

# **VITEEE 2009 Question Paper**

**Vellore Institute of Technology Engineering Entrance Examination**

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# SOLVED PAPER

VITEEE

2009

## PART - I (PHYSICS)

- When a wave traverses a medium the displacement of a particle located at  $x$  at a time  $t$  is given by  $y = a \sin (bt - cx)$ . Where  $a$ ,  $b$  and  $c$  are constants of the wave. Which of the following is a quantity with dimensions?
  - $\frac{y}{a}$
  - $bt$
  - $cx$
  - $c$
- A body is projected vertically upwards at time  $t = 0$  and is seen at a height  $H$  at time  $t$  second during its flight. The maximum height attained is ( $g$  is acceleration due to gravity)
  - $\frac{g(t^2 - t_1^2)}{8}$
  - $\frac{g(t_1^2 - t_2^2)}{4}$
  - $\frac{g(t_1^2 - t_2^2)}{8}$
  - $\frac{g(2t - t_1)^2}{4}$
- A particle is projected up from a point at an angle with the horizontal direction. At any time  $t$ , if  $p$  is the linear momentum,  $y$  is the vertical displacement,  $x$  is horizontal displacement, the graph among the following which does not represent the variation of kinetic energy KE of the particle is
 

(A)

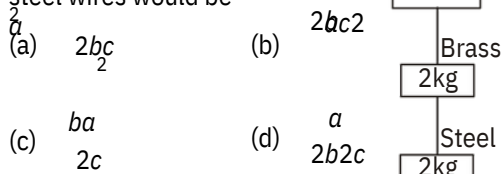
(B)

(C)

(D)

  - graph (A)
  - graph (B)
  - graph (C)
  - graph (D)
- OA is a horizontal pipe of length  $L$  and cross-sectional area  $A$  through which water is flowing at a certain rate. If the rate of flow of water through the pipe is increased to  $n$  times, the power of the motor is increased to  $P_1$ . The ratio of  $P_1$  to  $P_0$  is
  - $n : 1$
  - $n^2 : 1$
  - $n^3 : 1$
  - $n^4 : 1$
- A body of mass  $5 \text{ kg}$  moving with a velocity of  $10 \text{ m/s}$  collides elastically with another body at rest and continues to move in the original direction after collision with a velocity equal to  $\frac{1}{10}$ th of its original velocity. Then the mass of the second body is
  - $4.09 \text{ kg}$
  - $0.5 \text{ kg}$
  - $5 \text{ kg}$
  - $5.09 \text{ kg}$
- A particle of mass  $4 \text{ m}$  explodes into three pieces of masses  $m$ ,  $m$  and  $2m$ . The equal masses move along X-axis and Y-axis with velocities  $4 \text{ ms}^{-1}$  and  $6 \text{ ms}^{-1}$  respectively. The magnitude of the velocity of the heavier mass is
  - $\sqrt{17} \text{ ms}^{-1}$
  - $2\sqrt{13} \text{ ms}^{-1}$
  - $13 \text{ ms}^{-1}$
  - $\frac{\sqrt{13}}{2} \text{ ms}^{-1}$
- A body is projected vertically upwards from the surface of the earth with a velocity equal to half the escape velocity. If  $R$  is the radius of the earth, maximum height attained by the body from the surface of the earth is
  - $\frac{R}{6}$
  - $\frac{R}{3}$
  - $\frac{2R}{3}$
  - $R$
- The displacement of a particle executing SHM is given by
 
$$y = 5 \sin 4t$$
 If  $T$  is the time period and the mass of the particle is  $2 \text{ g}$ , the kinetic energy of the particle when  $t = T/4$  is given by
  - $0.4 \text{ J}$
  - $0.5 \text{ J}$
  - $3 \text{ J}$
  - $0.3 \text{ J}$

9. If the ratio of lengths, radii and Young's modulus of steel and brass wires shown in the figure are  $a$ ,  $b$  and  $c$  respectively, the ratio between the increase in lengths of brass and steel wires would be



10. A soap bubble of radius  $r$  is blown up to form a bubble of radius  $2r$  under isothermal conditions. If  $T$  is the surface tension of soap solution, the energy spent in the blowing

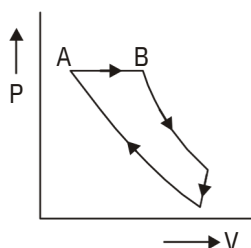
- (a)  $4\pi r^2 T$  (b)  $8\pi r^2 T$  (c)  $4\pi r^2 T$  (d)  $8\pi r^2 T$
11. Two soap bubbles of different radii are falling through air with terminal speed of  $6 \text{ cm s}^{-1}$ . If they coalesce to form one bigger drop, what will be the terminal speed of bigger drop? (Neglect the buoyancy of the air)

- (a)  $3 \text{ cm s}^{-1}$  (b)  $6 \text{ cm s}^{-1}$  (c)  $12 \text{ cm s}^{-1}$  (d)  $18 \text{ cm s}^{-1}$
12. A pendulum of length  $1 \text{ m}$  has a period of  $2 \text{ s}$  at  $20^\circ \text{C}$ . If it is used in a climate where the temperature averages to  $30^\circ \text{C}$ , how much time does the clock lose in each oscillation?

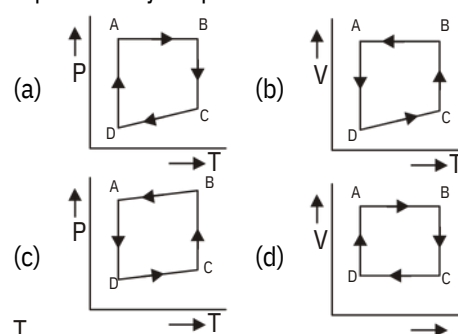
(For invar,  $\alpha = 9 \times 10^{-7} / ^\circ \text{C}$ ,  $g = \text{constant}$ )

- (a)  $2.25 \times 10^{-6} \text{ s}$  (b)  $2.5 \times 10^{-7} \text{ s}$  (c)  $5 \times 10^{-7} \text{ s}$  (d)  $1.125 \times 10^{-6} \text{ s}$
13. A piece of metal weighs  $45 \text{ g}$  in air and  $25 \text{ g}$  in a liquid of density  $1.5 \times 10^3 \text{ kg m}^{-3}$  kept at  $30^\circ \text{C}$ . When the temperature of the liquid is raised to  $40^\circ \text{C}$ , the metal piece weighs  $27 \text{ g}$ . The density of liquid of  $40^\circ \text{C}$  is  $1.25 \times 10^3 \text{ kg m}^{-3}$ . The coefficient of linear expansion of metal is

- (a)  $2 \times 10^{-5} / ^\circ \text{C}$  (b)  $3 \times 10^{-5} / ^\circ \text{C}$  (c)  $4 \times 10^{-5} / ^\circ \text{C}$  (d)  $5 \times 10^{-5} / ^\circ \text{C}$
14. A gas is subjected to a cyclic process as shown in the figure below



Which of the following curves represents the equivalent cyclic process?



An ideal gas is subjected to cyclic process involving four thermodynamic states, the amounts of heat ( $Q$ ) and work ( $W$ ) involved in each of these states are

- (a)  $Q_1 = 1400 \text{ J}$ ;  $W_1 = 630 \text{ J}$ ;  $Q_2 = -5500 \text{ J}$ ;  $W_2 = -1000 \text{ J}$ ;  $Q_3 = -3000 \text{ J}$ ;  $W_3 = -1200 \text{ J}$ ;  $Q_4 = 2500 \text{ J}$ ;  $W_4 = 1000 \text{ J}$

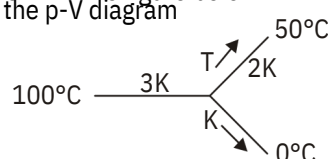
The ratio of the net work done by the gas to the total heat absorbed by the gas is  $x\%$ . The value of  $x$  is

- (a)  $500$ ;  $7.5\%$  (b)  $700$ ;  $10.5\%$  (c)  $1000$ ;  $21\%$  (d)  $1500$ ;  $15\%$

Two cylinders A and B fitted with pistons contain equal number of moles of an ideal monoatomic gas at  $400 \text{ K}$ . The piston of A is free to move while that of B is held fixed. Same amount of heat energy is given to the gas in each cylinder. If the rise in temperature of the gas in A is  $42 \text{ K}$ , the rise in temperature of the gas in B is

- (a)  $21 \text{ K}$  (b)  $35 \text{ K}$  (c)  $42 \text{ K}$  (d)  $70 \text{ K}$

Three rods of same dimensional have thermal conductivity  $3 \text{ K}$ ,  $2 \text{ K}$  and  $1 \text{ K}$ . They are arranged as shown in the figure below



Then, the temperature of the junction in steady state is

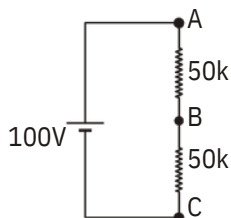
- (a)  $200/3^\circ \text{C}$  (b)  $100/3^\circ \text{C}$  (c)  $75^\circ \text{C}$  (d)  $50^\circ \text{C}$

18. Two sources A and B are sending notes of frequency 680 Hz. A listener moves from A and B with a constant velocity  $u$ . If the speed of sound in air is  $340 \text{ ms}^{-1}$ , what must be the value of  $u$  so that he hears 10 beats per second?
- (a)  $2 \text{ cm s}^{-1}$  (b)  $2 \text{ m s}^{-1}$  (c)  $2 \text{ km s}^{-1}$  (d)  $2 \text{ m s}^{-1}$
19. When a wire is stretched under the same tension. What fractional increase in the tension of one wire will lead to the occurrence of 6 beats per second when both wires vibrate simultaneously?
- (a) 0.0 (b) 0.0 (c) 1 (d) 2
20. In Young's double slit experiment, the intensities at two points P and Q are respectively  $I_1$  and  $I_2$ . The path difference between the two rays at P is  $\frac{1}{4}$  of the wavelength. The path difference between the two rays at Q is  $\frac{1}{2}$  of the wavelength. The ratio of  $I_1$  to  $I_2$  is
- (a) 2 (b)  $\frac{1}{2}$  (c) 4 (d) 16
21. In Young's double slit experiment, the 10th maximum of wavelength  $\lambda_1$  is at a distance of  $y_1$  from the central maximum. When the wavelength of the source is changed to  $\lambda_2$ , 5th maximum is at a distance of  $y_2$  from its central maximum. The ratio  $\frac{y_1}{y_2}$  is
- (a)  $\frac{2}{1}$  (b)  $\frac{2}{2}$  (c)  $\frac{1}{2}$  (d)  $\frac{2}{1}$
22. Four light sources produce the following four wave s:
- (i)  $y_1 = a \sin(\omega t)$   
 (ii)  $y_2 = a \sin(2\omega t)$   
 (iii)  $y_3 = a \sin(\omega t + \frac{\pi}{2})$   
 (iv)  $y_4 = a \sin(3\omega t)$
23. The two lenses of an achromatic doublet should have
- (a) equal powers (b) equal dispersive powers (c) equal ratio of their power and dispersive power (d) sum of the product of their powers and dispersive power equal to zero
24. Two bar magnets A and B are placed one over the other and are allowed to vibrate in a vibration magnetometer. They make 20 oscillations per minute when the similar poles of A and B are on the same side, while they make 15 oscillations per minute when their opposite poles lie on the same side. If  $M_A$  and  $M_B$  are the magnetic moments of A and B and if  $M_A$  and  $M_B$  is
- (a) 3 (b) 25 : 7 (c) 7 : 5 (d) 25 : 16
25. A bar magnet is 10 cm long is kept with its north (N)-pole pointing north. A neutral point is formed at a distance of 15 cm from each pole. Given the horizontal component of earth's field is 0.4 Gauss, the pole strength of the magnet is
- (a) 9 A-m (b) 6.75 A-m (c) 27 A-m (d) 1.35 A-m
26. An infinitely long thin straight wire has uniform linear charge density of  $\frac{1}{3} \text{ cm}^{-1}$ . Then, the magnitude of the electric intensity at a point 18 cm away is (given  $\epsilon_0 = 8.8 \times 10^{-12} \text{ CN}^2\text{m}^{-2}$ )
- (a)  $0.33 \times 10^{11} \text{ NC}^{-1}$  (b)  $3 \times 10^{11} \text{ NC}^{-1}$  (c)  $0.66 \times 10^{11} \text{ NC}^{-1}$  (d)  $1.32 \times 10^{11} \text{ NC}^{-1}$
27. Two point charges  $-q$  and  $+q$  are located at points  $(0, 0, -a)$  and  $(0, 0, a)$  respectively. The electric potential at a point  $(0, 0, z)$ , where  $z > a$  is
- (a)  $\frac{qa}{4\pi\epsilon_0 z^2}$  (b)  $\frac{q}{4\pi\epsilon_0 a}$  (c)  $\frac{2qa}{4\pi\epsilon_0(z^2 - a^2)}$  (d)  $\frac{2qa}{4\pi\epsilon_0(z^2 + a^2)}$

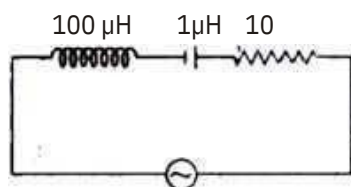
28. In the adjacent shown circuit, a voltmeter of internal resistance  $R$ , when connected across B and C reads

190 V. Neglecting the internal

resistance of the battery, the value of  $R$  is

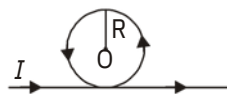


- (a) 100 k (b) 75  
(c) 50k (d) k
29. A cell in secondary circuit gives null deflection for 2.5 m length of potentiometer having 10 m length of wire. If the length of the potentiometer wire is increased by 1 m without changing the cell in the primary, the position of the null point now is
- (a) 3.5 m (b) 3 m  
(c) 2.75 m (d) 2.0 m
30. The following series L-C-R circuit, when driven by an emf source of angular frequency 70 kiloradians per second, the circuit effectively behaves like



- (a) purely resistive circuit  
(b) series R-L circuit  
(c) series R-C circuit  
(d) series L-C circuit with  $R = 0$
31. A wire of length  $l$  is bent into a circular loop of radius  $R$  and carries a current  $I$ . The magnetic field at the centre of the loop is  $B$ . The same wire is now bent into a double loop of equal radii. If both loops carry the same current  $I$  and it is in the same direction, the magnetic field at the centre of the double loop will be
- (a) Zero (b)  $2B$   
(c)  $4B$  (d)  $8B$
32. An infinitely long straight conductor is bent into the shape as shown below. It carries a current of

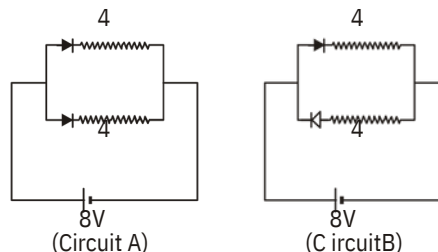
$I$  ampere and the radius of the circular loop is  $R$  metre. Then, the magnitude of magnetic induction at the centre of the circular loop is



- (a)  $\frac{0I}{2R}$  (b)  $\frac{0nI}{2R}$   
(c)  $\frac{0I}{2R} (1)$  (d)  $\frac{0I}{2R} (-1)$
33. The work function of a certain metal is  $3.31 \times 10^{-19}$  J. Then, the maximum kinetic energy of photoelectrons emitted by incident radiation of wavelength  $5000 \text{ \AA}$  is (Given,  $h = 6.62 \times 10^{-34} \text{ J-s}$ ,  $c = 3 \times 10^8 \text{ ms}^{-1}$ ,  $e = 1.6 \times 10^{-19} \text{ C}$ )
- (a) 2.48 (b) 0.41  
(c) 2.07 (d) 0.82
34. A photon of energy  $E$  ejects a photoelectron from a metal surface whose work function is  $W$ . If this electron enters into a uniform magnetic field of induction  $B$  in a direction perpendicular to the field and describes a circular path of radius  $r$ , then the radius  $r$  is given by, (in the usual notation)

