\right)^2}\right]$ is equal to :

(1) $\frac{9\pi^2}{8}$ (2) $\frac{11\pi^2}{10}$

(3) $\frac{3\pi^2}{2}$ (4) $\frac{5\pi^2}{9}$

Ans. (1)

Sol. $\lim_{x\to\frac{\pi}{2}}\frac{0-\left(\sin(2x)+\cos(x)\right)(3x^2)}{2\left(x-\frac{\pi}{2}\right)}$

$= \lim_{x\to\frac{\pi}{2}}\frac{-(2\sin x\cos x+\cos x)3x^2}{2\left(x-\frac{\pi}{2}\right)}$

$= \lim_{x\to\frac{\pi}{2}}\left[\frac{2\sin x\sin\left(\frac{\pi}{2}-x\right)}{2\left(\frac{\pi}{2}-x\right)}+\frac{\sin\left(\frac{\pi}{2}-x\right)}{2\left(\frac{\pi}{2}-x\right)}\right]3x^2$

$= \left(1+\frac{1}{2}\right)3\left(\frac{\pi}{2}\right)^2$

$= \frac{9\pi^2}{8}$

12. The sum of the coefficient of $x^{2/3}$ and $x^{-2/5}$ in the binomial expansion of $\left(x^{2/3}+\frac{1}{2}x^{-2/5}\right)^9$ is :

(1) 21/4 (2) 69/16

(3) 63/16 (4) 19/4

Ans. (1)

Sol. $T_{r+1} = {}^9C_r\left(x^{2/3}\right)^{9-r}\left(\frac{x^{-2/5}}{2}\right)^r$

$= {}^9C_r\left(\frac{1}{2}\right)^r x^{6-\frac{2r}{3}-\frac{2r}{5}}$

for coefficient of $x^{2/3}$, put $6-\frac{2r}{3}-\frac{2r}{5}=\frac{2}{3}$

$\Rightarrow r = 5$

$\therefore$ Coefficient of $x^{2/3}$ is $= {}^9C_5\left(\frac{1}{2}\right)^5$

For coefficient of $x^{-2/5}$, put $6-\frac{2r}{3}-\frac{2r}{5}=-\frac{2}{5}$

$\Rightarrow r = 6$

Coefficient of $x^{-2/5}$ is ${}^9C_6\left(\frac{1}{2}\right)^6$

Sum $= {}^9C_5\left(\frac{1}{2}\right)^5 + {}^9C_6\left(\frac{1}{2}\right)^6 = \frac{21}{4}$

13. Let $B=\begin{bmatrix}1 & 3\\ 1 & 5\end{bmatrix}$ and A be a 2×2 matrix such that $AB^{-1}=A^{-1}$. If $BCB^{-1}=A$ and $C^4+\alpha C^2+\beta I=O$, then $2\beta-\alpha$ is equal to :

(1) 16 (2) 2

(3) 8 (4) 10

Ans. (4)

Sol. $BCB^{-1}=A$

$\Rightarrow (BCB^{-1})(BCB^{-1})=A.A$

$\Rightarrow BCICB^{-1}=A^2$

$\Rightarrow BC^2B^{-1}=A^2$

$\Rightarrow B^{-1}(BC^2B^{-1})B=B^{-1}(A.A)B$

From equation (1)

$C^2=A.A^{-1}.B$

$C^2=B$

Also $AB^{-1}=A^{-1}$

$\Rightarrow AB^{-1}.A=A^{-1}A=I$

$\Rightarrow A(B^{-1}A)=I$

$B^{-1}A=A^{-1}$

Now characteristics equation of C^2 is

$|C^2-\lambda I|=0$

$|B-\lambda I|=0$

$$\Rightarrow \begin{vmatrix} 1-\lambda & 3 \\ 1 & 5-\lambda \end{vmatrix} = 0$$

$$\Rightarrow (1-\lambda)(5-\lambda)-3=0 \Rightarrow (\lambda^2-6\lambda+5)-3=0$$

$$\Rightarrow \lambda^2-6\lambda+2=0$$

$$\Rightarrow B^2-6B+2I=0$$

$$\Rightarrow C^2-6C+2I=0$$

$$\alpha=-6$$

$$\beta=2$$

$$\Rightarrow 2\beta-\alpha=4+6=10$$

14. If $\log_e y = 3\sin^{-1}x$, then $(1-x^2)y''-xy'$ at $x=\frac{1}{2}$ is equal to :

(1) $9e^{\pi/6}$ (2) $3e^{\pi/6}$

(3) $3e^{\pi/2}$ (4) $9e^{\pi/2}$

Ans. (4)

Sol. $\ln y = 3\sin^{-1}x$

$$\frac{1}{y}y' = 3\left(\frac{1}{\sqrt{1-x^2}}\right)$$

$$\Rightarrow y' = \frac{3y}{\sqrt{1-x^2}} \text{ at } x=\frac{1}{2}$$

$$\Rightarrow y' = \frac{3e^{\pi/2}}{\frac{\sqrt{3}}{2}} = 2\sqrt{3}e^{\pi/2}$$

$$\Rightarrow y'' = 3\left[\frac{\sqrt{1-x^2}\,y' - y\frac{1}{2\sqrt{1-x^2}}(-2x)}{(1-x^2)}\right]$$

$$\Rightarrow (1-x^2)y'' = 3\left[3y + \frac{xy}{\sqrt{1-x^2}}\right]$$

$$\Rightarrow \text{at } x=\frac{1}{2},\ y=e^{3\sin^{-1}\left(\frac{1}{2}\right)} = e^{\frac{3\pi}{6}} = e^{\frac{\pi}{2}}$$

$$(1-x^2)y''\Big|_{\text{at } x=\frac{1}{2}} = 3\left[3e^{\frac{\pi}{2}} + \frac{\frac{1}{2}e^{\frac{\pi}{2}}}{\frac{\sqrt{3}}{2}}\right]$$

$$= 3e^{\frac{\pi}{2}}\left[3+\frac{1}{\sqrt{3}}\right]$$

$$(1-x^2)y''-xy'\Big|_{\text{at } x=\frac{1}{2}}$$

$$= 3e^{\frac{\pi}{2}}\left[3+\frac{1}{\sqrt{3}}\right] - \left(\frac{1}{2}\right)\left(2\sqrt{3}e^{\frac{\pi}{2}}\right) = 9e^{\frac{\pi}{2}}$$

15. The integral $\int_{1/4}^{3/4} \cos\left(2\cot^{-1}\sqrt{\frac{1-x}{1+x}}\right)dx$ is equal to:

(1) −1/2 (2) 1/4

(3) 1/2 (4) −1/4

Ans. (4)

Sol. $I = \int_{1/4}^{3/4} \cos\left(2\cot^{-1}\sqrt{\frac{1-x}{1+x}}\right)dx$

$$\int_{1/4}^{3/4} \cos\left(2\tan^{-1}\sqrt{\frac{1+x}{1-x}}\right)dx$$

$$\int_{1/4}^{3/4} \frac{1-\tan^2\left(\tan^{-1}\sqrt{\frac{1+x}{1-x}}\right)}{1+\tan^2\left(\tan^{-1}\sqrt{\frac{1+x}{1-x}}\right)}dx$$

$$= \int_{1/4}^{3/4} \frac{1-\left(\frac{1+x}{1-x}\right)}{1+\left(\frac{1+x}{1-x}\right)}dx = \int_{1/4}^{3/4} \frac{-2x}{2}dx$$

$$= \int_{1/4}^{3/4} (-x)\,dx = \left(\frac{-x^2}{2}\right)_{1/4}^{3/4}$$

$$= -\frac{1}{2}\left(\frac{9}{16}-\frac{1}{16}\right)$$

$$= -\frac{1}{4}$$

١٦. Let a, ar, ar², ... be an infinite G.P. If $\sum_{n=0}^{\infty} ar^n = 57$ and $\sum_{n=0}^{\infty} a^3r^{3n} = 9747$, then a + 18r is equal to :

(١) ٢٧ (٢) ٤٦

(٣) ٣٨ (٤) ٣١

Ans. (٤)

Sol. $\sum_{n=0}^{\infty} ar^n = 57$

$a + ar + ar^2 + \ldots = 57$

$$\frac{a}{1-r} = 57 \ldots\ldots\ldots\ldots\ldots (I)$$

$$\sum_{n=0}^{\infty} a^3r^{3n} = 9747$$

$$a^3 + a^3r^3 + a^3r^6 + \ldots\ldots\ldots = 9746$$

$$\frac{a^3}{1-r^3} = 9746 \ldots\ldots\ldots\ldots (II)$$

$$\frac{(I)^3}{(II)} = \frac{\frac{a^3}{(1-r)^3}}{\frac{a^3}{1-r^3}} = \frac{57^3}{9717} = 19$$

On solving, $r = \frac{2}{3}$ and $r = \frac{3}{2}$ (rejected)

a = ١٩

$\therefore a + 18r = 19 + 18 \times \frac{2}{3} = 31$

١٧. If an unbiased dice is rolled thrice, then the probability of getting a greater number in the i^{th} roll than the number obtained in the $(i-1)^{th}$ roll, i = ٢, ٣, is equal to :

(١) ٣/٥٤ (٢) ٢/٥٤

(٣) ٥/٥٤ (٤) ١/٥٤

Ans. (٣)

Sol. Favourable cases = $^6C_3$

Total out comes = 6^3

Probability of getting greater number than previous one = $\frac{^6C_3}{6^3} = \frac{20}{216} = \frac{5}{54}$

١٨. The value of the integral $\int_{-1}^{2} \log_e\left(x+\sqrt{x^2+1}\right)dx$ is :

(١) $\sqrt{5}-\sqrt{2}+\log_e\left(\frac{9+4\sqrt{5}}{1+\sqrt{2}}\right)$

(٢) $\sqrt{2}-\sqrt{5}+\log_e\left(\frac{9+4\sqrt{5}}{1+\sqrt{2}}\right)$

(٣) $\sqrt{5}-\sqrt{2}+\log_e\left(\frac{7+4\sqrt{5}}{1+\sqrt{2}}\right)$

(٤) $\sqrt{2}-\sqrt{5}+\log_e\left(\frac{7+4\sqrt{5}}{1+\sqrt{2}}\right)$

Ans. (٢)

