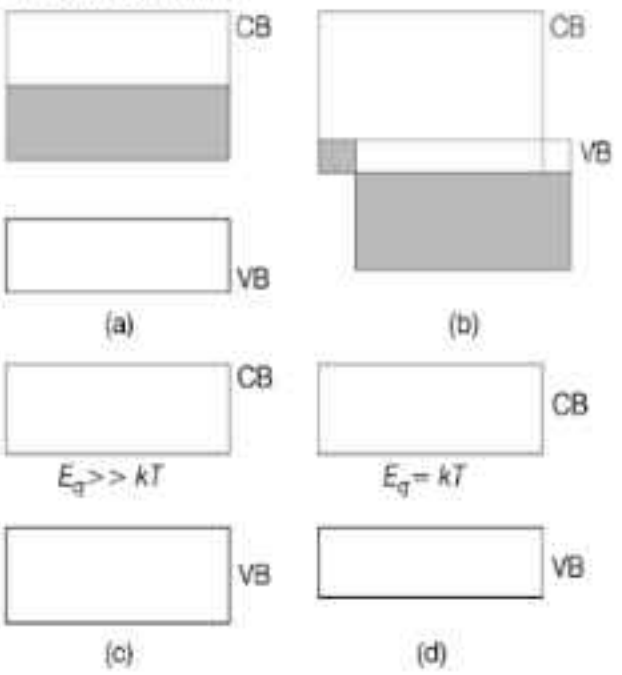
AMU

Engineering Entrance Exam

Solved Paper 2019

Physics

1. A setting sun appears to be at an altitude higher than it really is. This is because of
   (a) absorption of light (b) reflection of light
   (c) refraction of light (d) dispersion of light
2. In a Young's double slit experiment, the slit separation is 1mm and the screen is 1m from the slit. For a monochromatic light of wavelength 500 nm, the distance of 3rd minima from the central maxima is
   (a) 0.50 mm (b) 1.25 mm
   (c) 1.50 mm (d) 1.75 mm
3. Which of the energy band diagrams shown in the figure corresponds to that of a semiconductor?



4. Let i_e, i_c and i_b represent emitter current, collector current and the base current of a transistor, then
   (a) $i_c > i_e$ (b) $i_b > i_c$
   (c) $i_b > i_c$ (d) $i_e > i_c$
5. A radioactive substance has an average life of 5 hours. In a time of 5 hours
   (a) half of the active nuclei decay
   (b) less than half of the active nuclei decay
   (c) more than half of the active nuclei decay
   (d) all active nuclei decay
6. The magnetic field amplitude of an electromagnetic wave is $2 \times 10^{-7} T$. Its electric field amplitude if the wave is travelling in free space is
   (a) 6 Vm^{-1} (b) 60 Vm^{-1}
   (c) $\frac{10}{6}$ Vm^{-1} (d) None of these
7. The atomic number and the mass number of an atom remains unchanged when it emmits
   (a) a photon (b) a neutron,
   (c) β-particle (d) an α-particle
8. A small object placed on a rotating horizontal turn table just slips when it is placed at a distance of 4 cm from the axis of rotation, if the angular velocity of the turn-table is doubled, the object slips when its distance from the axis of rotation is
   (a) 1 cm (b) 2 cm (c) 4 cm (d) 8 cm

9. When two parallel wires carry currents in the same direction,
   (a) they attract each- other
   (b) they repel each other
   (c) magnetic forces on two wires are perpendicular to each other
   (d) they do not experience any magnetic force

10. The efficiency of Carnot engine is 0.6. It rejects 20J of heat to sink. The work done by the engine is
   (a) 20 J (b) 30 J (c) 33.3 J (d) 50 J

11. The dimension of K in the equation $W = \frac{1}{2}Kx^2$ is
   (a) $[M^1L^0T^{-2}]$ (b) $[M^0L^1T^{-1}]$
   (c) $[M^1L^1T^{-2}]$ (d) $[M^1L^0T^{-1}]$

12. Two bullets are fired simultaneously, horizontally and with different speeds from the same place. Which bullet will hit the ground first ?
   (a) The faster one
   (b) Depends on their mass
   (c) The slower one
   (d) Both will reach simultaneously

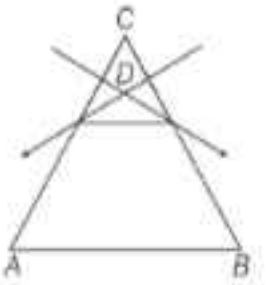
13. A plumb line is suspended from a ceiling of a car moving with' horizontal acceleration of a. What will be the angle of inclination with vertical ?
   (a) $\tan^{-1}\left(\frac{a}{g}\right)$ (b) $\tan^{-1}\left(\frac{g}{a}\right)$
   (c) $\cos^{-1}\left(\frac{a}{g}\right)$ (d) $\cos^{-1}\left(\frac{g}{a}\right)$

14. A body of mass 2 kg is kept by pressing to a vertical wall by a force of 100N. The friction between wall and body is 0.3. Then the frictional force is equal to
   (a) 6 N (b) 20 N
   (c) 600 N (d) 700 N

15. A sphere rolls down an inclined plane of inclination θ. What is the acceleration as the sphere reaches bottom ?
   (a) $\frac{5}{7}g\sin\theta$ (b) $\frac{3}{5}g\sin\theta$
   (c) $\frac{2}{7}g\sin\theta$ (d) $\frac{2}{5}g\sin\theta$

16. In a bicycle, the radius of rear wheel is twice the radius of front wheel. If r_f and r_r are the radius, v_f and v_r are the speeds of top most points of wheel, then
   (a) $v_r = 2v_f$ (b) $v_f = 2v_r$
   (c) $v_f = v_r$ (d) $v_f > v_r$

17. In the given figure, what is the angle of prism?



   (a) A (b) B (c) C (d) D

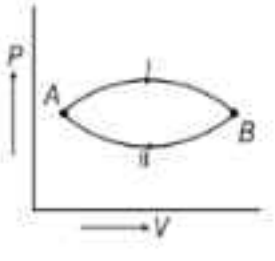
18. Moment of inertia of an object does not depend upon
   (a) mass of object (b) mass distribution
   (c) angular velocity (d) axis of rotation

19. A particle falls towards earth from infinity. Its velocity on reaching the earth would be
   (a) infinity (b) $\sqrt{2gR}$
   (c) $2\sqrt{gR}$ (d) zero

20. Universal gas constant is
   (a) C_p/C_V (b) $C_p - C_V$
   (c) $C_p + C_V$ (d) C_V/C_p

21. Two bodies of mass m and 4m have equal kinetic energy. What is the ratio of their momentum?
   (a) 1 : 4 (b) 1 : 2 (c) 1 : 1 (d) 2 : 1

22. A gas at state A changes to state B through path I and II shown in figure. The change in internal energy is ΔU_1 and ΔU_2, respectively. Then



   (a) $\Delta U_1 > \Delta U_2$ (b) $\Delta U_1 < \Delta U_2$
   (c) $\Delta U_1 = \Delta U_2$ (d) $\Delta U_1 = \Delta U_2 = 0$

23. According to Kepler's law, the time period of a satellite varies with its radius as
(a) $T^2 \propto R^3$ (b) $T^3 \propto R^2$
(c) $R^2 \propto \left(\frac{1}{R^3}\right)$ (d) $T^3 \propto \left(\frac{1}{R^2}\right)$

24. A particle is moving in a circle with uniform speed v. In moving from a point to another diametrically opposite point
(a) the momentum changes by mv
(b) the momentum changes by $2mv$
(c) the kinetic energy changes by $\left(\frac{1}{2}\right) mv^2$
(d) the kinetic energy changes by mv^2

25. n small metal drops of same size are charged to V volt each. If they coalesce to form a single large drop, then its potential will be
(a) $\frac{V}{n}$ (b) Vn (c) $Vn^{\frac{1}{3}}$ (d) $Vn^{\frac{2}{3}}$

26. In an adiabatic process, the state of a gas is changed from P_1, V_1, T_1 to P_2, V_2, T_2. Which of the following relation is correct?
(a) $T_1 V_1^{\gamma-1} = T_2 V_2^{\gamma-1}$ (b) $P_1 V_1^{\gamma-1} = P_2 V_2^{\gamma-1}$
(c) $T_1 P_1^{\gamma} = T_2 P_2^{\gamma}$ (d) $T_1 V_1^{\gamma} = T_2 V_2^{\gamma}$

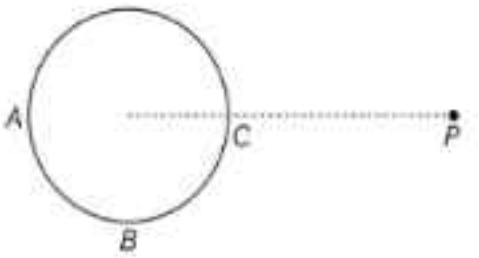
27. Minimum and maximum values of Possion's ratio for a metal lies between
(a) $-\infty$ to $+\infty$ (b) 0 to 1
(c) $-\infty$ to 1 (d) 0 to 0.5

28. A wire of diameter 1 mm breaks under a tension of 1000 N. Another wire of same material as that of the first one, but of diameter 2 mm breaks under a tension of
(a) 500 N (b) 100 N
(c) 1000 N (d) 4000 N

29. In a capillary tube experiment, a vertical, 30 cm long capillary tube is dipped in water. The water rises upto a height of 10 cm due to capillary action. If this experiment is conducted in a freely falling elevator, the length of the water column becomes
(a) 10 cm (b) 20 cm
(c) 30 cm (d) zero

30. An aeroplane gets its upward lift due to a phenomenon described by the
(a) Archimedes principle
(b) Bernoulli's principle
(c) Buoyancy principle
(d) Pascal law

31. A hollow conducting sphere is placed in an electric field produced by a point charge placed at P as shown in figure. Let V_A, V_B, V_C be the potentials at points A, B and C, respectively. Then



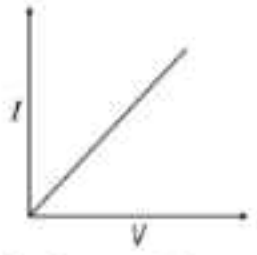
(a) $V_C > V_B$ (b) $V_B > V_C$
(c) $V_A > V_B$ (d) $V_A = V_C$

32. A parallel plate air capacitor is charged and then isolated. When a dielectric material is inserted between the plates of the capacitor, then which of the following does not change ?
(a) Electric field between the plates
(b) Potential difference across the plates
(c) Charge on the plates
(d) Energy stored in the capacitor

33. A spherical drop of mercury having a potential of 2.5 V is obtained as a result of merging 125 droplets. The potential of constituent droplets would be
(a) 1.0 V
(b) 0.5 V
(c) 0.2 V
(d) 0.1 V

34. A 10 micro farad capacitor is charged to 500 V and then its plates are joined together through a resistance of 10 ohm. The heat produced in the resistance is
(a) 500 J
(b) 250 J
(c) 125 J
(d) 1.25 J

35. *I-V* characteristic of a copper wire of length *L* and area of cross-section A is shown in figure. The slope of the curve becomes

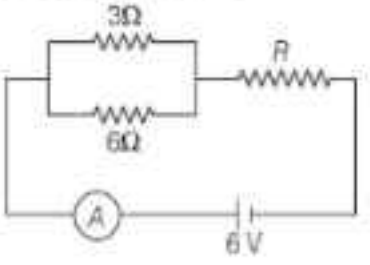


(a) more, if the experiment is performed at higher temperature
(b) more, if a wire of steel of same dimension is used
(c) more, if the length of the wire is increased
(d) less, if the length of the wire increased

36. The current in a conductor varies with time t as $I = 2t + 3t^2$, where I is in ampere and t in seconds. Electric charge flowing through a section of the conductor during $t = 2$ sec to $t = 3$ sec is

(a) 10C (b) 24C (c) 33 C (d) 44 C

37. If the ammeter in the given circuit reads 2 A, the resistance R is



(a) 1 Ω (b) 2 Ω
(c) 3 Ω (d) 4 Ω

38. Susceptibility of a diamagnetic substance is
(a) zero (b) negative
(c) less than 1 (d) greater than 1

39. A 50 ohm galvanometer gets full scale deflection when a current of 0.01 A pass through the coil. When it is converted to a 10 A ammeter, the shunt resistance is
(a) 0.01 Ω (b) 0.05 Ω
(c) 2000 Ω (d) 5000 Ω

40. A horizontal rod of length L rotates about a vertical axis with a uniform angular velocity ω. A uniform magnetic field B exists parallel to the axis of rotation. Then potential difference between the two ends of the rod is

(a) $\omega L^2 B$ (b) $\omega^2 LB$
(c) $\frac{1}{2}\omega L^2 B$ (d) $\frac{1}{2}\omega^2 LB$

41. A current of 10 A is passing through a long wire which has semicircular loop of the radius 20 cm as shown in the figure. Magnetic field produced at the centre of the loop is

(a) $10\pi\ \mu T$ (b) $5\pi\ \mu T$
(c) $4\pi\ \mu T$ (d) $2\pi\ \mu T$

42. An AC source is rated at 220 V, 50 Hz. The time taken for voltage to change from its peak value to zero is
(a) 50 sec (b) 0.02 sec
(c) 5 sec (d) 5×10^{-3} sec

43. An ideal transformer has 500 and 5000 turn in primary and secondary windings, respectively. If the primary voltage is connected to a 6 V battery, then the secondary voltage is
(a) zero (b) 60 V (c) 0.6 V (d) 6.0 V

44. Resistance in the two gaps of a meter bridge are 10 Ω and 30 Ω, respectively. If the resistances are inter changed, the balance point shifts by
(a) 33.3 cm (b) 66.67 cm
(c) 25 cm (d) 50 cm

45. When a plane electromagnetic wave enters a glass slab, then which of the following will not change ?
(a) Wavelength (b) Frequency
(c) Speed (d) Amplitude

46. A source emits a sound of frequency of 400 Hz, but the listener hears its to be 390 Hz. Then
(a) the listener is moving towards the source
(b) the source is moving towards the listener
(c) the listener is moving away from the source
(d) the listener has a defective ear

47. An open pipe resonates with a tuning fork of frequency 500 Hz. It is observed that two successive nodes are formed at distances 16 and 46 cm from the open end. The speed of sound in air in the pipe is
(a) 230 m/s
(b) 300 m/s
(c) 320 m/s
(d) 360 m/s

48. It takes 2.0 seconds for a sound wave to travel between two fixed points when the day temperature is 10°C. If the temperature rises to 30°C, the sound wave travels between the same fixed parts in
(a) 1.9 sec (b) 2.0 sec
(c) 2.1 sec (d) 2.2 sec

49. Two plane mirrors are inclined to each other at an angle of 60°. A point object is placed in between them. The total number of images produced by both the mirrors is
(a) 2 (b) 4 (c) 5 (d) 6

50. For a real object, which of the following can produce a real image?
(a) Plane mirror (b) Concave lens
(c) Convex mirror (d) Concave mirror

Chemistry

51. The standard e.m.f. for the cell reaction,
$2Cu^{+}(aq) \longrightarrow Cu(s) + Cu^{2+}(aq)$
is + 0.36 V at 298 K. The equilibrium constant of the reaction is
(a) 5×10^{6} (b) 1.4×10^{12}
(c) 7.4×10^{12} (d) 1.2×10^{6}

52. The standard e.m.f. of the cell,
$Cd(s) \mid CdCl_2(aq)\ (0.1M) \mid\mid AgCl(s) \mid Ag(s)$
in which the cell reaction is
$Cd(s) + 2AgCl(s) \longrightarrow 2Ag(s) + Cd^{2+}(aq) + 2Cl^{-}(aq)$
is 0.6915 *V* at 0°C and 0.6753 V at 25°C. The enthalpy change of the reaction at 25°C is
(a) – 176 kJ (b) – 234.7 kJ
(c) + 123.5 kJ (d) – 167.26 kJ