- (a)  $\frac{\sqrt{2m(E-W)_0}}{eB}$  (b)  $\frac{\sqrt{2m(E-W)_0}}{eB}$   
(c)  $\frac{\sqrt{2e(E-W)_0}}{mB}$  (d)  $\frac{\sqrt{2m(E-W)_0}}{eB}$

35. Two radioactive materials  $x_1$  and  $x_2$  have decay constants 10 and 11 respectively, if initially they have the same number of nuclei, then the ratio of the number of nuclei of  $x_1$  to that of  $x_2$  will be  $1/e$  after a time
- (a)  $(1/10)$  (b)  $(1/11)$   
(c)  $11/(10)$  (d)  $1/(9)$
36. Current flowing in each of the following circuit A and B respectively are



- (a) 1A, 2A (b) 2A, 1A  
(c) 4A, 2A (d) 2A, 4A
37. A bullet of mass  $0.02 \text{ kg}$  travelling horizontally with velocity  $250 \text{ ms}^{-1}$  strikes a block of wood of mass  $0.23 \text{ kg}$  which rests on a rough horizontal surface. After the impact, the block and bullet move together and come to rest after travelling a distance of  $40 \text{ m}$ . The coefficient of sliding friction of the rough surface is ( $g = 9.8 \text{ ms}^{-2}$ )  
(a)  $0.75$  (b)  $0.81$   
(c)  $0.51$  (d)  $0.30$
38. Two persons A and B are located in X-Y plane at the points  $(0, 0)$  and  $(0, 10)$  respectively. (The distances are measured in MKS unit). At a time  $t = 0$ , they start moving simultaneously with velocities  $v_a = 2 \text{ ms}^{-1}$  and  $v_b = 2 \text{ ms}^{-1}$  respectively. The time after which A and B are at their closest distance is  
(a)  $2.5 \text{ s}$  (b)  $4 \text{ s}$   
(c)  $1 \text{ s}$  (d)  $\frac{10}{2} \text{ s}$
39. A rod of length  $l$  is held vertically stationary with its lower end located at a point P, on the horizontal plane. When the rod is released to topple about P, the velocity of the upper end of the rod with which it hits the ground is  
(a)  $\sqrt{\frac{g}{l}}$  (b)  $\sqrt{3gl}$   
(c)  $3\sqrt{\frac{g}{l}}$  (d)  $\sqrt{\frac{3g}{l}}$
40. A wheel of radius  $0.4 \text{ m}$  can rotate freely about its axis as shown in the figure. A string is wrapped over its rim and a mass of  $4 \text{ kg}$  is hung. An angular acceleration of  $8 \text{ rad-s}^{-2}$  is produced in it due to the torque. Then, moment of inertia of the wheel is ( $g = 10 \text{ ms}^{-2}$ )  
(a)  $2 \text{ kg-m}^2$  (b)  $1 \text{ kg-m}^2$   
(c)  $\text{kg-m}^2$  (d)  $\text{kg-m}^2$
41. Identify the alkyne in the following sequence of reactions,  
Alkyne  $\xrightarrow{\text{H}_2, \text{ Lindlar's catalyst}}$  A  $\xrightarrow{\text{Ozonolysis}}$  B only  
Wacker Process  $\text{CH}_2 = \text{CH}_2$   
(a)  $\text{H}_3\text{C}-\text{C}\equiv\text{C}-\text{CH}_3$   
(b)  $\text{H}_2\text{C}=\text{C}=\text{CH}_2$   
(c) H  
(d)  $\text{HCC}-\text{CH}_2$   
Fluorination with dilute NaOH and forms a gaseous product A. The bond angle in the molecule of A is  
(a)  $104^\circ 40'$   
(b)  $103^\circ$   
(c)  $107^\circ$  (d)  $109^\circ 28'$
42. One mole of alkene X on ozonolysis gave one mole of acetaldehyde and one mole of acetone. The IUPAC name of X is  
(a) 2-methyl-2-butene (b) 2-methyl-1-butene  
(c) 2-butene (d) 1-butene
43. The number of p-d 'pi' bonds present in  $\text{XeO}_3$  and  $\text{XeO}_4$  molecules, respectively are  
(a) 3, (b) 4,  
(c) 4 (d) 2
44. The wavelengths of electron waves in two orbits is  $3:5$ . The ratio of kinetic energy of electrons will be  
(a)  $25:$  (b)  $5:3$   
(c)  $9:9:$  (d)  $3:5$
45. Which one of the following sets correctly represents the increase in the paramagnetic property of the ions?  
(a)  $\text{Cu}^{2+} > \text{V}^{2+} > \text{Cr}^{2+} > \text{Mn}^{2+}$   
(b)  $\text{Cu}^{2+} < \text{Cr}^{2+} < \text{V}^{2+} < \text{Mn}^{2+}$  (c)  $\text{V}^{2+} < \text{Cu}^{2+} < \text{Cr}^{2+} < \text{Mn}^{2+}$   
(d)  $\text{V}^{2+} < \text{Cr}^{2+} < \text{Mn}^{2+} < \text{Cu}^{2+}$
46. Electrons with a kinetic energy of  $6.023 \times 10^4 \text{ J/mol}$  are evolved from the surface of a metal, when it is exposed to radiation of wavelength of  $600 \text{ nm}$ . The minimum amount of energy required to remove an electron from the metal atom is  
(a)  $(2.3 \times 10^{-19} \text{ J})$   
(b)  $(1.9 \times 10^{-19} \text{ J})$   
(c)  $(1.6 \times 10^{-19} \text{ J})$   
(d)  $(1.2 \times 10^{-19} \text{ J})$

## PART - II (CHEMISTRY)

41. Given that  $H_f(\text{H}) = 218 \text{ kJ/mol}$ , express the H-H bond energy in kcal/mol.  
(a)  $52.15$  (b)  $911$   
(c)  $104$  (d)  $52153$
42. Identify the alkyne in the following sequence of reactions,  
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Wacker Process  $\text{CH}_2 = \text{CH}_2$   
(a)  $\text{H}_3\text{C}-\text{C}\equiv\text{C}-\text{CH}_3$   
(b)  $\text{H}_2\text{C}=\text{C}=\text{CH}_2$   
(c) H  
(d)  $\text{HCC}-\text{CH}_2$   
Fluorination with dilute NaOH and forms a gaseous product A. The bond angle in the molecule of A is  
(a)  $104^\circ 40'$   
(b)  $103^\circ$   
(c)  $107^\circ$  (d)  $109^\circ 28'$
43. One mole of alkene X on ozonolysis gave one mole of acetaldehyde and one mole of acetone. The IUPAC name of X is  
(a) 2-methyl-2-butene (b) 2-methyl-1-butene  
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44. The number of p-d 'pi' bonds present in  $\text{XeO}_3$  and  $\text{XeO}_4$  molecules, respectively are  
(a) 3, (b) 4,  
(c) 4 (d) 2
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(b)  $\text{Cu}^{2+} < \text{Cr}^{2+} < \text{V}^{2+} < \text{Mn}^{2+}$  (c)  $\text{V}^{2+} < \text{Cu}^{2+} < \text{Cr}^{2+} < \text{Mn}^{2+}$   
(d)  $\text{V}^{2+} < \text{Cr}^{2+} < \text{Mn}^{2+} < \text{Cu}^{2+}$
47. Electrons with a kinetic energy of  $6.023 \times 10^4 \text{ J/mol}$  are evolved from the surface of a metal, when it is exposed to radiation of wavelength of  $600 \text{ nm}$ . The minimum amount of energy required to remove an electron from the metal atom is  
(a)  $(2.3 \times 10^{-19} \text{ J})$   
(b)  $(1.9 \times 10^{-19} \text{ J})$   
(c)  $(1.6 \times 10^{-19} \text{ J})$   
(d)  $(1.2 \times 10^{-19} \text{ J})$

50. The type of bonds present in sulphuric anhydride are

- (a) 3 and three  $p-d$
- (b) 3, one  $p-p$  and two  $p-d$
- (c) 2 and three  $p-d$
- (d) 2 and two  $p-d$

51. In Gattermann reaction, a diazonium group is replaced by X using Y. X and Y are

- |            |             |
|------------|-------------|
| <u>X</u>   | <u>Y</u>    |
| (a) $Cl^-$ | $Cu/HCl$    |
| (b) $Cl$   | $CuCl_2$    |
| (c) $Cl^-$ | $CuCl/HCl$  |
| (d) $Cl_2$ | $Cu_2O/HCl$ |

52. Which pair of oxyacids of phosphorus contains

'P-H' bonds?

- (a)  $H_3PO_4$ ,  $H_3PO_3$
- (b)  $H_3PO_4$ ,  $H_3PO_2$
- (c)  $H_3PO_3$ ,  $H_3PO_2$
- (d)  $H_3PO_4$ ,  $H_3PO_2$

53. Dipole moment of  $HCl = 1.03$  D,  $HI = 0.38$  D. Bond length of  $HCl = 1.3$  Å and  $HI = 1.6$  Å. The ratio of fraction of electric charge, existing on each atom in  $HCl$  and  $HI$  is

- (a) 12 : 1
- (b) 2.7 :
- (c) 3.3 : 1
- (d) 11 :

54.  $SiCl_4$  on hydrolysis forms 'X'.  $X$  and  $HCl$ .

Compound 'X' loses water at  $1000^\circ C$  and gives 'Y'. Compounds 'X' and 'Y' respectively are

- (a)  $H_2SiCl_6$ ,  $SiO_2$
- (b)  $H_4SiO_4$ ,  $SiO_2$
- (c)  $SiO_2$ ,  $Si$
- (d)  $H_4SiO_4$ ,  $SiO_2$

55. 1.5 g of  $CdCl_2$  was found to contain 0.9 g of  $Cd$ . Calculate the atomic weight of  $Cd$ .

- (a) 118
- (b) 112
- (c) 106.5
- (d) 53.25

56. Aluminium reacts with  $NaOH$  and forms compound 'X'. If the coordination number of aluminium in 'X' is 6, the correct formula of X is

- (a)  $[Al(H_2O)_2(OH)_4]^-$
- (b)  $[Al(H_2O)_3(OH)_3]^-$
- (c)  $[Al(H_2O)_4(OH)_2]^-$
- (d)  $[Al(H_2O)_5(OH)]^-$

57. An ideal gas at  $27^\circ C$  and 1 atm pressure is

- (a)  $O_2$
- (b)  $CO_2$
- (c)  $CH_4$
- (d)  $H_2$

58. The order of decreasing ionization potential of the alkali metals is

The correct answer is

- (a) Both (A) and (R) are true and (R) is the correct explanation of (A)
- (b) Both (A) and (R) are true but (R) is not true
- (c) (A) is true but (R) is not true
- (d) Both (A) and (R) are false

59. How many mL of perhydrol is required to produce sufficient oxygen which can be used to completely convert 2 L of  $SO_2$  gas to  $SO_3$  gas?

- (a) 10 mL
- (b) 5 mL
- (c) 20 mL
- (d) 30 mL

60. The pH of a buffer solution decreases by 0.02 units when 0.12 g of acetic acid is added to 250 mL of a buffer solution of acetic acid and potassium acetate at  $27^\circ C$ . The buffer capacity of the solution is

- (a) 0.1
- (b) 10
- (c) 1
- (d) 0.4

61. Match the following

- |                  |                           |
|------------------|---------------------------|
| List I           | List II                   |
| (A) Felspar      | (I) $3Sb_2S_3$            |
| (B) Asbestos     | (II) $Al_2O_3 \cdot H_2O$ |
| (C) Pyrrargyrite | (III) $MgSO_4 \cdot H_2O$ |
| (D) Diaspore     | (IV) $KAlSi_3S_8O_3$      |
|                  | (V) $CaMg$                |

- |        |     |     |     |
|--------|-----|-----|-----|
| (A)    | (B) | (C) | (D) |
| (a) IV | V   | II  | I   |
| (b) IV | V   | I   | II  |
| (c) IV | I   | III | II  |
| (d) II | V   | IV  | I   |

62. Which one of the following order is correct for the first ionisation energies of the elements?

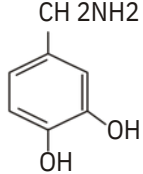
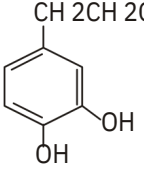
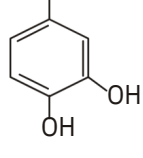
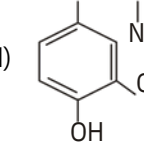
- (a)  $B < Be < N < O$
- (b)  $Be < B < N < O$
- (c)  $B < Be < O < N$
- (d)  $O < B < O < Be$

63. What are X and Y in the following reaction sequence?

- $CH_3OH \xrightarrow{Cl_2} X \xrightarrow{Cl_2} Y$
- (a)  $C_2H_5Cl$ ,  $CH_3CHO$
  - (b)  $CH_3CHO$ ,  $CH_3CO_2H$
  - (c)  $CH_3CHO$ ,  $CCl_3CHO$
  - (d)  $C_2H_5Cl$ ,  $CCl_3CHO$

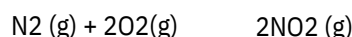
64. What are A, B, C in the following reactions?

- (i)  $(CH_3CO_2)_2Ca \rightarrow A$
- (ii)  $CH_3CO_2H \xrightarrow{HI} B$
- (iii)  $2CH_3CO_2H \xrightarrow{POCl_3} C$

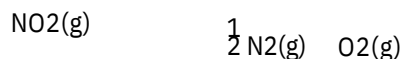
65. One per cent composition of an organic compound A is, carbon : 85.71% and hydrogen 14.29%. Its vapour density is 14. Consider the following reaction sequence
- A  $\xrightarrow{\text{Cl}_2/\text{H}_2\text{O}}$  B  $\xrightarrow[\text{(ii) H}_3\text{O}^+]{\text{(i) KCN / EtOH}}$  C
- Identify C.
- (a)  $\text{CH}_3-\text{CH}(\text{OH})-\text{CO}_2\text{H}$
- (b)  $\text{HO}-\text{CH}_2-\text{CH}_2-\text{CO}_2\text{H}$  (c)  $\text{HO}-\text{CH}_2-\text{CO}_2\text{H}$  (d)  $\text{CH}_3-\text{CH}_2-\text{CO}_2\text{H}$
66. How many tripeptides can be prepared by linking the amino acids glycine, alanine and phenyl alanine?
- (a) One (b) Three (c) Six (d) Twelve
67. A codon has a sequence of A and specifies a particular B that is to be incorporated into a C. What are A, B, C?
- A  $\xrightarrow{\text{B}}$  C
- (a) 3 bases amino acid carbohydrate  
 (b) 3 acids carbohydrate protein  
 (c) 3 bases protein amino acid  
 (d) 3 bases amino acid protein
68. Parkinson's disease is linked to abnormalities in the levels of dopamine in the body. The structure of dopamine is
- (a)  (b) 
- (c)  (d) 
69. During the depression in freezing point experiment, an equilibrium is established between the molecules of
- (a) liquid solvent and solid solvent  
 (b) liquid solute and solid solvent  
 (c) liquid solvent and solid solute  
 (d) liquid solute and solid solute
70. Consider the following reaction,
- $\text{CH}_3\text{CH}_2\text{CH}_2\text{COCl} + \text{AgCN} \xrightarrow{\text{EtOH}/\text{H}_2\text{O}} \text{X (major)}$
- Which one of the following statements is true for X?
- (I) It gives propionic acid on hydrolysis  
 (II) It has an ester functional group  
 (III) It has a nitrogen linked to ethyl carbon  
 (IV) It has a cyanide group
- (a) IV (b) III  
 (c) II (d) I
71. For the following cell reaction,
- $\text{Ag} | \text{Ag}^+ | \text{AgCl} | \text{Cl}^- | \text{Cl}_2, \text{Pt}$
- $G_f(\text{AgCl}) = -109 \text{ kJ/mol}$
- $G_f(\text{Cl}^-) = 129 \text{ kJ/mol}$
- $G_f(\text{Ag}) = 78 \text{ kJ/mol}$
- $E^\circ$  of the cell is (a) -0.60 V (b) 0.60 V  
 (c) 6.0 V (d) None of these
72. The synthesis of crotonaldehyde from acetaldehyde is an example of ..... reaction.
- (a) nucleophilic addition  
 (b) elimination  
 (c) electrophilic addition  
 (d) nucleophilic addition-elimination
73. At 25°C, the molar conductances at infinite dilution for the strong electrolytes NaOH, NaCl and BaCl<sub>2</sub> are  $248 \times 10^{-4}$ ,  $126 \times 10^{-4}$  and  $280 \times 10^{-4} \text{ Sm}^2 \text{ mol}^{-1}$  respectively, in  $\text{Sm}^2 \text{ mol}^{-1}$  is
- (a)  $52.4 \times 10^{-4}$  (b)  $524 \times 10^{-4}$   
 (c)  $402 \times 10^{-4}$  (d)  $262 \times 10^{-4}$
74. The cubic unit cell of a metal (molar mass = 63.55 g mol<sup>-1</sup>) has an edge length of 362 pm. Its density is 8.92 g cm<sup>-3</sup>. The type of unit cell is
- (a) primitive (b) face centred  
 (c) body centred (d) end centred



75. The equilibrium constant for the given reaction is 100.



What is the equilibrium constant for the reaction given below?