Sol. $I = \int_{-1}^{2} 1.\log_e\left(x+\sqrt{x^2+1}\right)dx$

$$= \left[x\log_e\left(x+\sqrt{x^2+1}\right)\right]_{-1}^{2} - \int_{-1}^{2}\left(\frac{1}{x+\sqrt{x^2+1}}\left(1+\frac{x}{\sqrt{x^2+1}}\right)\right)x\,dx$$

$$= \left[x\log_e\left(x+\sqrt{x^2+1}\right)\right] - \int_{-1}^{2}\frac{x}{\sqrt{x^2+1}}dx$$

$$= \left[x\log_e\left(x+\sqrt{x^2+1}\right) - \sqrt{x^2+1}\right]_{-1}^{2}$$

$$= \left(2\log_e\left(2+\sqrt{5}\right) - \sqrt{5}\right) - \left(-\log_e\left(-1+\sqrt{2}\right) - \sqrt{2}\right)$$

$$= \log_e\left(2+\sqrt{5}\right)^2 - \sqrt{5} + \log_e\left(\sqrt{2}-1\right) + \sqrt{2}$$

$$= \log_e\left(2+\sqrt{5}\right)^2 - \sqrt{5} + \log_e\left(\sqrt{2}-1\right) + \sqrt{2}$$

$\Rightarrow (\sqrt{2}+\sqrt{5})\log_e\left(\dfrac{(2+\sqrt{5})^2}{\sqrt{2}+1}\right)$

$\Rightarrow (\sqrt{2}+\sqrt{5})\log_e\left(\dfrac{9+4\sqrt{5}}{\sqrt{2}+1}\right)$

١٩. Let α, β; α > β, be the roots of the equation $x^2 - \sqrt{2}x - \sqrt{3} = 0$. Let $P_n = \alpha^n - \beta^n$, n ∈ N. Then $(11\sqrt{3} - 10\sqrt{2})P_{10} + (11\sqrt{2})P_{11} - 11P_{12}$ is equal to :

(١) $10\sqrt{2}P_9$

(٢) $10\sqrt{3}P_9$

(٣) $11\sqrt{2}P_9$

(٤) $11\sqrt{3}P_9$

Ans. (٢)

Sol. $x^2 - \sqrt{2}x - \sqrt{3} = 0 \left\langle \begin{matrix}\alpha \\ \beta\end{matrix}\right.$

$\alpha^{n+2} - \sqrt{2}\alpha^{n+1} - \sqrt{3}\alpha^n = 0$

and $\beta^{n+2} - \sqrt{2}\beta^{n+1} - \sqrt{3}\beta^n = 0$

Subtracting

$(\alpha^{n+2} - \beta^{n+2}) - \sqrt{2}(\alpha^{n+1} - \beta^{n+1}) - \sqrt{3}(\alpha^n - \beta^n) = 0$

$\Rightarrow P_{n+2} - \sqrt{2}P_{n+1} - \sqrt{3}P_n = 0$

Put n = 10

$P_{12} - \sqrt{2}P_{11} - \sqrt{3}P_{10} = 0$

n = 9

$P_{11} - \sqrt{2}P_{10} - \sqrt{3}P_9 = 0$

$11\sqrt{3}P_{10} + 11\sqrt{2}P_{11} - 11P_{12} - 10\sqrt{2}P_{10}$

$= 0 + 10\sqrt{3}P_9$

٢٠. Let $\vec{a} = 2\hat{i} + \alpha\hat{j} + \hat{k}$, $\vec{b} = -\hat{i} + \hat{k}$, $\vec{c} = \beta\hat{j} - \hat{k}$, where α and β are integers and αβ = −٦. Let the values of the ordered pair (α, β) for which the area of the parallelogram of diagonals $\vec{a}+\vec{b}$ and $\vec{b}+\vec{c}$ is $\frac{\sqrt{21}}{2}$, be (α١, β١) and (α٢, β٢).

Then $\alpha_1^2 + \beta_1^2 - \alpha_2\beta_2$ is equal to

(١) ١٧ (٢) ٢٤

(٣) ٢١ (٤) ١٩

Ans. (٤)

Sol. Area of parallelogram $= \frac{1}{2}|\vec{d}_1 \times \vec{d}_2|$

$A = \frac{1}{2}\left|(\vec{a}+\vec{b}) \times (\vec{b}+\vec{c})\right| = \frac{\sqrt{21}}{2}$

so, $\vec{a}+\vec{b} = \hat{i} + \alpha\hat{j} + 2\hat{k}$

$\vec{b}+\vec{c} = -\hat{i} + \beta\hat{j}$

$(\vec{a}+\vec{b}) \times (\vec{b}+\vec{c}) = \begin{vmatrix}\hat{i} & \hat{j} & \hat{k} \\ 1 & \alpha & 2 \\ -1 & \beta & 0\end{vmatrix}$

$= \hat{i}(-2\beta) - \hat{j}(2) + \hat{k}(\alpha+\beta)$

$\left|(\vec{a}+\vec{b}) \times (\vec{b}+\vec{c})\right| = \sqrt{4\beta^2 + 4 + (\alpha+\beta)^2} = \sqrt{21}$

$4\beta^2 + 4 + \alpha^2 + \beta^2 + 2\alpha\beta = 21$

$\alpha^2 + 5\beta^2 - 12 = 17$

$\alpha^2 + 5\beta^2 = 29$

and αβ = −٦

and given α, β are integers

so,

α = −٣, β = ٢

or

α = ٣, β = −٢

$(\alpha_1, \beta_1) = (-3, 2)$

$(\alpha_2, \beta_2) = (3, -2)$

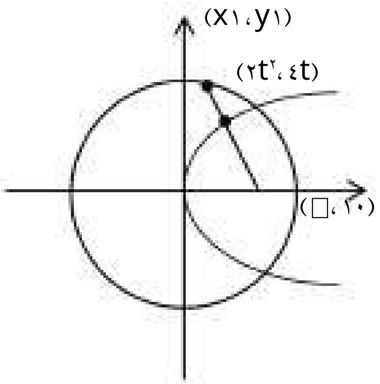
$\alpha_1^2 + \beta_1^2 - \alpha_2\beta_2 = 9 + 4 + 6 = 19$

SECTION-B

٢١. Consider the circle C : $x^2 + y^2 = ٤$ and the parabola P : $y^2 = ٨x$. If the set of all values of , for which three chords of the circle C on three distinct lines passing through the point (, ٠) are bisected by the parabola P is the interval (p, q), then $(٢q - p)^2$ is equal to ________.

Ans. (٨٠)

Sol.



$T = S_١$

$xx_١ + yy_١ = x_١^٢ + y_١^٢$

$x_١ = x_١^٢ + y_١^٢$

$(٢t^٢) = ٤t^٤ + ١٦t^٢$

$= ٢t^٢ + ٨$

$\frac{8 t^2}{٢}$

Also, $٤t^٤ + ١٦t^٢ - ٤ > ٠$

$t^٢ = -٢ + \sqrt{٥}$

$4 + 2\sqrt{٥}$

$(٨, ٤ + ٢\sqrt{٥})$

$(٢q - p)^٢ = ٨٠$

٢٢. Let the set of all values of p, for which $f(x) = (p^٢ - ٦p + ٨)(\sin^٢ ٢x - \cos^٢ ٢x) + ٢(٢ - p)x + ٧$ does not have any critical point, be the interval (a, b). Then ١٦ab is equal to ________.

Ans. (٢٥٢)

Sol. $f(x) = -(p^٢ - ٦p + ٨)\cos ٤n + ٢(٢ - p)n + ٧$

$f'(x) = +٤(p^٢ - ٦p + ٨)\sin ٤x + (٤ - ٢p) \neq ٠$

$\sin ٤x \neq \frac{٢p - ٤}{٤(p - ٤)(p - ٢)}$

$\sin ٤x \neq \frac{٢(p - ٢)}{٤(p - ٤)(p - ٢)}$

$p \neq 2$

$\sin 4x \neq \frac{١}{٢(p - ٤)}$

$\left|\frac{١}{٢(p - ٤)}\right| > ١$

on solving we get

$p \in \left(\frac{٧}{٢}, \frac{٩}{٢}\right)$

Hence $a = \frac{٧}{٢}, b = \frac{٩}{٢}$

$١٦ab = ٢٥٢$

٢٣. For a differentiable function f : IR → IR, suppose $f'(x) = ٣f(x) + \alpha$, where $\alpha \in IR$, $f(٠) = ١$ and

$\lim_{x \to -\infty} f(x) = ٧$. Then $٩f(-\log_e ٣)$ is equal to __________.

Ans. (٦١)

Sol. $\frac{dy}{dx} - ٣y = \alpha$

If $= e^{\int -٣dx} = e^{-٣x}$

$y \cdot e^{-٣x} = \int \alpha e^{-٣x} dx$

$y e^{-٣x} = \frac{\alpha e^{-٣x}}{-٣} + c$

$(\because e^{٣x})$

$y = \frac{\alpha}{-٣} + C e^{٣x}$

on substituting $x = ٠, y = ١$

$x \to -\infty, y = ٧$

we get $y = ٧ - ٦e^{٣x}$

$٩f(-\log_e ٣) = ٦١$

٢٤. The number of integers، between ١٠٠ and ١٠٠٠ having the sum of their digits equals to ١٤، is ________. Ans. (٧٠) N = a b c (i)

Sol.

All distinct digits

a + b + c = ١٤

a ١

b، c {٠ to ٩}

by hit & trial : ٨ cases

(٦، ٥، ٣) (٨، ٦، ٠) (٨، ٥، ١) (٩، ٤، ١)

(٧، ٦، ١) (٨، ٤، ٢) (٩، ٣، ٢)

(٧، ٥، ٢) ٥، ٠) ٢ same،

(٧، ٤، ٣) ١ diff ٢a + c =

(ii) ١٤ by values a = b ≠ c

:

(٣، ٨)

(٤، ٦) Total

(٥، ٤)

(٦، ٢) $\frac{٣!}{٢!}$ 5 1

(٧، ٠)

= ١٤ cases

(iii) all same :

٣a = ١٤

a = $\frac{١٤}{٣}$ × rejected

٠ cases

Hence، Total cases :

٨ × ٣! + ٢ × (٤) + ١٤

= ٤٨ + ٢٢

= ٧٠

٢٥. Let A = {(x، y) : ٢x + ٣y = ٢٣، x، y ∈ N} and B = {x : (x، y) ∈ A}. Then the number of one-one functions from A to B is equal to _______.

Ans. (٢٤)

Sol. ٢x + ٣y = ٢٣

x = ١ x y = ٧

= ٤ x = y = ٥

٧ x = ١٠ y = ٣

y = ١

A	B
(١، ٧)	١
(٤، ٥)	٤
(٧، ٣)	٧
(١٠، ١)	١٠

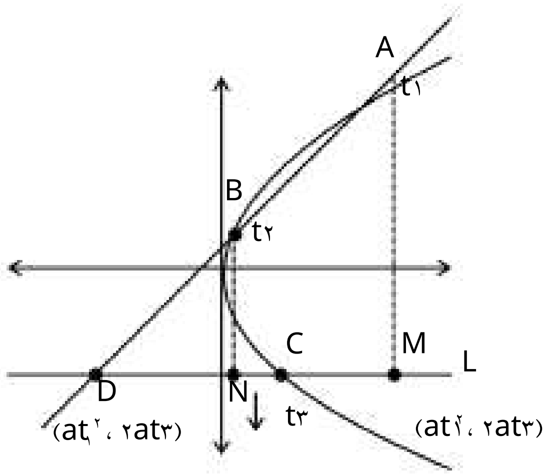
The number of one-one functions from A to B is equal to ٤!