53. Which of the following statement is true?
(a) The relative lowering of vapour pressure of a solution is equal to the mole fraction of the solute present in the solution
(b) Passage of solute molecules towards solution side through semipermeable membrane is osmosis.
(c) The boiling point of a solution is always lower than the solvent
(d) The boiling point of a liquid is the temperature at which its vapour pressure becomes equal to 260 mm

54. Which of the following statement is not true?
(a) The pressure of a gas is due to collision of the gas molecules with the walls of the container
(b) The molecular velocity of any gas is proportional to the square root of the absolute temperature
(c) The rate of diffusion of a gas is directly proportional to the density of the gas at constant pressure
(d) Kinetic energy of an ideal gas is directly proportional to the absolute temperature

55. The unit of second order reaction rate constant is;
(a) $L^{-1} \cdot mol \cdot s^{-1}$
(b) $L^{2} \cdot mol^{-2} \cdot s^{-1}$
(c) $L \cdot mol^{-1} \cdot s^{-1}$
(d) s^{-1}

56. Hess law states that,
(a) the standard enthalpy of an overall reaction is the sum of the enthalpy changes in individual reactions
(b) enthalpy of formation of a compound is same as the enthalpy of decomposition of the compound into constituent elements, but with opposite sign
(c) at constant temperature the pressure of a gas is inversely proportional to its volume
(d) the mass of a gas dissolved per litre of a solvent is proportional to the pressure of the gas in equilibrium with the solution

57. One gram of (*A*) decays by β-emission to 0.125 g in 200 years. The half-life period of the reaction is
(a) 0.014 years (b) 6.66 years
(c) 66.6 years (d) 666 years

58. Isotopes are
(a) atoms of different elements having same mass number
(b) atoms of same elements having same mass number
(c) atoms of same elements having different mass number
(d) atoms of different elements having same number of neutrons

59. Acid hydrolysis of sucrose is a
(a) pseudo first order reaction
(b) zero order reaction
(c) second order reaction
(d) unimolecular reaction

60. The product obtained after position emission from $_{31}Ga^{68}$ is
(a) $_{30}Ga^{68}$ (b) $_{30}Zn^{68}$ (c) $_{31}Zn^{69}$ (d) $_{31}Ga^{69}$

61. An aqueous solution in which the H^+ ion concentration is greater than 10^{-7} M is said to be
(a) acidic (b) alkaline
(c) neutral (d) None of these

62. In the hydrolysis of a salt of weak acid and weak base, the hydrolysis constant K_h is equal to
(a) $\frac{K_w}{K_b}$ (b) $\frac{K_w}{K_a}$
(c) $\frac{K_w}{K_a \cdot K_b}$ (d) $K_a \cdot K_b$

63. In the nuclear reaction
$_{13}Al^{27} + {}_2He^4 \longrightarrow {}_{14}X^{30} + {}_1H^1$, X is
(a) Si (b) Al
(c) Mg (d) P

64. What kind of a molecule $AlCl_3$ is?
(a) Bronsted acid (b) Lewis acid
(c) Lewis base (d) Bronsted base

65. How much $K_2Cr_2O_7$ (molecular weight = 294.19) is required to prepare one litre of 0.1 N solution ?
(a) 9.8036 g (b) 7.3548 g
(c) 3.6774 g (d) 4.903 g

66. A gas can expand from 100 mL to 250 mL under a constant pressure of 2 atm. The work done by gas is
(a) 30.38 joule (b) 25 joule
(c) 5 k joule (d) 16 joule

67. It the r.m.s. speed of a gaseous molecule is x m/sec at a pressure p atm, then what will be the r.m.s. speed at a pressure $2p$ atm and constant temperature?
(a) x (b) $2x$ (c) $4x$ (d) $x/4$

68. Ionic mobility of Ag^+ is
$(\lambda_{Ag^+} = 5 \times 10^{-1} \text{ohm}^{-1}\text{cm}^2\ \text{eq}^{-1})$
(a) 5.2×10^{-9} (b) 2.4×10^{-9}
(c) 1.52×10^{-9} (d) 8.25×10^{-9}

69. Which among the following is the strongest acid?
(a) HF (b) HCl (c) HBr (d) HI

70. What is the general outer electronic configuration of the coinage metals?
(a) $ns^2\ np^6$ (b) $(n-1)d^{10}\ ns^1$
(c) $(n-1)d^{10}\ ns^2$ (d) $(n-1)d^9\ ns^2$

71. How does the ionisation energy of 1st group elements vary?
(a) Increases down the group
(b) Decreases down the group
(c) Remains unchanged
(d) Variation is not regular

72. What is the oxidation number of chlorine in ClO_3^-?
(a) + 5 (b) + 3
(c) + 4 (d) + 2

73. What type of hybridisation takes place in the N atom of NH_3?
(a) sp^2 (b) sp^3
(c) dsp^2 (d) sp

74. What is the co-ordination number of Cl^- in NaCl crystal?
(a) 8 (b) 6 (c) 4 (d) 3

75. How many electrons are involved in oxidation of $KMnO_4$ in basic medium?
(a) 1 (b) 2 (c) 5 (d) 3

76. The magnetic moment of $K_3[Fe(CN)_6]$ is found to be 1.7 BM. How many unpaired electron(s) is/are present per molecule ?
(a) 1 (b) 2
(c) 3 (d) 4

77. Which among the following is an electron deficient compound ?
(a) NF_3 (b) PF_3
(c) BF_3 (d) AsF_3

78. Arrange the hydra-acids of halogens in increasing order of acidity.
(a) HF < HCl < HBr < HI
(b) HI < HBr < HCl < HF
(c) HF < HBr < HI < HCl
(d) HF < HI < HBr < HCl

79. What is the product of the reaction of H_2O_2 with Cl_2?
(a) O_2 + HOCl (b) HCl + O_2
(c) H_2O + HCl (d) HCl + H_2

80. Which of the following organo-silicon compound on hydrolysis will given a three dimensional silicon?
(a) R_3SiCl (b) $RSiCl_3$
(c) $SiCl_4$ (d) R_2SiCl_2

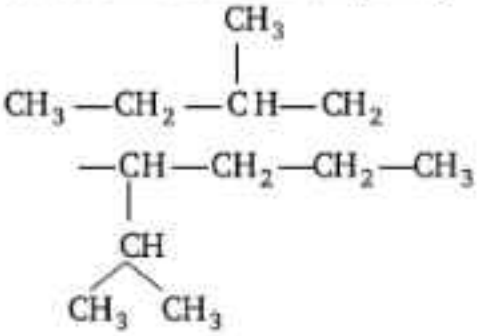
81. How can you synthesise nitric oxide in the laboratory?
(a) Zinc with cold and dilute HNO_3
(b) Zinc with concentrated HNO_3
(c) Copper with cold and dilute HNO_3
(d) Heating NH_4NO_3

82. Which colourless gas evolves when NH_4Cl reacts with zinc in a dry cell battery ?
(a) NH_3 (b) N_2 (c) H_2 (d) Cl_2

83. What is the nature of the bond between B and O in $(C_2H_5)_2OBH_3$?
(a) Covalent
(b) Co-ordinate covalent
(c) Ionic bond
(d) Banana shaped bond

84. An alkene gives two moles of HCHO, one mole of CO_2 and one mole of CH_3COCHO on ozonolysis. What is its structure ?
(a) $CH_2=C=CH-CH_2-CH_3$
(b) $CH_2=CH-\underset{\displaystyle CH_3}{\underset{|}{C}}H-CH=CH_2$
(c) $CH_2=C=\underset{\displaystyle CH_3}{\underset{|}{C}}-CH_3$
(d) $CH_2=CH=\underset{\displaystyle CH_3}{\underset{|}{C}}-CH=CH_2$

85. IUPAC name of the compound,



(a) 4-isopropyl, 6-methyl octane
(b) 3-methyl, 5-(1-methyl ethyl) octane
(c) 3-methyl, 5-isopropyl octane
(d) 6-methyl, 4-(1-methyl ethyl) octane

86. The order of melting point of *ortho*, *para*, *meta*-nitrophenol is
(a) $o > m > p$ (b) $p > m > o$
(c) $m > p > o$ (d) $p > o > m$

87. When $CHCl_3$ is boiled with NaOH, it gives
(a) formic acid
(b) trihydroxy methane
(c) acetylene
(d) sodium formate

88. Which of the following is an example of ketohexose?
(a) Mannose (b) Galactose
(c) Maltose (d) Fructose

89. When aniline is treated with sodium nitrite and hydrochloric acid at 0°C, it gives
(a) phenol and N_2
(b) diazonium salt
(c) hydrazo compound
(d) no reaction takes place

90. The key step in cannizzaro's reaction is the intermolecular shift of
(a) proton (b) hydrogen bond
(c) hydronium ion (d) hydrogen band

91. Aldehydes and ketones can be reduced to hydrocarbon by using
(a) $LiAlH_4$
(b) H_2 / pd - $BaSO_4$
(c) Na - Hg / HCl
(d) $NH_2—NH_2$ / C_2H_5ONa

92. Cinnamic acid is formed when $C_6H_5—CHO$ condenses with $(CH_3CO)_2O$ in presence of
(a) concentrated H_2SO_4
(b) sodium acetate
(c) sodium metal
(d) anhydrous $ZnCl_2$

93. What is the product of the reaction of phenol with $CHCl_3$ in aqueous NaOH and subsequent acid hydrolysis?
(a) Salicylic acid (b) Salicylaldehyde
(c) Benzoic acid (d) Benzaldehyde

94. On treatment with chlorine in presence of sunlight, toluene gives the product
(a) *o*-chloro toluene
(b) 2, 5-dichloro toluene
(c) *p*-chloro toluene
(d) benzyl chloride

95. Which of the following cycloalkane gives open chain compound, when reacts with bromine?
(a) Cyclopropane (b) Cyclopentane
(c) Cyclohexane (d) Cyclooctane

96. If the dipole moment of toluene and nitro-benzene are 0.43 D and 3.93 D, then what is the expected dipole moment of *p*-nitro toluene?
(a) 3.50 D (b) 2.18 D
(c) 4.36 D (d) 5.30 D

97. What is the product when 2-butyne is treated with liquid NH_3 in presence of lithium?
(a) *n*-butane (b) *cis*-2-butene
(c) *trans*-2-butene (d) 1-butene

98. In the dichlorination reaction of propane, mixture of products are obtained. How many isomers the mixture contains?
(a) 2 (b) 3
(c) 4 (d) 5

99. Cyclopentadienyl anion is
(a) aromatic (b) non-aromatic
(c) non-planar (d) aliphatic

100. The most common type of reaction in aromatic compounds is
(a) elimination reaction
(b) addition reaction
(c) electrophilic substitution reaction
(d) rearrangement reaction

Mathematics

101. The equation of the plane through the intersection of the planes $x + y + z = 1$ and $2x + 3y - z + 4 = 0$ and parallel to X-axis is
(a) $y - 3z + 6 = 0$
(b) $3y - z + 6 = 0$
(c) $y + 3z + 6 = 0$
(d) $3y - 2z + 6 = 0$

102. If the point (a, a) falls between the lines $|x + y| = 4$, then
(a) $|a| = 2$
(b) $|a| = 3$
(c) $|a| < 2$
(d) $|a| < 3$

103. If A, B, C, D are the points $(2, 3, -1)$, $(3, 5, -3)$, $(1, 2, 3)$, $(3, 5, 7)$ respectively, then the angle between AB and CD is

(a) $\frac{\pi}{2}$ (b) $\frac{\pi}{3}$

(c) $\frac{\pi}{4}$ (d) $\frac{\pi}{6}$

104. A five digits number is formed by writing the digits 1, 2, 3, 4, 5 in a random order without repetitions. Then the probability that the number is divisible by 4, is

(a) 3 /5 (b) 18/5

(c) 1/5 (d) 6/5

105. Two persons A and B take turns in throwing a pair of dice. The first person to throw 9 from both dice will be awarded the price. If A throws first, then the probability that B wins

(a) 9/17 (b) 8/17

(c) 8/9 (d) 1/9

106. The probability that in a year of the 22nd century chosen at random, then there will be 53 Sundays, is

(a) 3/28 (b) 2/28

(c) 7/28 (d) 5/28

107. The standard deviation of a variable x is 10. Then the standard deviation of $50 + 5x$ is

(a) 50 (b) 550

(c) 10 (d) 500

108. A real value of x will satisfy the equation $\left(\frac{3-4ix}{3+4ix}\right) = \alpha - i\beta$ (α, β are real), if

(a) $\alpha^2 + \beta^2 = -1$ (b) $\alpha^2 - \beta^2 = 1$

(c) $\alpha^2 + \beta^2 = 1$ (d) $\alpha^2 - \beta^2 = 1$

109. If ω is a complex cube root of unity, then the value of

$$\frac{p + q\omega + r\omega^2}{r + p\omega + q\omega^2} + \frac{p + q\omega + r\omega^2}{q + r\omega + p\omega^2}$$

($p, q, r \in R$) is equal to

(a) 0 (b) 1

(c) –1 (d) 2

110. If P, Q, R, S are represented by the complex numbers $4 + i$, $1 + 6i$, $-4 + 3i$, $-1 - 2i$ respectively, then $PQRS$ is a

(a) rectangle (b) square

(c) rhombus (d) parallelogram

111. If n is a positive integer, then $(1 + i)^n + (1 - i)^n$ is equal to

(a) $(\sqrt{2})^{n-2} \cos\left(\frac{n\pi}{4}\right)$

(b) $(\sqrt{2})^{n-2} \sin\left(\frac{n\pi}{4}\right)$

(c) $(\sqrt{2})^{n+2} \cos\left(\frac{n\pi}{4}\right)$

(d) $(\sqrt{2})^{n+2} \sin\left(\frac{n\pi}{4}\right)$

112. The number of ways in which 9 persons can be divided into three equal groups is

(a) 1680 (b) 840

(c) 560 (d) 280

113. A dictionary is printed consisting of 7 letters words only that can be made with a letters of word CRICKET. If the words are printed at the alphabetical order as in an ordinary dictionary, then the number of words before the word CRICKET is

(a) 530 (b) 480 (c) 531 (d) 481

114. If the sum of the coefficient in the expansion of $(x + y)^n$ is 1024, then the value of the greatest coefficient in the expansion is

(a) 356 (b) 252 (c) 210 (d) 120

115. An investigator interviewed 100 students to determine the performance of three drinks milk, coffee and tea. The investigator reported that 10 students take all three drinks milk, coffee and tea; 20 students take milk and coffee, 30 students take coffee and tea, 25 students take milk and tea, 12 students take milk only, 5 students take coffee only and 8 students take tea only. Then the number of students who did not take any of the three drinks is

(a) 10 (b) 20 (c) 25 (d) 30

116. Let $Y=\{1,2,3,4,5\}$, $A=\{1,2\}$, $B=\{3,4,5\}$ and ϕ denotes null set. If $(A\times B)$ denotes cartesian product of the sets A and B, then $(Y\times A)\cap(Y\times B)$ is
(a) Y (b) A (c) B (d) ϕ

117. Let $A=\{2,3,4,5,\ldots..,16,17,18\}$. Let $\approx$ be the equivalence relation on $A\times A$ cartesian product of A and A, defined by $(a,b)\approx(c,d)$ if $ad=bc$, then the number of ordered pairs of the equivalence class of $(3,2)$ is
(a) 4 (b) 5 (c) 6 (d) 7

118. It a, b be two fixed positive integers such that $f(a+x)=b+[b^3+1-3b^2f(x)+3b\{f(x)-\{f(x)\}^3\}]^{1/3}$ for all real x, then $f(x)$ is a periodic function with period
(a) a (b) $2a$ (c) b (d) $2b$

119. The domain of the function $f(x)=\log_{3+x}(x^2-1)$ is
(a) $(-3,-1)\cup(1,\infty)$
(b) $[-3,-1]\cup[1,\infty)$
(c) $(-3,-2)\cup(-2,-1)\cup(1,\infty)$
(d) $[-3,-2)\cup(-2,-1)\cup(1,\infty)$