- (a) 10 (b) 1  
(c) 0.1 (d) 0.01
76. For a first order reaction at 27°C, the ratio of time required for 75% completion to 25% completion of reaction is (a) 3.0  
(b) 2.303  
(c) 4.8 (d) 0.477
77. The concentration of an organic compound in chloroform is 6.15 g per 100 mL of solution. A portion of this solution in a 5cm polarimeter tube causes an observed rotation of  $-1.2^\circ$ . What is the specific rotation of the compound? (a)  $+12^\circ$   
(b)  $-3.9^\circ$   
(c)  $-39^\circ$  (d)  $+61.5^\circ$
78. 20 ml of 0.1 M acetic acid is mixed with 50 mL of potassium acetate.  $K_a$  of acetic acid =  $1.8 \times 10^{-5}$  at 27°C. Calculate concentration of potassium acetate if pH of the mixture is 4.8.  
(a) 0.1 M (b) 0.04 M  
(c) 0.4 M (d) 0.02 M
79. Calculate  $H^\circ$  for the reaction,  
 $\text{Na}_2\text{O}(\text{s}) + \text{SO}_3(\text{g}) \rightleftharpoons \text{Na}_2\text{SO}_4(\text{g})$   
given the following :
- (A)  $\text{Na}(\text{s}) + \text{H}_2\text{O}(\text{l}) \rightleftharpoons \text{NaOH}(\text{s}) + \frac{1}{2}\text{H}_2(\text{g})$   
 $H^\circ = -146 \text{ kJ}$
- (B)  $\text{Na}_2\text{SO}_4(\text{s}) + \text{H}_2\text{O}(\text{l}) \rightleftharpoons 2\text{NaOH}(\text{s}) + \text{SO}_3(\text{g})$   
 $H^\circ = +418 \text{ kJ}$
- (C)  $2\text{Na}_2\text{O}(\text{s}) + 2\text{H}_2(\text{g}) \rightleftharpoons 4\text{Na}(\text{s}) + 2\text{H}_2\text{O}(\text{l})$   
 $H^\circ = +259 \text{ kJ}$
- (a) +823 (b) -581  
(c) kJ - (d) kJ
80. Which of the following is the most effective in causing the coagulation of  $\text{As}_2\text{S}_3$  sol?  
(a) KCl (b)  $\text{AlCl}_3$   
(c)  $\text{MgSO}_4$  (d)  $\text{K}_3\text{Fe}(\text{CN})_6$

### PART - III (MATHEMATICS)

81. If  $f: [2, 3] \rightarrow \mathbb{R}$  is defined by  $f(x) = x^3 + 3x - 2$ , then the range  $f(x)$  is contained in the interval (a)  $[1, 12]$   
(b)  $[12, 34]$   
(c)  $[35, 50]$  (d)  $[-12, 12]$
82. The number of subsets of  $\{1, 2, 3, \dots, 9\}$  containing at least one odd number is  
(a) 324 (b) 39  
(c) 496 (d) 6
83. A binary sequence is an arrangement of 0's and 1's. The number of  $n$ -digit binary sequences which contain even number of 0's is  
(a)  $2^{n-1}$  (b)  $2^{n-1} - 1$   
(c)  $2^{n-1} + 1$  (d)  $2^{n-1} - 2$
84. If  $f(x) = \frac{2x^{3/2}}{3} - (325x)^{-1/5}$ , then  $f'(x)$  is approximately equal to  
(a)  $\frac{3231x}{64}$  (b)  $\frac{3132x}{64}$   
(c)  $\frac{3132x}{64}$  (d)  $\frac{12x}{64}$
85. The roots of  $(x-a)(x-a-1)(x-a-1)(x-a-2)(x-a)(x-a-2) = 0$  are always  
(a) imaginary  
(b) rational and equal  
(c) real and distinct  
(d) rational and equal
86. If  $f(x) = x^2 + 2x + 1$  and  $g(x) = x^2 + 3x + 2$ , then the roots of  $f(x) + g(x) + f(x) = 0$  are  
(a) real and distinct (b) imaginary  
(c) equal (d) rational and equal
87. If  $f(x) = 2x^4 - 13x^2 + ax + b$  is divisible by  $x^2 - 3x + 2$ , then  $(a, b)$  is equal to  
(a)  $(-9, -2)$  (b)  $(6, 4)$   
(c)  $(9, 2)$  (d)  $(4, 6)$
88. If  $x, y, z$  are all positive and  $a, b, c$  are the  $p$ th,  $q$ th and  $r$ th terms of a geometric progression respectively, then the value of the determinant  
$$\begin{vmatrix} \log x & p & 1 \\ \log y & q & 1 \\ \log z & r & 1 \end{vmatrix}$$
 equals  
(a)  $\log xyz$  (b)  $(p-1)(q-1)(r-1)$   
(c)  $pqr$  (d) 0

89. The locus of  $z$  satisfying the inequality

$$\left| \frac{z-2i}{2z-i} \right| < 1, \text{ where } z = x + iy, \text{ is}$$

- (a)  $x^2 + y^2 = 1$  (b)  $x^2 + y^2 = 1$   
 (c)  $x^2 + y^2 = 1$  (d)  $2x^2 + 3y^2 = 1$
90. If  $n$  is an integer which leaves remainder one when divided by three, then
- (a)  $\sqrt{3}^n (1 - \sqrt{3}i)^n$  equals  
 (a)  $-2n + 1$  (b)  $2n + 1$   
 (c)  $-(-2)^n$  (d)  $-2n$
91. The period of  $\sin 4x + \cos 4x$  is
- (a)  $\frac{\pi}{2}$  (b)  $\frac{\pi}{2}$  (c)  $\frac{\pi}{4}$  (d)  $\frac{\pi}{2}$
92. If  $3\cos x + 2\sin x$ , then the general solution of  $\sin 2x - \cos 2x = 2 \sin 2x$  is
- (a)  $n\pi + (-1)^n \frac{\pi}{2}, n \in \mathbb{Z}$   
 (b)  $2n\pi, n \in \mathbb{Z}$   
 (c)  $(4n+1)\pi, n \in \mathbb{Z}$   
 (d)  $(2n+1)\pi, n \in \mathbb{Z}$
93.  $\cos^{-1} \frac{1}{2} + 2\sin^{-1} \frac{1}{2} + 3\cos^{-1} \frac{1}{\sqrt{2}}$   
 $4\tan^{-1}(\frac{1}{1})$  equals
- (a)  $\frac{19}{12}$  (b)  $\frac{35}{12}$  (c)  $\frac{47}{12}$  (d)  $\frac{43}{12}$
94. In a  $\triangle ABC$   
 $(a+b+c)(b+c-a)(c+a-b)(a+b-c)$   
 $\frac{4a^2b^2c^2}{b^2c^2}$   
 equals  
 (a)  $\cos 2A$  (b)  $\cos 2B$   
 (c)  $\sin 2A$  (d)  $\sin 2B$
95. The angle between the lines whose direction cosines satisfy the equations  $l + m + n = 0$ ,  $l^2 + m^2 + n^2 = 0$  is
- (a)  $60^\circ$  (b)  $45^\circ$  (c)  $30^\circ$  (d)  $20^\circ$

96. If  $m_1, m_2, m_3$  and  $m_4$  are respectively the magnitudes of the vectors

$$a_1 = 2\hat{i} + \hat{j} + \hat{k}, a_2 = 3\hat{i} + 4\hat{j} + 4\hat{k},$$

$$a_3 = \hat{i} + \hat{j} + \hat{k}, \text{ and } a_4 = \hat{i} - \hat{j} + 3\hat{k},$$

then the correct order of  $m_1, m_2, m_3$  and  $m_4$  is

- (a)  $m_1 < m_2 < m_3 < m_4$   
 (b)  $m_3 < m_2 < m_1 < m_4$   
 (c)  $m_4 < m_3 < m_1 < m_2$   
 (d)  $m_3 < m_1 < m_4 < m_2$
97. If  $X$  is a binomial variate with the range  $\{0, 1, 2, 3, 4, 5, 6\}$  and  $P(X=2) = 4P(X=4)$ , then the parameter  $p$  of  $X$  is
- (a)  $\frac{1}{3}$  (b)  $\frac{1}{2}$  (c)  $\frac{2}{3}$  (d)  $\frac{3}{4}$
98. The area (in square unit) of the circle which touches the lines  $4x + 3y = 15$  and  $4x + 3y = 5$  is
- (a)  $\frac{4}{9}$  (b)  $\frac{3}{4}$   
 (c)  $\frac{2}{3}$  (d)  $\frac{1}{2}$
99. The area (in square unit) of the triangle formed by  $x + y + 1 = 0$  and the pair of straight lines  $x^2 - 3xy + 2y^2 = 0$  is
- (a)  $\frac{7}{12}$  (b)  $\frac{5}{12}$  (c)  $\frac{1}{12}$  (d)  $\frac{1}{6}$
100. The pairs of straight lines  $x^2 - 3xy + 2y^2 = 0$  and  $x^2 - 3xy + 2y^2 + x - 2 = 0$  form a
- (a) square but not rhombus  
 (b) rhombus  
 (c) parallelogram  
 (d) rectangle but not a square
101. The equations of the circle which pass through the origin and makes intercepts of lengths 4 and 8 on the  $x$  and  $y$ -axes respectively are
- (a)  $x^2 + y^2 + 4x + 8y = 0$   
 (b)  $x^2 + y^2 + 2x + 4y = 0$   
 (c)  $x^2 + y^2 + 8x + 16y = 0$   
 (d)  $x^2 + y^2 + x + y = 0$
102. The point  $(3, -4)$  lies on both the circles  $x^2 + y^2 - 2x + 8y + 13 = 0$  and  $x^2 + y^2 - 4x + 6y + 11 = 0$ . Then, the angle between the circles is
- (a)  $60^\circ$  (b)  $\tan^{-1} \frac{1}{2}$   
 (c)  $\tan^{-1} \frac{3}{5}$  (d)  $135^\circ$
103. The equation of the circle which passes through the origin and cuts orthogonally each of the circles  $x^2 + y^2 - 6x + 8 = 0$  and  $x^2 + y^2 - 2x - 2y - 7 = 0$  is

- (a)  $(3x^2 + 4y^2 + 8x - 12y - 9) = 0$  is a family of normals drawn to the parabola  $y^2 = 4x$  from the point (1, 0) is  $0$   $3x^2 + 3y^2 + 8x + 29y = 0$   $3x^2 + 3y^2 - 8x + 29y = 0$
104. If the circle  $x^2 + y^2 = a^2$  intersects the hyperbola  $xy = 6$  in four points  $(x_1, y_1)$  for  $i = 1, 2, 3$  and  $(x_4, y_4)$ , then  $y_1 + y_2 + y_3 + y_4$  equals  
(a) 4 (b) 1 (c) 2 (d) 3
105. The mid point of the chord  $4x - 3y = 5$  of the hyperbola  $2x^2 - 3y^2 = 12$  is  
(a)  $(0, \frac{5}{3})$  (b)  $(2, 1)$   
(c)  $(\frac{5}{4}, 0)$  (d)  $(\frac{11}{4}, 2)$
107. The perimeter of the triangle with vertices at (1, 0, 0), (0, 1, 0) and (0, 0, 1) is  
(a) 3 (b) 2  
(c)  $2\sqrt{2}$  (d)  $3\sqrt{2}$
108. If a line in the space makes angle  $\alpha$  with the coordinate axes, then  $\cos^2 \alpha + \cos^2 \beta + \cos^2 \gamma = \sin^2 \alpha + \sin^2 \beta + \sin^2 \gamma$  equals  
(a) -1 (b) 0 (c) 1 (d) 2
109. The radius of the sphere  $x^2 + y^2 + z^2 = 12x + 4y + 3z$  is  
(a) 4 (b) 5 (c) 6 (d) 52
110.  $\lim_{x \rightarrow \infty} x^{5x^2 - 3}$  equals  
(a) e (b)  $e^2$  (c)  $e^3$  (d)  $e^5$
111. If  $f: R \rightarrow R$  is defined by  

$$f(x) = \begin{cases} 2\sin x - \sin 2x, & \text{if } x \neq 0 \\ a, & \text{if } x = 0 \end{cases}$$
then the value of  $a$  so that  $f$  is continuous at 0 is  
(a) 2 (b) 1 (c) -1 (d) 0
112.  $x = \cos^{-1} \frac{1}{\sqrt{1+t^2}}$ ,  $y = \sin^{-1} \frac{t}{\sqrt{1+t^2}}$   $\frac{dy}{dx}$  is equal to  
(a) 0 (b)  $\tan t$   
(c) 1 (d)  $\sin t \cos t$
113.  $\frac{d}{dx} \tan^{-1} x = \log \frac{x+1}{x-1}$   
(a)  $\frac{1}{x^2+1}$  (b)  $\frac{1}{x^4+1}$   
(c)  $\frac{1}{x^2-1}$  (d)  $\frac{1}{x^4-1}$
114.  $y = e^{\sin x} (1+x^2)^{yn^2} (2n^{1/2})^{xy}$  is equal to  
(a)  $(n^2 - a^2)y_n$  (b)  $(n^2 - a^2)y_n$   
(c)  $(n^2 - a^2)y$  (d)  $(n^2 - a^2)y_n$
115. The function  $f(x) = x^3 - 2ax^2 + bx + ca$ ,  $a^2 = 3b$  has  
(a) one maximum value  
(b) one minimum value  
(c) no extreme value  
(d) one maximum and one minimum value
116.  $\int \frac{2 \sin 2x}{1 + \cos 2x} e^x dx$  is equal to  
(a)  $-e^x \cot x + c$  (b)  $e^x \cot x + c$   
(c)  $2e^x \cot x + c$  (d)  $-2e^x \cot x + c$
117. If  $I_n = \int \sin^n x dx$ , then  $n I_n - (n-1) I_{n-2}$  equals  
(a)  $\sin^{n-1} x \cos x$  (b)  $\cos^{n-1} x \sin x$   
(c)  $\sin^{n-1} x \cos x$  (d)  $\cos^{n-1} x \sin x$
118. The line  $x = \frac{\pi}{4}$  divides the area of the region bounded by  $y = \sin x$ ,  $y = \cos x$  and  $x$ -axis  $0 \leq x \leq \frac{\pi}{2}$  into two regions of areas  $A_1$  and  $A_2$ . Then  $A_1 : A_2$  equals  
(a) 4 : 1 (b) 3 : 1 (c) 2 : 1 (d) 1 : 1
119. The solution of the differential equation  $\sin(x-y) \tan(x-y) - 1 = \frac{dy}{dx}$  cosec  $(x+y) + \tan(x+y) = x + c$   
(a)  $x + \text{cosec}(x+y) = c$   
(b)  $x + \tan(x+y) = c$   
(c)  $x + \sec(x+y) = c$   
(d) If  $(\sim p \vee q)$  is false, the truth value of  $p$  and  $q$  are respectively  
(a) F, T (b) F, F (c) T, F (d) T, T

# SOLUTIONS

## PART - I (PHYSICS)

1. (d) Here,  $y = a \sin(bt - cx)$   
Comparing this equation with general wave equation  $y = a \sin \frac{2\pi}{T} t - \frac{2\pi}{\lambda} x$   
we get  $b = \frac{2\pi}{T}$ ,  $c = \frac{2\pi}{\lambda}$

(a) Dimensions of  $\frac{y}{a} = \frac{[L]}{[L]} = \text{Dimensionless}$

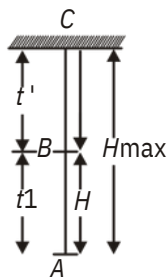
(b) Dimensions of  $bt = \frac{[L]}{[T]} t = \frac{[L]}{[T]} \frac{[T]}{[L]} = \text{Dimensionless}$

(c) Dimensions of  $cx = \frac{[L]}{[L]} x = \frac{[L]}{[L]} = \text{Dimensionless}$

(d) Dimensions of  $\frac{b}{c} = \frac{\frac{2\pi}{T}}{\frac{2\pi}{\lambda}} = \frac{\lambda}{T} = [LT^{-1}]$

2. (b) Let  $t'$  be the time taken by the body to fall from point C to B.