٢٦. Let A، B and C be three points on the parabola $y^٢ = ٦x$ and let the line segment AB meet the line L through C parallel to the x-axis at the point D. Let M and N respectively be the feet of the perpendiculars from A and B on L.

Then $\left(\frac{AM \cdot BN}{CD}\right)^2$ is equal to __________.

Ans. (٣٦)

Sol.



Sol.

$m_{AB} = m_{AD}$

$\frac{٢}{t_١ + t_٢} = \frac{٢a(t_١ - t_٣)}{at_١^٢ - ...}$

$at_١^٢ ... a(t_١^٢ + t_١t_٢ - t_١t_٣ - t_٢t_٣)$

$a(t_١t_٢ - t_٢t_٣ - t_١t_٣)$

AM = |٢a(t١ − t٣)| , BN = |٢a(t٢ − t٣)|

CD = |at٣² ...|

$\Rightarrow CD = \left|at_3^2 - a(t_1t_3 + t_2t_3 - t_1t_2)\right|$

$\Rightarrow \left|a\left(t_3^2 - t_1t_3 - t_2t_3 + t_1t_2\right)\right|$

$\Rightarrow a\left|t_3(t_3 - t_1) - t_2(t_3 - t_1)\right|$

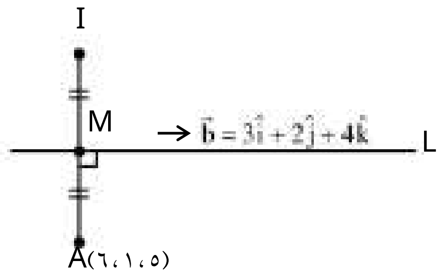
$\Rightarrow CD = a(t_3 - t_2)(t_3 - t_1)$

$\Rightarrow \left(\frac{AM\cdot BN}{CD}\right)^2 = \left(\frac{2a(t_1 - t_3)\cdot 2a(t_2 - t_3)}{a(t_3 - t_2)(t_3 - t_1)}\right)^2$

$\Rightarrow 16a^2 = 16 \times \frac{9}{4} = 36$

٢٧. The square of the distance of the image of the point (٦، ١، ٥) in the line $\frac{x-1}{3} = \frac{y}{2} = \frac{z-2}{4}$, from the origin is __________.

Ans. (٦٢)



Sol.

Let M(٣λ + ١، ٢λ، ٤λ + ٢)

$\overrightarrow{AM}\cdot\vec{b} = 0$

$\Rightarrow 9\lambda - 15 + 4\lambda - 2 + 16\lambda - 12 = 0$

$\Rightarrow 29\lambda = 29$

$\Rightarrow \lambda = 1$

M (٤، ٢، ٦)، I = (٢، ٣، ٧)

Required Distance $= \sqrt{4 + 9 + 49} = \sqrt{62}$

Ans. ٦٢

٢٨. If $\left(\frac{1}{\alpha+1} + \frac{1}{\alpha+2} + \ldots + \frac{1}{\alpha+1012}\right) - \left(\frac{1}{2\times1} + \frac{1}{4\times3} + \frac{1}{6\times5} + \ldots + \frac{1}{2024\times2023}\right) = \frac{1}{2024}$, then α is equal to–

Ans. (١٠١١)

Sol. $\left(\frac{1}{\alpha+1} + \frac{1}{\alpha+2} + \ldots + \frac{1}{\alpha+2012}\right)$

$-\left(\left(1 - \frac{1}{2}\right) + \left(\frac{1}{3} - \frac{1}{4}\right) + \ldots + \left(\frac{1}{2023} - \frac{1}{2024}\right)\right) = \frac{1}{2024}$

$\Rightarrow \left(\frac{1}{\alpha+1} + \frac{1}{\alpha+2} + \ldots + \frac{1}{\alpha+2012}\right)$

$-\left(\left(1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \ldots + \frac{1}{2023}\right)\right.$

$\left.- \frac{1}{2024} - 2\left(\frac{1}{2} + \frac{1}{4} + \ldots + \frac{1}{2022}\right)\right) = \frac{1}{2024}$

$\Rightarrow \left(\frac{1}{\alpha+1} + \frac{1}{\alpha+2} + \ldots + \frac{1}{\alpha+2012}\right) -$

$\left(\left(1 + \frac{1}{2} + \ldots + \frac{1}{2023}\right)\right.$

$\left.- \frac{1}{2024} - \left(1 + \frac{1}{2} + \ldots + \frac{1}{1011}\right)\right) = \frac{1}{2024}$

$\Rightarrow \frac{1}{\alpha+1} + \frac{1}{\alpha+2} + \ldots + \frac{1}{\alpha+2012}$

$= \frac{1}{1012} + \frac{1}{1013} + \ldots + \frac{1}{2023}$

$\Rightarrow \alpha = 1011$

٢٩. Let the inverse trigonometric functions take principal values. The number of real solutions of the equation $2\sin^{-1}x + 3\cos^{-1}x = \frac{2\pi}{5}$, is __________.

Ans. (٠)

Sol. $\Rightarrow 2\left(\frac{\pi}{2} - \cos^{-1}x\right) + 3\cos^{-1}x = \frac{2\pi}{5}$

$\Rightarrow \pi + \cos^{-1}x = \frac{2\pi}{5}$

$\Rightarrow \cos^{-1}x = -\frac{3\pi}{5}$

Not possible

Ans. ٠

30. Consider the matrices : $A = \begin{bmatrix} 2 & -5 \\ 3 & m \end{bmatrix}$, $B = \begin{bmatrix} 20 \\ m \end{bmatrix}$ and $X = \begin{bmatrix} x \\ y \end{bmatrix}$. Let the set of all m, for which the system of equations AX = B has a negative solution (i.e., x < 0 and y < 0), be the interval (a, b).

Then $8\int_a^b |A|\,dm$ is equal to ______________.

Ans. (450)

Sol. $A = \begin{bmatrix} 2 & -5 \\ 3 & m \end{bmatrix}$, $B = \begin{bmatrix} 20 \\ m \end{bmatrix}$

$X = \begin{bmatrix} x \\ y \end{bmatrix}$

$2x - 5y = 20$ ...(1)

$3x + my = m$ ...(2)

$\Rightarrow y = \dfrac{2m - 60}{2m + 15}$

$y < 0 \Rightarrow m \in \left(-\dfrac{15}{2}, 30\right)$

$x = \dfrac{25m}{2m + 15}$

$x < 0 \Rightarrow m \in \left(-\dfrac{15}{2}, 0\right)$

$\therefore m \in \left(-\dfrac{15}{2}, 0\right)$

$|A| = 2m + 15$

Now,

$8\int_{-\frac{15}{2}}^{0} (2m + 15)\,dm = 8\left(m^2 + 15m\right)_{-\frac{15}{2}}^{0}$

$= 8\left(0 - \left(\dfrac{225}{4} - \dfrac{225}{2}\right)\right)$

$= 8 \times \dfrac{225}{4} = 450$

PHYSICS TEST PAPER WITH SOLUTION

SECTION-A

٣١. A nucleus at rest disintegrates into two smaller nuclei with their masses in the ratio of ٢ : ١. After disintegration they will move :-

(١) In opposite directions with speed in the ratio of ١ : ٢ respectively

(٢) In opposite directions with speed in the ratio of ٢ : ١ respectively

(٣) In the same direction with same speed.

(٤) In opposite directions with the same speed.

Ans. (١)

Sol. By conservation of momentum

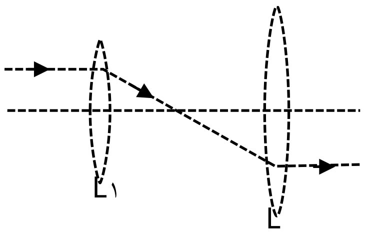
$p_i = p_f$

$O = m_١u_١ + m_٢u_٢$

$\frac{u_١}{u_٢}$ □□□$\frac{1}{2}$□ as $\frac{m_١}{m_٢}$ □ $\frac{٢}{١}$

move in opposite direction with speed ratio ١ : ٢ respectively.

٣٢. The following figure represents two biconvex lenses $L_١$ and $L_٢$ having focal length ١٠ cm and ١٥ cm respectively. The distance between $L_١$ & $L_٢$ is :

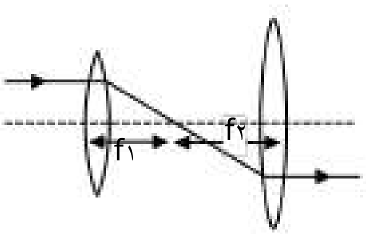


(١) ١٠ cm (٢) ١٥ cm

(٣) ٢٥ cm (٤) ٣٥ cm

Ans. (٣)

Sol.



$D = f_١ + f_٢ = ٢٥$ cm

Paraxial parallel rays pass through focus and ray from focus of convex lens will become parallel

٣٣. The temperature of a gas is –٧٨° C and the average translational kinetic energy of its molecules is K. The temperature at which the average translational kinetic energy of the molecules of the same gas becomes 2K is :

(١) –٣٩°C (٢) ١١٧°C

(٣) ١٢٧°C (٤) –٧٨°C

Ans. (٢)

Sol. K.E = $\frac{nfRT}{٢}$

T_i = –78°C □□□□□□□□□–78°C□□□□□□□K□

□ □□□□□□T

To double the K.E energy temp also become double

T_f = ٣٩٠ K

T_f = ١١٧°C

٣٤. A hydrogen atom in ground state is given an energy of ١٠.٢ eV. How many spectral lines will be emitted due to transition of electrons ?