120. The value of $\cot 70^\circ+4\cos 70^\circ$ is
(a) $1/\sqrt{3}$ (b) $\sqrt{3}$
(c) $2\sqrt{3}$ (d) $1/2$

121. The equation $\sin x+\sin y+\sin z=-3$ for $0\le x\le 2\pi$, $0\le y\le 2\pi$, $0\le z\le 2\pi$ has
(a) one solution
(b) two sets of solution
(c) four sets of solution
(d) no solution

122. If $\theta=\sin^{-1}x+\cos^{-1}x-\tan^{-1}x$, $x\ge 0$ then the smallest interval in which θ lies is
(a) $\frac{\pi}{2}\le\theta\le\frac{3\pi}{4}$ (b) $0\le\theta\le\frac{\pi}{4}$
(c) $-\frac{\pi}{4}\le\theta\le 0$ (d) $\frac{\pi}{4}\le\theta\le\frac{\pi}{2}$

123. Let A, B and C are the angles of a plain triangle and $\tan\left(\frac{A}{2}\right)=\frac{1}{3}$, $\tan\left(\frac{B}{2}\right)=\frac{2}{3}$. Then, $\tan\left(\frac{C}{2}\right)$ is equal to
(a) 7/9 (b) 2/9 (c) 1/3 (d) 2/3

124. If α,β $(\alpha\ne\beta)$ satisfies the equation $a\cos\theta+b\sin\theta=c$, then the value of $\tan\left(\frac{\alpha+\beta}{2}\right)$ is
(a) b/a (b) c/a (c) a/b (d) c/b

125. A ray of light passing through the point (1, 2) is reflected on the x-axis at a point P and passes through the point (5, 3), then the abscissa of a point P is
(a) 3 (b) 13/3
(c) 13/5 (d) 13/4

126. The equation $4x^2-24xy+11y^2=0$ represents
(a) two parallel lines
(b) two perpendicular lines
(c) two lines through the origin
(d) a circle

127. The length of the chord joining the points in which the straight line $\frac{x}{3}+\frac{y}{4}=1$ cuts the circle $x^2+y^2=\frac{169}{25}$ is
(a) 1 (b) 2 (c) 4 (d) 8

128. The normal to the parabola $y^2=8x$ at the point (2, 4) meets the parabola again at the point
(a) (−18, −12) (b) (−18, 12)
(c) (18, 12) (d) (18, −12)

129. If a bar of given length moves with its extremities on two fixed straight lines at right angles, then the locus of any point on bar marked on the bar describes a/an
(a) circle (b) parabola
(c) ellipse (d) hyperbola

130. The straight line $x+y=\sqrt{2}p$ will touch the hyperbola $4x^2-9y^2=36$ if
(a) $p^2=2$ (b) $p^2=5$
(c) $5p^2=2$ (d) $2p^2=5$

131. The function $f(x)=\frac{1-\sin x+\cos x}{1+\sin x+\cos x}$ is not defined at $x=\pi$. The value of $f(\pi)$, so that $f(x)$ is continuous at $x=\pi$, is
(a) −1/2 (b) 1/2
(c) −1 (d) 1

132. If $f(x) = \sin^2 x$ and the composite function $g(f(x)) = |\sin x|$, then the function $g(x)$ is equal to

(a) $\sqrt{x-1}$ (b) $\sqrt{x}$

(c) $\sqrt{x+1}$ (d) $-\sqrt{x}$

133. Area of the figure bounded by the curves $y = |x-1|$ and $y = 3-|x|$ is

(a) 1 sq units

(b) 2 sq units

(c) 3 sq units

(d) 4 sq units

134. Let $x = \left[\frac{a+2b}{a+b}\right]$ and $y = \frac{a}{b}$, where a and b are positive integers. If $y^2 > 2$, then

(a) $x^2 \le 2$ (b) $x^2 < 2$

(c) $x^2 > 2$ (d) $x^2 \ge 2$

135. $\int_0^1 \tan^{-1}\left(\frac{1}{x^2-x+1}\right) dx$ is

(a) $\log 2$ (b) $-\log 2$

(c) $\frac{\pi}{2} + \log 2$ (d) $\frac{\pi}{2} - \log 2$

136. The curves $x = \log(y+e)$ and $y = \log\left(\frac{1}{x}\right)$

(a) do not meet

(b) meet at one point

(c) meet at two points

(d) meet at more than two points

137. $\lim_{x\to 0} \frac{\cos(\sin x) - 1}{x^2}$ equals

(a) 1 (b) -1

(c) $1/2$ (d) $-1/2$

138. Let $\mathbf{a}, \mathbf{b}, \mathbf{c}$ be three vectors form $\mathbf{a} \times (\mathbf{b} \times \mathbf{c}) = (\mathbf{a} \times \mathbf{b}) \times \mathbf{c}$, if

(a) $\mathbf{b} \times (\mathbf{a} \times \mathbf{c}) = 0$ (b) $\mathbf{a} \times (\mathbf{b} \times \mathbf{c}) = 0$

(c) $\mathbf{c} \times \mathbf{a} = \mathbf{a} \times \mathbf{b}$ (d) $\mathbf{c} \times \mathbf{b} = \mathbf{b} \times \mathbf{a}$

139. If $\hat{\mathbf{i}}, \hat{\mathbf{j}}, \hat{\mathbf{k}}$ are unit vectors and $|\mathbf{a}| = a$, then the value of $|\hat{\mathbf{i}} \times \mathbf{a}|^2 + |\hat{\mathbf{j}} \times \mathbf{a}|^2 + |\hat{\mathbf{k}} \times \mathbf{a}|^2$ is

(a) a^2 (b) $3a^2$

(c) $2a^2$ (d) $4a^2$

140. If the area above the X-axis bounded by the curves $y = 2^{kx}$ and $x = 0$ and $x = 2$ is $\frac{3}{\log 2}$, then the value of k is

(a) $1/2$ (b) 1

(c) -1 (d) 2

141. The value of $\int_a^b \frac{x}{|x|} dx$, $a < b < 0$ is

(a) $-(|a| + |b|)$ (b) $|b| - |a|$

(c) $|a| - |b|$ (d) $|a| + |b|$

142. The value of

$$\int_{-2}^{2} \left[p \log\left(\frac{1+x}{1-x}\right) + q \log\left(\frac{1-x}{1+x}\right)^{-2} + r \right] dx$$

depends on

(a) the value of p

(b) the value of q

(c) the value of r

(d) the value of p and q

143. A curve having the condition that the slope of tangent at some point is two times the slope of the straight line joining the same point to the origin of co-ordinates, is a/an

(a) circle (b) ellipse

(c) parabola (d) hyperbola

144. If a is an arbitrary constant, then solution of the differential equation $\frac{dy}{dx} + \sqrt{\frac{1-y^2}{1-x^2}} = 0$ is

(a) $x\sqrt{1-y^2} + y\sqrt{1-x^2} = a$

(b) $y\sqrt{1-y^2} + x\sqrt{1-x^2} = a$

(c) $x\sqrt{1-y^2} - y\sqrt{1-x^2} = a$

(d) $y\sqrt{1-y^2} - x\sqrt{1-x^2} = a$

145. A particle is moving along the curve $x = at^2 + bt + c$. If $ac = b^2$, then the particle would be moving with uniform

(a) rotation

(b) velocity

(c) acceleration

(d) retardation

146. The solution of the equation $x^2 \frac{d^2y}{dx^2} = \log x$ when $x = 1$, $y = 0$ and $\frac{dy}{dx} = -1$ is

(a) $\frac{1}{2}(\log x)^2 + \log x$ (b) $\frac{1}{2}(\log x)^2 - \log x$

(c) $-\frac{1}{2}(\log x)^2 + \log x$

(d) $-\frac{1}{2}(\log x)^2 - \log x$

147. Let the unit vectors **a** and **b** be perpendicular to each other and the unit vector **c** be inclined at an angle θ to both **a** and **b**. If $\mathbf{c} = \alpha\mathbf{a} + \beta\mathbf{b} + \gamma(\mathbf{a} \times \mathbf{b})$, where α, β, γ are scalars, then

(a) $\alpha = \cot\theta, \beta = \sin\theta, \gamma^2 = \cos 2\theta$

(b) $\alpha = \cos\theta, \beta = \cos\theta, \gamma^2 = \cos 2\theta$

(c) $\alpha = \cos\theta, \beta = \sin\theta, \gamma^2 = \cos 2\theta$

(d) $\alpha = \sin\theta, \beta = \cos\theta, \gamma^2 = -\cos 2\theta$

148. If $y = \frac{1}{\sqrt{a^2 - b^2}} \cos^{-1}\left[\frac{a\cos(x-a) + b}{\theta}\right]$ where $\theta = a + b\cos(x - \alpha)$, then $\frac{dy}{dx}$ is equal to

(a) $1/\theta$ (b) $2/\theta$ (c) $1/\theta^2$ (d) $2/\theta^2$

149. Let K be a set of real number and $f : K \to R$ such that for all x and y $|f(x) - f(y)| \leq |x - y|^5$. If $f(3) = 7$, then the value of $f(9)$ is equal to

(a) 5 (b) 7 (c) 9 (d) 11

150. If $f(x) = \frac{1}{1-x}$ then the derivative of the composite function $f[f\{f(x)\}]$ is equal to

(a) 0 (b) 1/2 (c) 1 (d) 2

Answers

Physics

1.	(c)	**2.**	(b)	**3.**	(d)	**4.**	(d)	**5.**	(c)	**6.**	(b)	**7.**	(a)	**8.**	(a)	**9.**	(a)	**10.**	(b)
11.	(a)	**12.**	(d)	**13.**	(a)	**14.**	(b)	**15.**	(a)	**16.**	(c)	**17.**	(c)	**18.**	(c)	**19.**	(b)	**20.**	(b)
21.	(b)	**22.**	(c)	**23.**	(a)	**24.**	(b)	**25.**	(d)	**26.**	(a)	**27.**	(d)	**28.**	(d)	**29.**	(c)	**30.**	(b)
31.	(d)	**32.**	(c)	**33.**	(d)	**34.**	(d)	**35.**	(d)	**36.**	(b)	**37.**	(a)	**38.**	(b)	**39.**	(b)	**40.**	(c)
41.	(b)	**42.**	(d)	**43.**	(a)	**44.**	(d)	**45.**	(b)	**46.**	(c)	**47.**	(b)	**48.**	(a)	**49.**	(c)	**50.**	(d)

Chemistry

51.	(d)	**52.**	(d)	**53.**	(a)	**54.**	(c)	**55.**	(c)	**56.**	(a)	**57.**	(c)	**58.**	(c)	**59.**	(a)	**60.**	(b)
61.	(a)	**62.**	(c)	**63.**	(a)	**64.**	(b)	**65.**	(d)	**66.**	(a)	**67.**	(a)	**68.**	(a)	**69.**	(d)	**70.**	(b)
71.	(b)	**72.**	(a)	**73.**	(b)	**74.**	(b)	**75.**	(a)	**76.**	(a)	**77.**	(c)	**78.**	(a)	**79.**	(b)	**80.**	(b)
81.	(c)	**82.**	(c)	**83.**	(b)	**84.**	(d)	**85.**	(b)	**86.**	(b)	**87.**	(d)	**88.**	(d)	**89.**	(b)	**90.**	(b)
91.	(d)	**92.**	(b)	**93.**	(b)	**94.**	(d)	**95.**	(a)	**96.**	(c)	**97.**	(c)	**98.**	(c)	**99.**	(a)	**100.**	(c)

Mathematics

101.	(a)	**102.**	(a)	**103.**	(a)	**104.**	(c)	**105.**	(b)	**106.**	(d)	**107.**	(a)	**108.**	(c)	**109.**	(c)	**110.**	(b)
111.	(c)	**112.**	(a)	**113.**	(a)	**114.**	(b)	**115.**	(b)	**116.**	(d)	**117.**	(c)	**118.**	(b)	**119.**	(c)	**120.**	(b)
121.	(a)	**122.**	(b)	**123.**	(a)	**124.**	(a)	**125.**	(c)	**126.**	(c)	**127.**	(b)	**128.**	(d)	**129.**	(c)	**130.**	(d)
131.	(c)	**132.**	(b)	**133.**	(d)	**134.**	(b)	**135.**	(d)	**136.**	(b)	**137.**	(d)	**138.**	(a)	**139.**	(c)	**140.**	(b)
141.	(c)	**142.**	(c)	**143.**	(c)	**144.**	(a)	**145.**	(c)	**146.**	(d)	**147.**	(b)	**148.**	(a)	**149.**	(b)	**150.**	(c)

Hints & Solutions

Physics

1. *(c)* A setting sun appears to be at an altitude higher than it really is because of Refraction of light.

2. *(b)* Given, $d = 1\text{ mm} = 1\times10^{-3}\text{m}$, $D = 1\text{ m}$

   $\lambda = 500\text{ nm} = 500\times10^{-9}\text{m}$

   Distance of nth minima from central maxima $x_n = \dfrac{(2n-1)\lambda}{2}\dfrac{D}{d}$

   $$= \frac{(2\times3-1)\times500\times10^{-9}}{2}\times\frac{1}{1\times10^{-3}}$$

   $$= 2.5\times500\times10^{-6}$$

   $$= 12.5\times10^{-4}\text{m} = 1.25\text{ mm}$$

3. *(d)* Diagram (d) represents semiconductor because in s semiconductor, the forbidden energy gap between valence and conduction band is equal to the energy corresponding to room temperature.

4. *(d)* For transistor, $i_e = i_c + i_b$

   $i_c = 95\%$ of i_e

   $i_b = 5\%$ of i_e

   So, $i_e > i_c$

5. *(c)* Average life = 5 hour, in one average life, approximately 63% radioactive nuclei decay. Hence, in a time of 5 hour more than half of the active nuclei decay.

6. *(b)* The relation between electric field (E) and magnetic field (B) is $E = BC$

   Where, C is the speed of light.

   Given, $B = 2\times10^{-7}\ T$ and $C = 3\times10^{8}\text{m/c}$

   $\therefore \quad E = 2\times10^{-7}\times3\times10^{8}$

   $E = 60T\text{-m/s} \Rightarrow 60\ Vm^{-1}$

7. *(a)* When a γ-ray photon is emitted, then atomic number and mass number remains unchanged.

8. *(a)* The object will slip, if centripetal force acting on it is more than friction force.

   So, $mr\omega^2 > \mu mg;\ r\omega^2 \geq \mu g$

   $r\omega^2 =$ constant

   $$\Rightarrow \quad \frac{r_1}{r_2} = \left(\frac{\omega_2}{\omega_1}\right)^2 \Rightarrow \frac{4}{r_2} = \left(\frac{2\omega}{\omega}\right)^2 \Rightarrow r_2 = 1\text{ cm}$$

9. *(a)* Force between two current carrying parallel conductor per unit length,

   $$F = \frac{\mu_0}{2}\frac{2I_1I_2}{r}$$

   According to question,
   Current in two parallel wire is in same direction therefore they attract each-other.

10. *(b)* Efficiency = 0.6

    Heat given to sink, $Q_2 = 20\text{ J}$

    $$\eta = \frac{Q_1 - Q_2}{Q_1}$$

    $$0.6 = \frac{Q_1 - 20}{Q_1}$$

    $$0.6\,Q_1 = Q_1 - 20$$

    $$0.4\,Q_1 = 20$$

    $$Q_1 = 50\text{ J}$$

    Work done, $W = Q_1 - Q_2 = 50 - 20 = 30\text{ J}$

11. *(a)* $W = \dfrac{1}{2}kx^2$

    Writing the dimensions on both sides

    $[ML^2T^{-2}] = k[M^0L^2T^0]$

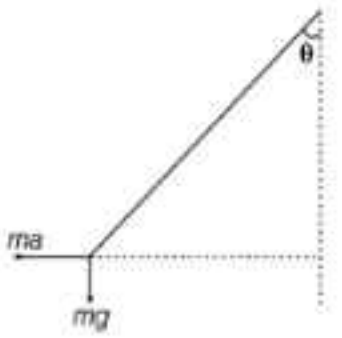
    $\therefore$ Dimension of $k = [MT^{-2}] = [ML^0T^{-2}]$

12. *(d)* Time taken by the bullets to reach at earth, $t = \dfrac{\sqrt{2h}}{g}$

    Hence, time does not depend on velocity, so both bullets will reach simultaneously.