Then  $t_1 = 2t' + t_2$   $t' = \frac{t_2 - t_1}{2}$  ... (i)



Total time taken to reach point C

$$T = t_1 + t' + t_1 = \frac{t_2 - t_1}{2} + t_1 + \frac{t_2 - t_1}{2} = t_2$$

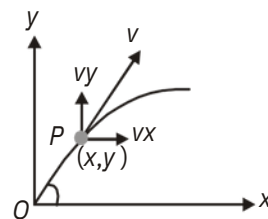
Maximum height attained

$$H_{\max} = \frac{1}{2} g (T)^2 = \frac{1}{2} g \frac{t_2^2}{2}$$

$$= \frac{1}{4} g (t_2 - t_1)^2$$

Or,  $H_{\max} = \frac{1}{8} g (t_2 - t_1)^2$

3. (a) Momentum,  $p = m v = v \frac{p}{m}$



Kinetic energy, KE

$$= \frac{1}{2} m v^2 = \frac{1}{2} m \frac{p^2}{m^2} = \frac{1}{2m} p^2$$

or,  $KE = \frac{1}{2m} p^2$  constant

Hence, the graph between KE and  $p^2$  will be a straight line passing through the origin.

$$\text{energy KE} = \frac{1}{2} m v^2$$

The velocity component at point P,

$$v_y = (u \sin \theta - gt) \text{ and } v_x = u \cos \theta$$

Resultant velocity at point P,

$$v = \sqrt{v_y^2 + v_x^2}$$

$$= \sqrt{(u \sin \theta - gt)^2 + (u \cos \theta)^2}$$

$$v = \sqrt{u^2 \cos^2 \theta + u^2 \sin^2 \theta - 2ugt \sin \theta + g^2 t^2}$$

$$= \sqrt{u^2 (\cos^2 \theta + \sin^2 \theta) - 2ugt \sin \theta + g^2 t^2}$$

$$= \sqrt{u^2 - 2ugt \sin \theta + g^2 t^2}$$

$$KE = \frac{1}{2}m(u^2 - g^2t^2 - 2ugt \sin \theta)$$

i.e., KE

Hence, graph will be parabolic intercept on y-axis.

Hence, the graph between KE and t.

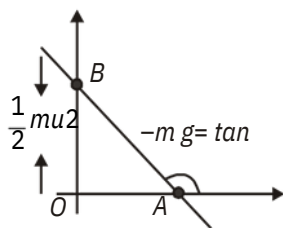
Now, in case of height

$$KE = \frac{1}{2}m(v^2) \text{ and } v^2 = (u^2 - 2gy)$$

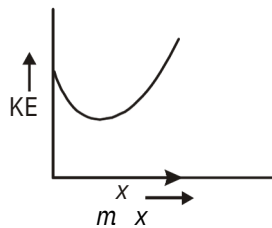
$$KE = \frac{1}{2}m(u^2 - 2gy)$$

$$KE = mgy + \frac{1}{2}mu^2$$

$$\text{Intercept on y-axis} = \frac{1}{2}mu^2$$



$$\text{Now, KE} = \frac{1}{2}mv^2$$



$$KE = \frac{1}{2} \frac{x^2}{t^2}$$

i.e., KE  $\propto x^2$ . Thus graph between KE and x will be parabolic.

4. (a) Power of motor initially =  $P_0$   
Let, rate of flow of motor =  $(x)$   
Since, power,

$$P_0 = \frac{\text{work}}{\text{time}} = \frac{mgy}{t} = mg \frac{y}{t}$$

$$\frac{y}{t} \cdot x = \text{rate of flow of water} = mgx \quad \dots(i)$$

If rate of flow of water is increased by  $n$  times, i.e.,  $(nx)$ .

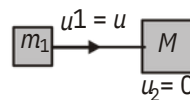
Increased power,

$$P_1 = \frac{mgy'}{t} = mg \frac{y'}{t} = nmgx \quad \dots(ii)$$

The ratio of power,

$$\frac{P_1}{P_0} = \frac{nmgx}{mgx}; P_1:P_0 = n:1$$

5. (a) Mass of the first body  $m_1 = 5 \text{ kg}$  and for elastic collision coefficient of restitution,  $e = 1$ .



Let initially body  $m_1$  moves with velocity  $v$

after collision velocity becomes  $\frac{u}{10}$ .

Let after collision velocity of  $M$  block becomes  $(v_2)$ .

By conservation of momentum

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$$

$$\text{or } 5u + 0 = 5 \cdot \frac{u}{10} + Mv_2$$

$$5u - \frac{u}{2} = Mv_2 \quad \dots$$

Since,  $v_1 = v_2 = e(u_1)$  (i)

$$\text{or } \frac{u}{10} = v_2 = 1(u) \text{ or } \frac{u}{10} = u - v_2$$

$$\text{or } \frac{11u}{10} = v_2 \quad \dots(ii)$$

Substituting value of  $2v$  in Eq. (i) from Eq. (ii)

$$5u - \frac{u}{2} = M \frac{11u}{10}$$

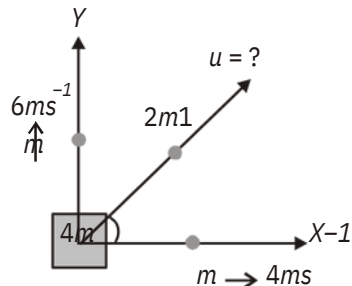
$$\text{or } 5 \cdot \frac{1}{2} = M \frac{11}{10} \Rightarrow M = \frac{9}{2} = 4.5 \text{ kg}$$

6. (c) Let third mass particle  $(2m)$  moves making angle with X-axis.

The horizontal component of velocity of

$2m$  mass particle =  $u \cos$

And vertical component =  $u \sin$



From conservation of linear momentum in X-direction

$$m_1 u_1 + m_2 u_2 = m_1 v_1 + m_2 v_2$$

$$0 = m \cdot 4 + 2m(u \cos 45^\circ)$$

$$\therefore 0 = 4m + 2m(u \cos 45^\circ) \quad \text{or} \quad 2u \cos 45^\circ = -4$$

Again, applying law of conservation of linear momentum in Y-direction

$$0 = m \cdot 6 + 2m(u \sin 45^\circ)$$

$$\frac{6}{2} = u \sin 45^\circ \quad \text{or} \quad 3 = u \sin 45^\circ \quad \dots (ii)$$

Squaring Eqs. (i) and (ii) and adding,

$$(4)^2 + (9) = u^2 \cos^2 45^\circ + u^2 \sin^2 45^\circ$$

$$u^2 (\cos^2 45^\circ + \sin^2 45^\circ) = 13$$

$$\text{or } 13 = u^2$$

$$u = \sqrt{13} \text{ ms}^{-1}$$

7. (b) Here, maximum height attained by a projectile

$$h = \frac{v^2 \sin^2 \theta}{2g} \quad \dots (i)$$

Velocity of body = half the escape velocity

$$\text{i.e., } v = \frac{v_e}{2}$$

$$\text{or } v = \frac{\sqrt{2gR}}{2} \quad v^2 = \frac{2gR}{4} \quad v^2 = \frac{gR}{2}$$

Now, putting value of  $v^2$  in Eq. (i), we get

$$\text{Height, } h = \frac{\frac{gR}{2} \sin^2 \theta}{2g} = \frac{gR}{4g} \sin^2 \theta = \frac{R}{4} \sin^2 \theta$$

8. (d) Particle executing SHM.

$$\text{Displacement } y = 5 \sin 4t \quad \dots (i)$$

Velocity of particle

$$\frac{dy}{dt} = 5 \cos 4t \cdot 4 = 20 \cos 4t$$

$$\text{Velocity at } t = \frac{T}{4}$$

$$\frac{dy}{dt} \bigg|_{t=\frac{T}{4}} = 20 \cos 4 \cdot \frac{T}{4} = 20 \cos T$$

$$\text{or } u = 20 \cos T \quad \dots (ii)$$

Comparing the given equation with standard equation of SHM.

$$y = a \sin \left( \frac{2\pi}{T} t \right)$$

$$\text{We get, } a = 5$$

$$\text{As } \frac{2\pi}{T} = 4 \quad T = \frac{2\pi}{4} = \frac{\pi}{2}$$

$$\text{or } T = \frac{\pi}{2}$$

Now, putting value of  $T$  in Eq. (ii), we get

$$u = 20 \cos \frac{\pi}{2} = 0$$

$$10\sqrt{3}$$

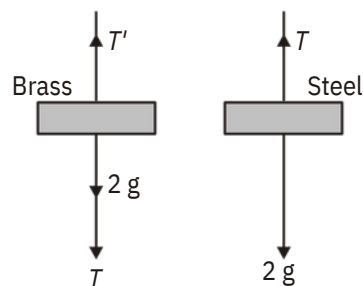
The kinetic energy of particle,

$$KE = \frac{1}{2} m u^2 = \frac{1}{2} \cdot 2103 \cdot (10\sqrt{3})^2$$

$$= 103100 \text{ J} \approx 0.31 \text{ MJ}$$

9. (d) Given,  $l_1 = 1 \text{ m}$ ,  $l_2 = 1 \text{ m}$ ,  $Y_1 = 1 \text{ cm}$ ,  $Y_2 = 1 \text{ cm}$

Free body diagram of the two blocks brass and steel are



Let Young's modulus of steel is  $Y_1$  and of brass is  $Y_2$ .

$$Y_1 = \frac{F_1 l_1}{A_1 \Delta l_1} \quad \dots(i)$$

$$\text{and } Y_2 = \frac{F_2 l_2}{A_2 \Delta l_2} \quad \dots(ii)$$

Dividing Eq. (i) by (ii),

$$\frac{Y_1}{Y_2} = \frac{F_1 l_1}{F_2 l_2} \frac{A_2 \Delta l_2}{A_1 \Delta l_1}$$

$$\text{or } \frac{Y_1}{Y_2} = \frac{F_1 A_2 l_1 \Delta l_2}{F_2 A_1 l_2 \Delta l_1} \quad \dots(iii)$$

Force on steel wire from free body diagram

$$T = F_1 = (2g) \text{ newton}$$

Force on brass wire from free body diagram

$$F_2 = T = 2g = (4g) \text{ newton}$$

Now, putting the value of  $F_1, F_2$ , in Eq. (iii), we get

$$\frac{Y_1}{Y_2} = \frac{2g}{4g} \frac{r_2^2}{r_1^2} \frac{l_1}{l_2} \frac{\Delta l_2}{\Delta l_1}$$

$$\text{or } \frac{1}{2} = \frac{1}{4} \frac{a}{b} \frac{l_2}{l_1}$$

$$\text{or } \frac{l_1}{l_2} = \frac{a}{2b}$$

10. (d) Initially area of soap bubble,  $A_1 = 4\pi r^2$   
Under isothermal condition radius becomes  $1.3r$ . (c)

$$\text{Area } A_2 = 4\pi(1.3r)^2 = 16\pi r^2$$

Increase in surface area

$$\Delta A = 2(A_2 - A_1) = 2(16\pi r^2 - 4\pi r^2) = 24\pi r^2$$

Energy spent,

$$W = T \Delta A = T \cdot 24\pi r^2 = 24\pi T r^2 \text{ J}$$

11. (c) Let radius of big drop =  $R$ .

$$\frac{4}{3}\pi R^3 = \frac{4}{3}\pi r^3 \cdot 8$$

$$R = 2r$$

Here  $r$  = radius of small drops.

Now, terminal velocity of drop in liquid

$$v_T = \frac{2}{9} \frac{r^2}{\eta} (\rho - \rho_l) g$$

where  $\eta$  is coefficient of viscosity and  $\rho_l$  is density of liquid.  
Terminal speed drop is  $6 \text{ cm s}^{-1}$

$$6 = \frac{2}{9} \frac{r^2}{\eta} (\rho - \rho_l) g \quad \dots(i)$$

Let terminal velocity becomes  $v'$  after coalesce, then

$$v' = \frac{2}{9} \frac{R^2}{\eta} (\rho - \rho_l) g \quad \dots(ii)$$

Dividing Eq. (i) by (ii), we get

$$\frac{6}{v'} = \frac{\frac{2}{9} \frac{r^2}{\eta} (\rho - \rho_l) g}{\frac{2}{9} \frac{R^2}{\eta} (\rho - \rho_l) g} = \frac{r^2}{R^2}$$

$$\text{or } v' = 24 \text{ cm s}^{-1}$$

(a) Time period of oscillation,

$$T = 2\pi \sqrt{\frac{l}{g}} \frac{dT}{T} \frac{1}{2} \frac{dl}{l}$$

$$\text{As, } \frac{dl}{l} = \frac{dT}{T}$$

$$\frac{dT}{T} = \frac{1}{2} \frac{dT}{T} \quad \frac{1}{2} \cdot 9 \cdot 10^{-7} \cdot (30 - 20)$$

$$= 4.5 \times 10^{-6}$$

$$\text{Loss in time} = 4.5 \times 10^{-6} \times 0.5$$

$$\text{Volume of the } m = e \cdot 2\pi \cdot 2\pi \cdot a \cdot \pi \cdot 310^\circ - C_6 \text{ s}$$

$$V_{30} = \frac{\text{loss of weight}}{\text{specific gravity} \times g}$$

$$\frac{(45 - 25)g}{1.5 \cdot g} = 13.33 \text{ cm}^3$$

Similarly, volume of metal at  $40^\circ\text{C}$

$$V_{40} = \frac{(45 - 27)g}{1.25 \cdot g} = 14.40 \text{ cm}^3$$

$$\text{Now, } V_{40} = V_{30} \left[ \frac{1 + \alpha_1 (t_2 - t_1)}{1 + \alpha_2 (t_2 - t_1)} \right]$$

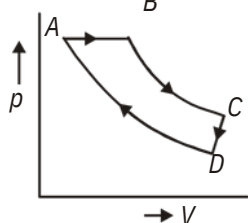
$$\text{or } \frac{V_{40}}{V_{30}} = \frac{14.40}{13.33} \left[ \frac{1 + \alpha_1 (40 - 30)}{1 + \alpha_2 (40 - 30)} \right]$$

$$= 8.03 \times 10^{-3}/^\circ\text{C}$$

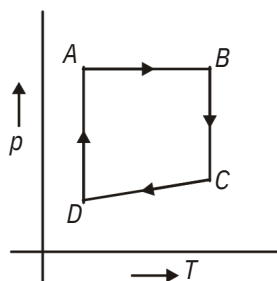
Coefficient of linear expansion of the metal

$$\frac{1}{3} \frac{8.03 \times 10^{-3}}{3} = 2.6 \times 10^{-3} / ^\circ\text{C}$$

14. (a) Process A → B → C → D → A is clockwise.  
During A → B, pressure is constant and  
B → C, process follows  $p \propto \frac{1}{V}$   
constant. During process C → D, both  $p$   
and  $V$  changes and process D → A  
follows  $p \propto \frac{1}{V}$  i.e.,  $T$  is constant.