(١) ٦ (٢) ٣

(٣) ١٠ (٤) ١

Ans. (٤)

Sol. Hydrogen will be in first excited state therefore it will emit one spectral line corresponding to transition b/w energy level ٢ to ١

٣٥. The magnetic field in a plane electromagnetic wave is $B_y = (٣.٥ \times ١٠^{-٧}) \sin(١.٥ \times ١٠^٣ x + ٠.٥ \times ١٠^{١١} t)$ T. The corresponding electric field will be

(١) $E_y = ١.١٧ \sin(١.٥ \times ١٠^٣ x + ٠.٥ \times ١٠^{١١} t) Vm^{-١}$

(٢) $E_z = ١٠٥ \sin(١.٥ \times ١٠^٣ x + ٠.٥ \times ١٠^{١١} t) Vm^{-١}$

(٣) $E_z = ١.١٧ \sin(١.٥ \times ١٠^٣ x + ٠.٥ \times ١٠^{١١} t) Vm^{-١}$

(٤) $E_y = ١٠.٥ \sin(١.٥ \times ١٠^٣ x + ٠.٥ \times ١٠^{١١} t) Vm^{-١}$

Ans. (٢)

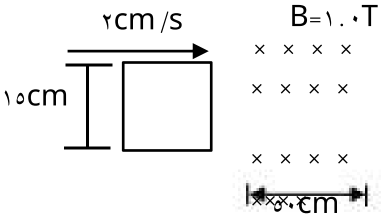
Sol. $E_\cdot = BC_\cdot$

$E_\cdot = ٣ \times ١٠^٨ \times (٣.٥ \times ١٠^{-٧}) \sin(١.٥ \times ١٠^٣ x + ٠.٥ \times ١٠^{١١} t)$

$E_\cdot = ١٠٥ \sin(١.٥ \times ١٠^٣ x + ٠.٥ \times ١٠^{١١} t) Vm^{-١}$

Data inconsistent while calculating speed of wave. You can challenge for data.

۳٦. A square loop of side ۱٥ cm being moved towards right at a constant speed of ۲ cm/s as shown in figure. The front edge enters the ٥۰ cm wide magnetic field at t = ۰. The value of induced emf in the loop at t = ۱۰ s will be :

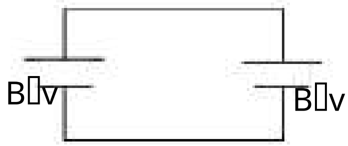


(۱) ۰.۳ mV (۲) ٤.٥ mV

(۳) zero (٤) ۳ mV

Ans. (۳)

Sol. At t = ۱۰ sec complete loop is in magnetic field therefore no change in flux



$e = \frac{d\phi}{dt} = 0$

e = ۰ for complete loop

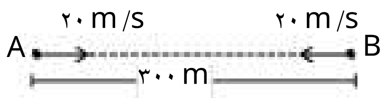
۳۷. Two cars are travelling towards each other at speed of ۲۰ m s⁻¹ each. When the cars are ۳۰۰ m apart, both the drivers apply brakes and the cars retard at the rate of ۲ m s⁻². The distance between them when they come to rest is :

(۱) ۲۰۰ m (۲) ٥۰ m

(۳) ۱۰۰ m (٤) ۲٥ m

Ans. (۳)

Sol.



$|u_{BA}| = 40\ ms^{-1}$

$|a_{BA}| = 4\ ms^{-2}$

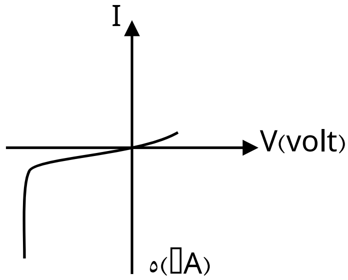
Apply $(v^2 = u^2 + 2as)_{relative}$

$0 = (40)^2 + 2(-4)(S)$

S = ۲۰۰ m

Remaining distance = ۳۰۰ − ۲۰۰ = ۱۰۰ m

۳۸. The I-V characteristics of an electronic device shown in the figure. The device is :



(۱) a solar cell

(۲) a transistor which can be used as an amplifier

(۳) a zener diode which can be used as voltage regula

(٤) a diode which can be used as a rectifier

Ans. (۳)

Sol. Theory

Zener diode used as voltage regulator

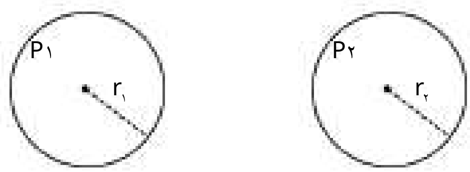
۳۹. The excess pressure inside a soap bubble is thrice the excess pressure inside a second soap bubble. The ratio between the volume of the first and the second bubble is :

(۱) ۱ : ۹ (۲) ۱ : ۳

(۳) ۱ : ۸۱ (٤) ۱ : ۲۷

Ans. (٤)

Sol.



$P_1 - P_0 = \frac{4T}{r_1}$ $P_2 - P_0 = \frac{4T}{r_2}$

$P_1 - P_0 = 3(P_2 - P_0)$

$\frac{4T}{r_1} = 3\frac{4T}{r_2}$

$r_2 = 3r_1$

$\frac{V_1}{V_2} = \frac{\frac{4}{3}\pi r_1^3}{\frac{4}{3}\pi r_2^3} = \frac{1}{27}$

٤٠. The de-Broglie wavelength associated with a particle of mass m and energy E is $h\sqrt{٢mE}$. The dimensional formula for Planck's constant is :

(١) [MLT]　　(٢) [MLT]

(٣) [MLT]　　(٤) [MLT]

Ans. (٣)

Sol. $\lambda = \frac{h}{\sqrt{٢mE}}$ or E = hν

[MLٖ٢T⁻٢] = h[T⁻١]

h = [MLٖ٢T⁻١]

٤١. A satellite of ١٠ kg mass is revolving in circular orbit of radius ٢R. If $\frac{١٠٤R}{٦}$ J energy is supplied to the satellite, it would revolve in a new circular orbit of radius :

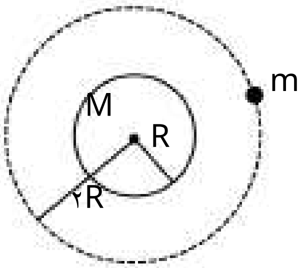
(use g = ١٠ m/s², R = radius of earth)

(١) ٢.٥ R　　(٢) ٣ R

(٣) ٤ R　　(٤) ٦ R

Ans. (٤)

Sol.



Total energy = $\frac{-GMm}{٢(٢R)}$

if energy = $\frac{١٠R}{٦}$ is added then

$\frac{-GMm}{٤R} + \frac{١٠٤R}{٦} = \frac{-GMm}{٢r}$

where r is new radius of revolving and g = $\frac{GM}{R^٢}$

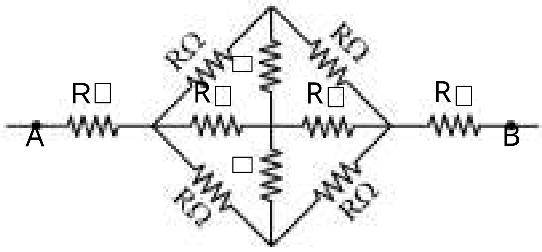
$-\frac{mgR}{٤} + \frac{١٠٤R}{٦} = -\frac{mgR^٢}{٢r}$ (m = ١٠ kg)

$-\frac{١٠^٣ \times ١٠ \times R}{٤} + \frac{١٠٤R}{٦} = -\frac{103 \times ١٠ \times R^٢}{٢r}$

$-\frac{١}{٤} + \frac{١}{٦} = -\frac{R}{٢r}$

r = ٦R

٤٢. The effective resistance between A and B, if resistance of each resistor is R, will be



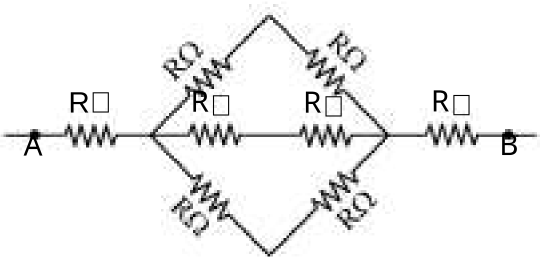
(١) $\frac{٢}{٣}$R　　(٢) $\frac{٨R}{٣}$

(٣) $\frac{٥R}{٣}$　　(٤) $\frac{٤R}{٣}$

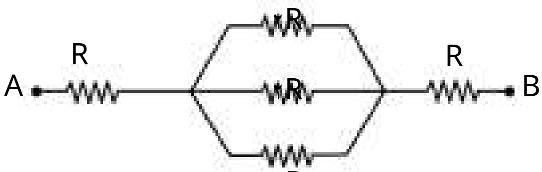
Ans. (٢)

Sol. From symmetry we can remove two middle resistance.

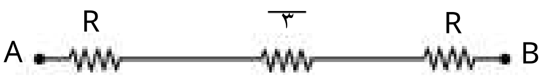
New circuit is



⇓

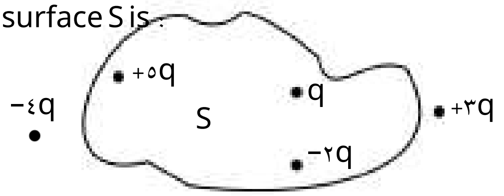


⇒ $\frac{٢R}{٣}$



⇒ $\frac{٨R}{٣}$

٤٣. Five charges +q، +٥q، −٢q، +٣q and −٤q are situated as shown in the figure. The electric flux due to this configuration through the surface S is :



(١) $\frac{٥q}{\varepsilon_0}$ (٢) $\frac{٤q}{\varepsilon_0}$

(٣) $\frac{٣q}{\varepsilon_0}$ (٤) $\frac{q}{\varepsilon_0}$

Ans. (٢)

Sol. As per gauss theorem،

$\phi = \frac{q_{in}}{\varepsilon_0} = \frac{q + (-٢q) + ٥q}{\varepsilon_0}$

$= \frac{٤q}{\varepsilon_0}$

٤٤. A proton and a deutron (q = +e، m = ٢.٠u) having same kinetic energies enter a region of uniform magnetic field $\vec{B}$، moving perpendicular to $\vec{B}$. The ratio of the radius r_d of deutron path to the radius r_p of the proton path is :

(١) ١ : ١ (٢) ١ : $\sqrt{٢}$

(٣) $\sqrt{٢}$: ١ (٤) ١ : ٢

Ans. (٣)

Sol. In uniform magnetic field،

$R = \frac{mv}{qB} = \frac{\sqrt{٢m(K.E)}}{qB}$

Since same K.E

$R \propto \frac{\sqrt{m}}{q}$

$\Rightarrow \frac{R_{deutron}}{R_{proton}} = \sqrt{\frac{m_d}{m_p}} \times \frac{q_p}{q_d}$

$\Rightarrow (\sqrt{2} : 1)$

$\Rightarrow r_d : r_p = \sqrt{٢} : ١$

٤٥. UV light of ٤.١٣ eV is incident on a photosensitive metal surface having work function ٣.١٣ eV. The maximum kinetic energy of ejected photoelectrons will be : (١) ٤.١٣ eV (٣) ٣.١٣ eV (٢) ١ eV (٤) ٧.٢٦ eV

Ans. (٢)

Sol. E_{photon} = (work function) + $K.E_{max}$

$٤.١٣\ eV = ٣.١٣\ eV + K.E_{max}$

$\Rightarrow K.E_{max} = ١\ eV$

٤٦. The energy released in the fusion of ٢ kg of hydrogen deep in the sun is E_H and the energy released in the fission of ٢ kg of ^{٢٣٥}U is E_U. The ratio $\frac{E_H}{E_U}$ is approximately :

(Consider the fusion reaction as $٤\,{}^{1}_{1}H + ٢e^- \rightarrow {}^{4}_{2}He + ٢\nu + ٦\gamma + ٢٦.٧MeV$، energy released in the fission reaction of ^{٢٣٥}U is ٢٠٠ MeV per fission nucleus and $N_A = ٦.٠٢٣\times١٠^{٢٣}$)

(١) ٩.١٣ (٢) ١٥.٠٤

(٣) ٧.٦٢ (٤) ٢٥.٦

Ans. (٣)

Sol. In each fusion reaction $٤\,{}^{1}_{1}H$ nucleus are used.