13. *(a)* Let the angle from the vertical be θ. The diagram showing the different forces is given

From the figure, $\tan\theta = \frac{a}{g}; \theta = \tan^{-1}\frac{a}{g}$



14. *(b)* Given, mass of body = 2 kg

According to question,

∵ Friction force due to 100 N

$$f = MR = 0.3 \times 100 = 30\text{ N}$$

but force due to gravitation,

$$F = mg = 2 \times 10 = 20\text{ N}$$

So, there is no acceleration in box i.e. $a = 0$

Hence, friction force will be $f = 20\text{ N}$

15. *(a)* Acceleration of the sphere

$$a = \frac{g\sin\theta}{1 + \frac{k^2}{r^2}}$$

∵ for sphere, $\frac{k^2}{r^2} = \frac{2}{5} = \frac{5}{7}g\sin\theta$

16. *(c)* Speeds at top most point of each wheel will equal and also it is equal to the speed of centre of mass.

17. *(c)* A prism is a homogeneous, transparent medium bounded by two plane surfaces inclined at an angle C with each other. These surfaces are called as refracting surfaces and the angle between them is called angle of prism C.

18. *(c)* Moment of inertia, $I = MR^2$

M = mass of object

R = distance of centre of mass from axis of rotation.

Hence, moment of inertia does not depend upon angular velocity.

19. *(b)* If a particle falls towards the earth from infinity, then velocity on reaching earth

$$v = \sqrt{2gR}$$

where, R = radius of earth.

20. *(b)* Universal gas constant $R = C_p - C_V$

Where, C_p and C_V are specific heat at constant pressure and constant volume respectively

21. *(b)* Momentum, $p = \sqrt{2mE_k}$

Both bodies have equal kinetic energy

So, $\frac{p_1}{p_2} = \sqrt{\frac{m_1}{m_2}}$

$\frac{p_1}{p_2} = \sqrt{\frac{m}{4m}}$ (∵ $m_1 = m$, $m_2 = 4m$, given)

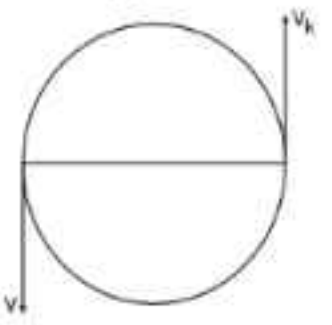
$p_1 : p_2 = 1 : 2$

22. *(c)* Change in internal energy of the system does not depend on path followed.

So, $\Delta U_1 = \Delta U_2$

23. *(a)* According to Kepler's law $T^2 \propto R^3$

24. *(b)* Initial velocity



Final velocity, $v_2 = -v$

Initial momentum, $p_1 = mv$

Final momentum, $p_2 = m(-v) = -mv$

Change in momentum, $\Delta p = p_1 - p_2$

$= mv - (-mv) = 2mv$

25. *(d)* Volume of big drop = n × volume of small drops

$$\frac{4}{3}\pi R^3 = n\frac{4}{3}\pi r^3$$

$$R = n^{1/3}r$$

Potential of each small drop $V = \frac{1}{4\pi\varepsilon_0}\frac{q}{r}$

Potential of big drop $V' = \frac{1}{4\pi\varepsilon_0}\frac{nq}{R}$

$$= \frac{1}{4\pi\varepsilon_0}\frac{nq}{n^{1/3}r} = n^{2/3}V$$

26. *(a)* For adiabatic process $P_1V_1^\gamma = P_2V_2^\gamma$

$$\frac{RT_1}{V_1}V_1^\gamma = \frac{RT_2}{V_2}V_2^\gamma$$

$$T_1V_1^{\gamma-1} = T_2V_2^{\gamma-1}$$

27. *(d)* Within the elastic limit, the ratio of lateral strain to the longitudinal strain is called poisson's ratio.

Practically, for metal, $0 < \sigma < 0.5$

28. *(d)* Breaking tension, $T \propto r^2$; $\frac{T_1}{T_2} = \left(\frac{r_1}{r_2}\right)^2$

According to question, $T_1 = 1000$N,

$rd_1 = 1$ mm, $d_2 = 2$mm

or $r_1 = 1/2$ mm, $r_2 = 1$mm

$$\frac{1000}{T_2} = \left(\frac{1/2}{2/2}\right)^2$$

$$\frac{1000}{T_2} = \left(\frac{1}{2}\right)^2 \Rightarrow T_2 = 4000\,\text{N}$$

29. *(c)* Height of liquid in capillary

$$T = \frac{rhdg}{2};\ h = \frac{2T}{rdg}$$

In freely falling elevator, gravitational acceleration will be zero.

$\therefore h = \infty$ i.e. the water will rise upto maximum available height, hence 30 cm.

30. *(b)* An aeroplane get upward lift due to a phenomenon described by Bernoulli's principle. Aerofoil is a solid object shaped to provide an upward. Dynamic lift as it moves horizontally through air. This upward force makes aeroplane fly.

31. *(d)* At each point on the surface of a conducting sphere, the potential is equal.

So, $V_A = V_B = V_C$

32. *(c)* Capacitor is first charged, then isolated, so the charge on the capacitor becomes constant.

33. *(d)* Potential of big drop, $V = n^{2/3}V'$

V' = potential of small drop

n = number of drops = 125

According to question, $V = 2.5V, n = 125$

$\therefore \quad 2.5 = (125)^{2/3}V'$

$2.5 = 25\,V'$

$V' = 0.1$ V.

34. *(d)* Energy stored in the capacitor

$$E = \frac{1}{2}CV^2$$

$$= \frac{1}{2}\times 10\times 10^{-6}\times (500)^2$$

$$= 5\times 10^{-6}\times 25\times 10^4 = 1.25\,\text{J}$$

Heat produced in resistance = energy stored

$= 1.25$ J

35. *(d)* Slope of graph $= \frac{I}{V} = \frac{1}{R}$

If experiment is performed at higher temperature, then resistance increase and hence slope decrease, so choice (a) is wrong.

Similarly, in choices (b) and (c), resistance increases.

But for choice (d), resistance R increases, so slope decreases.

36. *(b)* Given, current in a conductor, $I = 2t + 3t^2$

$\because \quad I = \frac{dq}{dt}$

$dq = Idt$

$$q = \int_2^3 Idt = \int_2^3 [2t + 3t^2]\,dt$$

$$= 2\left[\frac{t^2}{2}\right]_2^3 + 3\left[\frac{t^3}{3}\right]_2^3 = [t^2]_2^3 + [t^3]_2^3$$

$$= [(3)^2 - (2)^2] + [(3)^3 - (2)^3]$$

$$= (9-4) + (27-8) = 5 + 19 = 24\,\text{C}$$

37. *(a)* Equivalent resistance of 3Ω and 6Ω

$$R' = \frac{3\times 6}{3+6} \qquad \text{(in parallel)}$$

$$= \frac{18}{9} = 2\Omega$$

Total resistance = $2 + R$ (in series)

$$i = \frac{V}{R};\ 2 = \frac{6}{2+R}$$

$$2 + R = 3 \Rightarrow R = 1\ \Omega$$

38. *(b)* Diamagnetic substance have negative susceptibility ($X < 0$) while paramagnetic superparamagnetic and ferromagnetic substances have positive susceptibility ($X > 0$).

39. *(b)* Resistance of galvanometer $G = 50\,\Omega$

$I_g = 0.01$ A, $I = 10$ A

Shunt resistance, $S = \frac{I_g G}{I - I_g}$

$$= \frac{0.01 \times 50}{10 - 0.01} = 0.05\ \Omega$$

40. *(c)* A horizontal rod of length L rotates with a uniform angular velocity ω about a verticle axis in a uniform magnetic field (B) parallel to axis of rotation then, the potential difference between the two ends of the conductor $V = \frac{1}{2}\omega L^2 B$

41. *(b)* Magnetic field produced at the centre

$$B = \frac{1}{2}\left(\frac{\mu_0 I}{2r}\right)$$

According to question,

$I = 10A, r = 20\text{ cm} = 20 \times 10^{-2}$ m

$$= \frac{1}{2} \times \frac{4\pi \times 10^{-7} \times 10}{2 \times 20 \times 10^{-2}}$$

$$= \frac{\pi \times 10^{-6}}{20 \times 10^{-2}} = 5\pi \times 10^{-6}\ \text{T} = 5\pi\ \mu T$$

42. *(d)* Frequency, $n = 50$ Hz

$\therefore$ Time period, $T = \frac{1}{50}$ sec

Time taken for voltage to change from its peak value to zero $= \frac{T}{4}$

$$= \frac{1}{4 \times 50} = \frac{1}{200} = 5 \times 10^{-3}\ \text{sec}$$

43. *(a)* Transformer does not work in DC supply (battery supply) so, secondary voltage will be zero.

44. *(d)* From meter bridge formula

$$R_2 = R_1\left(\frac{100 - l}{l}\right)$$

$$30 = 10\left(\frac{100 - l}{l}\right)$$

$$3l = 100 - l \Rightarrow l = 25\text{ cm}$$

According to question,

Resistances are interchanged, i.e. $R_1 = 30\ \Omega$ and $R_2 = 1\ \Omega$ then $10 = 30\left(\frac{100 - l'}{l'}\right)$

$$l' = 300 - 3l'$$

$$l' = 75\text{ cm}$$

So, point shifts by, $l' - l = 75 - 25 = 50$ cm

45. *(b)* Frequency will not change with medium but wavelength and velocity decrease with the increase in refractive index.

46. *(c)* Real frequency, $n = 400$ Hz

Apparent frequency, $n' = 390$ Hz

$$n' < n$$

So, the distance between the source and listener increases or the listener is moving away from source.

47. *(b)* Position of first node = 16 cm

$$\frac{\lambda}{2} + e = 16\text{ cm} \qquad \ldots(i)$$

Where, e = end correction

Position of second node = 46 cm

$$\frac{\lambda}{2} + \frac{\lambda}{2} + e = 46\text{ cm} \qquad \ldots(ii)$$

From Eqs. (i) and (ii), $\frac{\lambda}{2} = 30$ cm

$$\lambda = 60\text{ cm}$$

$$= \frac{60}{100}\text{ m}$$

$\therefore$ Speed of sound, $v = n\lambda$

$= 500 \times \frac{60}{100}$ ($\because n = 500$Hz, Given)

$= 300$ m/s

48. *(a)* Velocity of sound $v \propto \sqrt{T}$

$$\text{Time } t \propto \frac{1}{\sqrt{v}}$$

$$\therefore \quad t \propto \frac{1}{\sqrt{T}}; \frac{t_1}{t_2} = \sqrt{\frac{T_2}{T_1}}$$

$$\frac{2}{t_2} = \sqrt{\frac{273+30}{273+10}}; \frac{2}{t_2} = \sqrt{\frac{303}{283}} = 1.03$$

$$t_2 = \frac{2}{1.03} = 1.9 \text{ sec}$$

49. *(c)* Number of images formed, $n = \frac{360^\circ}{\theta} - 1$

$$= \frac{360^\circ}{60} - 1 \quad (\because \theta = 60^\circ \text{ given})$$

$$= 6 - 1 = 5$$

50. *(d)* For a real object, Real image can be produced by concave mirror and converging lens, only if the object is placed further away from the mirror/lens than the focal point and this real image is inverted.

Chemistry

51. *(d)* Cell reaction $2Cu^{+}_{(aq)} \longrightarrow Cu_{(s)} + Cu^{2+}_{(aq)}$

According to Nernst equation

$$E = E^\circ - \frac{2.303RT}{nF} \log\left[\frac{M}{M^{n+}}\right]$$

where, E = Electrode potential of the metal

E° = Standard electrode potential

R = Gas constant (8.314) joules per mol per degree absolute)

T = Temperature on Kelvin scale

n = Number of electrons involved in the half-cell reaction

F = One Faraday (96500 coulombs)

at 298 K, $2.303\,RT/F = 0.059$

$$\therefore \quad E = E^\circ - \frac{0.059}{n} \log\left[\frac{M}{N^{n+}}\right]$$

$$= E^\circ - \frac{0.059}{n} \log\left[\frac{\text{Products}}{\text{Reactants}}\right]$$

$[\because E = 0 \text{ at equilibrium}]$

$$0 = E^\circ - \frac{0.059}{n} \log K_{eq}$$

$$\text{or} \quad \log K_{eq} = \frac{nE^\circ}{0.059}$$

$$\text{or} \quad \log K_{eq} = \frac{1 \times 0.36}{0.059} = 6.09$$

$$K_{eq} = 1.2 \times 10^6$$

52. *(d)* The cell reaction is

$$Cd_{(s)} + 2AgCl_{(s)} \rightarrow 2Ag_{(s)} + Cd^{2+}_{(aq)} + 2Cl^{-}_{(aq)}$$

$E_1 = 0.6915$ V at 0° C

$E_2 = 0.6753$ V at 25°C

$$\text{Now,} \quad \frac{\partial E_{cell}}{\partial T} = \frac{E_2 - E_1}{T_2 - T_1} = \frac{0.6753 - 0.6915}{298 - 273}$$

$$= -6.48 \times 10^{-4}$$

$$\Delta S = nF\left[\frac{\partial E_{cell}}{\partial T}\right]$$

Now, we put the value

$$\Delta S = 2 \times 96500\,(-6.48 \times 10^{-4}) = -125.064$$

We know that,

$$\Delta G = -nFE_{cell}$$

$$= -2 \times 96500 \times 0.6753$$

$$= -1.303 \times 10^5$$

As, $\Delta G = \Delta H - T\Delta S$

For calculating $\underset{(25^\circ C)}{\Delta H} = \Delta G + T\Delta S$

$$= -1.303 \times 10^5 + 298\,(-125.064 \text{ kJ})$$

$$\Delta H = -1.6726 \times 10^5 \text{ J} = -167.26 \text{ kJ}$$

53. *(a)* We know that, the relative lowering of vapour pressure of a solution is equal to the mole-fraction of the solute present in the solution $\frac{P_0 - P_S}{P_0} = x_1$

P_0 = Vapour pressure of solvent

P_S = Vapour pressure of solution

x_1 = Mole-fraction of solute

Thus, option a is true.

but the option (b), (c) and (d) are wrong.

(b) Because movement of solvent molecules from the lower concentration side to higher concentration side through semipermeable membrane is known as osmosis.

(c) According to elevation of b.p., the boiling point of solution is always higher than the solvent.

(d) The boiling point of a liquid is the temperature at which its vapour pressure becomes equal to atmospheric pressure (760 mm).

54. *(c)* It is wrong because the rate of diffusion of a gas is inversely proportional to the density of the gas at constant pressure.

$$r \propto \frac{1}{\sqrt{d}}$$

According to Graham's law the rate of diffusion is inversely proportional to square root of molecular weight and density.

$$r \propto \frac{1}{\sqrt{d}} \text{ and } r \propto \frac{1}{\sqrt{M}}$$

If one gas has rate of diffusion r_1 other gas has rate of diffusion r_2, density of gases are d_1 and d_2, then

$$\frac{r_1}{r_2} = \sqrt{\frac{d_2}{d_1}} \text{ or } \frac{r_1}{r_2} = \sqrt{\frac{M_2}{M_1}}$$

55. *(c)* According to second order reaction

$$2A \longrightarrow \text{Product}$$

rate of reaction $\frac{d[A]}{dt} \propto [A]^2$ or $\frac{dA}{dt} = k[A]^2$

k = reaction rate constant

then unit of second order reaction rate constant.

$$k = -\frac{d[A]}{dt} \times \frac{1}{[A]^2}$$

$$= \frac{1}{\text{s. mol L}^{-1}} \text{ or L mol}^{-1}\text{ s}^{-1}$$

56. *(a)* According to second law of thermo-chemistry the law states that the total heat change (ΔH) accompanying a chemical reaction is the same whether the reaction takes place in one or more steps.