Hence, equivalent cyclic process is as follows.



15. (b) From first law of thermodynamics  
 $Q = \Delta U + W$   
 $Q_1 = \Delta U_1 + W_1$   
 $6000 = 2500 + W_1$   
 $W_1 = 3500 \text{ J}$   
 $Q_2 = \Delta U_2 + W_2$   
 $5500 = 1000 + W_2$   
 $W_2 = 4500 \text{ J}$   
 $Q_3 = \Delta U_3 + W_3$   
 $3000 = 1200 + W_3$   
 $W_3 = 1800 \text{ J}$   
 $Q_4 = \Delta U_4 + W_4$   
 $3500 = 0 + W_4$   
 $W_4 = 3500 \text{ J}$   
For cyclic process  $\Delta U = 0$   
 $3500 + 4500 + 1800 + 3500 = 13300 \text{ J}$   
or  $13300 \text{ J}$

Efficiency,  $\frac{\text{output}}{\text{input}} \times 100$

$$\frac{W_1 + W_2 + W_3 + W_4}{Q_1 + Q_2 + Q_3 + Q_4} \times 100$$

$$\frac{(2500 + 1000 + 1200 + 700)}{(6000 + 3500 + 1000 + 10.5\%)} \times 100$$

$$\frac{5200}{9500} \times 100$$

16. (c) From first law of thermodynamics  
 $Q = \Delta U + W$   
For cylinder A pressure remains constant.  
Work done by a system

$$W = \int_1^2 p dV = \frac{R}{1} (T_1 - T_2)$$

For monoatomic gases,  $\gamma = \frac{5}{3}$

$$W = \frac{1}{\gamma - 1} R (T_1 - T_2) = \frac{3}{2} R (442 - 400) = 63R$$

or  $W = 63R$

But  $\Delta U = 0$ , for cylinder A

$$Q = 0 + 63R = 63R$$

For cylinder B volume is constant,

$$W = 0 \text{ and } Q = C_V \Delta T$$

For monoatomic gas

$$C_V = \frac{3}{2} R \quad Q = \frac{3}{2} R \Delta T$$

As heat given on both cylinder is same

$$63R = \frac{3}{2} R \Delta T \quad \Delta T = 42\text{K}$$

17. (a) From figure,  $H = H_1 + H_2$   
 $\frac{2 \times 3 \times 10^3 \times (100 - T)}{1} = \frac{50 \times 10^3 \times (T - 0)}{1}$   
 $300(3T - 2T) = 100(T - 0)$   
 $300T - 200T = 100T$   
 $100T = 0$   
 $T = 0^\circ\text{C}$

18. (b) Let listener go from A → B with velocity ( $v$ ).



And the apparent frequency of sound from source A by listener using Doppler's effect,



$$n' = n \frac{v + v_o}{v_s} \quad n' = 680 \frac{340 + u}{340}$$

The apparent frequency of sound from source B by listener

$$n'' = n \frac{v + v_o}{v_s} = 680 \frac{340 + u}{340}$$

Listener hear 10 beats per second.  
Hence,  $n'' - n' = 10$

$$680 \frac{340 + u}{340} - 680 \frac{340}{340} = 10$$

$$2(340 + u - 340) = 10$$

$$u = 2.5 \text{ ms}^{-1}$$

19. (b) When both the wires vibrate simultaneously, beats per second,  
 $n_1 - n_2 = 6$

$$\text{or } \frac{1}{2l} \sqrt{\frac{T}{m}} - \frac{1}{2l} \sqrt{\frac{T'}{m}} = 6$$

$$\frac{1}{2l} \sqrt{\frac{T}{m}} - \frac{1}{2l} \sqrt{\frac{T'}{m}} = 6$$

$$\frac{1}{2l} \sqrt{\frac{T}{m}} = 6006 \quad \frac{1}{2l} \sqrt{\frac{T'}{m}} = 606 \quad \dots(i)$$

Given that fundamental frequency

$$\frac{1}{2l} \sqrt{\frac{T}{m}} = 600 \quad \dots(ii)$$

Dividing Eq. (i) by (ii),

$$\frac{\frac{1}{2l} \sqrt{\frac{T}{m}}}{\frac{1}{2l} \sqrt{\frac{T'}{m}}} = \frac{6006}{600}$$

$$\sqrt{\frac{T}{T'}} = (1.01) \quad \frac{T}{T'} = (1.02)\%$$

$$T' = T(1.02)$$

Increase in tension

$$T' - T = 1.02T - T = (0.02T)$$

$$T = 0.02$$

Fringe width

$$\frac{D}{d}$$

20. (d)

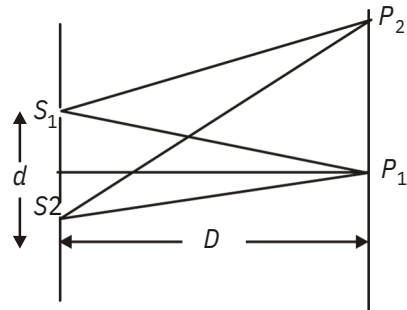
Let be the amplitude of the place where

constructive inference takes place.

The position of fringe at  $p_2$ .

$$x = \frac{nD}{d}$$

$$\text{Given, } n = \frac{1}{4}$$



$$\frac{D}{4d} = \frac{nD}{d} \quad \text{or } n = \frac{1}{4}$$

$$\frac{I_1}{I_2} = \frac{a^2}{a^2} = 16:1$$

21. (a) Position fringe from central maxima

$$y_1 = \frac{n_1 D}{d}$$

Given,  $n = 10$

$$y_1 = \frac{101D}{d} \quad \dots(i)$$

For second source

$$y_2 = \frac{52D}{d} \quad \dots(ii)$$

$$\frac{y_1}{y_2} = \frac{101D}{52D} = \frac{101}{52}$$

22. (c) Interference takes place between two waves having equal frequency and propagate in same direction.

Hence,  $y = 1 + \sin(\theta)$

$$y = 3 \quad \theta = 2$$

will give interference as the two waves have same frequency.

23. (d) The two lenses of an achromatic doublet should have, sum of the product of their powers and dispersive power = zero.

24. (b) Ratio of magnetic moments

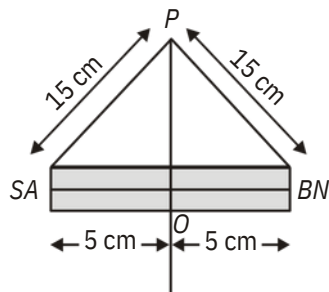
$$\frac{M_{BA}}{M} = \frac{Td}{2} \frac{T_s}{2} \frac{v_s}{2} \frac{v_d^2}{v_d^2}$$

$$\frac{T_1}{20} \frac{T_2}{15} \frac{v_1^2}{15^2} \frac{v_2^2}{20^2}$$

$$\frac{40}{15} \frac{225}{20} \frac{0}{40} \frac{225}{0}$$

$$MA : MB = 25 : 7$$

25. (d) Here, length of magnet = 10 cm =  $10 \times 10^{-2}$  m,  
 $r = 15 \times 10^{-2}$  m



$$OP = \sqrt{225 - 25} = \sqrt{200} \text{ cm}$$

Since, at the neutral point, magnetic field due to the magnet is equal to  $B_H$ ,

$$B_H = \frac{\mu_0}{4\pi} \frac{M}{(OP^2 + AO^2)^{3/2}}$$

$$0.4 \times 10^{-4} = \frac{\mu_0}{4\pi} \frac{M}{(200 \times 10^{-4} + 25 \times 10^{-4})^{3/2}}$$

$$\frac{0.4 \times 10^{-4}}{107} = (225 \times 10^{-4})^{3/2} M$$

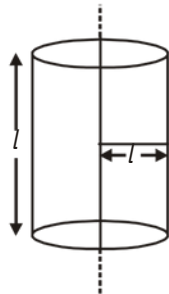
$$0.4 \times 103 = 106 (225)^{3/2} M$$

$$M = 1.35 \text{ A-m}$$

26. (a) Charge density or charge per unit length of long wire

$$\frac{1}{3\text{cm}}$$

$$3\text{cm} \text{ and } r = 18 \times 10^{-2} \text{ m}$$



According to Gauss theorem

$$E \cdot dS = \frac{q}{\epsilon_0}$$

$$E \cdot dS = \frac{q}{\epsilon_0} \text{ or } E \cdot 2\pi r l = \frac{q}{\epsilon_0}$$

$$E = \frac{q}{2\pi \epsilon_0 r l} = \frac{q/l}{2\pi \epsilon_0 r}$$

$$\frac{9 \times 10^9}{2\pi \times 0.18} \frac{1}{2\pi \times 0.18}$$

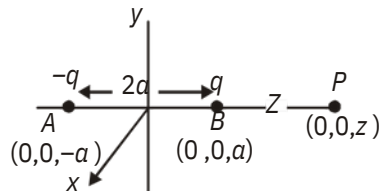
$$= 0.33 \times 10^{11} \text{ NC}^{-1}$$

27. (c) Potential at P due to (+q) charge of dipole

$$V_1 = \frac{1}{4\pi\epsilon_0} \frac{q}{(z-a)}$$

Potential at P due to (-q) charge of dipole

$$V_2 = \frac{1}{4\pi\epsilon_0} \frac{q}{(z+a)}$$



Total potential at P due to electric dipole

$$V = V_1 + V_2$$

$$\frac{1}{4\pi\epsilon_0} \frac{q}{(z-a)} - \frac{1}{4\pi\epsilon_0} \frac{q}{(z+a)}$$

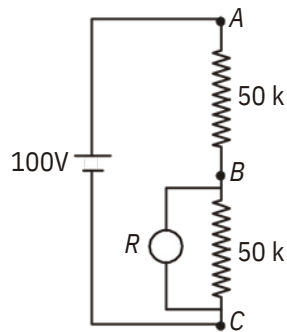
$$\frac{q}{4\pi\epsilon_0} \left( \frac{1}{(z-a)} - \frac{1}{(z+a)} \right)$$

$$\text{or } V = \frac{2qa}{4\pi\epsilon_0 (z^2 - a^2)}$$

28. (c) Internal resistance of voltmeter =  $R$ .  
 Effective resistance across B and C

$$\frac{1}{R'} = \frac{1}{R} + \frac{1}{50} \Rightarrow R' = \frac{50R}{50+R}$$

$$\text{or } R' = \frac{50R}{50+R}$$



According to Ohm's law,  $V = IR$

$$\text{or } \frac{100}{3} = I \frac{50R}{50 + R}$$

$$\text{or } \frac{100}{3} = I \frac{50R}{50 + R} \quad \dots(i)$$

Now, total resistance of circuit

$$R'' = 50 + \frac{50R}{50 + R}$$

$$\text{or } R'' = \frac{(2500 + 100R)}{(50 + R)}$$

Now,  $V = IR''$

$$100 = \frac{100}{3} = I \frac{50R}{50 + R} = \frac{2500 + 100R}{(50 + R)}$$

$$150R = 2500 + 100R \text{ or } R = 50k$$

29. (c) Here length of potentiometer wire,  $l = 10 \text{ m}$   
Resistance of potentiometer wire

$$R = \frac{l}{A} \text{ or } R = \frac{10}{A}$$

The value of 2.5 m length wire

$$R' = \frac{10}{A} \times 2.5 = \frac{2.5}{A}$$

$$\text{Potential, } V = I R' = I \frac{2.5}{A}$$

Again the length of potentiometer wire is increased by 1 m.

Resistance of null position

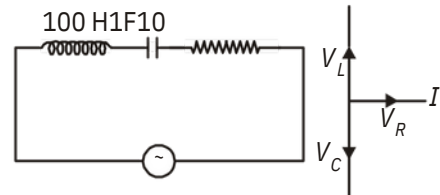
$$R'' = \frac{l}{11 A}$$

$$V' = IR'' \text{ and } V = V'$$

$$\frac{I}{A} \times \frac{2.5}{10} = \frac{l}{11 A} \times I$$

$$\text{or } \frac{2.5 \times 11}{10} = l = 2.75 \text{ m}$$

30. (c)



$$\text{Impedance, } Z = \sqrt{(X_L - X_C)^2 + R^2}$$

$$\text{or } Z = \sqrt{L^2 - \frac{1}{C^2} + R^2}$$

Inductive reactance

$$X_L = L = 70 \times 10^{-3} = 0.07 \text{ } \Omega$$

Capacitance reactance

$$X_C = \frac{1}{C} = \frac{1}{70 \times 10^{-6}} = 14285.7 \text{ } \Omega$$

$$\frac{1}{7 \times 10^{-2}} = \frac{10^2}{7} = \frac{100}{7}$$

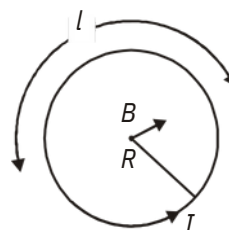
As  $X_C > X_L$

31. (c)

So, circuit behaves like R-C circuit.

At the centre of the loop, magnetic field

$$B = \frac{\mu_0 I}{4 R} \quad \dots(i)$$



For the wire which is looped double let radius becomes  $r$

$$\text{Then, } \frac{l}{2} = 2r \text{ or } \frac{l}{4} = r$$

$$B' = \frac{\mu_0 I}{4 r} = \frac{\mu_0 I}{4 \frac{l}{4}} = \frac{\mu_0 I}{l}$$

$$\text{or } B' = \frac{\mu_0 I}{4} \times \frac{l^2}{1} = \frac{\mu_0 I l^2}{4}$$

$$\text{or } B' = \frac{\mu_0}{4} \frac{Il}{l^2} \frac{16}{2} \quad \dots(\text{ii})$$

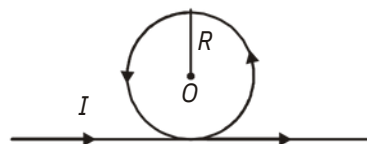
$$\text{Now, } B = \frac{\mu_0}{4} \frac{Il}{l^2} R \frac{1}{2} \quad \dots(\text{iii})$$

Dividing Eq. (ii) by (iii),

$$\frac{B'}{B} = \frac{\frac{\mu_0 Il}{4} \frac{16}{2}}{\frac{\mu_0 Il}{4} \frac{1}{2}} = 4$$

$$\text{or } B' = 4B$$

32. (c) Magnetic field due to long wire at O point



$$B_1 = \frac{\mu_0}{2} \frac{I}{R} \quad (\text{upward})$$

Magnetic field due to loop at O point

$$B_2 = \frac{\mu_0}{4} \frac{I}{R} \frac{2}{2}$$

$$B_2 = \frac{\mu_0}{2} \frac{I}{R} \quad (\text{in upward direction})$$

Resultant magnetic field at centre O  
 $B = B_1 + B_2$

$$B = \frac{\mu_0 I}{2} \left( \frac{1}{R} + \frac{1}{R} \right) T$$

33. (b) Work function  $W_0 = 3.31 \times 10^{-19} \text{ J}$

Wavelength of incident radiation

$$5000 \times 10^{-10} \text{ m}$$

According to Einstein's photoelectric equation  $E = W_0 + KE$

$$\frac{hc}{\lambda} = 3.31 \times 10^{-19} + KE$$

$$KE = \frac{6.62 \times 10^{-34}}{5000 \times 10^{-10}} - 3.31 \times 10^{-19}$$

$$= \frac{6.62 \times 10^{-34}}{5} - 3.31 \times 10^{-19}$$

$$= (-3.31 \times 1.324 \times 3) \times 10^{-19}$$

$$= (3.972 - 3.31) \times 10^{-19}$$

$$= 0.662 \times 10^{-19} \text{ J}$$

$$\text{or } E = \frac{0.662 \times 10^{-19}}{1.6 \times 10^{-19}} = 0.41 \text{ eV}$$

34. (d) From Einstein's photoelectric equation

$$E = W_0 + \frac{1}{2}mv^2 \text{ or } \sqrt{\frac{2(E - W_0)}{m}} = v$$

A charged particle placed in uniform magnetic field experience a force

$$F = evB = \frac{mv^2}{r}$$

$$\text{or } r = \frac{mv}{eB}$$

$$\text{or } r = \frac{m \sqrt{2(E - W_0)}}{eB} = \frac{\sqrt{2m(E - W_0)}}{eB}$$

35. (d) Here,  $N_1 = N_0 e^{-10t}$  and  $N_2 = N_0 e^{-t}$

$$\frac{N_1}{N_2} = \frac{1}{e} = e^{-1} = e^{-(10)t} = e^{-9t}$$

$$t = \frac{1}{9}$$

36. (c) Here current flows in circuit A as both  $p$ - $n$  junction diode act as forward biasing. Total resistance  $R$ .

$$\frac{1}{R} = \frac{1}{4} + \frac{1}{4} + \frac{1}{4} \quad \text{or } R = 2$$

According to Ohm's law

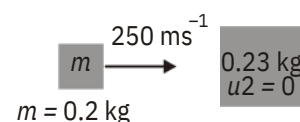
$$V = IAR$$

or  $8 = IA \times 2$  or  $IA = 4A$  In circuit B, lower  $p$ - $n$  junction diode is reverse biased. Hence, no current will flow but upper diode is forward biased so current can flow through it

$$V = IBR$$

$$\text{or } 8 = IB \times 4 \text{ or } I_B = 2A$$

37. (c) After collision the bullet and block move together and comes to rest after covering a distance of 40 m.