Energy released per Nuclei of ${}^{1}_{1}H = \frac{٢٦.٧}{٤}$ MeV

∴ Energy released by ٢ kg hydrogen (E_H)

$= \frac{٢٠٠٠}{١} \times N_A \times \frac{٢٦.٧}{٤}$ MeV

&

∴ Energy released by ٢ kg Vranium (E_V)

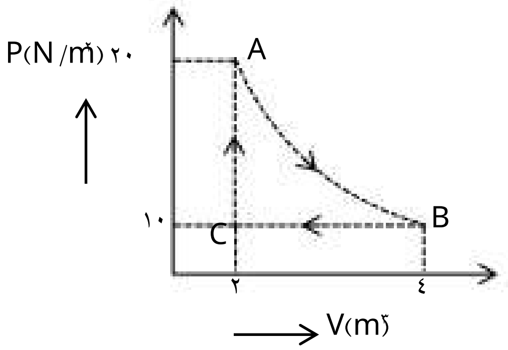
$= \frac{٢٠٠٠}{٢٣٥} \times N_A \times ٢٠٠$ MeV

So،

$\frac{E_H}{E_V} = \frac{٢٣٥ \times ٢٦.٧}{٤ \times ٢٠٠} = ٧.٨٤$

∴ Approximately close to ٧.٦٢

٤٧. A real gas within a closed chamber at ٢٧°C undergoes the cyclic process as shown in figure. The gas obeys $PV^{٣} = RT$ equation for the path A to B. The net work done in the complete cycle is (assuming R = ٨J/molK):



(١) ٢٢٥J (٢) ٢٠٥ J

(٣) ٢٠J (٤) −٢٠J

Ans. (٢)

Sol. $W_{AB} = \int PdV$ (Assuming T to be constant)

$= \int \frac{RTdV}{V^{٣}}$

$= RT \int V^{-٣} dV$

$= ٨ \times ٣٠٠ \times \left[\frac{١}{2 \times 4} - \frac{١}{٢٢}\right]$

$= ٢٢٥J$

$W_{BC} = \int PdV = ١٠(٢-٤) = -٢٠J$

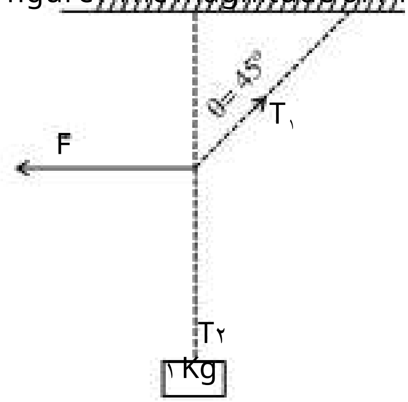
$W_{CA} = ٠$

$\therefore W_{cycle} = ٢٠٥J$

Note: Data is inconsistent in process AB.

So needs to be challenged.

٤٨. A ١ kg mass is suspended from the ceiling by a rope of length ٤m. A horizontal force 'F' is applied at the mid point of the rope so that the rope makes an angle of ٤٥° with respect to the vertical axis as shown in figure. The magnitude of F is:



(١) $\frac{١0}{\sqrt{٢}}$ N (٢) ١ N

(٣) $\frac{١}{١٠ \times \sqrt{٢}}$ N (٤) ١٠ N

Ans. (٤)

Sol. $T_{١}\sin ٤٥° = F$

$T_{١}\cos ٤٥° = T_{٢} = ١ \times g$

$\Rightarrow \tan ٤٥° = \frac{F}{g}$

$\Rightarrow F = ١٠N$

٤٩. A spherical ball of radius ١×١٠^{-٤} m and density $١٠^{٥}$ kg/m³ falls freely under gravity through a distance h before entering a tank of water. If after entering in water the velocity of the ball does not change, then the value of h is approximately:

(The coefficient of viscosity of water is ٩.٨×١٠^{-٦} N s/m²)

(١) ٢٢٩٦ m (٢) ٢٢٤٩ m

(٣) ٢٥١٨ m (٤) ٢٣٩٦ m

Ans. (٣)

Sol. $V_T = \frac{٢gR^{٢}}{٩} \frac{(\rho_B - \rho_L)}{\eta}$

$\Rightarrow V_T = \frac{٢}{٩} \times \frac{٩.٨ \times (10^{-٤})^{٢}}{10^{-٥}} (١٠٥ - ١٠٣)$

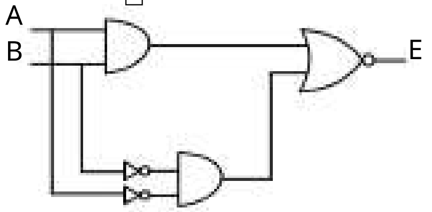
$\therefore V_T = ٢٢٤.٥$

when ball fall from height (h)

$V = \sqrt{٢gh}$

$h = \left(\frac{V^{٢}}{٢g}\right) = ٢٥١٨m$

٥٠.



A	B	E
٠	٠	١
٠	١	X
١	٠	Y
١	١	٠

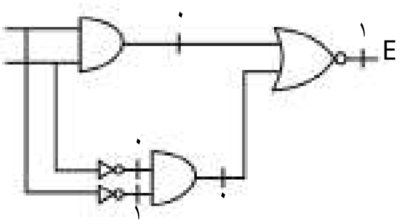
In the truth table of the above circuit the value of X and Y are : (١) ١, ١ (٣) ٠, ١

(٢) ١, ٠

(٤) ٠, ٠

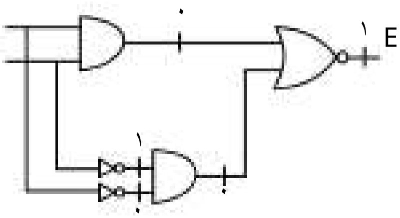
Ans. (١)

Sol. For x



X = ١

For y



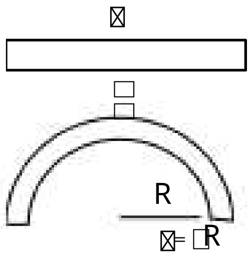
SECTION-B

٥١. A straight magnetic strip has a magnetic moment of ٤٤ Am. If the strip is bent in a semicircular shape, its magnetic moment will be ……… $Am^{٢}$

(Given $\pi = \frac{٢٢}{٧}$)

Ans. (٢٨)

Sol. Magnetic moment of straight wire = m × ℓ = ٤٤



Magnetic moment of arc

= m × ٢r

$= m \times \frac{٢\ell}{\pi}$

$= \frac{٤٤ \times ٢}{\pi} = \frac{٨٨}{\pi} = ٢٨$

٥٢. A particle of mass ٠.٥ kg executes simple harmonic motion under force F = −٥٠(Nm)x. The time period of oscillation is $\frac{x}{٣٥}$ s. The value of x is ……………..

(Given $\pi = \frac{٢٢}{٧}$)

Ans. (٢٢)

Sol. m = ٠.٥ kg

F = −٥٠ (x)

ma = (−٥٠x)

٠.٥ a = −٥٠x

a = (−١٠٠x)

$W^{٢} = ١٠٠ \Rightarrow w = ١٠$

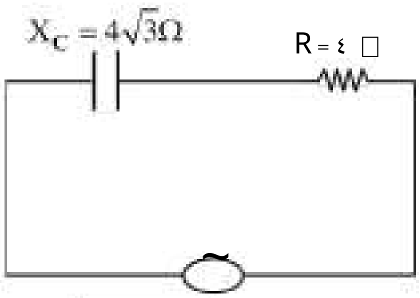
$T = \frac{٢\pi}{١٠} = \left(\frac{\pi}{٥}\right) = \frac{٢٢}{٧ \times ٥} = \left(\frac{٢٢}{٣٥}\right)$

$\frac{x}{٣٥} = \frac{٢٢}{٣٥}$ $x = ٢٢$

٥٣. A capacitor of reactance $4\sqrt{3}\,\Omega$ and a resistor of resistance ٤Ω are connected in series with an ac source of peak value $8\sqrt{2}$ V. The power dissipation in the circuit is ……………… W.

Ans. (٤)

Sol.



$Z = \sqrt{R^2 + X_C^2}$

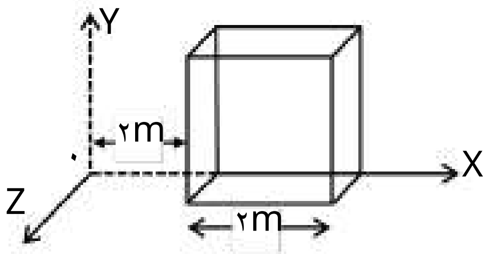
$Z = \sqrt{4^2 + (4\sqrt{3})^2} = 8\ \Omega$

$V_{rms} = \frac{V}{\sqrt{2}} = \frac{8\sqrt{2}}{\sqrt{2}} = 8V$

$I_{rms} = \frac{V_{rms}}{Z} = \frac{8}{8} = 1A$

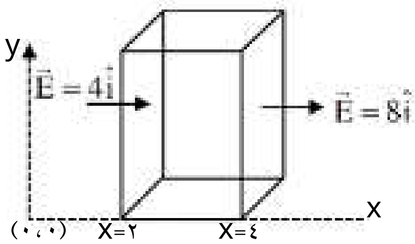
Power dissipated $= I_{rms}^2 \times R = 1 \times 4 = (4W)$

٥٤. An electric field $\vec{E} = (2x\hat{i})\,NC^{-1}$ exists in space. A cube of side ٢m is placed in the space as per figure given below. The electric flux through the cube is ……………… Nm/C.