It means that heat of a reaction depends only on the initial reactants and final products and not on intermediate products that may be formed.

Now, $\Delta H = \Delta H_1 + \Delta H_2 + \Delta H_3$

Enthalpy change in a reaction is always constant and independent of the path followed.

57. *(c)* Total amount of (A), $N_0 = 1\text{g}$

After β-emission the amount rest, $N = 0.125\text{ g}$

Total time = 200 yr

$$\frac{N}{N_0} = \left(\frac{1}{2}\right)^n$$

$$\frac{0.125}{1} = \left(\frac{1}{2}\right)^n$$

$$\frac{125}{1000} = \left(\frac{1}{2}\right)^n$$

$$\frac{1}{8} = \left[\frac{1}{2}\right]^n \text{ or } \left(\frac{1}{2}\right)^3 = \left(\frac{1}{2}\right)^n$$

$$n = \frac{T}{t_{1/2}} \Rightarrow 3 = \frac{200}{t_{1/2}} \qquad \because n = 3$$

$$t_{1/2} = \frac{200}{3} = 66.6\text{ yr}$$

58. *(c)* Isotopes are atoms of same elements having different mass number

Isobars are atoms of different elements having same mass number.

Isotones are atoms of different elements having same number of neutrons.

Nuclear isomers are atoms with the same atomic number and same mass number but different radioactive properties.

59. *(a)* Acid hydrolysis of sucrose is a pseudo unimolecular or pseudo first order reaction. Hydrolysis of sucrose in presence of mineral acid is a bimolecular reaction. But, as water is taken in large excess, so the rate of

reaction only depends upon concentration of sucrose. Hence, order of the reaction is one. Therefore, it is called as pseudo first order reaction.

60. *(b)* After emission of positron from ${}_{31}Ga^{68}$ then ${}_{31}Ga^{68} \longrightarrow {}_{30}Zn^{68} + {}_{1}e^{0}$

61. *(a)* We know that, the pH value of a solution is equal to the negative power to which 10 must be raised in order to express H^+ ion concentration, *i.e.*,

$$[H^+] = 10^{-pH}$$

The pH of a solution is the negative logarithm of its H^+ ion concentration.

$$pH = -\log [H^+]$$

The pH of a solution is logarithm of the reciprocal of H^+ ion concentration.

$$pH = \log \frac{1}{[H^+]}$$

Now, $[H^+] = [OH^-] = 1 \times 10^{-7}$, pH = 7

In an acidic solution $[H^+] > 10^{-7}$

pH is less than 7

In an alkaline solution $[H^+] < 10^{-7}$, pH is greater than 7.

So, an aqueous solution in which the H^+ ion concentration is greater than 10^{-7} M, the solution is acidic.

62. *(c)* According to hydrolysis of salts of a weak acid and a weak base,

Since, the hydrolysis of such salts will form weak acid and weak base, the solution will be neutral and its pH will be 7.

Then, $B^+A^- + H_2O \rightleftharpoons \underset{\text{Weak acid}}{HA} + \underset{\text{Weak base}}{BOH}$

$$K_h = \frac{[HA][BOH]}{[B^+][A^-]}$$

Now, let consider the dissociation of HA and *B*OH.

$$HA \rightleftharpoons H^+ + A^-$$

$$BOH \rightleftharpoons B^+ + OH^-$$

$$K_a = \frac{[H^+][A^-]}{[HA]}$$

$$K_b = \frac{[B]^+][OH^-]}{[HA][BOH^-]}$$

Also, we know that $K_w = [H^+][OH^-]$

Now, put the value of K_a and K_b in above equation.

$$K_h = \frac{K_w}{K_a \times K_b} = \frac{[H^+][OH^-][HA][BOH]}{[H^+][A^-][B^+][OH^-]}$$

Then $k_h = \frac{K_w}{K_a \times K_b}$

63. *(a)* In the nuclear reaction

${}_{13}Al^{27} + {}_{2}He^{4} \longrightarrow {}_{14}X^{30} + {}_{1}H^{1}$, *X* is Si, then

${}_{13}Al^{27} + {}_{2}He^{4} \longrightarrow {}_{14}Si^{30} + {}_{1}H^{1}$

64. *(b)* $AlCl_3$ is an electron deficient molecule, so it acts as a Lewis acid.

65. *(d)* $\because$ Equivalent weight of

$$K_2Cr_2O_7 = \frac{\text{Molecular weight of } K_2Cr_2O_7}{\text{Oxidation number of Cr}}$$

Oxidation number of Cr in $K_2Cr_2O_7$

$$2(+1) + 2(x) + 7(-2) = 0$$

$$2 + 2x - 14 = 0$$

$$2x = 12$$

$$x = 6$$

Equivalent weight of $K_2Cr_2O_7 = \frac{294.19}{6}$

$$= 49.08$$

$$\frac{\text{Weight of } K_2Cr_2O_7}{\text{Equivalent wt. } (E)} = \frac{N \times V}{1000}$$

$$w = 0.1 \times 1 \times 49.03$$

$$= 4.903 \text{ g}$$

66. *(a)* Given, $V_1 = 100$ mL, $V_2 = 250$ mL

Pressure, $p = 2$ atm $= 2 \times 1.01 \times 10^5$ Nm^{-2}

$\therefore$ Work done by the gas,

$$W = p\Delta V \text{ or } p(V_2 - V_1)$$

Put the value in given formula,

$$W = 2 \times 1.01 \times 10^5 (0.250 \times 10^{-3} - 0.100 \times 10^{-3})$$

$$= 2 \times 1.01 \times 10^5 \times 0.15 \times 10^{-3}$$

$$= 30.38 J$$

67. *(a)* Given, r.m.s speed of a gaseous molecule is x m/s at a pressure p atm.

We know that,

Kinetic theory of gas

$$\text{r.m.s. speed} = \sqrt{\frac{3RT}{M}}$$

We know $pV = RT$

$$\text{Then r.m.s. speed} = \sqrt{\frac{3pV}{M}}$$

As temperature is constant. So, pV is constant. Hence, r.m.s. speed is also constant. If the pressure is double at constant temperature, there is no change in r.m.s. speed.

68. *(a)* ∵ Ionic mobility

$$U_{Ag^+} = \frac{\text{Ionic conductance } (\lambda_{Ag^+})}{96500}$$

$$= \frac{5 \times 10^{-4}}{96500} = 5.2 \times 10^{-9} \text{ cm/s}$$

69. *(d)* According to Arrhenius concept, acids are those compounds which give hydrogen ions in aqueous solution and bases are those compounds which give hydroxide ions in solution.

Among halides of hydrogen intermolecular H—bonding is present. So, when we go top to bottom in halogen group, size of I^- ion increases and the inter molecular H—bonding becomes weak and easily gives H^+ in aqueous solution. So, it works as a strong acid. Acidity decreases in this order.

$$HI > HBr > HCl > HF$$

70. *(b)* Coinage metals are transition metals but they cannot work as transition metal because they have completely filled *d*-orbital.

Group IB elements are called coinage metals (Cu, Ag, Au). Their general outer electronic configuration is $(n-1)d^{10}ns^1$.

71. *(b)* Ionisation energy of 1st group elements decreases down the group because in groups from top to bottom atomic size increases. Due to increase in atomic size, the nuclear attraction of outer electron is reduced. They easily removed from valence orbital. So, ionisation energy is reduced from top to bottom in a group.

72. *(a)* Oxidation number of Cl in ClO_3^-.

$$ClO_3^- = -1$$

$$x + 3(-2) = -1;\ x = +6 - 1$$

$$x = +5$$

73. *(b)* In NH_3, the N atom contains one lone pair of electrons and three bond pairs in its valence shell. So, it shows sp^3 hybridisation.

Due to presence of one lone pair of electron, its shape deviate from tetrahedral because lone pair shows more repulsion that bond pairs.

As, $lp - lp > lp - bp > bp - bp$

So, its shape is pyramidal and angle 107°.

74. *(b)* NaCl has octahedral crystal system, therefore co-ordination number of each Na^+ and Cl^- ion is 6. Co-ordination number of atom is determined that the given atom surrounded by how many numbers of atom. In NaCl, each Na^+ ion is surrounded by 6 Cl^- ions and each Cl^- ions is surrounded by six Na^+ ions. So, they both show co-ordination number 6.

75. *(a)* Oxidation of $KMnO_4$ takes place in all the three medium acidic, basic and neutral. In all three medium the oxidation number is different. **In acidic medium**

$$2KMnO_4 + 3H_2SO_4 \longrightarrow K_2SO_4 + 2MnSO_4 + 3H_2O + 5O$$

The net reaction is $MnO_4^- \longrightarrow Mn^{2+}$

Change in oxidation numbers = 7 − 2 = 5

So, electrons involved = $5e^-$

In basic medium

$$2KMnO_4 + 2KOH \longrightarrow 2K_2MnO_4 + H_2O + O$$

The net reaction is

$$\overset{+7}{MnO_4^-} \longrightarrow \overset{+6}{MnO_4^{-2}}$$

Change in oxidation numbers = 7 − 6 = +1

So, electrons involved = $1e^-$

In neutral medium

$$2KMnO_4 + H_2O \longrightarrow 2KOH + 2MnO_2 + 3O$$

net reaction is

$$MnO_4^- \longrightarrow MnO_2$$

Change in oxidation numbers = 7 – 4 = + 3

So, electrons involved = $3e^-$

76. *(a)* Given, magnetic moment of

$$K_3[Fe(CN)_6] = 1.7 \text{ BM}$$

Magnetic moment = $\sqrt{n(n+2)}$

n = number of unpaired electrons present in molecule $1.7 = \sqrt{n(n+2)}$

$$-n^2 + 2n - 2.89 = 0$$

Then $n = 0.97$ or 1

77. *(c)* BF_3 is electron deficient compound because B has six electrons in outer most orbit. It has incomplete octet. So, it is an electron deficient molecule.

78. *(a)* Hydra-acids of halogens in increasing order of acidity.

$$HF < HCl < HBr < HI$$

All the strength of hydrogen bonding decreases in the order of

$$HF > HCl > HBr > HI$$

79. *(b)* $\overset{-1}{H_2}\overset{}{O_2} + \overset{0}{Cl_2} \longrightarrow 2\overset{-1}{H}\overset{}{Cl} + \overset{0}{O_2}$

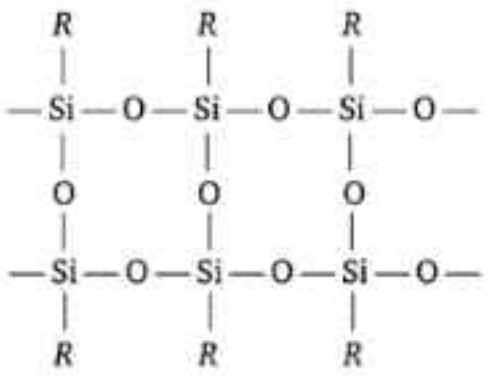
In is a redox reaction.

$\overset{-1}{H_2}O_2 \longrightarrow \overset{0}{O_2}$, oxidation reaction

$\overset{0}{Cl_2} \longrightarrow \overset{-}{Cl}$, reduction reaction

80. *(b)* $RSiCl_3 \xrightarrow{3H_2O} RSi(OH)_3 + 3HCl$

$RSi(OH)_3 \xrightarrow{\text{Polymerisation}}$



Three dimensional structure of silicon.

81. *(c)* NO (Nitric oxide) is synthesised in laboratory by copper with cold and dilute HNO_3.

$$3Cu + \underset{\text{Dil.}}{8HNO_3} \longrightarrow 3Cu(NO_3)_2 + \underset{\substack{\text{Nitric}\\\text{oxide}}}{2NO} + 4H_2O$$

By reacting Zn with cold and dil. HNO_3, NH_4NO_3 is formed.

$$\underset{\text{Dil. and cold}}{4Zn + 10HNO_3} \longrightarrow 4Zn(NO_3)_2 + 3H_2O + NH_4NO_3 + 2NO_2$$

By reacting Zn with conc. nitric acid NO_2 is formed. By heating NH_4NO_3, nitrous oxide is formed.

$$NH_4NO_3 \longrightarrow \underset{\substack{\text{Nitrous}\\\text{oxide}}}{N_2O} + 2H_2O$$

82. *(c)* H_2 gas is evolved, when Zn reacts with NH_4Cl in dry cell battery.

$$Zn + 2NH_4Cl \longrightarrow Zn(NH_3)_2Cl_2 + \underset{\substack{\text{Colourless}\\\text{gas}}}{H_2\uparrow}$$

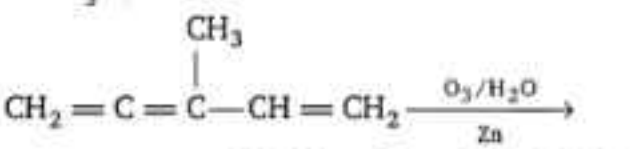
83. *(b)* Co-ordinate bond is formed.

$$(C_2H_5)_2O \longrightarrow BH_3$$

$(C_2H_5)_2O$ gives one lone pair of electron to BH_3. So, it is called electron pair donar and BH_3 is called electron pair acceptor.

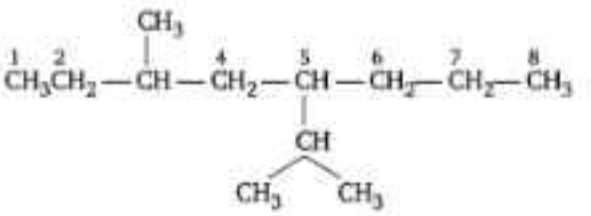
84. *(d)* On ozonolysis

$CH_2 = C = C - CH = CH_2$ gives two moles of HCHO, one mole of CO_2 one mole of CH_3COCHO.



$$2HCHO + CO_2 + CH_3COCHO$$

85. *(b)* IUPAC name of compound.



Straight chain which contains large number of side chains taken as parent chain and

counting starts from that side where the side chain is nearest. 3-methyl 5 (1-methyl ethyl) octane.

86. *(b)* Due to intra molecular hydrogen bonding in *ortho*-isomer, it has least m.p. Due to effective intermolecular hydrogen bonding in *para*-isomer, it has highest melting point among the isomers. So, the order is

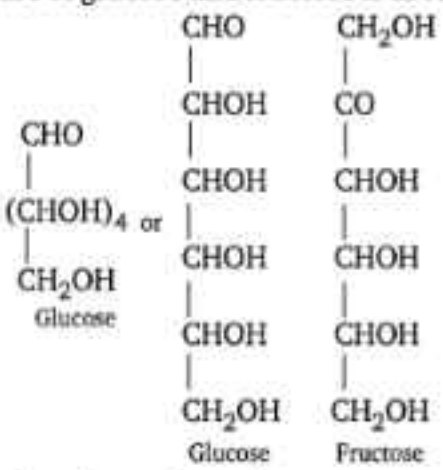
para (114°C) > *meta* (97°C) > *ortho* (45°C)

87. *(d)* $CHCl_3 + 4NaOH(aq) \longrightarrow \underset{\text{Sodium formate}}{HCOONa} + 3NaCl + 2H_2O$

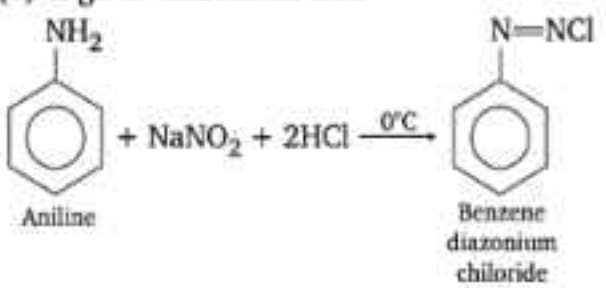
88. *(d)* Fructose, it has ketonic group. So, it is keto hexose sugar.