By conservation of momentum

$$m_1 u_1 + m_2 u_2 = m_1 v_1 + m_2 v_2$$

$$0.02 \times 250 + 0.23 \times 0 = 0.02v + 0.23v$$

$$5 = 0.25v \text{ or } v = 20 \text{ ms}^{-1}$$

Now, by conservation of energy

$$\text{or } \frac{1}{2} M v^2 = R d$$

$$\text{or } \frac{1}{2} \times 0.25 \times 400 = 0.25 \times 9.8 \times 40$$

$$\frac{200}{9.8 \times 40} = 0.51$$

38. (a) Let after the time ( $t$ ) the position of A is  $(0, vAt)$  and position of B =  $(vBt, 10)$ . Distance between them

$$y = \sqrt{(0 - vBt)^2 + (vAt - 10)^2}$$

$$\text{or } y^2 = (2t)^2 + (2t - 10)^2$$

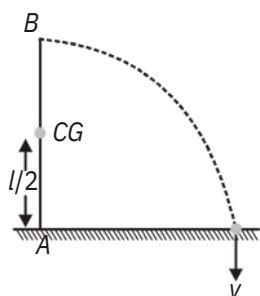
$$\text{or } y^2 = 4t^2 + 4t^2 - 40t + 100 = 8t^2 - 40t + 100$$

$$l = 8t^2 - 40t + 100$$

$$\text{Now, } \frac{dl}{dt} = (16t - 40) = 0$$

$$\text{or } t = \frac{40}{16} = 2.5 \text{ s} \quad \frac{d^2l}{dt^2} = 16 \text{ (+ve)}$$

39. (b) So,  $l$  will be minimum. Here, potential energy of the metre stick will be converted into rotational kinetic energy.



Because centre of gravity of stick lies at the middle of the rod,

$$\text{PE of metre stick} = \frac{mgl}{2}$$

$$\text{Rotational kinetic energy } E = \frac{1}{2} I \omega^2$$

$$I \text{ about point A} = \frac{ml^2}{3}$$

By law of conservation of energy

$$mg \frac{l}{2} = \frac{1}{2} I \omega^2 = \frac{1}{2} \frac{ml^2}{3} \frac{v_B^2}{l^2}$$

$$\text{By solving, we get } v_B = \sqrt{3gl}$$

40. (a) Given,  $r = 0.4 \text{ m}$ ,  
 $\omega = 8 \text{ rad s}^{-1}$ ,  
 $m = 4 \text{ kg}$ ,  $I = ?$

$$\text{Torque, } I \alpha = mgr$$

$$4 \times 10 \times 0.4 = I \times 8 \text{ or } I = \frac{16}{8} = 2 \text{ kg.m}^2$$

## PART - II (CHEMISTRY)

41. (c) Given :  $H_f(\text{H}) = 218 \text{ kJ/mol}$

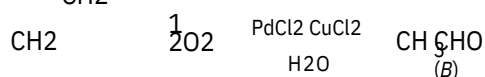
$$\text{i.e., } \frac{1}{2} \text{H}_2 \rightarrow \text{H}; \quad H = 218 \text{ kJ/mol}$$

$$2\text{H}; \quad H = 436 \text{ kJ/mol}$$

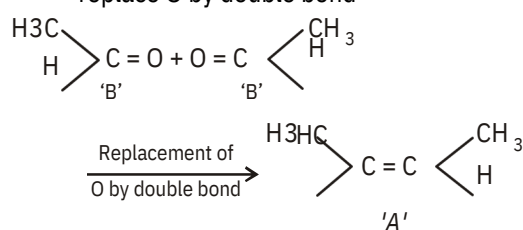
$$\frac{436}{4.18} = 104.3 \text{ kcal/mol}$$

Hence, H-H bond energy is 104.3 kcal/mol.

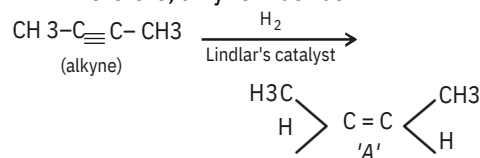
42. (a) In Wacker process, alkene is oxidised into aldehyde.



Since only alkenes produce aldehydes, on ozonolysis hence 'A' must be an alkene. Now to find the structure of alkene we should add two molecules of aldehyde and replace O by double bond

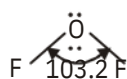


Therefore, alkyne must be





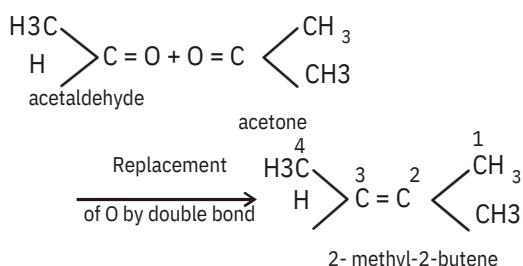
The structure of 'A' ( $\text{OF}_2$ ) is as



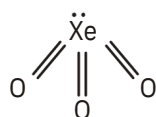
bonds made by O = 2

Due to repulsion between two lone pairs of electrons, its shape gets distorted. Therefore, the bond angle in the molecule is  $103^\circ$ .

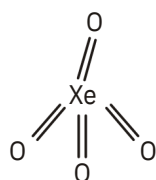
44. (a) To decide the structure of alkene that undergoes ozonolysis, add the products and replace O by double (=) bond. Thus,



45. (a) Structure of  $\text{XeO}_4$



3p-d pi bonds.  
Structure of  $\text{XeO}_4$



4p-d bonds.

46. (a) According to de-Broglie equation.

$$\frac{h}{mv} = \frac{h}{m\lambda} \quad \frac{h^2}{m\lambda^2}$$

or  $\frac{mv^2}{2} = \frac{h^2}{m\lambda^2}$

$$\text{KE (K)} = \frac{1}{2}mv^2$$

$$\text{KE (K)} = \frac{1}{2} \frac{h^2}{m\lambda^2}$$

$$\frac{K_1}{K_2} = \frac{\frac{1}{2}mv_1^2}{\frac{1}{2}mv_2^2} = \frac{v_1^2}{v_2^2} = \frac{52}{25:9}$$

$$K_1 : K_2$$

47. (c) Paramagnetic nature depends upon the number of unpaired electrons. Higher the number of unpaired electrons, higher the paramagnetic property will be.

$\text{Cu}^{2+} = [\text{Ar}] 3d^9$ , no. of unpaired electrons = 1

$\text{V}^{2+} = [\text{Ar}] 3d^3$ , no. of unpaired electrons = 3

$\text{Cr}^{2+} = [\text{Ar}] 3d^4$ , no. of unpaired electrons = 4

$\text{Mn}^{2+} = [\text{Ar}] 3d^5$ , no. of unpaired electrons = 5

Hence, correct order is  $\text{Mn}^{2+} > \text{Cr}^{2+} > \text{V}^{2+} > \text{Cu}^{2+}$

48. (a) 1 mol =  $6.023 \times 10^{23}$  atoms

KE of 1 mol =  $6.023 \times 10^4 \text{ J}$

or KE of  $6.023 \times 10^{23}$  atoms =  $6.023 \times 10^4 \text{ J}$

$$\text{KE of 1 atom} = \frac{6.023 \times 10^4}{6.023 \times 10^{23}} = 1.0 \times 10^{-19} \text{ J}$$

$$h\nu_{\text{energy}} = \frac{hc}{\lambda} = \frac{6.626 \times 10^{-34} \times 3 \times 10^8}{600 \times 10^{-9}} = 3.313 \times 10^{-19} \text{ J}$$

Now since Threshold energy

$$h\nu = \text{KE}$$

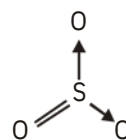
$$3.313 \times 10^{-19} = 1.0 \times 10^{-19}$$

=  $2.313 \times 10^{-19} \text{ J}$  Hence minimum amount of energy required to remove an electron from the metal ion will be  $2.313 \times 10^{-19} \text{ J}$ . The earth's thermosphere also includes the region of the atmosphere, called the *ionosphere*. The ionosphere is the region of the atmosphere that is filled with charged particles such as  $\text{O}^+$ ,  $\text{NO}^+$ . The high

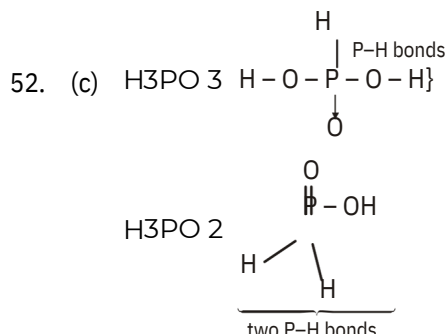
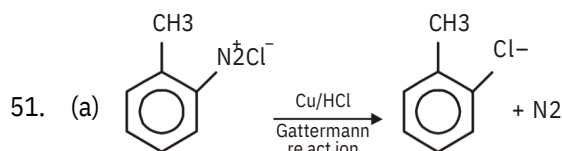
49. (a)

temperature in the thermosphere can cause molecules to ionize.

50. (b) The formula of sulphuric anhydride is  $\text{SO}_3$  and its structure is as follows :

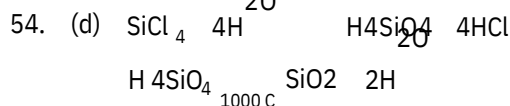


3, 1p-p, 2p-d bonds are present.



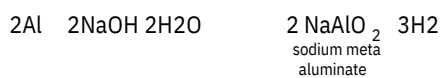
53. (c) Dipole moment,  $(\text{C}) \quad d$   
where,  $e$  = magnitude of electric charge  
 $d$  = distance between particles (or bond length)

$$\text{or } \frac{d}{\text{HCl}} = \frac{d_{\text{HCl}}}{d_{\text{HI}}} \quad \frac{1.03}{1.3} = \frac{1.6}{0.38} \quad 3.3 : 1$$



55. (c) Mass percent of Cd in  $\text{CdCl}_2$   
 $= \frac{0.9}{1.5} \times 100 = 60\%$

Mass percent of  $\text{Cl}_2$  in  $\text{CdCl}_2$   
 $= 100 - 60 = 40\%$   
40% part ( $\text{Cl}_2$ ) has atomic weight  
 $= 2 \times 35.5 = 71.0$   
60% part (Cd) has atomic weight  
 $= 106.5$

56. (c)   
Sodium metaaluminate, thus formed, is soluble in water and changes into the complex  $[\text{Al}(\text{HO})_2]^-$   
coordination number of Al is 6.

57. (b) Average kinetic energy per molecule

$$\frac{3}{2} kT$$

$$\text{or } \frac{3}{2} \times \frac{R}{2} \times T = \frac{3}{2} \times \frac{8.314}{2} \times 300$$

The species is  $\text{H}_2\text{O}$  and  $\text{H}_2\text{O}_2$  are known as

58. (c)

an oxidation state of  $\frac{1}{2}$  are super oxides and is represented as  $\text{O}_2^-$ . Usually these are formed by active metals such as  $\text{KO}_2$ ,  $\text{RbO}_2$  and  $\text{CsO}_2$ . For the salts of larger anions (like  $\text{O}_2^{2-}$ ), lattice energy increases

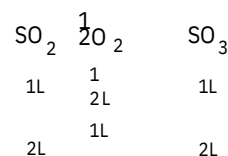
in a group. Since, lattice energy is the driving force for the formation of an ionic compound and its stability, the stability of the superoxides from 'K' to 'Cs' also increases.

59. (a) 30% solution of  $\text{H}_2\text{O}_2$  is known as perhydrol.

$\text{H}_2\text{O}_2$  decomposes as



2 volumes of  $\text{H}_2\text{O}_2$  solution give 1 volume of  $\text{O}_2$  at STP. 30%  $\text{H}_2\text{O}_2$  solution means 30 mL of  $\text{H}_2\text{O}_2$  in 100 mL of solution. On decomposition, it gives 15 mL of  $\text{O}_2$ .



Since, 100 mL of oxygen is obtained by 1 mL of  $\text{H}_2\text{O}_2$

1000 mL of oxygen will be obtained by

$$\frac{1}{100} \times 1000 \text{ mL of } \text{H}_2\text{O}_2 = 10 \text{ mL of } \text{H}_2\text{O}_2$$

60. (d) Buffer capacity,  $\frac{dC_{\text{HA}}}{dpH}$

where,  $dC_{\text{HA}}$  = no. of moles of acid added per litre

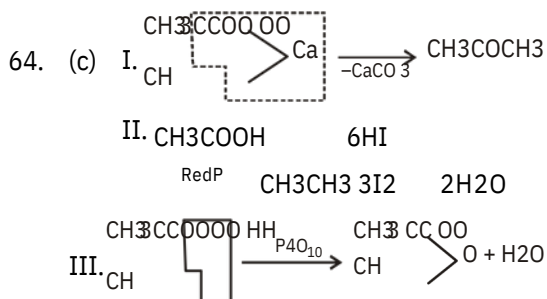
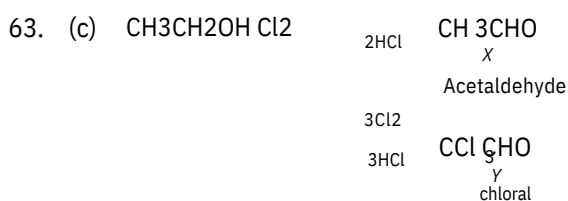
$dpH$  = change in pH

$$\frac{dC_{\text{HA}}}{dpH} = \frac{\text{moles of acetic acid}}{\text{volume}}$$

$$= \frac{0.12/60}{250/1000} = \frac{1}{125}$$

$$\frac{1}{125} \times \frac{1}{0.02} = 0.4$$

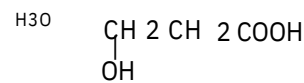
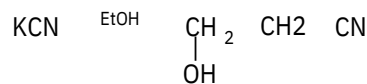
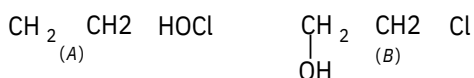
61. (b) (A) Felspar (orthoclase) ( $\text{KAlSi}_3\text{O}_8$ ) (B) Asbestos ( $\{\text{CaMg}_3(\text{SiO}_3)_4\}$ ) (C) Pyrrargyrite (Ruby silver) ( $\text{Ag}_2\text{S}$ ) (D) Diaspore ( $\text{Al}_2\text{O}_3 \cdot \text{H}_2\text{O}$ ) Along a period first ionisation energy increases. Thus, the first IE of the elements of the second period should follow the order  $\text{Be} < \text{B} < \text{N} < \text{O}$  But in practice. The first IE of these elements follows the order  $\text{B} < \text{Be} < \text{O} < \text{N}$  The lower IE of B than that of Be is because in B ( $1s^2, 2s^2 2p^1$ ), electron is to be removed from  $2p$  which is easy while in Be ( $1s^2, 2s^2$ ), electron is to be removed from  $2s$  which is difficult. The low IE of O than that of N is because of the half-filled  $2p$  orbitals in N ( $1s^2, 2s^2 2p^3$ ).