Ans. (١٦)

Sol.



$E = 2x\hat{i}$

$\phi = \vec{E}.\vec{A}$

$\phi_{in} = -4 \times 4 = -16\,Nm^2/c$

$\phi_{out} = 8 \times 4 = 32\,Nm^2/c$

$\phi_{net} = \phi_{in} + \phi_{out} = -16 + 32 = 16\,Nm^2/c$

٥٥. A circular disc reaches from top to bottom of an inclined plane of length l. When it slips down the plane, if takes t s. When it rolls down the plane then it takes $\left(\frac{\alpha}{2}\right)^{1/2} t$ s, where α is ………….

Ans. (٣)

Sol. For slipping

$a = g\sin\theta$

$\ell = \frac{1}{2}at^2 \Rightarrow t = \sqrt{\frac{2\ell}{g\sin\theta}}$

For rolling

$a' = \frac{g\sin\theta}{1 + \frac{k^2}{R^2}} \quad \left(k = \frac{R}{\sqrt{2}}\right)$

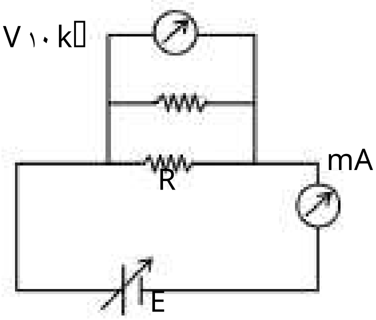
$\Rightarrow a' = \frac{2g\sin\theta}{3}$

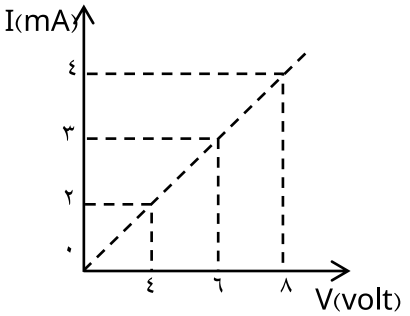
$\ell = \frac{1}{2}a't'^2$

$\Rightarrow t' = \sqrt{\frac{6\ell}{2g\sin\theta}} = \sqrt{\frac{3}{2}}\sqrt{\frac{2\ell}{g\sin\theta}}$

$\Rightarrow \alpha = 3$

٥٦. To determine the resistance (R) of a wire, a circuit is designed below. The V-I characteristic curve for this circuit is plotted for the voltmeter and the ammeter readings as shown in figure. The value of R is ……… Ω.





Ans. (٢٥٠٠)

Sol. $Req = \frac{١٠٤R}{١٠٤ + R}$

E = ٤V, I = ٢mA

$I = \frac{E}{Req} \Rightarrow ٢ \times ١٠^{-٣} = \frac{٤(١٠٤ + R)}{١٠٤R}$

⇒ ٢٠R = ٤٠٠٠٠ + ٤R

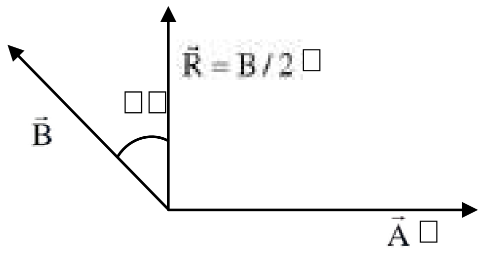
١٦R = ٤٠٠٠٠

R = ٢٥٠٠Ω

٥٧. The resultant of two vectors A and B is perpendicular to A and its magnitude is half that of B. The angle between vectors A and B is …………

Ans. (١٥٠)

Sol.



$B\cos\theta = \frac{B}{٢}$

θ = ٦٠°

So, angle between A & B is ٩٠° + ٦٠° = ١٥٠°

٥٨. Monochromatic light of wavelength ٥٠٠ nm is used in Young's double slit experiment. An interference pattern is obtained on a screen When one of the slits is covered with a very thin glass plate (refractive index = ١.٥), the central maximum is shifted to a position previously occupied by the ٤th bright fringe. The thickness of the glass-plate is ……………… μm.

Ans. (٤)

Sol. (μ − ١) t = nλ

$(١.٥ - ١)t = ٤ \times ٥٠٠ \times ١٠^{-٩}$ m

$t = ٤٠٠٠ \times ١٠^{-٩}$ m

t = ٤ μm

٥٩. A force $(٣x^٢ + ٢x - ٥)$ N displaces a body from x = ٢ m to x = ٤ m. Work done by this force is ………J.

Ans. (٥٨)

Sol. $W = \int_{x_١}^{x_٢} F\,dx$

$W = \int_{٢}^{٤} (٣x^٢ + ٢x - ٥)\,dx$

$W = [x^٣ + x^٢ - ٥x]_{٢}^{٤}$

W = (٦٠ − ٢) J = ٥٨ J

٦٠. At room temperature (٢٧°C), the resistance of a heating element is ٥٠Ω. The temperature coefficient of the material is ٢.٤×١٠^{-٤} °C^{-1}. The temperature of the element, when its resistance is 62 Ω, is …………. °C.

Ans. (١٠٢٧)

Sol. R = R₀(١ + αΔT)

$٦٢ = ٥٠ (١ + ٢.٤ \times ١٠^{-٤} \Delta T)$

ΔT = ١٠٠٠°C

T = ١٠٢٧°C

CHEMISTRY

TEST PAPER WITH SOLUTION

SECTION–A

٦١. The candela is the luminous intensity، in a given direction، of a source that emits monochromatic radiation of frequency 'A' × ١٠ hertz and that has a radiant intensity in that direction of ١/'B' watt per steradian. 'A' and 'B' are respectively

(١) ٥٤٠ and ١/٦٨٣

(٢) ٥٤٠ and ٦٨٣

(٣) ٤٥٠ and ١/٦٨٣

(٤) ٤٥٠ and ٦٨٣

Ans. (٢)

Sol. The candela is the luminous intensity of a source that emits monochromatic radiation of frequency radiation of frequency ٥٤٠ × ١٠ Hz and has a radiant intensity in that direction of ١/٦٨٣ w/sr. It is unit of Candela.

٦٢. The correct stability order of the following resonance structures of $CH_٣$–CH = CH–CHO is

$CH_٣$–CH–CH=C–H (I) ⟷ $CH_٣$–CH–CH=C–H (II)

⟷ $CH_٣$–CH=CH–C–H (III)

(١) II < III < I

(٢) III < II < I

(٣) I < II < III

(٤) II < I < III

Ans. (٢)

Sol. $CH_٣$–CH=CH–CH=O (III)

Non Polar R.S.

More No of covalent bond

$CH_٣$–CH–CH=CH–O (II)

Having –ve charge on more electronegative atom

$CH_٣$–CH–CH=CH–O (I)

Having –ve charge on less electronegative atom

Stability order III < II < I

٦٣. Total number of stereo isomers possible for the given structure:

Br Br

(١) ٨ (٢) ٢

(٣) ٤ (٤) ٣

Ans. (١)

Sol.

Br

Br

Br

There are three stereo center

So No of stereoisomer = $٢^٣$ = ٨

٦٤. The correct increasing order for bond angles among $BF_٣$، $PF_٣$ and $C\ell F_٣$ is:

(١) $PF_٣ > BF_٣ > C\ell F_٣$

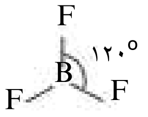
(٢) $BF_٣ > PF_٣ > C\ell F_٣$

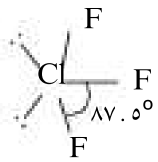
(٣) $C\ell F_٣ > PF_٣ > BF_٣$

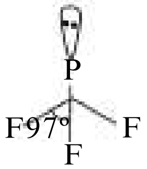
(٤) $BF_٣ = PF_٣ > C\ell F_٣$

Ans. (٣)

Sol.







Order of bond angle is

ClF٣ > PF٣ > BF٣

٦٥. Match List I with List II

LIST-I (Test)		LIST-II (Observation)	
A.	Br٢ water test	I.	Yellow orange or orange red precipitate formed
B.	Ceric ammonium nitrate test	II.	Reddish orange colour disappears
C.	Ferric chloride test	III.	Red colour appears
D.	٢,٤-DNP test	IV.	Blue, Green, Violet or Red colour appear

Choose the correct answer from the options given below: (١) A-I, B-II, C-III, D-IV (٢) A-II, B-III, C-IV, D-I (٣) A-III, B-IV, C-I, D-II (٤) A-IV, B-I, C-II, D-III

Ans. (٢)

Sol. (A) Br٢ water test is test of unsaturation in which reddish orange colour of bromine water disappears.

(B) Alcohols given Red colour with ceric ammonium nitrate.

(C) Phenol gives Violet colour with natural ferric chloride.

(D) Aldehyde & Ketone give Yellow/Orange/Red Colour compounds with ٢,٤-DNP i.e., ٢,٤-Dinitrophenyl hydrazine.

٦٦. Match List I with List II

LIST-I (Cell)		LIST-II (Use/Property/Reaction)	
A.	Leclanche cell	I.	Converts Energy of combustion into electrical energy
B.	Ni-Cd cell	II.	Does not involve any ion in solution and is used in hearing aids
C.	Fuel cell	III.	Rechargeable
D.	Mercury cell	IV.	Reaction at anode Zn → Zn+٢ + ٢e−

Choose the correct answer from the options given below: (١) A-I, B-II, C-III, D-IV (٢) A-III, B-I, C-IV, D-II (٣) A-IV, B-III, C-I, D-II (٤) A-II, B-III, C-IV, D-I

Ans. (٣)

Sol. A-IV, B-III, C-I, D-II

٦٧. Match List I with List II

LIST-I		LIST-II	
A.	K٢[Ni(CN)٤]	I.	sp³
B.	Ni(CO)٤	II.	sp³d²
C.	[Co(NH٣)٦]Cl٣	III.	dsp²
D.	Na٣[CoF٦]	IV.	d²sp³

Choose the correct answer from the options given below:

(١) A-III, B-I, C-II, D-IV

(٢) A-III, B-II, C-IV, D-I

(٣) A-I, B-III, C-II, D-IV

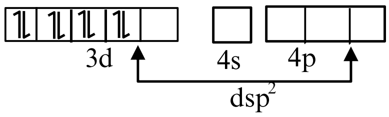
(٤) A-III, B-I, C-IV, D-II

Ans. (٤)

Sol. (A) $K_2[\overset{+2}{Ni}(CN)_4]$

Ni^{+2} : [Ar] 3d ٤s (CN is S.F.L)

Pre hybridization state of Ni

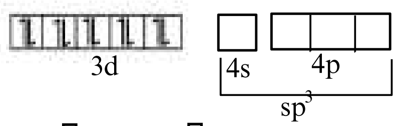


(B) [Ni(CO)٤]

Ni : [Ar] ٣d٤s

CO is S.F.L, so pairing occur

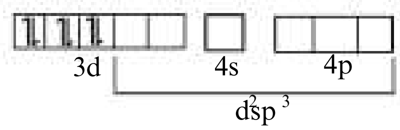
Pre hybridization state of Ni



(C) [Co(NH٣)٦]Cl٣

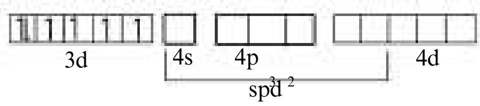
Co٣ : [Ar] ٣d٦ ٤s٠

With Co^{3+}, NH3 act as S.F.L



(d) Na٣[CoF٦]

Co٣ : [Ar] ٣d (F⁻ W.F.L)



٦٨. The coordination environment of Ca ion in its complex with EDTA is :

(١) tetrahedral

(٢) octahedral

(٣) square planar

(٤) trigonal prismatic

Ans. (٢)

Sol. EDTA Hexadentate ligand

[Ca(EDTA)]

So Coordination environment is octahedral

٦٩. The incorrect statement about Glucose is :

(١) Glucose is soluble in water because of having aldehyde functional group

(٢) Glucose remains in multiple isomeric form in its aqueous solution

(٣) Glucose is an aldohexose

(٤) Glucose is one of the monomer unit in sucrose

Ans. (١)

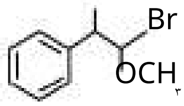
Sol. Glucose is soluble in water due to presence of alcohol functional group and extensive hydrogen bonding.