$\underset{\text{Sucrose}}{C_{12}H_{22}O_{11}} + H_2O \xrightarrow[\text{Hydrolysis}]{}$ Glucose (aldohexose sugar) (With aldehyde group) + Fructose (ketohexose sugar) (With ketone group)

Structure of glucose and fructose is as follows



89. *(b)* It gives diazonium salt.



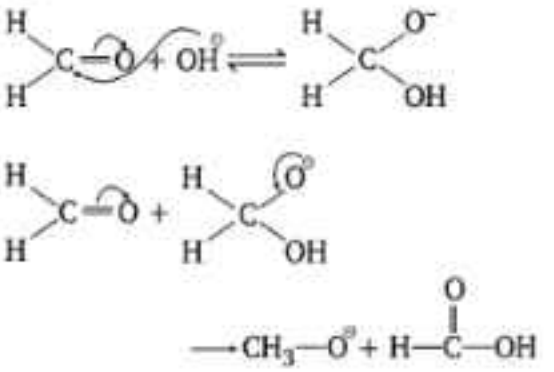
$+ 2H_2O + NaCl$

It is known as diazotisation reaction.

90. *(b)* Cannizzaro reaction

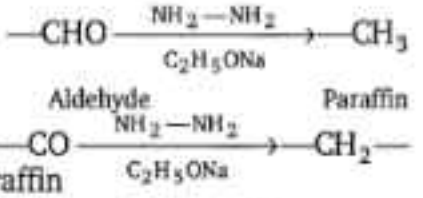
$HCHO + NaOH \longrightarrow CH_3OH + HCOONa$

This reaction takes place by those compounds which has no α-H atom. Intermolecular shift of hydride ion is key step of Cannizzaro reaction

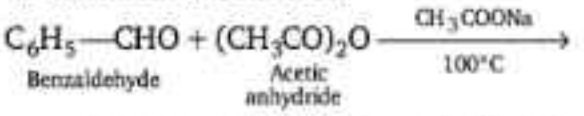


91. *(d)* By $NH_2—NH_2/C_2H_5ONa$, aldehyde and ketones are reduced with hydrazine $NH_2—NH_2$ and C_2H_5ONa to give hydrocarbon (paraffins).

This reaction is called Wolf-Kishner reaction.



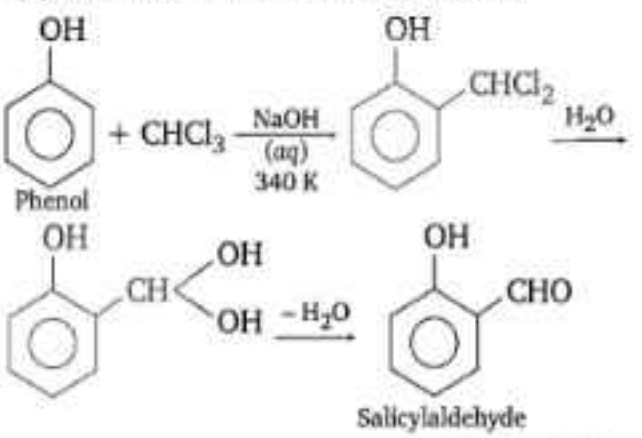
92. *(b)* When C_6H_5CHO condenses with $(CH_3CO)_2O$ in presence of sodium acetate then cinnamic acid is formed. This reaction is called Perkin reaction.



$C_6H_5CH = CHCOOH + CH_3COOH$

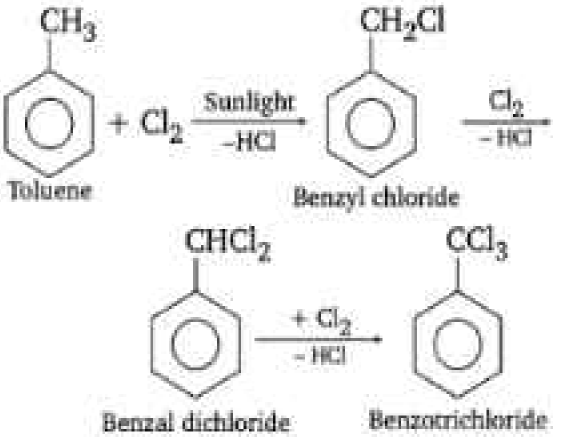
Cinnamic acid, Acetic acid

93. *(b)* This is Reimer-Tiemann reaction.

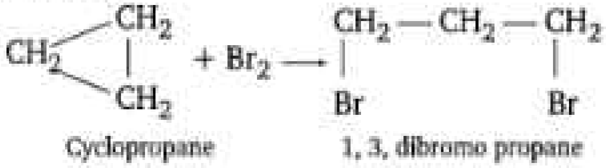


$+ NaCl + H_2O$

94. *(d)* In presence of sunlight toluene undergoes aliphatic substitution with chlorine and give benzyl chloride, benzal dichloride and benzo trichloride.



95. *(a)* Cyclopropane is the most unstable cyclic compound. So-with bromine it gives an open chain compound 1, 3-dibromo propane.

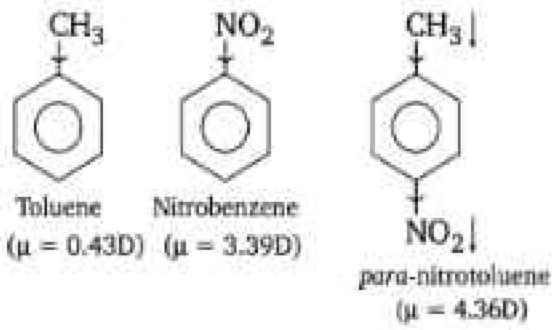


96. *(c)* Methyl group has *+I* effect and $—NO_2$ group has *–I* effect.

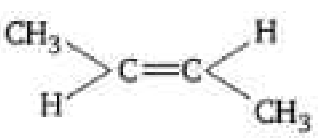
Therefore, in *p*-nitro toluene the dipole moments of $—CH_3$ and $—NO_2$ groups act in the same direction.

So, the resultant dipole moment is additive.

i.e. 3.93 + 0.43 = 4.36 debye



97. *(c)* $\underset{\text{2-butyne}}{CH_3—C \equiv CH} + 2[H] \xrightarrow[\text{liq.NH}_3]{\text{Li}}$



(*trans*-2-butene)

But, in presence of Lindlar's catalyst (Pd - $CaCO_3$) 2-butyne reacts with hydrogen giving *cis*-2-butene.

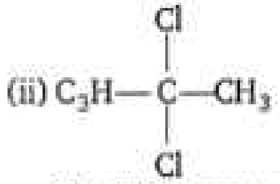
$\underset{\text{2-butyne}}{CH_3—C \equiv CH} + H_2 \xrightarrow{\text{Pd.-CaCO}_3}$

(*cis*-2-butene)

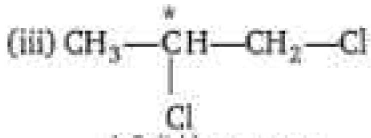
98. *(c)* There are four isomers obtained.

(i) $CH_3—CH_2—CH_3 + Cl_2 \longrightarrow$

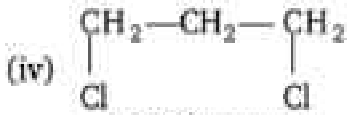
$\underset{\text{1, 1– dichloro propane}}{CH_3—CH_2—CHCl_2} + 2HCl$

(ii) 

2, 2– dichloro propane

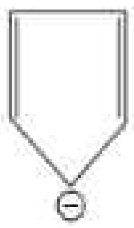
(iii) 

1, 2-dichloro propane (optical active)

(iv) 

1, 3-dichloro propane

99. *(a)*



Cyclopenta dienyl anion

According to Huckel's rule, total number of π-electrons inside the ring $(4n + 2)\pi$ electrons $= (4 \times 1 + 2) = 6\pi$ electrons

So, it is aromatic.

100. *(c)* Due to presence of delocalised π-electrons in the aromatic compounds, the electron density is maximum inside the ring. Therefore, aromatic compounds undergo electrophilic substitution reaction and resistant to addition reactions.

Mathematics

101. *(a)* Given that, equation of planes are

$$x + y + z = 1 \quad \ldots(i)$$

and $2x + 3y - z + 4 = 0 \quad \ldots(ii)$

$\therefore$ The equation of the plane through the intersection of the planes (i) and (ii) is

$$(x + y + z - 1) + k(2x + 3y - z + 4) = 0 \quad \ldots(iii)$$

$$\Rightarrow x(1 + 2k) + y(1 + 3k) + z(1 - k) - 1(1 - 4k) = 0$$

$$\Rightarrow \quad x(1 + 2k) = 0$$

[$\because$ plane is parallel to X-axis]

$$\Rightarrow \quad 1 + 2k = 0 \Rightarrow k = -\frac{1}{2}$$

Put $k = -1/2$ in equation (iii), we get the required equation of plane.

$$2(x + y + z - 1) - 2x - 3y + z - 4 = 0$$

$$\Rightarrow \quad 2x + 2y + 2z - 2 - 2x - 3y + z - 4 = 0$$

$$\Rightarrow \quad -y + 3z - 6 = 0$$

$$\Rightarrow \quad y - 3z + 6 = 0$$

102. *(a)* Give that, $|x + y| = 4$ If point (a, a) lies between the lines, then $|a + a| = 4$

$$|a| = 2$$

103. *(a)* We have, co-ordinates of A, B, C and D are $(2, 3, -1)$, $(3, 5, -3)$, $(1, 2, 3)$ and $(3, 5, 7)$ respectively.

D.R.'s of $AB = \{(3 - 2), (5 - 3), (-3 + 1)\}$

$$= \{1, 2, -2\}$$

D.R.'s of $CD = \{(3 - 1), (5 - 2), (7 - 3)\}$

$$= (2, 3, 4)$$

$\therefore$ Angle between AB and CD is given by

$$\cos\theta = \frac{a_1a_2 + b_1b_2 + c_1c_2}{\sqrt{a_1^2 + b_1^2 + c_1^2}\sqrt{a_2^2 + b_2^2 + c_2^2}}$$

$$= \frac{2 \times 1 + 3 \times 2 + (-2) \times 4}{\sqrt{1 + 4 + 4}\sqrt{4 + 9 + 16}}$$

$$\Rightarrow \quad \cos\theta = \frac{8 - 8}{3\sqrt{29}} = 0 \Rightarrow \theta = \frac{\pi}{2}$$

104. *(c)* Total number of ways to form the numbers of five digits with 1, 2, 3, 4, 5 are $= {}^5P_5 = 5! = n(S)$

Total number of numbers which are divisible by 4, $n(E) = 3! \times 4 = 4!$

$\therefore$ Required probability $= \dfrac{n(E)}{n(S)}$

$$= \frac{4!}{5!} = \frac{4 \times 3 \times 2 \times 1}{5 \times 4 \times 3 \times 2 \times 1} = \frac{1}{5}$$

105. *(b)* $n(S) = 36$.

The probability of throwing 9 with two dice

$$\{(3, 6), (4, 5), (5, 4), (6, 3)\} = \frac{4}{36} = \frac{1}{9}$$

$\therefore$ probability for not throwing $9 = 1 - \dfrac{1}{9} = \dfrac{8}{9}$

$\therefore$ If A is to win, he should throw dice alternately (*i.e.*, 1st, 3rd and so on) to get 9 and obviously for B is to win. He should throw dice in 2nd, 4th and so on.

$\therefore$ Probability that 'B' wins

$$= \left(\frac{8}{9}\right)^1 \cdot \frac{1}{9} + \left(\frac{8}{9}\right)^3 \cdot \frac{1}{9} + \ldots$$

$$= \frac{\left(\frac{8}{9}\right) \times \frac{1}{9}}{1 - \left(\frac{8}{9}\right)^2} = \frac{8}{17}$$

106. *(d)* Probability of being 53 Sundays in a leap year is $\dfrac{2}{7}$ and probability of 53 Sundays in a non-leap year is $\dfrac{1}{7}$.

$\therefore$ Required probability of that in a year of the 22nd century chosen at random having 53 Sundays $= \dfrac{3}{4} \times \dfrac{1}{7} + \dfrac{1}{4} \times \dfrac{2}{7} = \dfrac{3}{28} + \dfrac{2}{28} = \dfrac{5}{28}$

107. *(a)* Given that, the standard deviation (S.D.) of the variable x is 10.

Standard deviation of $50 + 5x = 5|\sigma_x|$

$$= 50 \qquad [\because \sigma_x = 10]$$

108. *(c)* We have, $\left(\dfrac{3 - 4ix}{3 + 4ix}\right) = \alpha - i\beta, \alpha, \beta \in R$

$$\Rightarrow \quad \alpha - i\beta = \left(\frac{3 + i(-4x)}{3 + i(4x)}\right)$$

Now, taking modulus on both sides, we get

$$|\alpha + i(-\beta)| = \left|\frac{3 + i(-4x)}{3 + i(4x)}\right|$$

$$= \frac{|3 + i(-4x)|}{|3 + i(4x)|}$$

$$\Rightarrow \quad \alpha^2 + \beta^2 = \frac{9 + 16x^2}{9 + 16x^2} \Rightarrow \alpha^2 + \beta^2 = 1$$

109. *(c)* Given expression is

$$\frac{p + q\omega + r\omega^2}{r + p\omega + q\omega^2} + \frac{p + q\omega + r\omega^2}{q + r\omega + p\omega^2}$$

$$= \frac{\omega(p + q\omega + r\omega^2)}{(q\omega + r\omega^2 + p)} + \frac{\omega^2(p + q\omega + r\omega^2)}{(r\omega^2 + q\omega + p)}$$

$$= \omega + \omega^2 = -1$$

110. *(b)* We have,

$P = 4 + i, Q = 1 + 6i, R = -4 + 3i, S = -1 - 2i$

Now,

$|PQ| = \sqrt{(4-1)^2 + (1-6)^2} = \sqrt{9 + 25} = \sqrt{34}$

$|QR| = \sqrt{(1+4)^2 + (6-3)^2} = \sqrt{25 + 9} = \sqrt{34}$

$|RS| = \sqrt{(-4+1)^2 + (3+2)^2} = \sqrt{9 + 25} = \sqrt{34}$

$|SP| = \sqrt{(-1-4)^2 + (-2-1)^2} = \sqrt{25 + 9} = \sqrt{34}$

$\Rightarrow |PQ| = |QR| = |RS| = |SP|$

$\therefore$ It is a square.

111. *(c)* $(1 + i)^n + (1 - i)^n$

$$= 2^{n/2}\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right)^n + 2^{n/2}\left(\cos\frac{\pi}{4} - i\sin\frac{\pi}{4}\right)^n$$

$$= 2^{n/2}\left[\cos\frac{n\pi}{4} + i\sin\frac{n\pi}{4} + \cos\frac{n\pi}{4} - i\sin\frac{n\pi}{4}\right]$$

$$= 2^{n/2}\left[2\cos\frac{n\pi}{4}\right]$$

$$= 2^{\frac{n}{2}+1}\cos\left(\frac{n\pi}{4}\right) = 2^{\frac{(n+2)}{2}}\cos\left(\frac{n\pi}{4}\right)$$

$$= (\sqrt{2})^{n+2}\cos\left(\frac{n\pi}{4}\right)$$

112. *(a)* Since each group has 3 persons hence, required number of ways

$$= \frac{9!}{(3!)^3} = \frac{362880}{6 \times 6 \times 6} = 1680$$

113. *(a)* Given that the word is CRICKET.

Total number of letters are 7 out of which two letters 'C' are count as one.