65. (b)  $\text{C} = 85.71\% = \frac{85.71}{12} = 7.14$   
 $\text{H} = 14.29\% = \frac{14.29}{1} = 14.29$   
 Empirical formula =  $\text{CH}_2$   
 and, empirical formula weight =  $12 + 2 = 14$   
 Now since molecular formula weight =  $2 \times \text{vapour density}$   
 $= 2 \times 14 = 28$

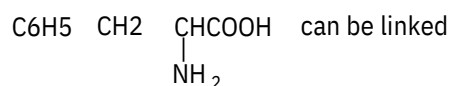
$$n = \frac{28}{14} = 2$$

$$\text{Molecular formula} = (\text{CH}_2)_2 = \text{C}_2\text{H}_4$$



(c)

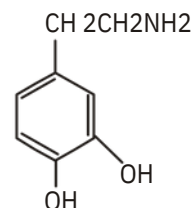
66. (c) Tripeptides are amino acids polymers in which three individual amino acid units, called residues, are linked together by amide bonds.  
 glycine ( $\text{NH}_2 - \text{CH}_2 - \text{COOH}$ )  
 $(\text{CH}_3 - \text{CH}(\text{COOH}) - \text{NH}_2)$



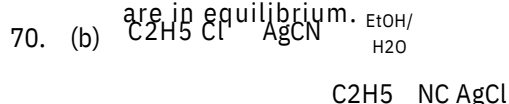
in six different ways.

67. (d) A codon is a specific sequence of three adjacent bases on a strand of DNA or RNA that provides genetic code information for a particular amino acid.

68. (c) The IUPAC name of dopamine is 2-(3,4-dihydroxyphenyl) ethylamine and its structure is as follows:

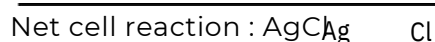


69. (a) Freezing point of a substance is the temperature at which the solid and the liquid forms of the substance are in equilibrium.



(N-linked to ethyl carbon)

71. (a) For the given cell,  
 $\text{Ag} | \text{Ag}^+ | \text{AgCl} | \text{Cl}^- | \text{Cl}_2, \text{Pt}$   
 The cell reactions are as follows





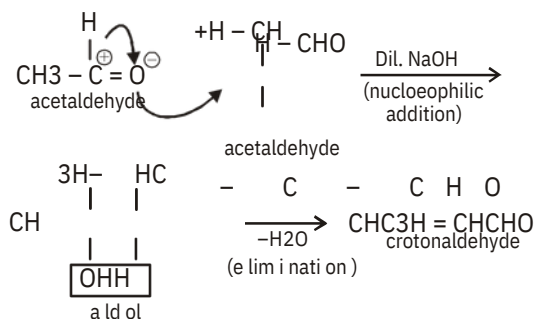
$$\Delta G_{\text{reaction}} = \Delta G_{\text{O}} - \Delta G_{\text{R}} \\ = (78 - 129) - (-109) \\ = +58 \text{ kJ/mol}$$

$$\Delta G_{\text{O}} = nFE_{\text{O}}$$

$$58103 \text{ J} = 196500 E_{\text{cell}}^{\circ}$$

$$E_{\text{cell}}^{\circ} = \frac{581000}{96500} = 0.6 \text{ V}$$

72. (d) Crotonaldehyde is produced by the aldol condensation of acetaldehyde.



73. (b)  $\text{BaCl}_2 + 2\text{NaOH} \rightarrow \text{Ba(OH)}_2 + 2\text{NaCl}$

$$m\text{Ba(OH)}_2 = m\text{BaCl}_2 - 2m\text{NaOH} + 2m\text{NaCl}$$

$$= 280 \times 10^{-4} + 2 \times 248 \times 10^{-4} - 2 \times 126 \times 10^{-4}$$

$$= (280 + 496 - 252) \times 10^{-4}$$

Density,  $d = 524 \times 10^{-4} \text{ Sm}^2 \text{ mol}^{-1}$

74. (b)  $\frac{MZ}{N_0 a^3}$

where, Z = number of atoms in unit cell

$$Z = \frac{d N_0 a^3}{M}$$

$$= \frac{8.92 \times 6.023 \times 10^{23} \times (362 \times 10^{-10})^3}{63.55}$$

$$= 4.0$$

Thus, metal has face centred unit cell.

75. (c)  $\text{N}_2 + 2\text{O}_2 \rightarrow 2\text{NO}_2$

$$K = \frac{[\text{NO}_2]^2}{[\text{N}_2][\text{O}_2]^2}$$

$$\text{or } 100 = \frac{[\text{NO}_2]^2}{[\text{N}_2][\text{O}_2]^2} \quad \dots(i)$$

Again,  $[\text{NO}_2] = \frac{1}{2} \text{N}_2 + \text{O}_2$

$$K_2 = \frac{[\text{N}_2][\text{O}_2]^2}{[\text{NO}_2]^2}$$

$$\text{or } K_2 = \frac{[\text{N}_2][\text{O}_2]^2}{[\text{NO}_2]^2} \quad \dots(ii)$$

Eqs. (i)  $\times$  (ii), we get  $100 K_2^2 = 1$

$$K_2^2 = \frac{1}{100} \text{ or } K_2 = \frac{1}{10} = 0.1$$

76. (c) For a first order reaction,

$$t = \frac{2.303}{k} \log_{10} \frac{a}{a-x}$$

Let initial amount of reactant is 100.

$$\frac{t_1}{t_2} = \frac{\log_{10} 100}{\log_{10} \frac{100-75}{100}} \quad [\text{remains constant}]$$

$$\frac{100}{100} = \frac{25}{100}$$

$$\frac{\log_{10} 25}{\log_{10} 75} = \frac{\log_{10} 4}{\log_{10} 3}$$

$$\frac{\log_{10} 4}{\log_{10} 3} = \frac{0.6020}{0.4771}$$

$$\frac{2 \times 0.3010}{2 \times 0.3010} = 0.4771$$

$$0.6020 = 4.81$$

$$0.1249$$

77. (c)  $\left[ \text{I} \right]_{\text{observed}} = \frac{1.2}{5 \times 6.15} = \frac{39}{1000}$

78. (a) Let the concentration of potassium acetate is x. According to Henderson's equation,

$$\text{pH} = \text{pKa} + \log \frac{[\text{salt}]}{[\text{acid}]}$$

$$4.8 = \log(1.8 \times 10^{-5}) + \log \frac{x \times 50}{20 \times 0.1 \text{ M}}$$

$$4.8 - 4.74 = \log 25x$$

$$\text{or } \log 25x = 0.06$$

$$25x \quad 1.148$$

$$x \quad 0.045M$$

$$6$$

79. (b) By '2A — B', we get



$$H \quad \frac{2 \times 146 + 259 + 418}{2}$$

$$H \quad 580.5 \text{ kJ}$$

80. (a) According to Hardy Schulze rule, greater the valency of the coagulating ion, greater is its coagulating power. Thus, out of the given,  $AlCl_3$  is most effective for causing coagulation of  $As_2S_3$  sol.

### PART - III (MATHEMATICS)

81. (b) Given,  $f(x) = x^3 - 3x^2$

$$f'(x) = 3x^2 - 3$$

$$\text{Put } f'(x) = 0 \quad 3x^2 - 3 = 0$$

$$x^2 = 1$$

$f(x)$  is either increasing or decreasing.

$$\text{At } x = 2, f(2) = 2^3 + 3(2) - 2 = 12$$

$$\text{At } x = 3, f(3) = 3^3 + 3(3) - 2 = 34$$

$$f(x) = [12, 34].$$

82. (c) The total number of subsets of given set is  $2^9 = 512$

Even numbers are  $\{2, 4, 6, 8\}$ .

Case I : When selecting only one even number =  ${}^4C_1 = 4$

Case II : When selecting only two even numbers =  ${}^4C_2 = 6$

Case III : When selecting only three even numbers =  ${}^4C_3 = 4$

Case IV : When selecting only four even numbers =  ${}^4C_4 = 1$

Required number of ways

$$= 512 - (4 + 6 + 4 + 1) - 1 = 496$$

[Here, we subtract 1 for due to the null set]

83. (a) The required number of ways = The even number of 0's i.e.,  $\{0, 2, 4, 6, \dots\}$

$$\frac{n}{1} - \frac{n!}{2!(n-2)!} - \frac{n!}{4!(n-4)!}$$

$${}^nC_0 - {}^nC_2 - {}^nC_4 - \dots - 2n^1$$

$$84. (a) 1 - \frac{2x}{3}^{3/2} (32 - 5x)^{1/5}$$

$$1 - \frac{3}{2} \cdot \frac{2x}{3} (32)^{1/5} - 1 \cdot \frac{5}{32} x^{1/5}$$

(neglecting higher powers of  $x$ )

$$[1 - x]^{1/2} = 1 - \frac{1}{2}x - \frac{1}{8}x^2$$

(neglecting higher powers of  $x$ )

$$\frac{1}{2}(1 - x)^{1/2} = \frac{x}{32}$$

$$\frac{x(32 - x)}{64} = \frac{32 - 31x}{64}$$

(neglecting  $x^2$  term)

$$85. (c) (x - a)(x - a - 1) = (x - a - 1)(x - a - 2)$$

Let  $x - a = t$ , then

$$t(t - 1) = (t - 1)(t - 2) \quad t(t - 2) = 0$$

$$t^2 - t = t^2 - 3t + 2 \quad t^3 - 2t = 0$$

$$3t^2 - 6t + 2 = 0$$

$$t = \frac{6 \pm \sqrt{36 - 24}}{2(3)} = \frac{6 \pm 2\sqrt{3}}{2(3)}$$

$$x - a = \frac{3 \pm \sqrt{3}}{3}$$

$$x - a = \frac{3 \pm \sqrt{3}}{3}$$

Hence,  $x$  is real and distinct.

86. (b)  $f(x) = x^2 - ax + b$  has imaginary roots.

$$\text{Discriminant, } D = 0 \quad a^2 - 4b = 0$$

$$f'(x) = 2x - a$$

$$f''(x) = 2$$

$$\text{Also, } f(x) = f'(x) = f''(x) = 0 \quad \dots(i)$$

$$x^2 - ax + b = 2x - a = 2 = 0$$

$$x^2 - (a - 2)x + b - a + 2 = 0$$

$$x = \frac{(a - 2) \pm \sqrt{(a - 2)^2 - 4(b - a + 2)}}{2}$$

$$\frac{(a - 2) \pm \sqrt{a^2 - 4b - 4}}{2}$$

Since,  $a^2 - 4b < 0$

$$a^2 - 4b = 4 < 0$$

Hence, Eq. (i) has imaginary roots.

87. (c)  $f(x) = 2x^4 + 13x^2 + ax + b$  is divisible by  $(x - 2)(x + 1)$ .

$$\begin{aligned} f(2) &= 2(2)^4 + 13(2)^2 + a(2) + b = 0 \\ 2a + b &= -20 \quad \dots(i) \end{aligned}$$

$$\text{and } f(1) = 2(1)^4 + 13(1)^2 + a + b = 0$$

$$a + b = -11 \quad \dots(ii)$$

On solving Eqs. (i) and (ii), we get  $a = 9, b = -20$

88. (d) Let  $a$  and  $R$  be the first term and common ratio of a GP respectively.

$$\text{So, } T_p = aR^{p-1} = x$$

$$T_q = aR^{q-1} = y \text{ and } T_r = aR^{r-1} = z$$

$$\log x = \log a + (p-1)\log R$$

$$\log y = \log a + (q-1)\log R$$

$$\text{and } \log z = \log a + (r-1)\log R$$

$$\begin{vmatrix} \log x & p-1 & 1 \\ \log y & q-1 & 1 \\ \log z & r-1 & 1 \end{vmatrix} = \begin{vmatrix} \log a + (p-1)\log R & p-1 & 1 \\ \log a + (q-1)\log R & q-1 & 1 \\ \log a + (r-1)\log R & r-1 & 1 \end{vmatrix}$$

$$\begin{vmatrix} \log a & p-1 & 1 \\ \log a & q-1 & 1 \\ \log a & r-1 & 1 \end{vmatrix} = \begin{vmatrix} (p-1)\log R & p-1 & 1 \\ (q-1)\log R & q-1 & 1 \\ (r-1)\log R & r-1 & 1 \end{vmatrix}$$

$$\log a \begin{vmatrix} 1 & p-1 & 1 \\ 1 & q-1 & 1 \\ 1 & r-1 & 1 \end{vmatrix} + \log R \begin{vmatrix} p-1 & 1 & 1 \\ q-1 & 1 & 1 \\ r-1 & 1 & 1 \end{vmatrix} = 0$$

$$(C_2 - C_3 - C_1)$$

89. (c)  $= 0 + 0 = 0$  (two columns are identical)  
Let  $z = x + iy$

$$\text{Given: } \left| \frac{z-2i}{2z-i} \right| = 1$$

$$\frac{\sqrt{(x)^2 + (y-2)^2}}{\sqrt{(2x)^2 + (2y-1)^2}} = 1$$

$$x^2 + y^2 - 4y + 4 = 4x^2 + 4y^2 - 4y + 1$$

$$3x^2 + 3y^2 - 3x^2 - y^2 = 1$$

$$90. (c) (1 - \sqrt{3}i)^n + (1 + \sqrt{3}i)^n$$

$$2 \left[ \frac{1 - \sqrt{3}i}{2} \right]^n + 2 \left[ \frac{1 + \sqrt{3}i}{2} \right]^n$$

$$(2 - 2i)^n + (2 + 2i)^n$$

$$(2)^n [(2 - 2i)^n + (2 + 2i)^n]$$

$$(2)^n [2^n (1 - i)^n + 2^n (1 + i)^n]$$

$$(2)^n (2^n) [(1 - i)^n + (1 + i)^n]$$

$$91. (d) \text{ Let } f(x) = \sin^4 x \cos^4 x$$

$$= (\sin^2 x \cos^2 x)^2 = \frac{1}{4} (1 - \cos^2 2x)^2$$

$$= \frac{1}{4} (1 - 2\cos^2 2x + \cos^4 2x)$$

$$= \frac{1}{4} (1 - 2\cos^2 2x + \cos^4 2x)$$

$$\text{Period of } f(x) = \frac{2\pi}{4} = \frac{\pi}{2}$$

$$92. (c) \sin^2 x \cos^2 x = \frac{1}{4} (1 - \cos^2 2x)$$

$$= \frac{1}{4} (1 - \cos^2 2x) = \frac{1}{4} (1 - \cos^2 2x)$$

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$$93. (d) \cos^{-1} \frac{1}{2} = \frac{\pi}{3}, \sin^{-1} \frac{1}{2} = \frac{\pi}{6}, \cos^{-1} \frac{1}{\sqrt{2}} = \frac{\pi}{4}$$

$$\cos^{-1} \frac{1}{2} + \sin^{-1} \frac{1}{2} + \cos^{-1} \frac{1}{\sqrt{2}} = \frac{\pi}{3} + \frac{\pi}{6} + \frac{\pi}{4} = \frac{3\pi}{4}$$

$$4 \tan^{-1} \left( \frac{1}{2} \right)$$

$$\frac{3}{4} + \frac{3}{4} + \frac{3}{4} = \frac{9}{4}$$

94. (c) We know that,  $2s = a + b + c$   
 $(a+b+c)(b+c-a)(c+a-b)(a+b-c)$   
 $4b^2c^2$

$$2s(2s-2a)(2s-2b)(2s-2c)$$

$$\frac{4b^2c^2}{bc} \cdot \frac{b^2c^2}{bc}$$

$$4\cos^2 \frac{A}{2} \sin^2 \frac{A}{2} \sin^2 \frac{A}{2}$$

95. (c)  $lmn \neq 0, l, m, n$

$$2m^2n^2 = 0$$

$$\left( \frac{m}{m^2} \right)^2 \left( \frac{n}{n^2} \right)^2 = 0$$

$$2mn = 0$$

$$2m(m+n) = 0$$

$$m = 0 \text{ or } m+n = 0$$

If  $m = 0$ , then  $l = -n$

$$\frac{l}{1} = \frac{m}{0} = \frac{n}{1}$$

and if  $m+n=0$   $m = -n$ , then  $l = 0$

$$\frac{l}{0} = \frac{m}{1} = \frac{n}{-1}$$

ie.,  $(l, m, n) = (1, 0, 1)$

and  $(l, m, n) = (0, 1, -1)$

$$\cos \frac{0 \cdot 0 \cdot 1}{\sqrt{1} \cdot 0 \cdot 1 \sqrt{0 \cdot 1 \cdot 1}} = \frac{1}{2}$$

$$3$$

96. (a)  $m = |a_1| \sqrt{2^2 (1)^2 (1)^2} = \sqrt{6}$

$$1 = |a_2| \sqrt{3^2 (4)^2 (4)^2} = \sqrt{41}$$

$$m = |a_3| \sqrt{1^2 (1)^2 (1)^2} = \sqrt{3}$$

and  $m_4 = |a_4| \sqrt{(1)^2 (3)^2 (1)^2} = \sqrt{11}$

$$m^3 m^1 = m^4 m^2$$

97. (a) Here,  $n = 6$   
 According to the question

$${}^6C_2 q^4 = 4 {}^6C_4 p^4 q^2$$

$$q^2 = 4p^2 \quad (1-p)^2 = 4p^2$$

$$3p^2 - 2p - 1 = 0$$

$$(p-1)(3p+1) = 0$$

$$p = \frac{1}{3} \quad (p \text{ cannot be negative})$$

98. (d) Given lines are parallel.