Glucose exist is open chain as well as cyclic forms in its aqueous solution.

Glucose having ٦C atoms so it is hexose and having aldehyde functional group so it is aldose.

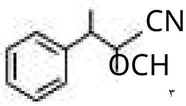
Thus, aldohexose.

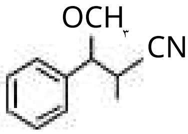
Glucose is monomer unit in sucrose with fructose.

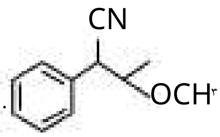
٧٠. 

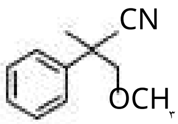
KCN (alc) → Major Product 'P'

In the above reaction product 'P' is

(١) 

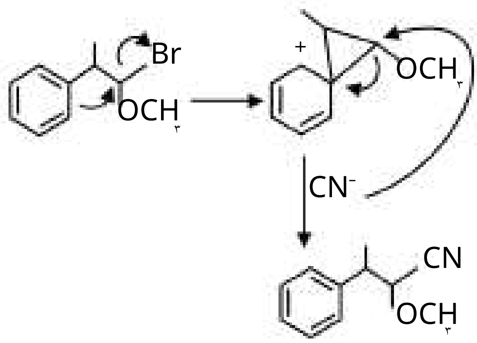
(٢) 

(٣) Ans. 

(٤) 

(١)

Sol.



Due to NGP effect of phenyl ring Nucleophilic substitution of Br will occurs.

٧١. Which of the following compound can give positive iodoform test when treated with aqueous KOH solution followed by potassium hypoiodite.

(١) $CH_3CH_2-\overset{O}{\overset{\|}{C}}-CH_2CH_3$

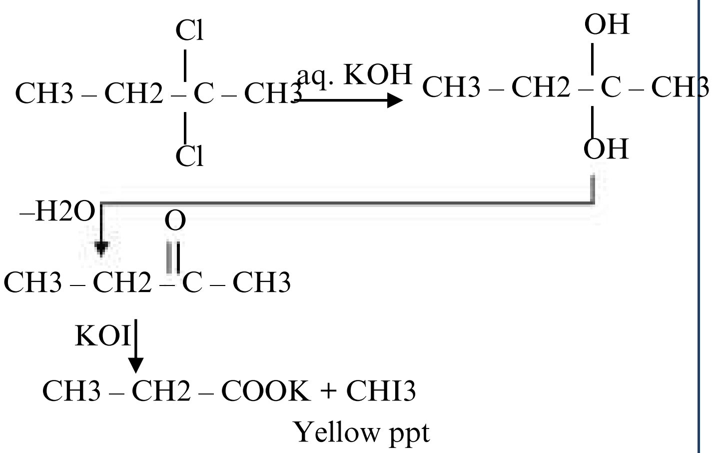
(٢) $CH_3CH_2-\underset{Cl}{\overset{Cl}{C}}-CH_3$

(٣) $CH_3CH_2CH_2CHO$

(٤) $CH_3CH_2-CH-CH_2$ (epoxide, O bridging)

Ans. (٢)

Sol.



٧٢. For a sparingly soluble salt AB_2, the equilibrium concentrations of A^{2+} ions and B^- ions are 1.2×10^{-4} M and 0.24×10^{-3} M, respectively. The solubility product of AB_2 is :

(١) 0.069×10^{-12}

(٢) 6.91×10^{-12}

(٣) 27.6×10^{-12}

(٤) 27.65×10^{-12}

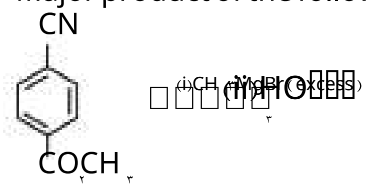
Ans. (٢)

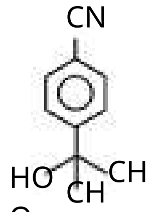
Sol. $AB_{2(s)} \rightleftharpoons A^{2+}_{(aq)} + 2B^-_{(aq)}$

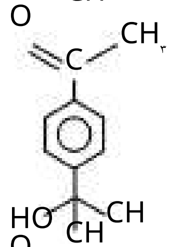
$K_{sp} = [A^{2+}][B^-]^2 = (1.2 \times 10^{-4})(0.24 \times 10^{-3})^2$

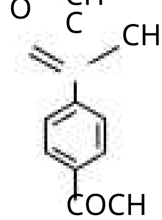
$= 6.91 \times 10^{-12} M^3$

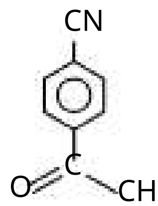
٧٣. Major product of the following reaction is



(١)


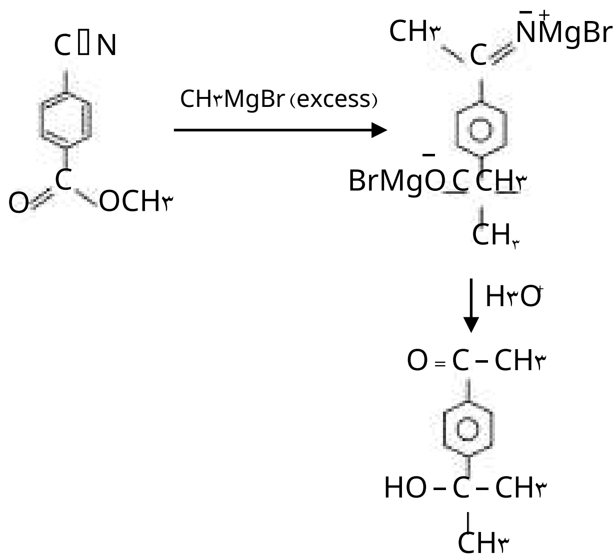
(٢)


(٣)


(٤)


Ans. (٢)

Sol.



٧٤. Given below are two statements :

Statement I : The higher oxidation states are more stable down the group among transition elements unlike p-block elements.

Statement II : Copper can not liberate hydrogen from weak acids.

In the light of the above statements, choose the correct answer from the options given below :

(١) Both Statement I and Statement II are false

(٢) Statement I is false but Statement II is true

(٣) Both Statement I and Statement II are true

(٤) Statement I is true but Statement II is false

Ans. (٣)

Sol. On moving down the group in transition elements, stability of higher oxidation state increases, due to increase in effective nuclear charge.

$E^o_{Cu^{2+}/Cu}$ = ٠.٣٤ V

$E^o_{H^+/H_2}$ = ٠

SRP : Cu^{2+} > H^+

Cu can't liberate hydrogen gas from weak acid.

٧٥. The incorrect statement regarding ethyne is

(١) The C–C bonds in ethyne is shorter than that in ethene

(٢) Both carbons are sp hybridised

(٣) Ethyne is linear

(٤) The carbon-carbon bonds in ethyne is weaker than that in ethene

Ans. (٤)

Sol. The carbon-carbon bonds in ethyne is stronger than that in ethene.

(H–C≡C–H) Ethyne is linear and carbon atoms are SP hybridised.

٧٦. Match List I with List II

List-I (Element)		List-II (Electronic Configuration)	
A.	N	I.	[Ar] ٣d¹⁰ ٤s² ٤p⁵
B.	S	II.	[Ne] ٣s² ٣p⁴
C.	Br	III.	[He] ٢s² ٢p³
D.	Kr	IV.	[Ar] ٣d¹⁰ ٤s² ٤p⁶

Choose the correct answer from the options given below :

(١) A-IV, B-III, C-II, D-I

(٢) A-III, B-II, C-I, D-IV

(٣) A-I, B-IV, C-III, D-II

(٤) A-II, B-I, C-IV, D-III

Ans. (٢)

Sol. (A) $_{٧}N$: [He] ٢s² ٢p³

(B) $_{١٦}S$: [Ne] ٣s² ٣p⁴

(C) $_{٣٥}Br$: [Ar] ٣d¹⁰ ٤s² ٤p⁵

(D) $_{٣٦}Kr$: [Ar] ٣d¹⁰ ٤s² ٤p⁶

٧٧. Match List I with List II

List-I		List-II	
A.	Melting point [K]	I.	Tl > In > Ga > Al > B
B.	Ionic Radius [M³⁺/pm]	II.	B > Tl > Al ≈ Ga > In
C.	ΔiH١ [kJ mol⁻¹]	III.	Tl > In > Al > Ga > B
D	Atomic Radius [pm]	IV.	B > Al > Tl > In > Ga

Choose the correct answer from the options given below :

(١) A-III, B-IV, C-I, D-II

(٢) A-II, B-III, C-IV, D-I

(٣) A-IV, B-I, C-II, D-III

(٤) A-I, B-II, C-III, D-IV

Ans. (٣)

Sol. Melting point : B > Al > Tl > In > Ga

Ionic radius (M^{3+}/pm) : Tl > In > Ga > Al > B

$(\Delta H_{IE})_1$ (kJ mol⁻¹) : B > Tl > Al > Ga > In

Atomic radius (in pm) : Tl > In > Al > Ga > B

٧٨. Which of the following compounds will give silver mirror with ammoniacal silver nitrate?

(A) Formic acid

(B) Formaldehyde

(C) Benzaldehyde

(D) Acetone

Choose the correct answer from the options given below :

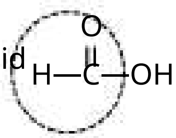
(١) C and D only

(٢) A, B and C only

(٣) A only

(٤) B and C only

Ans. (٢)

Sol. Apart from aldehyde, Formic acid 

also gives silver mirror test with ammonical silver nitrate.