$\therefore$ Required number of ways of words before the word CRICKET $= 5! \times 4 + 2 \times 4! + 2!$

$$= 480 + 2 \times 4 \times 3 \times 2 + 2$$

$$= 480 + 48 + 2 = 530$$

114. *(b)* Given that the sum of the coefficient in the expansion of $(x + y)^n = 1024$

i.e., $2^n = 1024$

$\Rightarrow$ $2^n = 2^{10} \Rightarrow n = 10$

Since, n is even, hence greatest coefficient

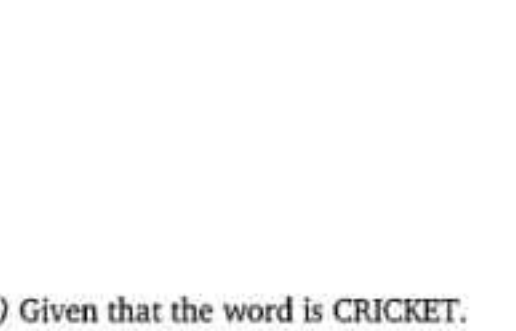
$= {}^nC_{n/2} = {}^{10}C_5$

$$= \frac{10!}{5! \times 5!} = \frac{10 \times 9 \times 8 \times 7 \times 6}{5 \times 4 \times 3 \times 2 \times 1} = 252$$

115. *(b)* Total drink = 3 (*i.e.,* milk, coffee, tea).

10 students take all 3 drinks

i.e. $n(M \cap C \cap T) = 10$



Only 20 students take milk and coffee

i.e., $n(M \cup C) = 20$

and 30 students take coffee and tea

i.e., $n(C \cup T) = 30$

and 25 students take milk and tea

i.e., $n(M \cup T) = 25$

12 students take milk only

i.e., $n(M) = 12$

5 students take coffee only

i.e., $n(C) = 5$

and 8 students take tea only

i.e., $n(T) = 8$

Total number of students who take any of the drink is 80.

$\therefore$ The number of students who did not take any of three drinks = 100 – 80 = 20

116. *(d)* Given that, $Y = \{1, 2, 3, 4, 5\}$

$$A = \{1, 2\}$$

$$B = \{3, 4, 5\}$$

$\therefore$ $(Y \times A) = \{(1, 1), (1, 2), (2, 1), (2, 2),$

$(3,1), (3,2)(4,1), (4,2), (5,1), (5,2)\}$

and $(Y \times B) = \{(1, 3), (1, 4), (1, 5), (2, 3),$ $(2, 4), (2, 5), (3, 3), (3, 4), (3, 5), (4, 3),$ $(4, 4), (4, 5), (5, 3), (5, 4), (5, 5)\}$

Now, $(Y \times A) \cap (Y \times B) = \phi$ (null set)

117. *(c)* Given that,

$A = \{2, 3, 4, 5, \ldots\ldots\ldots, 16, 17, 18\}$

and $(a, b) = (c, d)$

$\therefore$ Equivalence class of (3, 2) is

$$\{(a, b) \in A \times A = (a, b)\, R\, (c, d)\}$$

$$= \{(a, b) \in A \times A : 2a = 3b\}$$

$$= \left\{(a, b) \in A \times A : b = \frac{2}{3}a\right\}$$

$$\left\{\left(a, \frac{2}{3}a\right) : a \in A \times A\right\}$$

$= \{(3, 2), (6, 4), (9, 6), (12, 8), (15, 10), (18, 12)\}$

$\therefore$ Number of ordered pairs of the equivalence class = 6

118. *(b)* Given that,

$$f(a + x) = b + [b^3 + 1 - 3b^2 f(x) + 3b\{f(x)\} - \{f(x)\}^3]^{1/3}$$

where $a, b \in I^+$

$$\Rightarrow \quad f(a + x) = b + (1 + \{b - f(x)\}^3)^{1/3}$$

$$\Rightarrow f(a + x) - b = \{1 - \{f(x) - b\}^3\}^{1/3}$$

$$\Rightarrow \quad \phi(a + x) = \{1 - \{\phi(x)\}^3\}^{1/3}$$

[where $\phi(x) = f(x) - b$]

$$\phi(x + 2a) = \{1 - \{\phi(x + a)\}^3\}^{1/3} = \phi(x)$$

$$\Rightarrow \quad f(x + 2a) - b = f(x) - b$$

$$\Rightarrow \quad f(x + 2a) = f(x)$$

$\therefore$ $f(x)$ is periodic with period $2a$.

119. *(c)* Given that, $f(x) = \log_{3+x}(x^2 - 1)$

The given function is defined when $x^2 - 1$ is greater than zero

i.e., $x^2 - 1 > 0 \Rightarrow x^2 > 1$

$\Rightarrow$ $-1 > x > 1$, and $3 + x > 0$

$\Rightarrow$ $-3 < x$ and $x \neq -2$

$\therefore$ domain of the function is

$$D_f = (-3, -2) \cup (-2, -1) \cup (1, \infty)$$

120. *(b)* Given, $\cot 70^\circ + 4\cos 70^\circ$

$$= \frac{\cos 70^\circ}{\sin 70^\circ} + 4\cos 70^\circ$$

$$= \frac{\cos 70^\circ + 4\sin 70^\circ \cos 70^\circ}{\sin 70^\circ}$$

$$= \frac{\cos 70^\circ + 2\sin(180^\circ - 40^\circ)}{\sin 70^\circ}$$

$$= \frac{\cos(90^\circ - 20^\circ) + 2\sin 40^\circ}{\sin 70^\circ}$$

$$= \frac{\sin 20^\circ + \sin 40^\circ + \sin 40^\circ}{\sin 70^\circ}$$

$$= \frac{2\sin 30^\circ \cos 10^\circ + \sin 40^\circ}{\sin 70^\circ}$$

$$= \frac{2\sin 30^\circ \cos(90^\circ - 80^\circ) + \sin 40^\circ}{\sin 70^\circ}$$

$$= \frac{\sin 80^\circ + \sin 40^\circ}{\sin 70^\circ}$$

$$= \frac{2\sin 60^\circ \cos 20^\circ}{\sin 70^\circ}$$

$$= \frac{2\left(\frac{\sqrt{3}}{2}\right)\sin 70^\circ}{\sin 70^\circ} = \sqrt{3}$$

121. *(a)* Given that, $\sin x + \sin y + \sin z = -3$

$\because$ values of x, y and z lie between 0 to 2π i.e., $x, y, z \in [0, 2\pi]$

$\because$ The minimum value of sin is – 1.

$\therefore$ In between 0 to 2π, the given equation is satisfied at $x = \frac{3\pi}{2}$, $y = \frac{3\pi}{2}$, $z = \frac{3\pi}{2}$ and having only one solution

i.e., $\sin\frac{3\pi}{2} + \sin\frac{3\pi}{2} + \sin\frac{3\pi}{2} = -3$

$$\Rightarrow \quad 3\sin\frac{3\pi}{2} = -3$$

$$\Rightarrow \quad -1 = -1$$

$$\left[\because \sin\frac{3\pi}{2} = -1\right]$$

122. *(b)* We have, $\theta = \sin^{-1}x + \cos^{-1}x - \tan^{-1}x$

$$= \frac{\pi}{2} - \tan^{-1}x$$

$$\left[\because \sin^{-1}x + \cos^{-1}x = \frac{\pi}{2}\right]$$

$$= \cot^{-1}x$$

since $\quad 0 \le x \le 1$

therefore $\quad 0 \le \theta \le \frac{\pi}{4}$.

123. *(a)* We know, $A + B + C = \pi$

$$\Rightarrow \quad \frac{A+B+C}{2} = \frac{\pi}{2}$$

$$\Rightarrow \quad \frac{A+B}{2} = \frac{\pi}{2} - \frac{C}{2}$$

$$\Rightarrow \quad \tan\left(\frac{A+B}{2}\right) = \tan\left(\frac{\pi}{2} - \frac{C}{2}\right) = \cot\frac{C}{2}$$

$$\Rightarrow \quad \frac{\tan\frac{A}{2} + \tan\frac{B}{2}}{1 - \tan\frac{A}{2}\tan\frac{B}{2}} = \cot\frac{C}{2}$$

$$\left[\because \tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}\right]$$

$$\Rightarrow \quad \frac{\frac{1}{3} + \frac{2}{3}}{1 - \frac{1}{3}\times\frac{2}{3}} = \cot\frac{C}{2}$$

$$\left[\because \tan\frac{A}{2} = \frac{1}{3}, \tan\frac{B}{2} = \frac{2}{3}\text{ (given)}\right]$$

$$\Rightarrow \quad \frac{\frac{3}{3}\times 9}{(9-2)} = \cot\frac{C}{2}$$

$$\Rightarrow \quad \cot\frac{C}{2} = \frac{9}{7} \Rightarrow \tan\frac{C}{2} = \frac{7}{9}$$

124. *(a)* We have,

$$a\cos\theta + b\sin\theta = c \qquad \ldots(i)$$

$\because$ α and $\beta(\alpha \neq \beta)$ satisfy the Eq. (i)

$$\Rightarrow \quad a\cos\alpha + b\sin\alpha = c \qquad \ldots(ii)$$

and $\quad a\cos\beta + b\sin\beta = c \qquad \ldots(iii)$

Now, subtracting Eq. (iii) from Eq. (ii), we get

$$a\cos\alpha + b\sin\alpha - a\cos\beta - b\sin\beta = 0$$

$$\Rightarrow a(\cos\alpha - \cos\beta) + b(\sin\alpha - \sin\beta) = 0$$

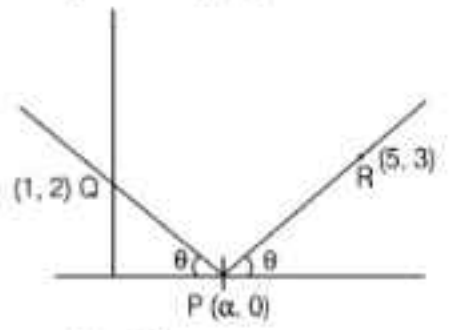
$$\Rightarrow a(\cos\alpha - \cos\beta) = -b(\sin\alpha - \sin\beta)$$

$$\Rightarrow \quad a\sin\frac{\alpha+\beta}{2} = -b\left[-\cos\left(\frac{\alpha+\beta}{2}\right)\right]$$

$$\Rightarrow \quad a\sin\frac{(\alpha+\beta)}{2} = b\cos\frac{(\alpha+\beta)}{2}$$

$$\Rightarrow \quad \tan\left(\frac{\alpha+\beta}{2}\right) = \frac{b}{a}$$

125. *(c)* Here in the figure it is shown that a ray right passing through the point $Q(1, 2)$ and reflected from a point $P(\alpha, 0)$ on X-axis towards points $R(5, 3)$



$\therefore$ slope of incident ray

(*i.e.*, before reflection)

$$\Rightarrow \quad \tan(\pi - \theta) = \frac{0-2}{\alpha - 1} \Rightarrow -\tan\theta = \frac{-2}{\alpha - 1}$$

$$\Rightarrow \quad \tan\theta = \frac{2}{\alpha - 1} \qquad \ldots(i)$$

Similarly, slope of reflected ray

(*i.e.*, after reflection)

$$\Rightarrow \quad \tan\theta = \frac{3}{5-\alpha} \qquad \ldots(ii)$$

From Eq. (i) and (ii), we get

$$\Rightarrow \quad \frac{2}{\alpha - 1} = \frac{3}{5-\alpha}$$

$$\Rightarrow \quad 10 - 2\alpha = 3\alpha - 3 \Rightarrow \alpha = 13/5$$

126. *(c)* Given that, $4x^2 - 24xy + 11y^2 = 0$

On comparing with $ax^2 + 2hxy + by^2 = 0$,

we get $a = 4$, $b = 11$ and $h = -12$

$\therefore$ using $h^2 - ab = (-12)^2 - 4 \times 11$

$= 144 - 44 = 100$

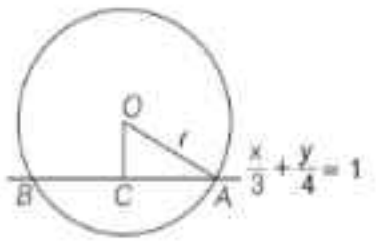
$\therefore$ The two lines represented by given equation will be real and distinct which represent a pair of straight lines passing through the origin.

127. *(b)* We have, equations of line and circle are

$$\frac{x}{3} + \frac{y}{4} = 1 \qquad \ldots(i)$$

and $$x^2 + y^2 = \frac{169}{25}$$

$$\Rightarrow \quad 25x^2 + 25y^2 = 169 \qquad \ldots(ii)$$



$\therefore$ Length of the perpendicular from centre (0, 0) of circle to the line (i) = 12/5 and radius of the circle = 13/5

$\therefore$ Required length $= 2\sqrt{\frac{169}{25} - \frac{144}{25}} = 2$

128. *(d)* Given equation is $y^2 = 8x \qquad \ldots(i)$

$$2y\frac{dy}{dx} = 8 \Rightarrow \frac{dy}{dx} = \frac{4}{y}$$

$$\Rightarrow \quad \left(\frac{dy}{dx}\right)_{(2,4)} = 1$$

$\therefore$ The equation of normal at the point (2, 4) and slope -1, is $y - 4 = -1(x - 2)$

$$\Rightarrow \quad x + y - 6 = 0 \qquad \ldots(ii)$$

$\therefore$ This line intersect the Eq. (i). Solving Eqs. (i) and (ii), we get

$$(6 - x)^2 = 8x$$

$$\Rightarrow \quad x^2 + 36 - 12x = 8x$$

$$\Rightarrow \quad x^2 - 20x + 36 = 0$$

$$\Rightarrow \quad x = 2, 18$$

Point $x = 2$ is already taken, now

$$x = 18 \Rightarrow y = \pm 12$$

Here, we take $y = -12$ because second point will be below the X-axis.

$\therefore$ Required point is $(18, -12)$.

129. *(c)* Obviously ellipse because the normal and tangent at point P of an ellipse bisect the internal and external angles between the focal distance of the point.

130. *(d)* Given that, equation of line and hyperbola are

$$x + y = \sqrt{2}p \qquad \ldots(i)$$

and $$4x^2 - 9y^2 = 36 \qquad \ldots(ii)$$

Condition of any line to touch any hyperbola is $c^2 = a^2 m^2 - b^2 \qquad \ldots(iii)$

From Eqs. (i) and (ii), we have

$$m = -1, c = \sqrt{2}\,p, a^2 = 9, b^2 = 4$$

$$\therefore \quad c^2 = a^2m^2 - b^2$$

$$\Rightarrow \quad (\sqrt{2}p)^2 = 9 \times (-1)^2 - 4$$

$$\Rightarrow \quad 2p^2 = 9 - 4 \Rightarrow 2p^2 = 5$$

131. *(c)* Given that,

$$f(x) = \frac{1 - \sin x + \cos x}{1 + \sin x + \cos x}$$

$$\Rightarrow f(x) = \frac{(1 + \cos x) - \sin x}{(1 + \cos x) + \sin x}$$

$$= \frac{2\cos^2(x/2) - 2\sin(x/2)\cos(x/2)}{2\cos^2(x/2) + 2\sin(x/2)\cos(x/2)}$$

$$= \frac{\cos(x/2) - \sin(x/2)}{\cos(x/2) + \sin(x/2)}$$

$$= \frac{1 - \tan(x/2)}{1 + \tan(x/2)} = \tan\left(\frac{\pi}{4} + \frac{x}{2}\right)$$

at $\quad x = \pi$

$$\Rightarrow f(\pi) = -\cot\frac{\pi}{4} = -1$$

132. *(b)* We have,

$$f(x) = \sin^2 x$$

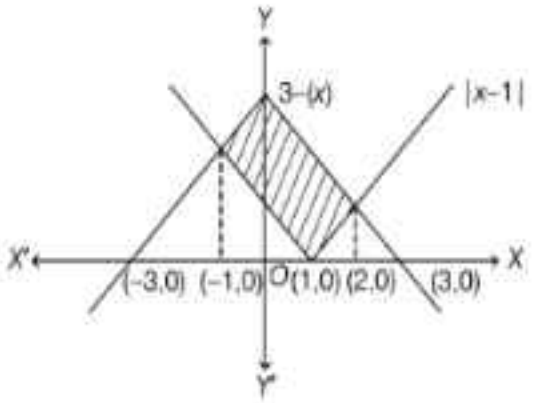
and $$g(f(x)) = |\sin x|$$

$$\Rightarrow \quad g(\sin^2 x) = |\sin x|$$

$$\Rightarrow \quad g(\sin^2 x) = \sqrt{\sin^2 x}$$

$$\therefore \quad g(x) = \sqrt{x}$$

133. *(d)* Given that,



$$y = |x-1| \qquad \ldots(i)$$

and $$y = 3-|x| \qquad \ldots(ii)$$

Solving the Eqs. (i) and (ii)

if $x \geq 1$, then $3 - x = x - 1 \Rightarrow x = 2$

if $0 \leq x \leq 1$, then $3 - x = 1 - x$

(which is not possible)

Also, if $x < 0$, then $3 + x = 1 - x \Rightarrow x = -1$

Thus, required area

$$= \int_{-1}^{2} (3 - |x| - |x-1|)\, dx$$

$$= \int_{-1}^{0} [3 + x - (1-x)]\, dx + \int_{0}^{1} [(3-x) - (1-x)]\, dx + \int_{1}^{2} [(3-x) - (x-1)]\, dx$$

$$= \int_{-1}^{0} (2 + 2x)\, dx + 2\int_{0}^{1} 1\, dx + \int_{1}^{2} (4 - 2x)\, dx$$

$$= [2x + x^2]_{-1}^{0} + [4x - x^2]_{1}^{2}$$

$$= 1 + 2 + 1 = 4 \text{ sq units}$$

134. *(b)* Given that, $x = \left[\frac{a+2b}{a+b}\right]$ and $y = \frac{a}{b}$

$$\therefore \quad x = \frac{a+2b}{a+b} = \frac{\frac{a}{b}+2}{1+\frac{a}{b}}$$

$$= 1 + \frac{1}{\frac{a}{b}+1} \Rightarrow x = 1 + \frac{1}{y+1}$$

$$[\because y = \frac{a}{b} \text{ and } y^2 > 2 \text{ (given)}]$$

which shows $x^2 < 2$.