$$d = \frac{15 \cdot 5 - 10}{\sqrt{4^2 + 3^2}} = \frac{10}{5}$$

$$d = 2 \text{ diameter of the circle}$$

$$\text{Radius of circle} = 1$$

$$\text{Area of circle} = r^2 = \text{sq unit}$$

99. (c)  $x^2 - 2xy + xy - 2y^2 = 0$

$$(x-2y)(x-y) = 0$$

$$x = 2y, x = y \quad \dots(i)$$

$$\text{Also, } x + y + 1 = 0 \quad \dots(ii)$$

On solving Eqs. (i) and (ii), we get

$$A = \frac{2}{3}, \frac{1}{3}, B = \frac{1}{2}, \frac{1}{2}, C(0, 0)$$

$$\text{Area of } \triangle ABC = \frac{1}{2} \begin{vmatrix} 2 & \frac{1}{3} & 1 \\ \frac{1}{3} & 1 & 1 \\ 0 & 0 & 1 \end{vmatrix}$$

$$\frac{1}{2} \begin{vmatrix} 1 & 1 & 1 \\ 2 & 3 & 6 \\ 2 & 6 & 12 \end{vmatrix}$$

100. (c) Given pair of lines are

$$x^2 - 3xy + 2y^2 = 0$$

$$\text{and } x^2 - 3xy + 2y^2 - x + 2 = 0$$

$$(x-2y)(x-y) = 0$$

$$\text{and } (x-2y-2)(x-y-1) = 0$$

$$x-2y=0, x-y=0$$

$$\text{and } x-2y-2=0, x-y-1=0$$

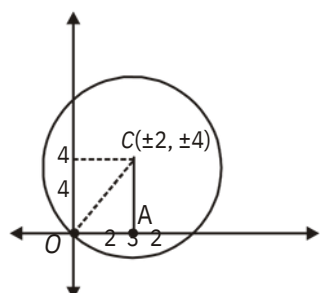
$$\text{The lines } x-2y=0, x-2y-2=0 \text{ and}$$

$$x-y=0, x-y-1=0 \text{ are parallel.}$$

Also, angle between  $x-2y=0$  and  $x-y=0$  is not  $90^\circ$ .

It is a parallelogram.

101. (a) In  $\triangle OAC$ ,  $OC^2 = 2^2 + 4^2 = 20$



Required equation of circle is

$$(x-2)^2 + (y-4)^2 = 20$$

$$x^2 + y^2 - 4x - 8y = 0$$

102. (d) Given circles are

$$x^2 + y^2 - 2x - 8y - 13 = 0$$

$$\text{and } x^2 + y^2 - 4x - 6y - 11 = 0$$

Here  $C_1(1, 4)$ ,  $C_2(2, 3)$

$$r_1 = \sqrt{1^2 + 4^2} = 5$$

$$\text{and } r_2 = \sqrt{2^2 + 3^2} = \sqrt{13}$$

So, d

$$C_1C_2 = \sqrt{(2-1)^2 + (3-4)^2} = \sqrt{2}$$

$$\cos \theta = \frac{d^2 + r_1^2 - r_2^2}{2r_1r_2} = \frac{2 + 25 - 13}{2 \cdot 5 \cdot \sqrt{13}} = \frac{10}{10\sqrt{13}} = \frac{1}{\sqrt{13}}$$

135

103. (b) Let the required equation of circle be

$$x^2 + y^2 + 2gx + 2fy = 0$$

The above circle cuts the given circles orthogonally.

$$2(3g) + f(0) + 8 - 2g = \frac{8}{3}$$

$$\text{and } 2g + 2f = 7$$

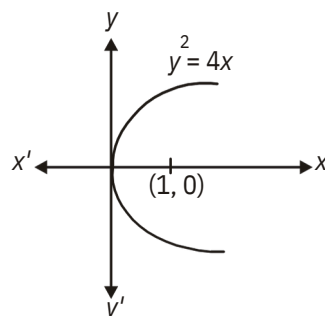
$$2f = 7 - \frac{8}{3} = \frac{14}{3}$$

Required equation of circle is

$$x^2 + y^2 + \frac{8}{3}x + \frac{29}{3}y = 0$$

$$\text{or } 3x^2 + 3y^2 + 8x + 29y = 0$$

104. (b) Given curve is  $y^2 = 4x$ .



Also, point (1, 0) is the focus of the parabola. It is clear from the graph that only one normal is possible.

105. (a)  $4y^2$

$$y^2(a^2 - y^2) = c^4$$

$$y^4 - a^2y^2 + c^4 = 0$$

Let  $y_1, y_2, y_3$  and  $y_4$  are the roots.

$$y_1 + y_2 + y_3 + y_4 = 0$$

106. (b) Solving  $4x - 3y = 5$  and  $2x^2 - 3y^2 = 12$

$$2 \cdot \frac{5 + 3y}{4} - 3y^2 = 12$$

$$\frac{(25 - 9y^2 - 30y)}{4} - 3y^2 = 12$$

$$15y^2 - 30y - 71 = 0$$

$$y = \frac{30 \pm \sqrt{900 - 4260}}{30} = 1 \pm \frac{\sqrt{3360}}{30}$$

$$\text{Also, } 2x^2 - 3 \cdot \frac{5^2}{3} = 12$$

$$10x^2 - 40x - 61 = 0$$

$$x = \frac{40 \pm \sqrt{1600 - 4 \cdot 10 \cdot 61}}{2 \cdot 10}$$

$$\frac{40 \pm \sqrt{840}}{20} = 2 \pm \frac{\sqrt{840}}{20}$$

$$\text{Points are } A = 2 + \frac{\sqrt{840}}{20}, 1 + \frac{\sqrt{3360}}{30}$$

$$\text{and } B = 2 - \frac{\sqrt{840}}{20}, 1 - \frac{\sqrt{3360}}{30}$$

Mid point of AB is (2, 1).

107. (d) Let  $A = (1, 0, 0)$ ,  $B = (0, 1, 0)$  and  $C = (0, 0, 1)$

$$\text{So, } AB = \sqrt{(0-1)^2 + (1-0)^2 + (0-0)^2} = \sqrt{2}$$

$$BC = \sqrt{(0-0)^2 + (1-0)^2 + (0-1)^2} = \sqrt{2}$$

$$\text{and } CA = \sqrt{(1-0)^2 + (0-0)^2 + (0-1)^2} = \sqrt{2}$$

Perimeter of triangle

$$= AB + BC + CA = \sqrt{2} + \sqrt{2} + \sqrt{2} = 3\sqrt{2}$$

108. (c)  $\cos^2 x + \sin^2 x = 1$

$$\begin{aligned} & \cos^2 x + \sin^2 x = 1 \\ & (\cos^2 x + \sin^2 x) \cos^2 x = \cos^2 x \\ & (\cos^2 x + \sin^2 x) \sin^2 x = \sin^2 x \\ & \cos^2 x + \cos^2 x + \sin^2 x = 1 \end{aligned}$$

109. (a) Given equation of sphere is  $x^2 + y^2 + z^2 - 12x - 4y - 3z = 0$

Centre of sphere is  $(6, 2, \frac{3}{2})$ .

$$\text{Radius of sphere} = \sqrt{(6)^2 + (2)^2 + \left(\frac{3}{2}\right)^2}$$

$$= \sqrt{36 + 4 + \frac{9}{4}} = \sqrt{\frac{169}{4}} = \frac{13}{2}$$

110. (c)  $\lim_{x \rightarrow \infty} \frac{x^5 - x^3}{x^2} = \lim_{x \rightarrow \infty} \frac{5x^4 - 3x^2}{2x} = \lim_{x \rightarrow \infty} \frac{20x^3 - 6x}{2} = \lim_{x \rightarrow \infty} \frac{60x^2 - 6}{2} = \lim_{x \rightarrow \infty} \frac{120x - 6}{2} = \lim_{x \rightarrow \infty} \frac{120}{2} = 60$

111. (d)  $f(x) = \begin{cases} 2\sin x - \sin 2x, & \text{if } x < 0 \\ 2x \cos x, & \text{if } x = 0 \\ 2\sin x, & \text{if } x > 0 \end{cases}$

$$\lim_{x \rightarrow 0} f(x) = \lim_{x \rightarrow 0} \frac{\sin 2x}{2x \cos x} = 0 \text{ form } \frac{0}{0}$$

$$\lim_{x \rightarrow 0} \frac{2\cos 2x}{2(\cos x - x \sin x)} = \frac{2\cos 0}{2(1 - 0)} = 1$$

$$\lim_{x \rightarrow 0} \frac{2}{2(1 - \frac{2}{0})} = 0$$

Since,  $f(x)$  is continuous at  $x = 0$

$$f(0) = \lim_{x \rightarrow 0} f(x) = a = 0$$

112. (c) We have,  $x = \cos^{-1} \frac{1}{\sqrt{1-t^2}}$

$$\text{and } y = \sin^{-1} \frac{t}{\sqrt{1-t^2}}$$

$$x = \tan^{-1} t \text{ and } y = \tan^{-1} t$$

$$y = x \quad \frac{dy}{dx} = 1$$

113. (b)  $\frac{d}{dx} \tan^{-1} x = \frac{1}{1+x^2} \log \frac{x+1}{x-1} = \frac{1}{x^4-1}$

On integrating both sides, we get

$$\tan^{-1} x = \log \frac{x+1}{x-1}$$

$$\frac{1}{2} \left( \frac{1}{x^2-1} - \frac{1}{x^2+1} \right) dx$$

$$\tan^{-1} x = \log \frac{x+1}{x-1}$$

$$\frac{1}{4} \log \frac{x+1}{x-1} = \frac{1}{4} \log \frac{12 \tan^{-1} x + 1}{12 \tan^{-1} x - 1}$$

$$a = \frac{1}{2}, b = \frac{1}{4}$$

$$a = 2b = \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4}$$

114. (c)  $y = e^{\sin x}$

On differentiating w.r.t.  $x$ , we get

$$y_1 = e^{\sin x} \cos x = \frac{1}{\sqrt{1-x^2}}$$

$$y_1 \sqrt{1-x^2} = ay$$

$$(1-x^2) y_1^2 = a^2 y^2$$

Again differentiating w.r.t.  $x$ , we get

$$(1-x^2) 2y_1 y_{11} - 2xy_1^2 = 2ay_1 y_2$$

$$(1-x^2) y_{11} - xy_1^2 = ay_2$$

Using Leibnitz's rule,

$$(1-x)^2 y_{n+2} - nC_1 y_{n+1}(2x) - nC_2 y_n(2) = 0 \quad 118. (d)$$

$$xy_{n+1} - nC_1 y_n(2x) - nC_2 y_{n-1}(2) = 0$$

$$(1-x^2)y_{n+2} - xy_{n+1} = 0$$

$$y_n[n(n-1) - n^2] = 0$$

$$(1-x^2)y_{n+2} - (2n-1)xy_{n+1} - (n^2-a^2)y_n = 0$$

115. (c) Given,  $f(x) = x^3 - ax^2 + bx - c$ ,  $a^2 = 3b$

$$f'(x) = 3x^2 - 2ax + b$$

Put  $f'(x) = 0$

$$3x^2 - 2ax + b = 0$$

$$x = \frac{2a \pm \sqrt{4a^2 - 12b}}{6}$$

$$x = \frac{2a \pm 2\sqrt{a^2 - 3b}}{6}$$

Since,  $a^2 = 3b$ ,

$$3b = 3b,$$

$x$  has an imaginary value.

Hence, no extreme value of  $x$  exists.

116. (a) Let  $I = \int \frac{2 \sin 2x}{1 + \cos 2x} e^x dx$

$$= \int \frac{2 \sin x \cos x}{1 + \cos^2 x} e^x dx$$

$$= \int \frac{2 \sin x}{1 + \cos^2 x} \cot x e^x dx$$

$$= \int \cot x e^x (x \cot x) e^x dx$$

$$= \int \cot x e^x dx$$

$$= \int \cot x e^x dx$$

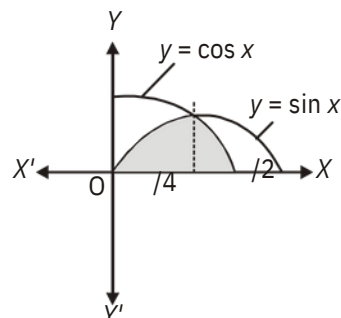
117. (c) We know that, if

In  $\int \sin^n x dx$ , then

$$I_n = \frac{\sin^{n-1} x \cos x}{n} - \frac{n-1}{n} I_{n-2}$$

where  $n$  is a positive integer.

$$n I_n = (n-1) I_{n-2} + \sin^{n-1} x \cos x$$



Area,  $A_1 = \int_0^{\pi/4} \sin x dx$

$$= [\cos x]_0^{\pi/4} = 1 - \frac{1}{\sqrt{2}}$$

$$= \frac{\sqrt{2} - 1}{\sqrt{2}}$$

and area,  $A_2 = \int_{\pi/4}^{\pi/2} \cos x dx$

$$= [\sin x]_{\pi/4}^{\pi/2} = 1 - \frac{1}{\sqrt{2}} = \frac{\sqrt{2} - 1}{\sqrt{2}}$$

$$A_1 : A_2 = \frac{\sqrt{2} - 1}{\sqrt{2}} : \frac{\sqrt{2} - 1}{\sqrt{2}} = 1 : 1$$

119. (b)  $\frac{dy}{dx} = \sin(x-y) \tan(x-y) = 1$

Put  $x + y = z$

$$1 = \frac{d}{dy} \frac{dz}{dx} = \frac{dz}{dx} \frac{dz}{dy} = 1$$

$$\frac{dz}{dx} = \frac{dz}{dz} \frac{dz}{dy} = 1$$

$$x = \cos z, \quad \frac{dz}{dx} = \frac{dz}{dz} = 1$$

Putting  $\sin z = t$ ,  $\cos z dz = dt$ , we have

$$\frac{1}{t^2} dt = \frac{dx}{x} \Rightarrow \frac{1}{t} = \frac{x}{c} \Rightarrow \frac{1}{\sin z} = \frac{x}{c} \Rightarrow \frac{1}{\sin(x-y)} = \frac{x}{c}$$

120. (c)  $p \rightarrow (\sim p \rightarrow q)$  is false means  $p$  is true and  $\sim p \rightarrow q$  is false.

$p$  is true and both  $\sim p$  and  $q$  are false.

$p$  is true and  $q$  is false.