٧٩. Which out of the following is a correct equation to show change in molar conductivity with respect to concentration for a weak electrolyte, if the symbols carry their usual meaning :

(١) $\Lambda_m^2 C - K_a \Lambda_m^{\circ 2} + K_a \Lambda_m \Lambda_m^{\circ} = 0$

(٢) $\Lambda_m = \Lambda_m^{\circ} - A C^{\frac{1}{2}} = 0$

(3) $\Lambda_m - \Lambda_m^{\circ} + A C^{\frac{1}{2}} = 0$

(٤) $\Lambda_m^2 C + K_a \Lambda_m^{\circ 2} - K_a \Lambda_m \Lambda_m^{\circ} = 0$

Ans. (١)

Sol. $HA(aq) \rightleftharpoons H^+(aq) + A^-(aq)$

$K_a = \frac{\alpha^2 C}{1-\alpha}$

$\alpha^2 C + K_a \alpha - K_a = 0$

$\left(\frac{\Lambda_m}{\Lambda_m^{\circ}}\right)^2 C + K_a \frac{\Lambda_m}{\Lambda_m^{\circ}} - K_a = 0$

$\Lambda_m^2 C + K_a \Lambda_m \Lambda_m^{\circ} - K_a \Lambda_m^{\circ 2} = 0$

٨٠. The electronic configuration of Einsteinium is :

(Given atomic number of Einsteinium = ٩٩)

(١) [Rn] $5f^{13} 6d^0 7s^2$ (٢) [Rn] $5f^{11} 6d^0 7s^2$

(٣) [Rn] $5f^{12} 6d^0 7s^2$ (٤) [Rn] $5f^{10} 6d^0 7s^2$

Ans. (٢)

Sol. Einsteinium (atomic No = ٩٩) : [Rn] $5f^{11} 6d^0 7s^2$

SECTION-B

٨١. Number of oxygen atoms present in chemical formula of fuming sulphuric acid is __________.

Ans. (٧)

Sol. Fuming sulphuric acid is a mixture of

conc. $H_2SO_4 + SO_3$ Or $H_2S_2O_7$

So, Number of Oxygen atoms = ٧

٨٢. A transition metal 'M' among Sc, Ti, V, Cr, Mn and Fe has the highest second ionisation enthalpy. The spin only magnetic moment value of M^+ ion is ________ BM (Near integer)

(Given atomic number Sc : ٢١, Ti : ٢٢, V : ٢٣, Cr : ٢٤, Mn : ٢٥, Fe : ٢٦)

Ans. (٦)

Sol. Among given metals, Cr has maximum IE_2 because Second electron is removed from stable configuration $3d^5$

Cr^+ : [Ar] $3d^5 4s^0$

∴ No of unpaired e^- in Cr^+ is ٥, n = ٥

So, Magnetic moment $= \sqrt{n(n+2)}$ B.M

$= \sqrt{5(5+2)} =$ ٥.٩٢ BM ≈ ٦

٨٣. The vapour pressure of pure benzene and methyl benzene at ٢٧°C is given as ٨٠ Torr and ٢٤ Torr، respectively. The mole fraction of methyl benzene in vapour phase، in equilibrium with an equimolar mixture of those two liquids (ideal solution) at the same temperature is _____ (nearest integer)

Ans. (٢٣)

Sol. Xmethylbenzene = ٠.٥

$$Y_{methylbenzene} = \frac{P_{methylbenzene}}{P_{total}}$$

$$Y_{methylbenzene} = \frac{٠.٥ \times ٢٤}{٠.٥ \times ٨٠ + ٠.٥ \times ٢٤}$$

$$= \frac{١٢}{٤٠ + ١٢} = ٠.٢٣ = ٢٣ \times ١٠^{-٢}$$

٨٤. Consider the following test for a group-IV cation.

$M^{2+} + H_2S \rightarrow$ A (Black precipitate) + byproduct

A + aqua regia → B + NOCl + S + H2O

$B + KNO_2 + CH_3COOH \rightarrow C$ + byproduct

The spin only magnetic moment value of the metal complex C is _________BM.

(Nearest integer)

Ans. (٠)

Sol. $Co^{2+} + H_2S \rightarrow CoS \downarrow$ (Black)

(A)

CoS + Aqua-regia → $Co^{2+}(aq) + NOCl + S + H_2O$

(A) (B)

$Co^{2+}(aq) + KNO_2 + CH_3COOH$

→

$K_3[Co(NO_2)_6] + NO + S + H_2O$

In $K_3[Co(NO_2)_6]$ ⇒ Co^{3+} : $3d^6$ 4s

Co^{3+} : d^2sp^3 Hybridisation

Number of unpaired e^- = ٠

Magnetic moment = $\sqrt{n(n+2)}$ = ٠ B.M

٨٥. Consider the following first order gas phase reaction at constant temperature $A(g) \rightarrow 2B(g) + C(g)$

If the total pressure of the gases is found to be ٢٠٠ torr after ٢٣ sec. and ٣٠٠ torr upon the complete decomposition of A after a very long time، then the rate constant of the given reaction is _____ $\times 10^{-2}\ s^{-1}$ (nearest integer)

[Given : log10(2) = 0.301]

Ans. (٣)

Sol. $A(g) \rightarrow 2B(g) + C(g)$

$P_{23} = P_0 + 2x = ٢٠٠$

$P_\infty = 3P_0 = ٣٠٠$

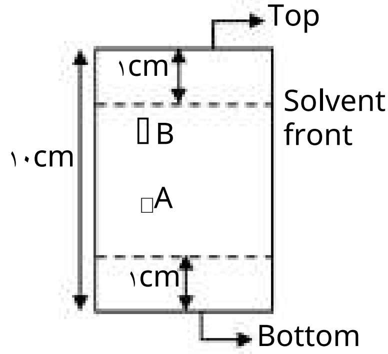
P_0 = ١٠٠

$$K = \frac{1}{t} \ln \frac{P_\infty - P_0}{P_\infty - P_t}$$

$$K = \frac{٢.٣}{٢٣} \log \frac{٣٠٠ - ١٠٠}{٣٠٠ - ٢٠٠}$$

$$= \frac{٢.٣ \times ٠.٣٠١}{٢٣} = ٠.٠٣٠١ = ٣.٠١ \times ١٠^{-٢}\ sec^{-1}$$

٨٦.



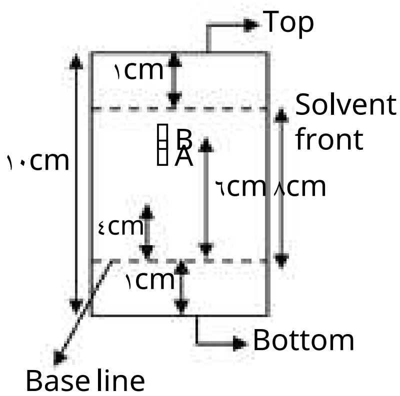
In the given TLC، the distance of spot A & B are ٥ cm & ٧ cm، from the bottom of TLC plate، respectively.

Rf value of B is x × 10^{-1} times more than A. The value of x is_____.

Ans. (١٥)

Sol.

$R_f = \frac{\text{Distance moved by substance from base line}}{\text{Distance moved by solvent from base line}}$



$(R_f)_A = \frac{٤}{٨}$ $\quad (R_f)_B = \frac{٦}{٨}$

$\frac{(R_f)_B}{(R_f)_A} = \frac{٦}{٨} \times \frac{٨}{٤}$

(Rf)B = ١.٥ (Rf)A

x = ١٥

٨٧. Based on Heisenberg's uncertainty principle، the uncertainty in the velocity of the electron to be found within an atomic nucleus of diameter $١٠^{-١٥}$ m is ______ $\times ١٠^{٩}$ $ms^{-١}$ (nearest integer)

[Given : mass of electron = ٩.١×١٠^{-٣١} kg،

Plank's constant (h) = ٦.٦٢٦×١٠^{-٣٤} Js]

(Value of π = ٣.١٤)

Ans. (٥٨)

Sol. $m\Delta V.\Delta x \geq \frac{h}{4\pi}$

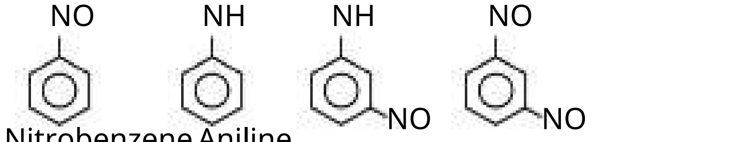
$\Delta V \geq \frac{٦.٦٢٦ \times ١٠^{-٣٤}}{٩.١ \times ١٠^{-٣١} \times ١٠^{-١٥} \times ٤ \times ٣.١٤}$

$\geq ٥٧.٩٧ \times ١٠^{٩}$ m/sec

٨٨. Number of compounds from the following which cannot undergo Friedel-Crafts reactions is : ______
toluene، nitrobenzene، xylene، cumene، aniline، chlorobenzene، m-nitroaniline، m-dinitrobenzene

Ans. (٤)

Sol. Compounds which can not undergo Friedel Crafts reaction are



Nitrobenzene Aniline m-nitroaniline m-dinitrobenzene

٨٩. Total number of electron present in (π^*) molecular orbitals of O_2، O_2^+ and O_2^- is ______.

Ans. (٦)

Sol. O_2(١٦e) → $\sigma 1s^2 \sigma^* 1s^2 \sigma 2s^2 \sigma^* 2s^2 \sigma 2p_z^2 \pi 2p_x^2 = \pi 2p_y^2 \pi^* 2p_x^1 = \pi^* 2p_y^1$

Number of e^- present in π^* of O_2 = ٢

Number of e^- present in π^* of O_2^+ = ١

Number of e^- present in π^* of O_2^- = ٣

So total e^- in π^* = ٢ + ١ + ٣ = ٦

٩٠. When ΔH_{vap} = ٣٠ kJ/mol and ΔS_{vap} = ٧٥ J $mol^{-1}K^{-1}$ then the temperature of vapour، at one atmosphere is ______ K.

Ans. (٤٠٠)

Sol. At equilibrium $\Delta G_{P,T}$ = ٠

$\Delta H_{vap} = T\Delta S_{vap}$

٣٠ × ١٠٠٠ = T × ٧٥

T = ٤٠٠ K