135. *(d)* Let $I = \int_0^1 \tan^{-1} \frac{1}{(x^2 - x + 1)}\, dx$

$$= \int_0^1 \tan^{-1} \left(\frac{x-(x-1)}{1+x(x-1)}\right)$$

$$= \int_0^1 \tan^{-1} x\, dx - \int_0^1 \tan^{-1} (x-1)\, dx$$

$$= 2\int_0^1 \tan^{-1} x\, dx$$

$$\left[\because \int_0^1 \tan^{-1} (x-1)\, dx = -\int_0^1 \tan^{-1} x\, dx\right]$$

$$= 2\left[x \tan^{-1} x - \int \frac{x}{1+x^2}\, dx\right]_0^1$$

$$= 2\left[x \tan^{-1} x - \frac{1}{2} \log (1+x^2)\right]_0^1$$

$$= \frac{\pi}{2} - \log 2$$

136. *(b)* Given that, equation of curves are

$$x = \log (y + e) \qquad \ldots(i)$$

and $$y = \log \left(\frac{1}{x}\right) \qquad \ldots(ii)$$

$\therefore$ From Eq. (i), we get

$$e^x = (y + e) \qquad \ldots(iii)$$

Put the value of 'y' from Eq. (ii) in Eq. (iii), we get

$$e^x = \log \left(\frac{1}{x}\right) + e = \log x^{-1} + e$$

$$= -\log x + e$$

$$\Rightarrow \quad \log x = -e^x + e \Rightarrow x = 1 \therefore y = 0$$

$\therefore$ Point is (1, 0).

$\therefore$ Curves meet at one point.

137. *(d)* $\lim_{x \to 0} \frac{\cos (\sin x) - 1}{x^2}$

Now, using L-Hopital's rule,

$$= \lim_{x \to 0} \frac{-\sin (\sin x) \cdot \cos x}{2x}$$

Again using L-Hopital's rule

$$= \lim_{x \to 0} \frac{-[-\sin (\sin x) \times \sin x + \cos^2 x \times \cos (\sin x)]}{2}$$

$$= -\frac{1}{2}$$

138. *(a)* Given that, $\mathbf{a}, \mathbf{b}, \mathbf{c}$ are three vectors and

$\mathbf{a} \times (\mathbf{b} \times \mathbf{c}) = (\mathbf{a} \times \mathbf{b}) \times \mathbf{c}$

$\Rightarrow \quad (\mathbf{a} \cdot \mathbf{c})\mathbf{b} - (\mathbf{a} \cdot \mathbf{b})\mathbf{c} = (\mathbf{a} \cdot \mathbf{c})\mathbf{b} - (\mathbf{b} \cdot \mathbf{c})\mathbf{a}$

$\Rightarrow \quad -(\mathbf{a} \cdot \mathbf{b})\mathbf{c} = -(\mathbf{b} \cdot \mathbf{c})\mathbf{a}$

$\Rightarrow (\mathbf{b} \cdot \mathbf{c})\mathbf{a} - (\mathbf{b} \cdot \mathbf{a})\mathbf{c} = 0$

$\Rightarrow \quad \mathbf{b} \times (\mathbf{a} \times \mathbf{c}) = 0$

139. *(c)* Given that $|\mathbf{a}| = a$

Let $\quad \mathbf{a} = a_1 \hat{\mathbf{i}} + a_2 \hat{\mathbf{j}} + a_3 \hat{\mathbf{k}}$

$\therefore \quad a^2 = a_1^2 + a_2^2 + a_3^2$

Now $\quad \hat{\mathbf{i}} \times \mathbf{a} = \begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ 1 & 0 & 0 \\ a_1 & a_2 & a_3 \end{vmatrix}$

$= -a_3 \hat{\mathbf{j}} + a_2 \hat{\mathbf{k}}$

$|\hat{\mathbf{i}} \times \mathbf{a}|^2 = a_3^2 + a_2^2$

Similarly, $\quad |\hat{\mathbf{j}} \times \mathbf{a}|^2 = a_1^2 + a_3^2$

and $\quad |\hat{\mathbf{k}} \times \mathbf{a}|^2 = a_1^2 + a_2^2$

$\therefore \quad |\hat{\mathbf{i}} \times \mathbf{a}|^2 + |\hat{\mathbf{j}} \times \mathbf{a}|^2 + |\hat{\mathbf{k}} \times \mathbf{a}|^2$

$= a_1^2 + a_2^2 + a_2^2 + a_3^2 + a_3^2 + a_1^2$

$= 2(a_1^2 + a_2^2 + a_3^2) = 2a^2$

140. *(b)* Given that the area bounded by the curves $y = 2^{kx}$ and $x = 0$, $x = 2$ is $\dfrac{3}{\log 2}$

$$A = \int_0^2 y\,dx = \frac{3}{\log 2}$$

$$\Rightarrow \quad \int_0^2 2^{kx}\,dx = \frac{3}{\log 2}$$

$$\Rightarrow \quad \left[\frac{2^{kx}}{\log_e 2}\right]_0^2 = \frac{3}{\log 2}$$

$$\Rightarrow \frac{2^{2k}}{\log_e 2} - \frac{1}{\log_e 2} = \frac{3}{\log 2}$$

$$\Rightarrow \quad 2^{2k} - 1 = 3$$

$$\Rightarrow \quad 2^{2k} = 2^2 \Rightarrow 2k = 2 \Rightarrow k = 1$$

141. *(c)* Given that,

$$I = \int_a^b \frac{x}{|x|}\,dx, \forall\ a < b < 0$$

or $\quad a < 0, b < 0$

$\therefore$ By using the property of definite integrals, we get

$$I = \int_a^b \frac{x}{|x|}\,dx = \int_a^b -1\,dx$$

$$\Rightarrow \quad I = -[x]_a^b = [-b + a]$$

$$= |a| - |b| \qquad (\because a < b < 0)$$

142. *(c)* We have,

$$I = \int_{-2}^{2} \left[p \log\left(\frac{1+x}{1-x}\right) + q \log\left(\frac{1-x}{1+x}\right)^{-2} + r \right] dx$$

Since, $\log\left(\dfrac{1+x}{1-x}\right)$ is odd function

$$\therefore \quad \int_{-2}^{2} \log\left(\frac{1+x}{1-x}\right) dx = 0$$

$$\Rightarrow \quad I = \int_{-2}^{2} r\,dx = r[x]_{-2}^{2} = r(2+2) = 4r$$

143. *(c)* Slope of tangent at some point

= 2 × (Slope of the straight line passing through same point)

i.e., $\quad \dfrac{dy}{dx} = 2\dfrac{y}{x} \Rightarrow \int \dfrac{dy}{y} = 2\int \dfrac{dx}{x}$

$$\Rightarrow \quad \log y = 2 \log x + \log c$$

$$\Rightarrow \quad \log y = \log x^2 + \log c$$

$$\Rightarrow \quad y = c\,x^2$$

which represent a parabola of the form

$$x^2 = 4ay.$$

144. *(a)* Given that,

$$\frac{dy}{dx} + \frac{\sqrt{1-y^2}}{1-x^2} = 0$$

$$\Rightarrow \quad \int \frac{dy}{\sqrt{1-y^2}} = -\int \frac{dx}{\sqrt{1-x^2}}$$

$$\Rightarrow \quad \sin^{-1} y = -\sin^{-1} x + \sin^{-1} a$$

$$\Rightarrow \sin^{-1}[x\sqrt{1-y^2} + y\sqrt{1-x^2}] = \sin^{-1} a$$

$$\Rightarrow \quad x\sqrt{1-y^2} + y\sqrt{1-x^2} = a$$

145. *(c)* Given that, $x = at^2 + bt + c$

On differentiating w.r. to 't' we get

(speed) $\quad \dfrac{dx}{dt} = 2at + b$

Again differentiating, we get

(acceleration) $\frac{d^2x}{dt^2} = 2a$

$\therefore$ The particle would be moving with uniform acceleration.

146. *(d)* Given that, $x^2 \frac{d^2y}{dx^2} = \log x$

$$\Rightarrow \quad \frac{d^2y}{dx^2} = \frac{\log x}{x^2}$$

On integrating, we get

$$\frac{dy}{dx} = \frac{-(\log x + 1)}{x} + c$$

$$\therefore \quad \left(\frac{dy}{dx}\right)_{(1,0)} = -1$$

$$\Rightarrow \quad -1 = \frac{-1}{1} + c$$

$$\Rightarrow \quad c = 0$$

$$\therefore \quad \frac{dy}{dx} = -\frac{(\log x + 1)}{x} + 0$$

Again integrating,

$$\Rightarrow \quad y = -\frac{1}{2}(\log x)^2 - \log x + c_1$$

at $x = 1, y = 0$

$$\Rightarrow \quad a = 0$$

$$\therefore \quad y = -\frac{1}{2}(\log x)^2 - \log x$$

147. *(b)* $\because$ **a** and **b** are perpendicular to each other.

$$\mathbf{a} \cdot \mathbf{b} = 0$$

And **c** is inclined at an angle θ to both **a** and **b**.

i.e., $\mathbf{a} \cdot \mathbf{c} = \mathbf{b} \cdot \mathbf{c} = \cos\theta$

Now, $\mathbf{c} = \alpha\,\mathbf{a} + \beta\mathbf{b} + \gamma(\mathbf{a} \cdot \mathbf{b})$

$$\mathbf{a} \cdot \mathbf{c} = \alpha\,\mathbf{a} \cdot \mathbf{a} + \beta\,\mathbf{a} \cdot \mathbf{b} + \gamma\,\mathbf{a} \cdot (\mathbf{a} \cdot \mathbf{b})$$

$\cos\theta = \alpha + 0 + 0$ $\quad (\because \mathbf{a} \cdot \mathbf{b} = 0)$

and $\mathbf{b} \cdot \mathbf{c} = \alpha\,\mathbf{a} \cdot \mathbf{b} + \beta\,\mathbf{b} \cdot \mathbf{b} + \gamma(\mathbf{a} \cdot \mathbf{b}) \cdot \mathbf{b}$

$$\Rightarrow \quad \cos\theta = \beta$$

and γ^2 may be $\cos 2\theta$.

148. *(a)* Given that

$$y = \frac{1}{\sqrt{a^2 - b^2}} \cos^{-1}\left[\frac{a\cos(x-\alpha) + b}{\theta}\right]$$

$$\Rightarrow \cos\sqrt{a^2 - b^2}\, y = \frac{a\cos(x-\alpha) + b}{\theta} \quad \ldots(i)$$

and $\theta = a + b\cos(x - \alpha)$ $\quad \ldots(ii)$

Differentiating Eq. (ii) w.r.t. x, we get

$$\frac{d\theta}{dx} = -b\sin(x-\alpha) \quad \ldots(iii)$$

Differentiating Eq. (i) w.r.t. x, we get

$$-\sin(\sqrt{a^2 - b^2}\,y)\sqrt{a^2 - b^2}\,\frac{dy}{dx}$$

$$= \frac{\theta[-a\sin(x-\alpha)] - (a\cos(x-\alpha) + b)\frac{d\theta}{dx}}{\theta^2}$$

$$= \frac{\theta(-a\sin(x-\alpha)) + (a\cos(x-\alpha) + b)(-b\sin(x-\alpha))}{\theta^2}$$

$$= \sin(x-\alpha)\frac{[-a^2 - ab\cos(x-\alpha) + ab\cos(x-\alpha) + b^2]}{\theta^2}$$

$$= \frac{\sin(x-\alpha)(b^2 - a^2)}{\theta^2}$$

$$\Rightarrow \frac{dy}{dx} = \frac{(a^2 - b^2)\sin(x-\alpha)}{\sqrt{a^2 - b^2}\sin(\sqrt{a^2 - b^2})y}\left(\frac{1}{\theta^2}\right)$$

$$= \frac{\sqrt{a^2 - b^2}\sin(x-\alpha)}{\theta^2\sqrt{(1 - \cos^2\sqrt{a^2 - b^2}\,y)}}$$

$$= \frac{\sqrt{a^2 - b^2}\sin(x-\alpha)}{\theta^2\sqrt{\frac{\theta^2 - (a\cos(x-\alpha) + b)^2}{\theta^2}}}$$

$$= \frac{\sqrt{a^2 - b^2}\sin(x-\alpha)}{\theta\, a^2 + (b^2\cos^2(x-\alpha) + 2ab\cos(x-\alpha) - (a^2\cos^2(x-\alpha) + b^2 + 2ab\cos(x-\alpha))}$$

$$= \frac{\sqrt{a^2 - b^2}\sin(x-\alpha)}{\theta\sqrt{a^2 - b^2}\sin(x-\alpha)} \Rightarrow \frac{dy}{dx} = \frac{1}{\theta}$$

149. *(b)* We have,

$$|f(x)-f(y)|\le|x-y|^5$$

$$\Rightarrow \frac{|f(x)-f(y)|}{|x-y|}\le|x-y|^4$$

Put $x = y + h$

$$\therefore \quad \frac{|f(y+h)-f(y)}{|h|}\le|h|^4$$

$$\Rightarrow \quad \left|\frac{f(y+h)-f(y)}{h}\right|\le|h|^4$$

$$\Rightarrow \quad \lim_{h\to0}\left|\frac{f(y+h)-f(y)}{h}\right|\le\lim_{h\to0}|h|^4$$

$$\Rightarrow \quad |f'(y)|\le 0$$

$$\Rightarrow \quad |f'(y)| = 0 \qquad [\because |x|\ge 0]$$

$$\Rightarrow \quad f'(y) = 0$$

$$\Rightarrow \quad f(y) = \text{Constant}$$

$$\therefore \quad f(9) = f(3) = 7$$

150. *(c)* We have,

$$f(x)=\frac{1}{1-x}$$

$$f[f(f(x))] = f\left[f\left(\frac{1}{1-x}\right)\right]$$

$$= f\left(\frac{1}{1-\frac{1}{1-x}}\right) = f\left[\frac{1-x}{-x}\right]$$

$$= \frac{1}{1+\left(\frac{1-x}{x}\right)}$$

$$\Rightarrow \quad f[f\{f(x)\}] = x$$

$\therefore$ The derivative of composite function is equal to 1.